47. *On the Seismic Vibrations of a Gozyûnotô (Pagoda).*

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1. *Introduction.*

The question why a *gozyûnotô* (5-storied Japanese Pagoda), notwithstanding its slender tower-like construction, is earthquake-proof seems to have received not little attention of investigators. Omori[1] studied the vibrations of several *gozyûnotôs* in Japan, and Muto[2] later attempted to explain their earthquake-proof properties. Of the various possible explanations, either that bases on the deformation damping of the structure or on the dissipation of vibrational energy, seems most plausible, particularly owing to the relative safety of the tower under resonance conditions. Although Muto,[2] in his study of the towers, applied the idea of deformation damping, due originally to us,[3] his idea is not very well adapted to the problem because of the improbability of such damping feature's predominating in the vibration in question. Taniguti,[4] on the other hand, with our dissipation theory in mind, believed that the enormous damping in the vibration of structures generally is due to deformation damping in the foundations of such structures, but our opinion is that damping of this kind is involved in the dissipation of vibrational energy into the ground, because deformation damping is a phenomenon that takes place when the vibrational energy has already been dissipated.

Mere application of the idea of dissipation of vibrational energy does not solve the whole problem. Our close investigation into the subject showed however that the resistance of the tower in question to seismic vibration is due to the smallness of its effective height and also to the accompanying marked dissipation of vibrational energy in the

1) F. OMORI, *Bull. Earthq. Inv. Comm.*, **9** (1918~21), 110~152.

2) K. MUTO, *Proc. World Eng. Congr., Tokyo, 1929*, **3**.

3) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **3** (*1929*), 50. K. SEZAWA's papers published thereafter relating to the material damping were merely applications of the idea shown in *Bull.* **3** under consideration to various similar problems.

4) T. TANIGUTI, verbally, at a meeting in which the problems of earthquake-proof construction were discussed.

form of elastic waves.

Some hold that the resistibility of the tower might be due in part to the *sinbasira*, a kind of central post within the tower, in some cases hanging from the top of the tower and serving as an oscillating damper. Investigators without full knowledge of the nature of the oscillating damper are liable to believe that the solid surface friction caused by the massive oscillator resists tearing action in any joint within the structure. Our studies however show that the oscillator under consideration, instead of participating in any reduction of the seismic vibration, acts sometimes quite contrarily.

Section 2 of the present paper discusses the case in which the tower is free from the effects of the *sinbasira*, but its vibrational energy is dissipated into the ground; Section 3 the case in which the tower vibration is affected by the pendulum action of the *sinbasira* but without energy dissipation, and Section 4 the case in which the tower vibration is under the influence of the *sinbasira* as well as of the energy dissipation.

2. *The condition under the influence of energy dissipation, but not of pendulum action.*

Careful examination of the structure of a *gozyûnotô* showed that its effective height is fairly small. An extremely rigid structural block that forms a part of each roof rests between the successive purely columned structures of relatively small rigidity. This condition corresponds to the case of a tall structure with rigid floors that was dealt with in our previous paper,[5] the formulae from which we shall use here. For example, from (13′), (19) for the case in which a structure is subjected to horizontal ground vibrations due to the vertical incident transverse waves, we get

$$u_0 = 2\cos pt, \tag{1}$$

and we have a horizontal displacement of the structure such that

$$u' = 4\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos k'(x+l)\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{2}$$

where

$$P = 2\Gamma_1\cos k'l + \frac{3G\varepsilon}{\mu l}\Lambda_1 k'l\sin k'l,$$

5) K. SEZAWA and K. KANAI, Section 3 of the paper "Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure," *Bull. Earthq. Res. Inst.*, **14** (1936), 167.

$$\left.\begin{aligned}
&Q=2\,\Gamma_2\cos k'l+\frac{3G\varepsilon}{\rho l}\Delta_2 k'l\sin k'l\,,\\
&\Gamma_1=3\left(\frac{\lambda}{\mu}+2\right)+\nu(k'l)^2\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right),\\
&\Gamma_2=\sqrt{\nu}\,(k'l)\left\{3\left(\frac{\lambda}{\mu}+2+\sqrt{\frac{\lambda}{\mu}+2}\right)+\nu(k'l)^2\left(\sqrt{\frac{\lambda}{\mu}+2}-2\right)\right\},\\
&\Delta_1=\left(\frac{2\lambda}{\mu}+5\right)-\nu(k'l)^2\,,\\
&\Delta_2=\sqrt{\nu}\,(k'l)\left(2\sqrt{\frac{\lambda}{\mu}+2}+1\right),\\
&\nu=\rho G\varepsilon^2/\rho'\mu l^2\,.
\end{aligned}\right\}\qquad(3)$$

As the meanings of these notations have been given in the paper cited, they will not be repeated here.

It is now possible to calculate the theoretical values of the displacements of the tower in seismic vibrations, provided we have the structural dimensions as well as the physical constants of the tower, besides data regarding the nature of the ground.

We shall take the case of the Ueno gozûnotô[6] in Tokyo as an example. The important numerical constants are as follows:

Total height of the tower body $=79{\cdot}5$ shaku $=24{\cdot}095$ m,

Effective height of the tower body (estimated) $=l=25{\cdot}0$ shaku $=7{\cdot}575$ m,

Height of each elementary elastic part of the tower body $=l/5=5$ shaku $=1{\cdot}515$ m,

ε for each column $=1{\cdot}5/2$ shaku $=0{\cdot}2273$ m,

j (roughly) $=0{\cdot}5$ shaku $=0{\cdot}1515$ m,

E (assumed) $=100.10^3$ kg/cm$^2=10^9$ kg/m^2
$\qquad= 9{\cdot}8.10^9$ kg mass/(m sec^2),

$G=12{\cdot}4\,Ej^2/l_1^2=1{\cdot}215.\ 10^9$ kg mass/(m sec^2),

$T=$ period of natural vibration without dissipation, (assumed), $=1$ sec,

$G/\rho'=(4l/T)^2=30{\cdot}3^2$ m^2/sec^2,

$N=$ number of columns excepting the central post $=16$,

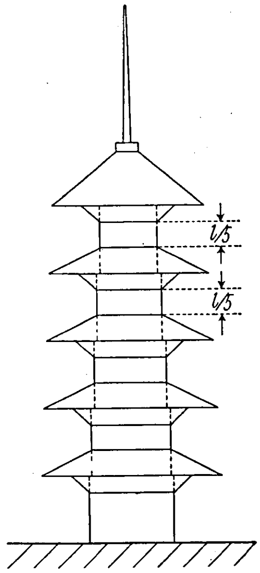

Fig. 1.

6) F. OMORI, *loc.cit.* 1).

$a = N\pi\varepsilon^2 = 2{\cdot}596\ \mathrm{m}^2$,
$\sqrt{\mu/\rho}$ (assumed) $= 100$ m/sec,
ρ (assumed) $= 2.10^3$ kg mass/m^3,
μ (calculated) $= 2.10^7$ kg mass/(m sec^2).
From these values of the constants we obtain

$$
\begin{aligned}
&k'l = p/4\,, \\
&\Gamma_1 = 9 \quad 0{\cdot}0^4 2172\, p^2\,, \\
&\Gamma_2 = 0{\cdot}03226\, p - 0{\cdot}0^8 3153\, p^3\,, \\
&P = (18 - 0{\cdot}0^4 4344\, p^2)\cos p/4 \quad (9{\cdot}573\, p - 0{\cdot}0^5 707\, p^3)\sin p/4\,, \\
&Q = (0{\cdot}06452\, p - 0{\cdot}0^8 6306\, p^3)\cos p/4 + 0{\cdot}01388\, p^2 \sin p/4\,. \qquad (4)
\end{aligned}
$$

From (2), (4) it is possible to get the resonance curve of the seismic vibrations of the tower. Fig. 2 indicates the ratio of the maximum horizontal displacement of the tower (not restricted to the part $x = -l$), u', to that of the ground, u_0, for any vibration frequency p. This figure also indicates the number that is roughly proportional to the ratio of the maximum bending moment induced in the structure to that of the maximum acceleration of free ground, that is, ground on which there is no tower standing.

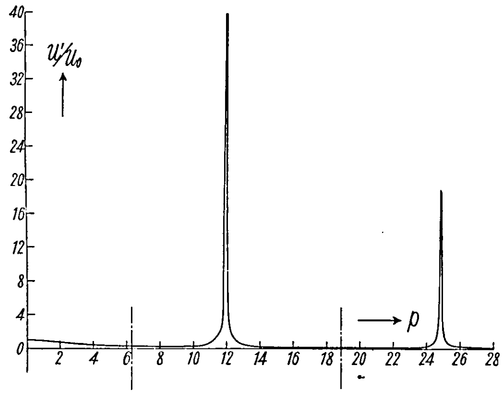

Fig. 2.

Although the natural frequency shown by the present resonance curve is somewhat too large, the character of the dissipation is not greatly affected.

This result shows that the dissipation of vibrational energy is not so marked as was expected, the maximum displacement of the tower at resonance condition being about fourty times that of one at zero frequency vibration. Since however the effective height of the tower is only 7·575 m, the tower under consideration is virtually nothing more than a sort of a low house, suggesting that the induced bending moments at any vibration frequency are exceedingly small, being sometimes smaller than that of a 2-storied house. Thus, even the sharp peak of the curve under resonance condition is of no great importance.

3. *The condition under pendulum action, and free from energy dissipation.*

Let the vibratory motion of the tower and the central post be

$$\left.\begin{aligned} u' &= A\cos(k'x+B)\cos pt\,, \\ \xi &= \xi_0 \cos pt \end{aligned}\right\} \qquad (5)$$

respectively, where $k'=p\sqrt{\rho'/G}$. Then the displacement of the tower at $x=0$ is the same as that of the ground, namely

$$u'=2\cos pt\,, \qquad (6)$$

whereas the condition at $x=-l$ is

$$Ga\frac{\partial u_1'}{\partial x}=\frac{Mg\xi_0}{L/2}, \qquad (7)$$

where $u_1'=A\cos(k'x+B)_{x=-l}$, $a=N\pi\varepsilon^2$; and M, $L/2$ are the mass and the pendulum length of the central post respectively. The dynamical relation between u' and ξ can be obtained by the equation of motion of the centre of gravity of the central post, namely,

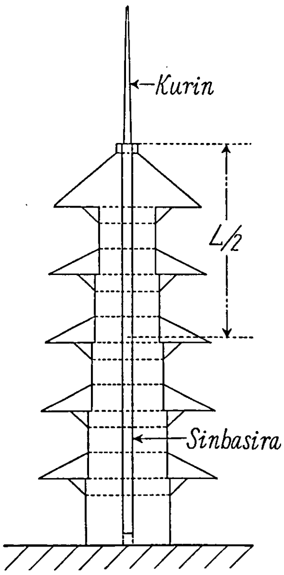

Fig. 3.

$$\frac{d^2\xi}{dt^2}+\frac{2g}{L}\xi=-\frac{d^2u'}{dt^2}. \qquad (8)$$

From (5), (8) we get

$$\xi_0=\frac{p^2u_1'}{2g/L-p^2}. \qquad (9)$$

Substituting u', ξ in (5) in conditions (6), (7), we obtain

$$u'=\frac{2(n_0{}^2-p^2)\cos(k'x+B)}{\cos B\sqrt{(n_0{}^2-p^2)^2+\left(\dfrac{2Mgp^2}{LGak'}\right)^2}}\cos pt\,, \qquad (10)$$

$$\xi=\frac{2p^2}{\cos B\sqrt{(n_0{}^2-p^2)^2+\left(\dfrac{2Mgp^2}{LGak'}\right)^2}}\cos pt\,, \qquad (11)$$

where $n_0^2=2\,g/L$ and

$$B=-\left[k'l+\tan^{-1}\left\{\frac{2Mgp^2}{LGak'(n_0^2-p^2)}\right\}\right]. \tag{12}$$

In this case

$$L/2=79{\cdot}5/2 \text{ shaku}=12{\cdot}05\text{ m}, \qquad M=5{\cdot}95.10^3\text{ kg mass}.$$

Substituting these values as well as the numerical values shown in the preceding section, we obtain

$$\left.\begin{aligned} u'&=\frac{2\,(0{\cdot}8132-p^2)\cos\left(\dfrac{px}{30{\cdot}29}+B\right)\cos pt}{\cos B\sqrt{(0{\cdot}8132-p^2)^2+(0{\cdot}0^4 4646p)^2}}, \\ \xi&=\frac{2\,p^2\cos pt}{\cos B\sqrt{(0{\cdot}8132-p^2)^2+(0{\cdot}0^4 4646p)^2}}, \\ B&=\frac{p}{4}+\tan^{-1}\frac{0{\cdot}0^4 4646}{0{\cdot}8132-p^2}, \end{aligned}\right\} \tag{13}$$

x being in meters. The result is plotted in Fig. 4, in which the full and broken lines indicate $u'_{\max}$, and ξ_0 respectively. Although displacement u'_1 vanishes at a frequency that is equal to the natural frequency of the central post, namely $p=n_0=\sqrt{0{\cdot}8132}$, a new resonance condition manifests itself near $p=n_0$, namely at frequency $p=\sqrt{0{\cdot}8132}-0{\cdot}0^5 533$, although the original resonance conditions of the tower itself still exist, the corresponding frequencies changing but very slightly. It will be seen from this result that the pendulum action of the central post rather defeats the intended object of the post which is to prevent seismic vibration of the tower, excepting however increase in statical frictional force at any joint of the structure due to the weight of the post.

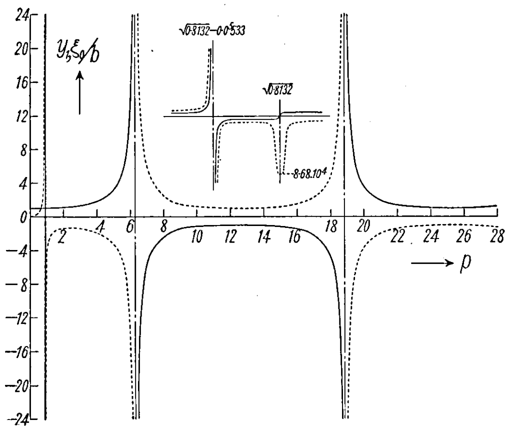

Fig. 4.

4. *The condition under the influence of both energy dissipation and pendulum action.*

While the conditions at $x=0$ are the same as those shown in the previous paper[7], the condition at $x=-l$ is that as given by (7) in the last section. The final result shows that if the ground vibration is expressed by

$$u_0=2\cos pt, \tag{1'}$$

the vibrations of the tower and the central post may be written in the forms

$$u'=4\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left\{(n_0^2-p^2)\cos k'(x+l)+\gamma p^2\sin k'(x+l)\right\}$$
$$\cdot\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{14}$$

$$\xi=4p^2\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{15}$$

where

$$\left.\begin{aligned}P&=\left\{2(n_0^2-p^2)\Gamma_1-\frac{3k'G\varepsilon\gamma p^2\Lambda_1}{\mu}\right\}\cos k'l\\&\quad+\left\{2\gamma p^2\Gamma_1+\frac{3k'G\varepsilon(n_0^2-p^2)\Lambda_1}{\mu}\right\}\sin k'l,\\Q&=\left\{2(n_0^2-p^2)\Gamma_2-\frac{3k'G\varepsilon\gamma p^2\Lambda_2}{\mu}\right\}\cos k'l\\&\quad+\left\{2\gamma p^2\Gamma_2+\frac{3k'G\varepsilon(n_0^2-p^2)\Lambda_2}{\mu}\right\}\sin k'l,\end{aligned}\right\} \tag{16}$$

and $\gamma=2Mg/Gak'L$, the meaning of Γ_1, Γ_2, Λ_1, Λ_2, n_0, k', ν being shown in the preceding two sections.

In the present case (14), (15) become

$$\left.\begin{aligned}P&=\{2(n_0^2-p^2)\Gamma_1-0{\cdot}0^46358p^2\Lambda_1\}\cos k'l\\&\quad+\{0{\cdot}0^49292p\Gamma_1+1{\cdot}3675p(n_0^2-p^2)\Lambda_1\}\sin k'l,\\Q&=\{2(n_0^2-p^2)\Gamma_2-0{\cdot}0^46358p^2\Lambda_2\}\cos k'l\\&\quad+\{0{\cdot}0^49292p\Gamma_2+1{\cdot}3675p(n_0^2-p^2)\Lambda_2\}\sin k'l.\end{aligned}\right\} \tag{17}$$

The result of the calculation, particularly that for u'_{max}, is plotted

7) *loc. cit.* 5).

in Fig. 5. In this case too, displacement $u_1'=u'(x=-l)$ vanishes at $p=\sqrt{0{\cdot}8132}$. But, the additional resonance condition due to the central post appears at $p=\sqrt{0{\cdot}8132}-0{\cdot}0^5525$.

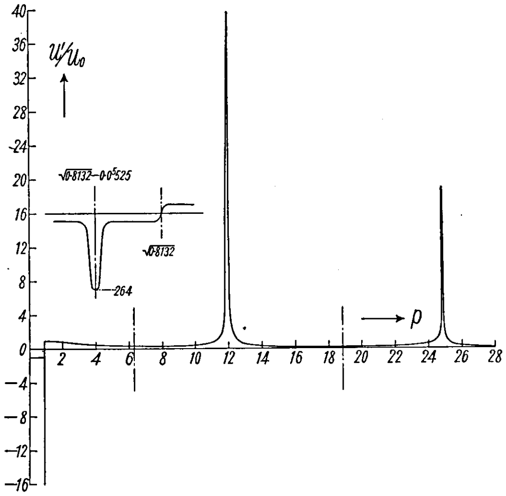

Fig. 5.

5. *Concluding remarks.*

From the present investigation it has been ascertained that the gozyûnotô is earthquake-proof solely because of the fact that the effective height of the elastic part of its structure is extremely small, the induced stress due to the ground vibration being consequently very small. The relatively large stress under resonance condition is still restricted to a certain limiting value due to the fairly large dissipation of the vibrational energy into the ground. The central post, even in the hanging state, has no effect in preventing large vibrations, although it rather gives rise to an additional resonance condition of the tower.

Nevertheless, in order to make the central post effective on the vibration damping of the tower, it is rather advisable to insert some viscous damper between the central post and the tower body, its full character being now investigated.

47. 五重塔の耐震性

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五重塔が見たところ細長いのに拘らず耐震性があるために以前から多くの人の注意をひいた．例へば大森博士の數多くの振動測定や武藤博士の推論などはそのあらはれであるとしてもよい．耐震性のある理由として或人は心柱が振子の作用と鉛直壓力とを與へるからであるといひ，他の人はその構造上から材料を破壞しないけれども構造や基礎の振動減衰摩擦が多いからであるとしてゐるが，何れも研究して見れば見る程これ等の特異性がわからなくなるものである．

筆者等の研究によると，五重塔は構造力學的には決して細長いものではなく，震動問題として取扱つて見ると彈性抵抗から考へても外力の剪斷力から考へても普通の二階建より低い場合にしか當つてをらぬことがわかつたのである．從て筆者等の研究にかゝる震動逸散性がなくても破壞力が極めて少いことがわかつた．又，共振に當る場合の强い筈の震動性は，構造が効果的に低いことから當然逸散性の影響を受けて，これ亦可なり少くなることが知られたのである．

心柱の振子の作用は昔からの構造のままでは共振を更に一つ增加する以外何等の利益もないことがわかつた．卽ちそのまゝの狀態では寧ろ風力に對して効果があるものである．心柱を耐震の目的に役立たせるには心柱と五重塔の間に適當な粘性摩擦を設けることである．如何なる粘性摩擦が適當であるかは目下研究中であり，近く報告できる筈である．