49. *Preliminary Notes on Experimental Studies on the Plastic Deformation of Soil.*

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1. Introduction. In investigating the conspicuous subsidences of the earth's surface in certain restricted areas, as those that occurred in the Kôtô district in the city of Tokyo[1] and in low ground in the city of Osaka[2], the necessity was felt for studying the physical properties of the soils composing the surface layers, the contraction of which seems to have been an important factor in bringing about the remarkable subsidence.

The writer describes in this paper some of these preliminary experiments that were made for studying the mechanical properties of soils. On carrying out the experiments, however, it was found that not only are the test pieces of the soils deformed elastically, but that time played a very important rôle in the progress of deformation under statical loading, even when the soil was perfectly dry. A non-linear relation between the applied load and the deformation obtained as the result of these experiments was therefore explained tentatively by assuming a visco-elastic or plastic property of the soil.

2. Soils Prepared for the Experiments. The soils used in the present experiments were taken from various depths from where a pier of the Komatugawa-basi (bridge)[3] is built.

Sieving showed the soil to be composed of particles of various sizes. Since the most important information sought by sieving is the percentage of the finer grained soil particles, which may be gel-forming material if the soil mass contains a considerable amount of water and becomes a colloidal gel, it was decided to use Stoke's method of sieving. The composition of the soil is thus separated into three components, i. e., (I) that which deposits in a very short time after the soil is mixed with an excessive quantity of water, (II) that which deposits from the re-

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[1] N. MIYABE, *Bull. Earthq. Res. Inst.*, **10** (1932), 844; **13** (1935), 763; etc.

[2] A. IMAMURA, *Proc. Imp. Acad.*, **9** (1933), 378; **11** (1935), 186; etc.

[3] Komatugawa-basi is a bridge built over the Arakawa (river), connecting Kameido and Komatugawa, in the eastern part of the city of Tokyo.

sidual suspension during the next 40 minutes, and (III) that which deposits during the next 10 hours.

The percentages of the components of the soil grains, i. e., the so-called mechanical ratios, are obtained as shown in Table I for soils from different depths.

Table I. Mechanical Ratios of Soils from Different Depths.

Sample	Depth	I deposites	II deposites	III deposites	Residuals
A	− 3·3 m	35%	39%	5%	1%
B	−19·0 m	2%	78%	12%	8%
C	−27·5 m	73%	7%	3%	17%

N. B. Height of this point about 1 m above mean sea-level.

3. Apparatus and Results of Experiments. The experimental study of the mechanical properties of soil was made by two different methods, i. e., by bending and by torsion. For the bending tests, the apparatus shown in Fig. 1 was used. To the soil test piece, which is supported at two edges A, A′, a load is applied to its middle P. The tilt of the surface of the test piece at point A″ was measured for different loads by means of scale S and the mirror M.

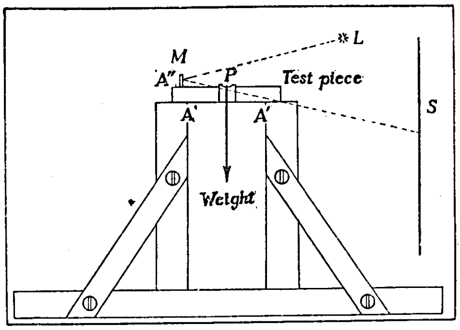

Fig. 1. Apparatus for bending test.
(M: Mirror; S: Scale;
L: Light source.)

The apparatus for torsion test is shown in Fig. 2. In the case of torsion tests, the soil test piece is clamped at one end to B and the other end fixed to B′, the centre of wheel S being supported at V by a pivot. The wheel is rotated by weights as shown in Fig. 2, and the test piece consequently subjected to torsional deformation.

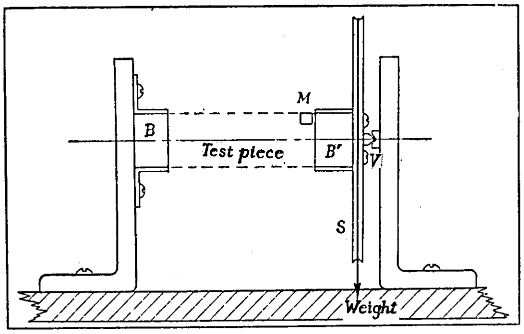

Fig. 2. Apparatus for torsion test.
(M: Mirror; V: Pivot;
S: Wheel.)

The soil test pieces each from a different depth, are formed into rectangular bars of approximate dimensions, 2 × 3 × 10 cm, for use in the bending tests, and 3×3×6 cm for use in the trosion tests. Two series of tests were made in the present experiments, the one being the series of tests for wet soils, and the other for perfectly dry soils. Perfect dryness of soil was obtained by exposing the test pieces *in situ* for several days and weighed at times, untill the dacrease in weight diminished.

The results showed that the differences in the mechnical ratios and in the methods of experiments, i. e., bending or torsion, do not greatly affect the relation between deformation and applied load. On the other hand, as shown in Figs. 3~8, the difference in water contents, as may be expected, was the greatest factor affecting the load-deformation relation of soils.

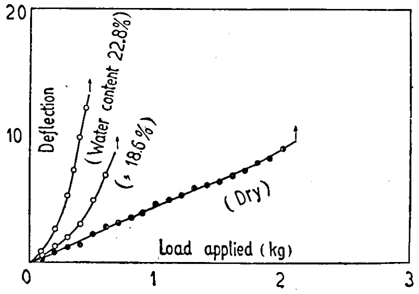

Fig. 3. Load-deformation in the case of bending of wet and dry state of soil A.

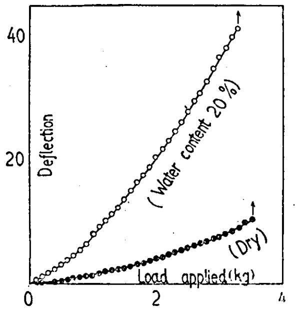

Fig. 4. Load-deformation relation in the case of bending of wet and dry state of soil B.

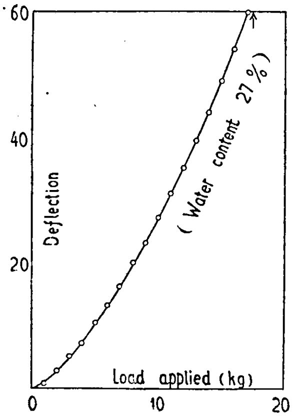

Fig. 5. Load-deformation relation in the case of bending of wet state of soil C.

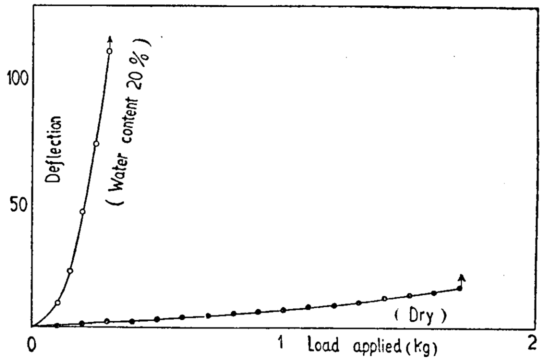

Fig. 6. Load-deformation relation in the case of torsion of wet and dry state of soil A.

It was noticed that the deformation differed in mode with the loads applied according to whether they were bending tests or torsion tests and also with the kind of soil tested, that is, from different depths. These differences, if real, may be attributed to differences in the mechanical properties of the soils as well as to instrumental errors due to imperfections in the apparatus used, whence it follows that in the present results we can deal only with the effects of the water content under the applied loads.

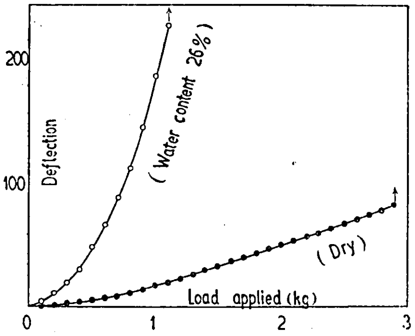

Fig. 7. Load-deformation relation in the case of torsion of wet and dry state of soil B.

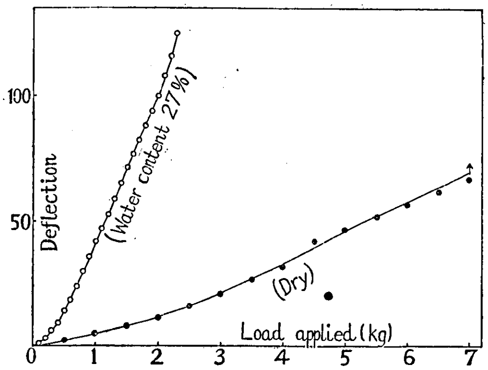

Fig. 8. Load-deformation relation in the case of torsion of wet and dry state of soil C.

4. Relaton between Deformation and the Loads Applied.

The relation beween the deformations and the loads applied, though not linear as shown in Figs. 3~8, can be explained in more ways than one. The fact, that deformation E and the applied load S are not in linear relation, may in one way be elucidated by assuming the relation $E=NS^n$. where n is greater than unity, as in numerous cases of deformation of the soft materials[4]. However it may not in order to invoke such a relation to explain the results of the present experiments, seeing that the test pieces of soil showed permanent deformation when the load was applied.

Soil-like material may therefore be regarded as deforming in such a way that the deformation is composed of elastic deformation and deformation due to viscous flow[5]. For such visco-elastic deformation, the relation between deformation and applied load is expressed in two different forms, the one by a formula introduced by Voigt and the other

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[4] BACH and BAUMANN, "Elastizität und Festigkeit", S-36.

[5] R. K. SCHOFIELD and G. W. S. BLAIR, *Proc. Roy. Soc.*, A-**138** (1932), 707; **139** (1933), 557; **141** (1933), 72.

by a formula due to Maxwell.

To explain the results of the present experiments, it seems convenient to use Maxwell's formula, which is given in the form

$$\frac{de}{dt}=\frac{1}{n}\frac{dS}{dt}+\frac{1}{\eta}S,$$

where $\frac{de}{dt}$ is the rate (velocity) of deformation, S the stress, and n, η are the elastic constant and the coefficient of viscosity respectively.

It is of course a debatable question whether or not deformation of soil can rightly be regarded as being of the nature of visco-elastic deformation. It is well known that soil-like substances holding varying quantities of sandy materials differ in their resistance to shearing deformation under varying normal pressures, which makes it seem that the frictional resistance between the particles composing the soil mass is the most important factor in their resistance to external forces. Therefore, a theoretical treatment of deformation of such material is best developed rather on such lines as those taken by R. Becker in his investigations⁶⁾, in which the deformation of crystalline aggregates and the formations of slip lines are discussed statistically. In this paper however such phases of the subject are not dealt with: in being merely an attempt to explain the non-linear relation between deformation and applied loads from the stand point of visco-elasticity.

Since we are to use Maxwell's formula, it may perhaps be worth while to show a mode of deformation of soil under constant load, i. e., the time effect on deformation.

A sample of soil was subjected to deformation as described in the preceding paragraph under a constant load and the deformation plotted against time, the results being shown in Figs. 9～12. Although as

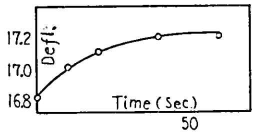

Fig. 9. Progress in torsional deformation with time (dry soil, under the statical load of 1·0 kg.).

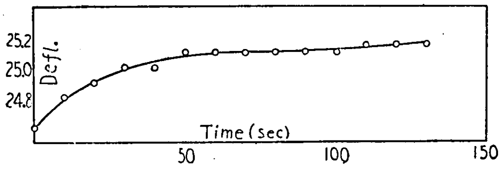

Fig. 10. Progress in torsional deformation with time (dry soil, under the statical load of 1·5 kg.).

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6) R. Becker, *Phys. ZS.*, **26** (1925), 919.

shown in these figures, the deformation progresses, though very slowly, with time, it does not tend to any asymptotic value, from which we may tentatively explain the mode of deformation of soils as follows, on the basis of visco-elasticity.

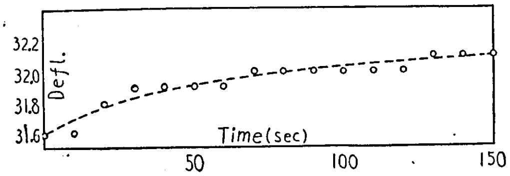

Fig. 11. Progress in torsional deformation with time (dry soil, under the station load of 2·0 kg.).

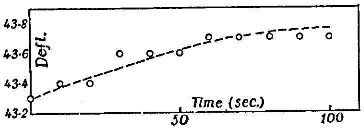

Fig. 12. Progress in torsional deformation with time (dry soil, under the statical load of 3·0 kg.).

To use Maxwell's formula in explaining the results of the present experiments, it is expressed in the form of finite differences, as

$$\Delta e=\frac{1}{n}\Delta S+\frac{1}{\eta}S\Delta t,$$

where Δe is the amount of deformation caused by application of the load ΔS and lapse of time Δt. After adding unit load m times with a constant time interval, the resultant deformation E is expressed by

$$E=\sum_m \Delta e=\frac{1}{n}\sum_m \Delta S+\frac{1}{\eta}\sum S\Delta t.$$

It is evident that $\sum_m \Delta S=S$ and $\sum_m S\Delta t$ may be calculated as follows;

$$\sum_m S\Delta t=S_0\Delta t+\{S_0+(\Delta S)_1\}\Delta t+\{S_0+(\Delta S)_1+(\Delta S)_2\}\Delta t+\cdots\cdots\cdots$$
$$=mS_0\Delta t\Delta+t\{(m-1)(\Delta S)_1+(m-2)(\Delta S)_2+\cdots\cdots\cdots\}.$$

Assuming that $S_0=0$, i. e., no load was applied initially, and that ΔS, Δt are constant, we have

$$\sum_m S\Delta t=\{(m-1)+(m-2)+\cdots\cdots\cdots+1\}\Delta S\cdot\Delta t$$
$$=\frac{m(m-1)}{2}\Delta S\cdot\Delta t=\frac{1}{2}(S\cdot m\Delta t-S\Delta t).$$

Since in plotting the result of the present experiment in an $S-E$ diagram, the addition of unit load ΔS corresponds to lapse of time Δt, we have $m\Delta t=AS$, the equation being finally obtained in the form

$$E=\left(\frac{1}{n}-\frac{\Delta t}{2\eta}\right)S+\frac{A}{2\eta}S^2,$$

which may suitably represent the experimental results obtained above.

5. Effect of Water Content on Elasticity and Viscosity. The relation between E and S deduced form Maxwell's formula is, as obtained in the preceding paragraph, written in the form

$$E = aS + bS^2,$$

where

$$a = \frac{1}{n} - \frac{\Delta t}{2\eta},$$

$$b = \frac{A}{2\eta}$$

Hence, constants a and b that were determined from actual data by the method of least squares may represent some measure of the elastic constant and the coefficient of viscosity of the visco-elastic soils.

In a case of torsion test, these constants were determined for dry and wet soils separately with the following results:

$$\begin{aligned} a &= -5{\cdot}6 \times 10^{-3} \text{ for dry soil} \\ &= -1{\cdot}4 \times 10^{-1} \text{ for wet soil}, \\ b &= 1{\cdot}3 \times 10^{-5} \text{ for dry soil} \\ &= 1{\cdot}8 \times 10^{-3} \text{ for wet soil}. \end{aligned}$$

As the scale for the deformation was quite arbitrary, the absolute values of these constants could not be obtained.

From the results obtained above, we can say that the viscosity of the soil, i. e., the value corresponding to the reciprocal of the value of b, is greater in dry than in wet soil.

Although it is also noticed that the order of magnitude of constant a varies parallel to b, it does not always indicate that the elastic constant of the soil parallel to the variation of viscosity. Since the constant a is composed of an elastic constant and the coefficient of viscosity, as in the above expression, $\frac{1}{n}$ could not contribute much to changes in the value of a even though considerable changes occurred in the value of n, in the case when $\frac{1}{n} \ll \frac{\Delta t}{2\eta}$. The negative value of a obtained in the foregoing examples suggests that these facts should be taken into consideration.

6. Hysteresis. In the case of a torsion test for dry soil, deformation due to increasing and decreasing loads were measured. As might be expected, the curve representing the relation between deformation and load in this case is a hysteresis loop as shown in Fig. 13.

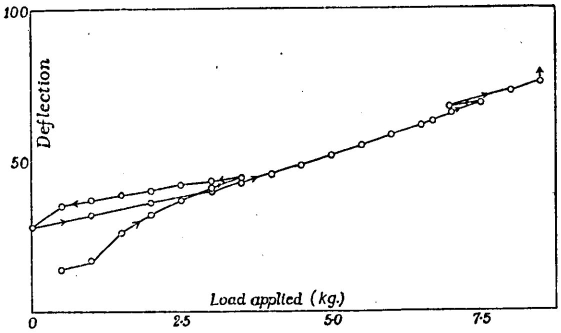

Fig. 13. Hysteresis in load-deformation relation in the case of torsion test of dry soil.

It may also be noticed that in deformation due to decreasing load after it had been deformed by an increasing load, for several tens of seconds, the soil shows no sign of plastic flow, whereas it does so in the case of deformation due to increasing load, as shown in Fig. 14, in which the deformation is plotted against the time that elapsed.

The hysteresis in deformation due to alternate applications of increasing and decreasing loads may be explained in the way as has been developed by Mr. Iida[7] in studying hysteresis in the deformation of soft materials.

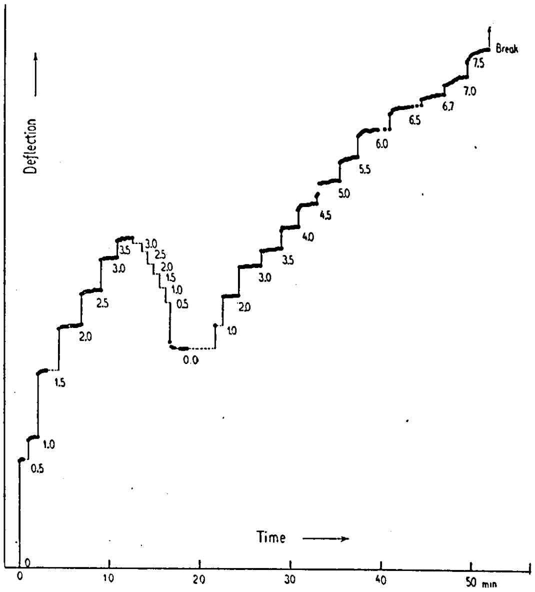

Fig. 14. Time-deformation relation under various statical loads, in the case of torsion of dry soil. (numerical figures denote the load for corresponding deformation).

7. Ultimate Strength of Soil. As shown in the results of the present experiments, deformation of the soil was visco-elastic, that is, the relation between deformation and applied load was not at all linear. The critical load for breaking of the test piece of soil however was

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[7] K. Iida, *Bull. Earthq. Res. Inst.*, **13** (1935), 198.

determined rather clearly.

At the moment of breaking of the test piece of dry soil, visible cracks begin to develop and the velocity of deformation increases abruptly.

The surface of break that developed in the test piece of dry soil seems to have a regular orientation. It was however unfortunate that we could not make systematic studies in this respect. In the case of wet soil, the velocity of deformation of the test piece seems to increase in the neighbourhood of the ultimate load for break. This fact suggest that several micro-cracks, although invisible, would have developed in any case before the test piece broke down. For the breaking strengths of soils, we obtained the values as shown in Table II.

Table II. Strength of Soil.

Bending			Torsion		
Sample	Water content	Critical Load $\mathrm{kg/cm^{-2}}$	Sample	Water content	Critical Load $\mathrm{kg/cm^{-2}/cm}$
A	22·8%	0·06	A	20∼17%	0·0068
A	18·6″	0·09	A	0″	0·028
A	0·0″	0·41	B	28∼26″	0·021
B	20·0″	0·62	B	0″	0·080
B	0·0″	0·85	C	28∼27″	0·057
C	28·0″	0·27	C	0″	0·209

The breaking strength of soil seems, as shown in Table II, to differ somewhat according to the size of the particles composing it. The above results however are not so reliable owing to scantiness of data and to the fact that the experimental apparatus was not complete in every respect.

8. Concluding Remark. In the above mentioned preliminary notes on experiments in connection with deformation of soils, the modes of deformations within the limit of their breaking strengths are explained by assuming a visco-elastic property for the soil as expressed by Maxwell's formula.

As to the points left unexplained, of which there are a number, it is hoped that it will be possible to deal with them in a future investigation, particularly as there may be some points in the physical and chemical properties of soil that could be discussed in connection with its colloidal nature. The results of such studies will be closely related to a number of problems in geophysical phenomena, for instance, oscillations of surface layers, the remarkable local subsidences as referred to at the beginning of this paper, etc.

In conclusion, the writer wishes to express his sincere thanks to the council of the Hattori Hôkôkwai for aid received for studying the crustal deformation in the Kôtô region, by means of which the present experiments were carried out. The writer's best thanks are also due to Mr. Umezawa, civil engineer, for his kindness in placing samples of soils at the writer's disposal.

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49. 土のプラスティック變形に關する實驗（序報）

地震研究所　宮部直巳

東京市內小松川に於ける架橋工事場に於いて種々の深さから採取した土の試片に就き，撓曲及び振りの試驗を行ひ，その結果にあらはれた土の變形に關する性質について述べた．

荷重—變形の關係が直線的でないことは從來知られてゐる所と全く同じであるが，之を解釋するのに，Maxwell の粘彈性式を用ひた．Maxwell の式から導かれる結果は

$$E = aS + bS^2$$

であつて，非直線的な荷重—變形の關係を解釋するに都合がよい．又，試片の含水量の多少によつて上式の a，b 等が變化する點についても一顧を費した．

以上の如き結果は土のコロイド的性質に關係する所が多いやうに思はれる．