52. *Notes on the Origins of Earthquakes.*

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1. *Introduction*

It is well known that a study of the initial motions of earthquakes is of utmost importance in understanding the mechanism of the earthquake waves from the seismic origin.

Nowadays, the initial motions of the S waves are also invoked in obtaining knowledge regarding seismic origins, notwithstanding the difficulty of determining the phases on the seismograms. It may moreover, prove advantageous in connection with this problem to study the surface waves at the times of shallow earthquakes as also the ScS waves, which are the reflected waves at the margin of the central core of the globe in the case of deep-focus earthquakes.

The pull and push waves in the P phase were known since the time of the late Prof. F. Omori,[1] but it was the late Prof. T. Shida who first showed some excellent examples of the geographical distribution of the pull and the push waves and discussed the mechanisms of seismic origins.

In accordance with the results of later studies of shallow earthquakes, the pull and push waves were grouped separately into four quadrants divided by two lines(nodal) intersected perpendicularly at the epicenters. To explain these phenomena, the late Prof. T. Shida offered formations of fissures at the seismic origins. Against this idea, Prof. S. T. Nakamura[2] proposed the fault theory on the ground that frequently one of the nodal lines fairly coincided with faults observed in the epicentral regions. Very interesting distributions of the initial motions were found by Mr. K. Tanahasi[3] at the time of the deep-focus earthquake of June 2, 1931, in the central part of the main island of Japan.

In this case, the boundaries between the pull and the push waves were well expressed by a pair of parabolas. Later studies of Mr. K.

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1) F. OMORI, *Bull. Earthq. Invest. Commit.*, **1** (1907), 145~154.

2) S. T. NAKAMURA, *Kishô-Shûsi*, **37** (1918), 390~401; **38** (1919), 395~404; [ii], **1** (1923), 1~13.

3) K. TANAHASI, *Umi to Sora*, **11** (1931), 277~288.

Sagisaka[4] and Mr. M. Takehana[5], showed that even in the case of deep-focus earthquakes there are conical types as well as quadrant types among the lines dividing the regions where pull waves or push waves were observed.

Prof. M. Ishimoto[6] after carefully examining cases in which conical types were observed, found that the push waves were always on the conical surfaces with their vertices at the hypocentre. He also expressed the distribution of the pull and the push waves projected on the spherical surface with its centre at the seismic origin by the simple zonal harmonic functions, $P_0(\cos\theta)$ and $P_2(\cos\theta)$, and further pointed out, in the case of shallow earthquakes, instances in which the nodal lines of the initial motions assumed peculiar forms. He explained these cases by refraction of the P waves at the boundary between the Mohorovicic layer and the substratum; and that otherwise the nodal lines might assume simple conical forms.

Mr. T. Minakami,[7] in his paper on the origins of shallow earthquakes, satisfactorily explained the geographical distribution of initial motions in a number of earthquakes, assuming that the pull waves were concentrated in cones with their vertices at the seismic origins and that the solid angles of these cones were all $\pi/2$. Mr. K. Fukutomi[8], after a study of the earthquakes that occurred in the Kwanto district, obtained the interesting result in the case of shallow earthquakes, the directions of the axes of the nodal cones of the initial motions were horizontal, while in the case of rather deep earthquakes their directions were indefinite.

Valuable investigations were next made by the able seismologists Dr. H. Kawasumi[9] and Dr. K. Honda[10] regarding the sense and magnitude of the P phases and the S phases of some deep earthquakes. Dr. K. Honda explained the observed P and S phases of some deep-focus earthquakes, assuming that the normal stress on the spherical surface with its centre at the origin might be expressed by a simple spherical harmonic function ($P_2^1(\cos\theta)$), taking into consideration the effects of the reflection of the waves at the earth's surface and the effects due

4) K. SAGISAKA, *Geophys. Mag.*, **3** (1930), 165~176.
5) M. TAKEHANA, *Kensin-Jihô*, **7** (1933).
6) M. ISHIMOTO, *Bull. Earthq. Res Inst.*, **10** (1932), 449~471; *Proc. Imp. Acad., Japan*, **8** (1932), 36.
7) T. MINAKAMI, *Bull. Earthq. Res. Inst.*, **13** (1935), 114~129.
8) T. FUKUTOMI, *Bull. Earthq. Res. Inst.*, **11** (1933), 510~529.
9) H. KAWASUMI, *Bull. Earthq. Res. Inst*, **11** (1933), 403~453; **12** (1934), 660~705.
10) H. HONDA, *Geophys. Mag.*, **8** (1934), 153~164; **8** (1935), 327~332.

to the refraction of the waves in the earth's crust. In these investigations the observed fact that the periods of the P phases differ from those of the S phases were not taken into considerations. We may, with Mr. T. Ishikawa,[11] conclude that the wave lengths of the P waves and S waves are almost equal. But we do not yet know the mechanism by which such waves are radiated from the seismic origin.

2. We are now thinking of the distribution of the initial motions as we know them to be on the earth's surface as distributions on a spherical surface with the seismic origin as its centre.

As Prof. T. Matuzawa has already pointed out, it is possible to imagine various kinds of mechanisms for the origin of an earthquake corresponding to any one type of distribution of initial motions. We shall take for example the case in which the conical type of distribution of the initial motions is observed.

a) We can explain these observed distributions of initial motions by assuming that the internal pressure on the spherical surface with its centre at the hypocentre is expressed by $P_0(\cos\theta)$ and $P_2(\cos\theta)$. Changing the ratio in the combination of the quadruple source P_2 and the single source P_0, we can obtain an arbitrary solid angle of the cone.

b) We shall suppose that a combination of a pair of doublet forces acting along one axis and a single source. Changing the proportion of the doublet to the sink, or the negative single source, we can obtain an arbitrary solid angle of the cone.

c) Let us suppose that three pairs of these doublets are co-acting at the seismic origin, and that one of the two pairs of the doublets in the same phase lies on top of the other, each of which is acting along the axis perpendicular to each other, and that the last pair of the doublets in the opposite phase, perpendicularly to the plane, on which are the above two pairs of doublets, also lie, one pair on top of the other. If we take the intensities of the last pair of doublets to be different from those of the other two pairs, we can obtain an arbitrary solid angle of the cone.

Now what we can deduce from the observations of seismic waves is limited to variations in the stresses on a certain closed surface taken at the seismic origin. As to what occurs in that closed surface we can only imagine. For example, the stress distribution on a certain closed surface may assume more or less complicated features as the result of the co-action of a number of single sources, positive or nega-

11) T. ISHIKAWA, *Kishô-Shûsi*, [ii], **10** (1932), 260.

tive, within that surface, owing to the interference of waves from each source and the reflections of waves from that closed surface.

3. Let us suppose a sphere at the seismic origin and consider the effect of variation in internal pressure within it.

Actually, it is natural to assume that the shape of the magma pocket may be more or less flat, rather than a sphere, but for simplicity we here assume that the cavity that is filled with magma is spherical. Should the vapour pressure within that spherical cavity increase, yielding of the surrounding medium will follow, and should the pressure exceed the limit of the breaking strength at some point on the spherical surface, the magma will break out through that weakened part. The seismic waves that are generated then might be assumed to correspond to those that are generated through the action of a pair of doublets acting along an axis. The normal stresses on the spherical surface in that case may be expressed by $P_2(\cos\theta)+\frac{1}{2}P_0(\cos\theta)$ or $\cos^2\theta$.

The pressure diminution and the reduction in volume of the magma within the cavity accompanied by the eruption of magma may act as a negative single source. The total volume of the magma might remain constant during its eruption as is generally the case when plastic material change their shape. To satisfy the condition that the volume of the magma shall be constant during its deformation (outflow) we have to assume a sink $(-\frac{1}{2}P_0)$ co-acting with a pair of the doublets. At all events, we may conclude that seismic waves that are generated in this case correspond to that case in which the normal stresses may be expressed by $P_2(\cos\theta)$.

Since the normal component displacement that results from the internal pressure $P_2(\cos\theta)$ on the spherical surface, the radius of which is a, are expressed by

$$u_1+u_2=-P_2(\cos\theta)e^{ipt}\left[\frac{A}{h^2}\frac{d}{dr}\frac{H^{(2)}_{\frac{5}{2}}(hr)}{\sqrt{r}}+12\frac{B}{k^2}\frac{1}{r^{3/2}}H^{(2)}_{\frac{5}{2}}(kr)\right]_{r=a},$$

where u_1 and u_2 corresponds to the dilatational and distortional waves respectively, the azimuthal distributions are also expressed by $P_2(\cos\theta)$.

We shall next consider the proportion of the amounts of the positive displacements to those of negative displacements.

Since $$P_2(\cos\theta)=\frac{1}{2}(3\cos^2\theta-1)$$

and
$$\int_0^{\frac{\pi}{2}} P_2(\cos\theta) 2\pi a^2 \sin\theta \, d\theta = 0,$$

the positive and the negative parts cancel each other, so that the volume remains constant during deformation.

As it is assumed that the pressure distributions are $P_2(\cos\theta)$, the sum of the positive and negative parts in the pressure change is zero.

The fact that the distribution of the initial motions of many earthquakes are fairly well expressed by $P_2(\cos\theta)$ may partly be due to cause just mentioned. Even in the case just considered, the proportion of the single source to the quadruple source may be influenced by a number of causes, and there will be a case in which the distributions of the initial motions may be expressed by $P_2(\cos\theta) - \frac{1}{4}P_0(\cos\theta)$ and in which the solid angle of the cone is $\frac{\pi}{2}$. And since, at any rate, the single source $P_0(\cos\theta)$ is the simplest, and may be assumed to be the one that is most likely to occur, it is plausible to assume that the single source may generally be found accompanied by complex sources, such as $P_2(\cos\theta)$ or $P_2^1(\cos\theta)$.

4. We shall next consider the case in which the normal stresses at the spherical surface are distributed in a simple manner and can be expressed by a zonal harmonic series. Bearing in mind the principle of the "conservation of momentum", the normal stresses may be expressed by harmonics of even order, namely,

$$\text{normal stress} = a_0 P_0(\cos\theta) + a_2 P_2(\cos\theta) + a_4 P_4(\cos\theta) + \cdots.$$

We shall here study the case in which the normal stress can be expressed by the first two terms. The general solution has been already obtained by Prof. K. Sezawa.[12]

Displacements due to dilatational waves in an isotropic elastic solid body are expressed by

$$u_{1,n} = -\frac{A_n}{h^2} P_n(\cos\theta) \frac{d}{dr} \frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}} e^{ipt},$$

$$v_{1,n} = -\frac{A_n}{h^2} \frac{dP_n(\cos\theta)}{d\theta} \frac{1}{r^{3/2}} H^{(2)}_{n+\frac{1}{2}}(hr) e^{ipt}.$$

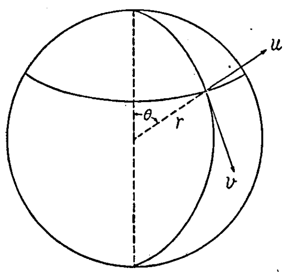

Fig. 1.

Displacements due to distortional waves are expressed by

12) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **2** (1927), 13.

$$u_{2,n} = -2B_n \frac{n(n+1)}{k^2} P_n(\cos\theta) \frac{1}{r^{3/2}} H^{(2)}_{n+\frac{1}{2}}(kr) e^{ipt},$$

$$v_{2,n} = -\frac{2B_n}{k^2} \frac{dP_n(\cos\theta)}{d\theta} \frac{1}{r} \frac{d}{dr} \sqrt{r}\, H^{(2)}_{n+\frac{1}{2}}(kr) e^{ipt},$$

where $h^2 = \dfrac{\rho p^2}{\lambda + 2\mu}$, $k^2 = \dfrac{\rho p^2}{\mu}$,

$\dfrac{2\pi}{p}$ = period of oscillations,

u, v=radial and transverse component of displacement respectively,
ρ=density of isotropic solid,
λ, μ=Lame's elastic constants.

The surface $r=a$ being assumed to be free from tangential traction and to be under normal pressure $p_n P_n(\cos\theta) e^{ipt}$, the equations

$$\widehat{rr} = \lambda\Delta + 2\mu \frac{\partial u}{\partial r} = p_n P_n(\cos\theta) e^{ipt},$$

$$\frac{\widehat{r\theta}}{\mu} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta} = 0,$$

in which Δ is the dilatation, $u=u_{1,n}+u_{2,n}$ and $v=v_{1,n}+v_{2,n}$ must hold on that surface.

Satisfying the above boundary conditions, we have

$$\frac{B_n}{A_n} = \frac{-\dfrac{k_2}{h^2}\left[\dfrac{d}{dr}\dfrac{H_{n+\frac{1}{2}}(hr)}{r^{3/2}}\right]_{r=a}}{\left[\dfrac{1}{\sqrt{r}}\dfrac{d^2}{dr^2}H_{n+\frac{1}{2}}(kr) - \dfrac{1}{r^{3/2}}\dfrac{d}{dr}H_{n+\frac{1}{2}}(kr) + \dfrac{1}{r^{5/2}}\left(n(n+1) - \dfrac{4}{5}\right)H_{n+\frac{1}{2}}(kr)\right]_{v=a}}$$

$$\equiv R,$$

$$A_n\left[\lambda \frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}} - \frac{2u}{h^2}\frac{d^2}{dr^2}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}} - 4\mu R \frac{n(n+1)}{k^2}\frac{d}{dr}\frac{1}{r^{3/2}} \cdot H^{(2)}_{n+\frac{1}{2}}(kr)\right]_{r=a} = p_n.$$

Henceforth, we shall consider only the case $\lambda=\mu$, or the Poisson ratio $\sigma=\frac{1}{4}$.

(1) The single source, in which the normal pressure at the origin is expressed by $P_0(\cos\theta)e^{ipt}$.

In this case we have

$$A_0=\frac{p_0}{\mu a^{-\frac{1}{2}}}\Bigg/\left[(3h^2a^2-4)\,H_{\frac{1}{2}}^{(2)}(ha)+4haH_{\frac{1}{2}}^{(2)}(ha)\right]\equiv a_0+ib_0\,.$$

The radial component of the displacement of the dilataional wave is given by

$$u_{1,0}=-\frac{a_0+ib_0}{h^2}P_0(\cos\theta)\frac{d}{dr}\frac{H_{\frac{1}{2}}^{(2)}(hr)}{\sqrt{r}}e^{ipt}$$

$$=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_0(\cos\theta)\left\{\left(a_0+\frac{b_0}{hr}\right)\cos(pt-hr)\right.$$

$$\left.+\left(\frac{a_0}{hr}-b_0\right)\sin(pt-hr)\right\},$$

and at a distant point from the origin as compared with the wave length, we have

$$hr=2\pi\frac{r}{L}\gg 1 \text{ (where } L \text{ is the wave length),}$$

$$u_{1,0}=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_0(\cos\theta)\left\{a_0\cos(pt-hr)-b_0\sin(pt-hr)\right\}$$

$$=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_0(\cos\theta)\sqrt{a_0^2+b_0^2}\cos\left(pt-hr+\tan^{-1}\frac{b_0}{a_0}\right).$$

The constants a_0 and b_0 are given by the equations

$$a_0=\frac{p^0}{\mu a^{-\frac{1}{2}}}\frac{a}{a^2+b^2},\quad b_0=\frac{p_0}{\mu a^{-\frac{1}{2}}}\frac{-b}{a^2+b^2},$$

where

$$a=(3h^2a^2-4)J_{\frac{1}{2}}(ha)+4haJ_{-\frac{1}{2}}(ha),$$

$$b=(3h^2a^2-4)J_{-\frac{1}{2}}(ha)-4haJ_{\frac{1}{2}}(ha),$$

and $ha=2\pi\frac{a}{L}$ is the ratio of the radius of the seismic origin to the wave length of the seimic waves generated from the origin.

The relations between the amplitude of $u_{1,0}$ at a distant point and ha are shown graphically in Fig. 2. As will be seen from the figure, the amplitude decreases as the wave length diminishes.

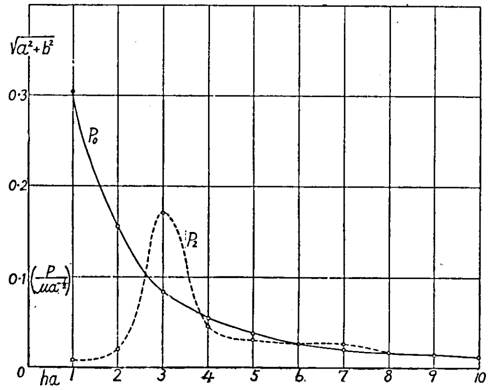

Fig. 2.

(2) The quadruple source in which the normal pressure at the origin is expressed by $P_2(\cos\theta)e^{ipt}$.

In this case we have

$$A_2=\frac{p_2}{\mu a^{-\frac{1}{2}}}\Bigg/\Bigg[(3h^2a^2-24)H^{(2)}_{\frac{5}{2}}(ha)+4haH^{(2)}_{\frac{3}{2}}(ha)$$
$$-24\,U\Big\{-4H^{(2)}_{\frac{5}{2}}(ka)+ka\,H^{(2)}_{\frac{3}{2}}(ka)\Big\}\Bigg]\equiv a_2+ib_2,$$

where

$$U=\frac{h^2}{k^2}R\,.$$

The radial component of the displacement of the dilatational wave is given by

$$u_{1,2}=-\frac{a_2+ib_2}{h^2}P_2(\cos\theta)\frac{d}{dr}\frac{H^{(2)}_{\frac{5}{2}}(hr)}{\sqrt{r}}e^{ipt}$$
$$=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_2(\cos\theta)\Bigg[\Big\{a_2\Big(\frac{9}{h^2r^2}-1\Big)-\frac{b_2}{hr}\Big(-\frac{9}{h^2r^2}+4\Big)\Big\}\cos(pt-hr)$$
$$-\Big\{b_2\Big(\frac{9}{h^2r^2}-1\Big)+\frac{a_2}{hr}\Big(-\frac{9}{h^2r^2}+4\Big)\Big\}\sin(pt-hr)\Bigg],$$

and at a great focal distance as compared with the wave length, we have

$$u_{1,2}=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_2(\cos\theta)\left\{a_2\cos(pt-hr)-b_2\sin(pt-hr)\right\}$$

$$=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}P_2(\cos\theta)\sqrt{a_2^2+b_2^2}\cos\left(pt-hr+\tan^{-1}\frac{b_2}{a_2}\right).$$

The colatitudinal component of the displacement of the dilatational wave is given by

$$v_{2,2}=-\frac{2A_2R}{k^2}\frac{dP_2(\cos\theta)}{d\theta}\frac{1}{r}\frac{d}{dr}\sqrt{r}\,H^{(2)}_{\frac{5}{2}}(kr)e^{ipt}$$

$$=-\frac{2}{h^2}\frac{1}{r}\sqrt{\frac{2}{\pi}}k^{\frac{1}{2}}\frac{dP_2(\cos\theta)}{d\theta}\left[\left\{(a_2\alpha-b_2\beta)\left(\frac{6}{k^2r^2}-1\right)-(a_2\beta+b_2\alpha)\right.\right.$$

$$\left.\cdot\left(-\frac{6}{k^3r^3}+\frac{3}{kr}\right)\right\}\cos(pt-kr)-\left\{(a_2\beta+b_2\alpha)\left(\frac{6}{k^2r^2}-1\right)\right.$$

$$\left.\left.+(a_2\alpha-b_2\beta)\left(-\frac{6}{k^3r^3}+\frac{3}{kr}\right)\right\}\sin(pt-kr)\right],$$

and at a great focal distance as compared with the wave length, we have

$$v_{2,2}=-\frac{2}{h^2}\frac{1}{r}\sqrt{\frac{2}{\pi}}k^{\frac{1}{2}}\frac{dP_2(\cos\theta)}{d\theta}\left\{(a_2\beta+b_2\alpha)\sin(pt-kr)\right.$$

$$\left.-(a_2\alpha-b_2\beta)\cos(pt-kr)\right\}$$

$$=-\frac{2}{h^2}\frac{1}{r}\sqrt{\frac{2}{\pi}}k^{\frac{1}{2}}\frac{dP_2(\cos\theta)}{d\theta}\sqrt{(a_2\beta+b_2\alpha)^2+(a^2\alpha-b_2\beta)^2}$$

$$\cdot\sin\left\{pt-kr-\tan^{-1}\frac{a_2\alpha-b_2\beta}{a_2\beta+b_2\alpha}\right\}.$$

Satisfying the boundary conditions at the seismic focus, the constants a_2, b_2, α and β are given by the following equations:

$$R=\frac{k^2}{h^2}U=\frac{k^2}{h^2}(\alpha+i\beta),$$

$$\alpha=-\frac{aa'+bb'}{a'^2+b'^2},\ \beta=\frac{a'b+ab'}{a'^2+b'^2},$$

where

$$\begin{cases} a = -4J_{\frac{5}{2}}(ha) = haJ_{\frac{3}{2}}(ha) \\ b = J_{-\frac{5}{2}}(ha) + haJ_{-\frac{3}{2}}(ha) \\ a' = (16-k^2a^2)J_{\frac{5}{2}}(ka) - 2kaJ_{\frac{3}{2}}(ka) \\ b' = (16-k^2a^2)J_{-\frac{5}{2}}(ka) + 2kaJ_{-\frac{3}{2}}(ka), \end{cases}$$

and $$A_2 = a_2 + ib_2 = \left(\frac{p_2}{\mu a^{-\frac{1}{2}}}\frac{a''}{a''^2+b''^2}\right) + i\left(\frac{p_2}{\mu a^{-\frac{1}{2}}}\frac{b''}{a''^2+b''^2}\right),$$

where

$$a'' = (3h^2a^2-24)J_{\frac{5}{2}}(ha) + 4haJ_{\frac{3}{2}}(ha)$$
$$-24\alpha\{-4J_{\frac{5}{2}}(ka) + kaJ_{\frac{3}{2}}(ka)\} - 24\beta\{4J_{-\frac{5}{2}}(ka) + kaJ_{-\frac{3}{2}}(ka)\},$$
$$b'' = -(3h^2a^2-24)J_{-\frac{5}{2}}(ha) + 4haJ_{-\frac{3}{2}}(ha)$$
$$+24\beta\{-4J_{\frac{5}{2}}(ka) + kaJ_{\frac{3}{2}}(ka)\} - 24\alpha\{4J_{-\frac{5}{2}}(ka) + kaJ_{-\frac{3}{2}}(ka)\}.$$

The amplitudes of the radial component of the displacement of the dilatational wave are plotted against ha in Fig. 2. As will be seen from the figure, the curve has a maximum at $ha=3$, exhibiting a spectrum-like feature.

At a point distant from the origin as compared with the wave length, both dilatational and distortional waves have only the radial and the colatitudinal component of displacement, respectively.

In that case, the proportion of the displacement of the distortional to that of the dilatational wave is represented in Fig. 3, omitting the term relating to the azimuthal difference between the two waves.

It will be seen from the figure that, as the wave length diminishes compared with the size of the seismic origin, S waves appear less intensely than P waves.

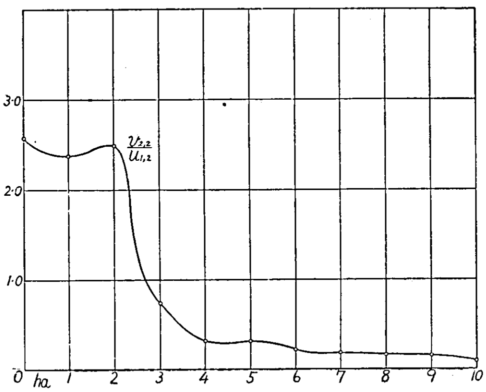

Fig. 3.

(3) The case in which the normal pressure is expressed by two zonal harmonics $P_0(\cos\theta)$ and $P_2(\cos\theta)$.

In this case, the radial component of the displacement at a distant point is given by

$$u_{1,0}+u_{1,2}=-\sqrt{\frac{2}{\pi}}\frac{1}{h^{3/2}}\frac{1}{r}\left[\left\{a_0P_0(\cos\theta)+a_2P_2(\cos\theta)\right\}\cos(pt-hr)\right.$$
$$\left.-\left\{b_0P_0(\cos\theta)+b_2P_2(\cos\theta)\right\}\sin(pt-hr)\right].$$

(a) We shall first take the case in which a pair of the doublets act along an axis. Here the normal pressure on the surface of the spherical cavity is given by

$$\left\{-p'P_2(\cos\theta)-\frac{1}{2}p'P_0(\cos\theta)\right\}e^{ipt}.$$

The change in the azimuthal distributions of the push and the pull waves as the value of ha varies is shown in Fig. 4. In the figure, the axis $\theta=0$ is taken upwards and the amount of the displacement is represented by the vector from the centre. The push and the pull waves are represented by (+) and (−) respectively. As will be seen from the figure, so long as the wave length is greater than the dimension of the seismic origin, we shall see the push waves in all directions. If the wave length were to become comparable with the diameter of the seismic origin, we shall also see pull waves as well as push waves, but the amplitudes of the former will be less than those of the latter.

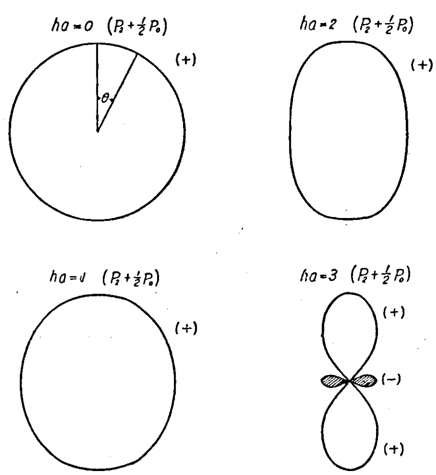

Fig. 4.

(b) We shall next consider the case in which the normal pressure on the surface of the spherical cavity is expressed by $(\cos^2\theta-\frac{1}{2})$, that is, the magnitude of the normal pressure at the axis equals that of the normal pressure at direction perpendicular to the axis, but the senses of the two are opposite to each other.

In this case, the normal pressure can be expressed by

$$\left\{-p'P_2(\cos\theta)+\frac{1}{4}p'P_0(\cos\theta)\right\}e^{ipt}.$$

The azimuthal distributions of the push and the pull waves are shown in Fig. 5. As will be seen from the figure, so long as the wave length is greater than the dimension of the seismic origin, the displacements due to the term $P_0(\cos\theta)$ will be far greater than those due to the

Table I.

hr	$\mu a^{-\frac{1}{2}}\alpha_0/p_0$		$\mu a^{-\frac{1}{2}}\alpha_2/p_2$	
	$ha=1$	$ha=2$	$ha=1$	$ha=2$
1	0·4312	——	0·0755	——
2	0·3409	0·1743	0·0122	0·0323
3	0·3212	0·1643	0·0080	0·0211
4	0·3142	0·1606	0·0077	0·0204
5	0·3110	0·1589	0·0097	0·0204
6			0·0078	
∞	0·3048	0·1559	0·0080	0·0211

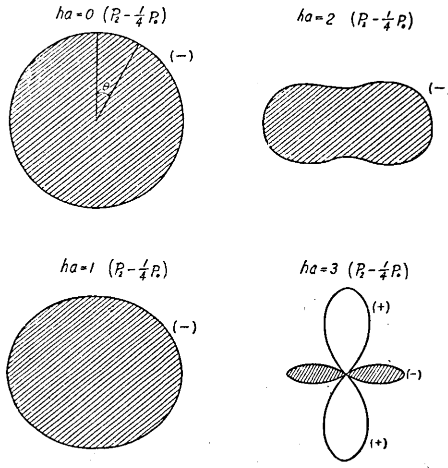

Fig. 5.

term $P_2(\cos\theta)$, so that we shall see pull waves regardless of the azimuth. Should the wave length become comparable with the dimension of the seismic origin we shall see push waves as well as pull waves.

The foregoing arguments are valid only for great focal distances. Nevertheless, as will be seen from Table I, the values

$$\alpha_0=\sqrt{\left(\frac{a_0}{hr}-b_0\right)^2+\left(a_0+\frac{b_0}{hr}\right)^2}$$

in the case of P_0 and

$$\alpha_2=\sqrt{\left\{a_2\left(\frac{9}{h^2r^2}-1\right)-\frac{b_2}{hr}\left(-\frac{9}{h^2r^2}+4\right)\right\}^2+\left\{b\left(\frac{9}{h^2r^2}-1\right)+\frac{a_2}{hr}\left(-\frac{9}{h^2r^2}+4\right)\right\}^2}$$

in the case of P_2 approach values at infinity the moment we recede a few wave lengths from the focus. The foregoing arguments also apply fairly well to neighbourhoods of the seismic origin.

Now, generally speaking, should there be the term $P_0(\cos\theta)$, however small, besides $P_2(\cos\theta)$, in the normal pressure, the displacements due to the single source will predominate so long as the wave length is greater than the dimension of the seismic origin, but when the wave length becomes comparable with the dimension of the origin, the displacements due to P_0 and P_2 become of the same order of magnitude (see Fig. 2), so that we shall find that the pull and the push waves are equal in magnitude.

We shall now consider the free oscillation of an isotropic elastic sphere. In the $P_0(\cos\theta)$ type oscillation, proper oscillation takes place when $ha=2{\cdot}57$, $6{\cdot}07$, ... and in the $P_2(\cos\theta)$ type (spheroidal type) of oscillation, proper oscillation occurs when $ha=1{\cdot}52$.

Then, if the sphere taken at the seismic origin makes its proper oscillation, it emits seismic waves having wave lengths comparable with the dimension of the origin, and the distribution of the initial motions becomes almost what we see in actual earthquakes.

We shall next examine the orders of magnitude of the wave lengths of the initial motions. Here we exclude severe earthquakes, for the reason that in their case it is plausible to assume that the forces are applied to the earth's surface. According to Mr. T. Suzuki,[13] the observed periods at Tokyo and at Mt. Kiyosumi of the initial motions of sensible earthquakes that have occurred in the Kwanto district are mostly less than one second. The periods of the initial motions observed at Mt. Kiyosumi, situated at the south-eastern corner of the Kwanto district, are often 0·5 sec. (See Table II.)

Table II.

Period	Frequency
0·1 sec	0
0·2	0
0·3	4
0·4	8
0·5	6
0·6	5
0·7	1
0·8	4
0·9	2
1·0	1
1·1	1
1·2	1
1 3	1

If then we assume the diameter of a seismic origin to equal the wave length the magnitude of the dimension of the seismic origin works out to about 3 km, assuming that the velocity of the dilatational waves is about 6 km/sec. Further, according to studies made by Dr. N. Nasu on the observed data of the Ito earthquake swarms that occurred in 1930, the dimensions of these seismic origins might be less than 1 km. The periods of the waves were then about 0·15∼0·20 sec and the velocity of the dilatational waves was about 3·5 km/sec, whence the wave length was determined to be about 500∼700 m. By assuming that the dimension of the seismic origin and the wave length are of the same order of magnitude, what has just been said is consistent with the findings of Dr. N. Nasu.

Dr. H. Honda, in his paper already referred to, assumed that the dimension of a seismic origin was infinitesimal compared with the wave lengths of the seismic waves generated from that orgin. In the deep-

13) T. SUZUKI, *Bull. Earthq. Res. Inst.*, **10** (1932), 517; *Jisin*, **8** (1936), 72∼83.

focus earthquake of June 2, 1929, the observed directions of the initial motions of the S waves were fully in accord with his theory, but the observed magnitudes were only about a half of the calculated values. This discrepancy may be obviated by assuming that the dimensions of the seismic origin were moderate compared with the wave lengths.

At any rate, it is unnatural to assume that waves, the wave lengths of which are infinitely greater than its dimension, are radiated from a seismic origin.

5. We shall next consider the case in which a pair of the doublets act on two points separated from each other along an axis.

The radial component of displacement due to the doublet is given by

$$u_{1,1} = -\frac{A}{h^2} P_1(\cos\theta) \frac{d}{dr} \frac{H_{\frac{3}{2}}^{(2)}(hr)}{\sqrt{r}} e^{ipt}$$

$$= -\frac{1}{h^{3/2}} \frac{1}{r} \sqrt{\frac{2}{\pi}} P_1(\cos\theta) \left[\left\{ \frac{2a_1}{hr} - b_1\left(1-\frac{2}{h^2r^2}\right) \right\} \cos(pt-hr) \right.$$

$$\left. - \left\{ \frac{2b_1}{hr} + a_1\left(1-\frac{2}{h^2r^2}\right) \right\} \sin(pt-hr) \right],$$

and at a distant point, we have

$$u_{1,1} = -\frac{1}{h^{3/2}} \frac{1}{r} \sqrt{\frac{2}{\pi}} P_1(\cos\theta) \sqrt{a_1^2+b_1^2} \cos\left\{ pt-hr+\tan^{-1}\frac{a_1}{-b_1} \right\}.$$

The constants a_1 and b_1 are determined by the boundary conditions on the spherical surface at the seismic origin.

For simplicity, we next put

$$A = -\sqrt{\frac{2}{\pi}} \frac{\sqrt{a_1^2+b_1^2}}{h^{3/2}}, \quad \varepsilon = \tan^{-1}\frac{a_1}{-b_1},$$

and we have

$$u_{1,1} = -\frac{A}{r} P_1(\cos\theta) \cos(pt-hr+\varepsilon)$$

$$= -\frac{A}{r} \cos\theta \cos(pt-hr+\varepsilon).$$

Assuming that the two doublets act on two points, s_1 and s_2, separated from each other by d along an axis, the radial component of displacement of the dilatational wave is given by

$$u_{1,1}=-\frac{A}{r}\cos\theta\cos(pt-hr+\varepsilon)$$

$$+\frac{A}{r}\cos\theta\cos(pt-hr+\varepsilon+\varphi)$$

$$=-2\frac{A}{r}\cos\theta\sin\frac{\varphi}{2}\sin\left(pt-hr+\varepsilon+\frac{\varphi}{2}\right).$$

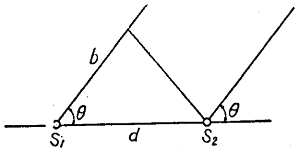

Fig. 6.

From the relation

$$\varphi=\frac{2\pi b}{L}=\frac{2\pi d}{L}\cos\theta\text{(in which } L=\text{wave length)},$$

we have

$$u_{1,1}=-2\frac{A}{r}\cos\theta\sin\left(\frac{\pi d}{L}\cos\theta\right)\sin\left(pt-hr+\varepsilon+\frac{\pi d}{L}\cos\theta\right).$$

The azimuthal distributions of the initial motions in cases in which the distances between the two points differ are shown in Fig. 7. As will be seen from the figure, so long as the distances between the two points are less than the wave lengths, we shall simply have push waves, regardless of the azimuths. Should the distance become comparable with the wave length, we shall have pull waves as well as push waves, and the features of the azimuthal distributions of these waves will become fairly complex.

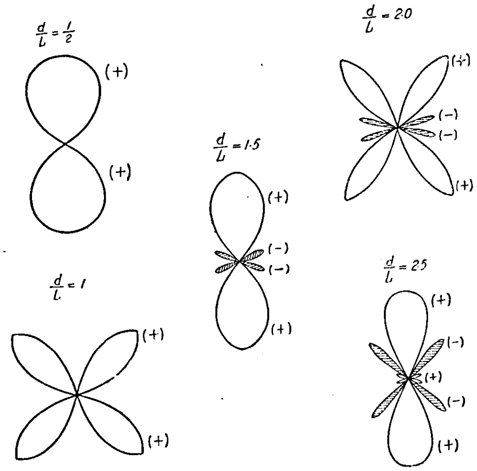

Fig. 7.

6. Model experiments on initial motions by artificial earthquakes were conducted by the writer[14] as a preliminary step to this study.

The case of a weight being allowed to fall on the earth's surface or into a pit dug in the earth corresponds to the source of the doublet. When the weight falls into the pit, we observe pull waves in the neighbourhood of the pit and push waves at a distance. This case may be applied to volcanic earthquakes generated by reactions of the erupted material and of the air waves.

14) W. INOUYE and H. KIMURA, *Bull. Earthq. Res. Inst.*, **13** (1935), 194.

When explosives are fired underground, the initial motions are all push waves. If the explosives are packed between covers (see Fig. 8.) and then fired, we also observe push waves in all azimuths, although here the displacements in direction *aa* are greater than those in direction *bb*. These facts may be explained on the assumption that the single source co-acted with the quadruple source.

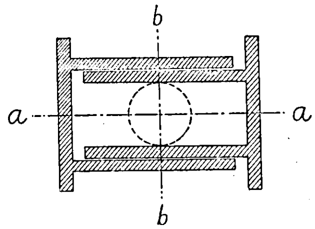

Fig. 8.

7. Summary.

1) The author has made a theoretical study of the initial motions of dilatational waves generated from a seismic origin, in which the normal pressure on the surface of the spherical cavity can be expressed by the sum of two zonal harmonics $P_0(\cos\theta)$ and $P_2(\cos\theta)$.

It is shown that so long as the wave length exceeds the dimension of the focus, we only observe the push and the pull waves according as the single source is the source or the sink, respectively. If the wave length becomes comparable with the diameter of the seimic origin, the displacements of the dilatational waves due to the single source turn out to be of the same order of magnitudes as those due to the quadruple source, so that we observe both pull and push waves.

2) It is shown that it is not unreasonable to assume that the mechanisms above stated can be applied to the origins in the case of actual earthquakes. If so, then in order to be consistent with the observed distributions of the initial motions, we shall have to assume that the dimensions of the seismic origins are comparable with the wave lengths of the seismic waves radiated from these origins.

Our next object for study should be the polarizations in the propagations of very short seismic waves that accompany earthquake sounds, and which cause anomalies in the distributions of the felt areas, owing probably to the short wave lengths compared with the dimensions of the seismic origins.

Finally, the author's cordial thanks are due to Prof. M. Ishimoto, Prof. K. Sezawa, Dr. N. Nasu, Dr. G. Nishimura, Prof. F. Kishinouye, and Prof. R. Takahasi for their kind advices.

52. 發震機構に就いて

地震研究所 井上宇胤

1. 石本所長はその「地震初動方向分布より震源に四重源の推定」なるエボツクメイキングの論文中に於て，初動方向の分布は帯球函數の $P_0(\cos\theta)$ と $P_2(\cos\theta)$ の和で現はし得る事を示されてゐる．

然し此の初動方向分布から震源の發震機構は一義的には定まらぬものである．

其の中の一つの解釋となり得る場合として球形震源を考へ，其の球面上の壓力變化の方位的分布が P_0 と P_2 の重合に依つて現はし得る場合を數理的に取扱つて見た．

2. 此の計算に依ると，波長が震源の大きさに比較して相當長い場合には單源が正の單源であるか負の單源であるかに從つて，方向に無關係に押し波或は引き波のみを觀測する事となる．

然し波長が震源の大きさと同程度に成つて來ると，單源から出る縱波の振幅と四重源から出る縱波の振幅とが同程度に成つて來て，從つて押し波引き波が同程度に現はれる事になる．此の樣な發震機構を考へる事は餘り無理で無い事を多少論じておいたが，實際に此の樣な場合があるとすると，震源の大きさはそれから出る波の波長と同程度であると考へるのが自然である事となる．