58. *Determination of Elastic Constants of Soils by means of Vibration Methods.*

Part I. Young's Modulus.

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1. Introduction.

Since the research of Prof. F. Omori[1] it has been noticed that earthquake motions show different properties according to the geologic or topographic conditions of the regions in which the seismic disturbances are experienced. Recently the same subject was also studied by means of the acceleration seismograph, and the proper period of the ground was determined.[2]

When an earthquake takes place and the seismic waves produced at the hypocenter arrive at the surface of the earth's crust, it is considered that they cause an excitement of the frequent vibration due to the proper elastic properties of the surface layer of each place. This kind of vibration is frequent mostly in the earthquake motion. We wish to explain the reason why each region has its proper period of vibration. It is believed that the vibration of the surface layer, in which the moduli of elasticity are comparatively small, resolves into the proper vibration of each place. It will thus be seen that most of the vibrations of the surface layer are included in the earthquake motion. In order to know the properties of the earthquake motion, a study of the properties of soils in the surface layer of the earth's crust seems to be one of the most important matters of investigation. Therefore we intend to investigate the properties of soils, the present experiments being the first step in the direction above pointed out.

It is difficult to ascertain distinctly the properties of soils in the surface layer owing to such various causes as heterogeneity, moisture

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1) F. OMORI, *Pub. Earthq. Inv. Com.*, **10** and **11** (1902).

2) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **10** (1932), 171; **12** (1934), 234; **13** (1935), 592; **14** (1936), 240.

T. SAITA and M. SUZUKI, *ditto*, **12** (1934), 517.

N. NASU and T. HAGIWARA, *ditto*, **14** (1936), 290.

content, pressure, etc. In order to discover the complicated properties of surface layers, homogeneous soils were selected as being readily accessible. For the purpose of investigation of the distinct properties of soils the present experiments were carried out in the laboratory by using the boring-matters taken out in the natural state from the sub-surface soil of some regions.

Studies of the elastic properties of soils have in the past been made in our country,[3)] but most of them have been carried out by the static method. Studies of soils by means of the dynamic method do not seem to have been made. Since soils are known as visco-elastic substances,[4)] the results obtained by each method, that is, static or dynamic method, are considered to be different from each other. In the present experiments we investigated the elastic properties of soils by means of the dynamic method, and determined the velocity of the longitudinal wave in the soils. From the velocity thus obtained, Young's modulus of soils was calculated. According to the results of these experiments it was found that the moduli of elasticity of soils obtained by the present experiments are in good agreement with those obtained by the field measurements of the velocity of elastic waves generated by falling bodies.[5)]

2. Mechanical Analysis of Soils.

By means of the method of mechanical analysis adopted by the

Table I. Mechanical Analysis of Soils.

Source of soil	Mesh number (Tyler's standard sieves)						Silt grain size 0·05~0·01 mm %	Clay grain size 0·01~0 mm %	Porosity %	Density app.	Density real	Mois. Cont. %
	8	14	28	48	100	200						
Maru-no-uti	% 0·58	% 0·40	% 0·60	% 1·18	% 1·50	% 2·34	24·92	68·48	73·15	1·46	2·660	50·0
Hongô	2·10	3·76	7·14	8·38	11·24	1·76	21·84	34·78	73·85	1·44	2·632	52·3
No. 1	0	0·02	0·14	0·32	2·04	17·60	46·20	33·60	51·50	1·838	2·717	24·15
No. 2	0	0	0·06	0·48	1·36	2·02	11·94	84·14	41·93	2·06	2·551	28·09

3) See *Bull. Geotechnical Committee, Government Railways*, **1** (1931); **2** (1932); **3** (1934); **4** (1936); (in Japanese).

A. Takata, *Publ. Res. Office of Public Works, Department of Home Affairs*, **20** (1931), 1, (in Japanese).

4) M. Ishimoto and K. Iida, *Bull. Earthq. Res. Inst.*, **14** (1936), 534, (in Japanese).

5) N. Nasu, T. Hagiwara, and S. Omote, *ditto* **14** (1936), 560, (in Japanese).

Geotechnical Committee of the Government Railways of Japan, the mechanical analysis of the sedimentary soils to be tested was carried out. The results are shown in Table I. The sedimentary soils to be tested were of 5 kinds, that is, the soils taken from underground in Hongô, Imperial University, Maru-no-uti,[6] Komatugawa,[7] and two other kinds No. 1 and No. 2.[8] The two last-named soils have been used for the other experiments. On the basis of the results of mechanical analysis, the soils were divided into 4 classes, that is, clay, silt, silty-clay and loam. The soils taken from Maru-no-uti and Komatugawa belong to the class of silty-clay and silt respectively and that taken from Hongô belongs to the class of loam. The other two soils belong to the class of clay and loam.

3. Theoretical Treatment.

Take the x- and y-axis horizontally and the z axis vertically upwards, the origin being at the bottom of a prismatic bar (Fig. 1). The equation of motion for the longitudinal wave in the bar, is expressed by

$$\rho\frac{\partial^2\zeta}{\partial t^2}=E\frac{\partial^2\zeta}{\partial z^2}+\gamma\frac{\partial^3\zeta}{\partial z^2\partial t},\ ^{9)} \qquad (1)$$

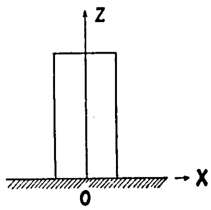


Fig. 1.

in which ζ is the displacement in the direction of z, t the time, ρ the density, and E, γ Young's modulus, the coefficient of solid viscosity (or internal friction) respectively.

Put

$$\zeta=Ae^{ifz-i\omega t}, \qquad (2)$$

then substitute this in (1), under the following boundary conditions of free vibration of clamped- and free-end bar, the height of which is h,

$$\left.\begin{aligned} z=0;&\quad \zeta=0,\\ z=h;&\quad \frac{\partial\zeta}{\partial z}=0, \end{aligned}\right\} \qquad (3)$$

we have $\omega=\dfrac{\gamma f^2}{2\rho}\pm i\sqrt{\dfrac{Ef^2}{\rho}-\left(\dfrac{\gamma f^2}{2\rho}\right)^2}$, and

6) See T. SAITA and M. SUZUKI, *Bull. Earthq. Res. Inst.*, **12** (1934), 517.

7) See N. MIYABE, *Bull. Earthq. Res. Inst.*, **14** (1936), 543.

8) See M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **14** (1936), 534.

9) The similar equation has already been quoted by T. Fukutomi, *Bull. Earthq. Res. Inst.*, **12** (1934), 498.

$$\zeta=Ae^{-\frac{\gamma f^2}{2\rho}t}\cos\left\{\sqrt{\frac{Ef^2}{\rho}-\left(\frac{\gamma f^2}{2\rho}\right)^2}\,t-\alpha\right\}\sin fz, \tag{4}$$

$$f=\frac{2r-1}{2h}, \qquad r=1,\ 2,\ldots.$$

where $\frac{2\pi}{f}$ is the wave length and A is the constant.

Next we consider the forced vibration of this system. We assume the motion of the floor, on which the prismatic bar is placed, as

$$\zeta=Ce^{ipt}, \tag{5}$$

then the particular solution of (1) may be written

$$\zeta=CD(z)e^{ipt}, \tag{6}$$

where D is a normal function. Substituting this in (1), we get

$$\left.\begin{aligned}&\frac{\partial^2 D}{\partial z^2}=-f'^2 D,\\ &f'^2=\frac{p^2}{\frac{E}{\rho}+ip\frac{\gamma}{\rho}}.\end{aligned}\right\} \tag{7}$$

Put

$$\varepsilon=\frac{1}{2}\frac{\gamma}{\rho}f^2, \quad n^2=\frac{E}{\rho}f^2, \tag{8}$$

then we obtain

$$f'=\frac{fp}{(n^4+4p^2\varepsilon^2)^{\frac{1}{4}}}\left\{\sqrt{\frac{1}{2}\left(1+\frac{n^2}{\sqrt{n^4+4p^2\varepsilon^2}}\right)}-i\sqrt{\frac{1}{2}\left(1-\frac{n^2}{\sqrt{n^4+4p^2\varepsilon^2}}\right)}\right\}.$$

For the sake of simplicity we write this equation in the form of f_1-if_2. (9)

As the boundary conditions are such that

$$\left.\begin{aligned}&z=0; \quad D=1,\\ &z=h; \quad \frac{\partial D}{\partial z}=0,\end{aligned}\right\} \tag{10}$$

we substitute these conditions in (7), and get

$$D=\cos f'z+\tan f'h\sin f'z, \tag{11}$$

or

$$D=\cos f_1 z \cosh f_2 z+\frac{1}{\vartheta}(\sin f_1 h \cos f_1 h \sin f_1 z \cosh f_2 z$$
$$-\cos f_1 z \sinh f_2 z \sinh f_2 h \cosh f_2 h)+i\Big\{\sin f_1 z \sinh f_2 z$$
$$-\frac{1}{\vartheta}(\sin f_1 h \cos f_1 h \cos f_1 z \sinh f_2 z+\sin f_1 z \cosh f_2 z \sinh f_2 h \cosh f_2 h)\Big\}, \quad (12)$$

where, $\vartheta=\cos^2 f_1 h \cosh^2 f_2 h+\sin^2 f_1 h \sinh^2 f_2 h.$

Substituting (12) in (6), we obtain

$$\left.\begin{aligned}
&\zeta=C(\zeta_1+i\zeta_2)e^{ipt},\\
&\zeta_1=\cos f_1 z \cosh f_2 z+\frac{1}{\vartheta}(\sin f_1 h \cos f_1 h \sin f_1 z \cosh f_2 z\\
&\qquad-\cos f_1 z \sinh f_2 z \sinh f_2 h \cosh f_2 h),\\
&\zeta_2=\sin f_1 z \sinh f_2 z-\frac{1}{\vartheta}(\sin f_1 h \cos f_1 h \cos f_1 z \sinh f_2 z\\
&\qquad+\sin f_1 z \cosh f_2 z \sinh f_2 h \cosh f_2 h).
\end{aligned}\right\} \quad (13)$$

The displacement of forced vibration in the bar is given by this equation. The ratio of its amplitude to that of motion of floor is also given by

$$R=\sqrt{\zeta_1^2+\zeta_2^2}. \quad (14)$$

The relation between R and $u=\frac{n}{p}=\frac{T_p}{T}$ in the case $z=h$ is shown in Fig. 2, in which T_p, T being the period of forced vibration and of the bar respectively and k the damping ratio of the bar deduced from the equation

$$\log_e k=\frac{\pi}{\sqrt{n^2/\varepsilon^2-1}}. \quad (15)$$

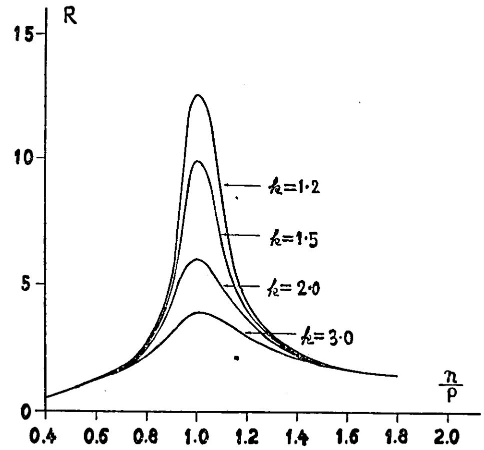


Fig. 2. Relation between R and $\frac{n}{p}$ in damped longitudinal vibration of the bar at $z=h$.

As will be seen from Fig. 2, R takes the maximum value when n/p becomes 1. It is believed that the state in which R becomes great is the phenomenon of resonance. Consequently, in resonance, the amplitude of vibration of the bar becomes the greatest and the period of forced vibration coincides with that of free vibration of the bar. Therefore the period of free vibration of the bar can be determined in the case of forced vibration. The period of free vibration of the bar, that is, the period of resonance vibration, T, is given by

$$T = \frac{4h}{(2r-1)\sqrt{\frac{E}{\rho} - \frac{(2r-1)^2\pi^2\gamma^2}{16\rho^2h^2}}}. \tag{16}$$

An example of the relation between the fundamental period T and the height h is shown in Fig. 3, in which we take T as ordinate and h as abscissa. The larger the value of the solid viscosity coefficient, the more the line in question curves and deviates from the ideal condition. When the soild viscosity coefficient attains a certain value, the proper period T becomes infinitely great. In Fig. 4, however, we take

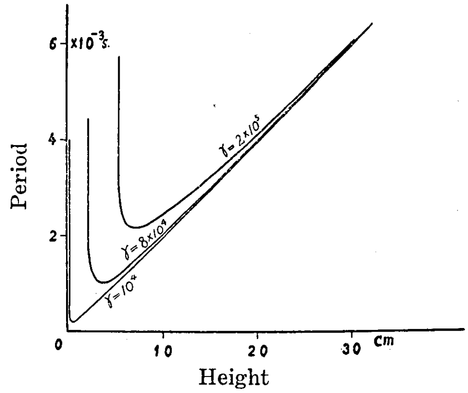


Fig. 3. Relation between the fundamental period of resonance vibration, T, and the height of the bar, h. In this case

$\rho=1{\cdot}5, \quad \sqrt{\frac{E}{\rho}}=200$ m/sec.

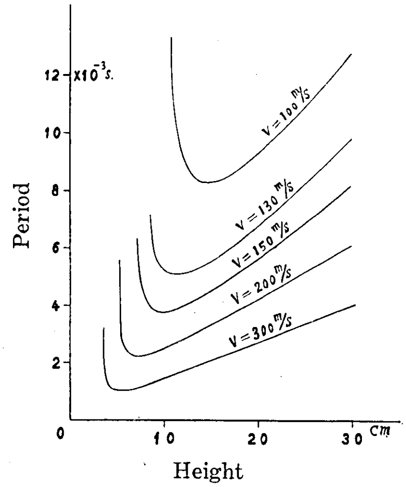


Fig. 4. Relation between the fundamental resonance period T and the height h. In this case

$\rho=1{\cdot}5, \quad \gamma=2\times10^5$.

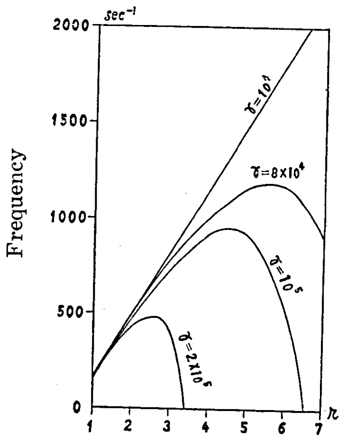


Fig. 5. Relation between the resonance frequency of the bar and the order r of its partials. In this case

$\rho=1{\cdot}5$, $v=200$m/sec, $h=30$cm.

the velocity $v=\sqrt{\frac{E}{\rho}}$ as a parameter, and the solid viscosity coefficient γ is kept constant.

Further, we consider the relation between the frequency of the vibration, N, and the order of partials. Fig. 5 is the diagram showing this relation. In the diagram the values of r ($r=1, 2, 3, \ldots\ldots$) are taken as abscissa and the corresponding values of N as ordinate. These values of N against r are combined with a curve in order to determine distinctly the relation between them. The frequency of the resonance vibration decreases either slowly or somewhat rapidly, with the increse in the number of r according to the magnitude of γ. As will be seen from the figure, we have only 3 partials when $\gamma=2\times10^5$.

4. Experiments.

(a) Experimental apparatus.

The photographs of our apparatus used in the present experiments are shown in Figs. 6 and 7, and also a schematic diagram of the arrangement of the apparatus is shown in Fig. 8. As will be seen from these figures, the principal parts of this apparatus consist of a vibrating iron plate, on which the soil specimen S stands upright, and a connection circuit of a motor-generator $M\text{-}D_1$, that produces variable frequencies of electric current to provoke the vibration of the iron plate. The idea[10] of our experiment is to give the various frequency of vibration at the foot of the soil specimen, and to find the fundamental resonance frequency and then the elastic constant of the soils computed therefrom. The particular characteristics of our apparatus are described in the following:

Vibrating plate. We have adopted a circular steel disc P_1 of 23·7 cm in diameter; the middle concentric part of which is thinner than its boundary part. This thin part, 11·0 cm in diameter and 0·5 cm in thickness, of course, serves as a vibrating portion; naturally, the soil specimen stands upright on this vibrator. We have moreover employed another iron disc P_2 which is about 10 cm below the disc P_1 and firmly clamped to it by the aid of 4 brass bolts. We have inserted a solid cylindrical iron stick between these two discs, fixing to the lower and not touching the upper. This iron stick has been magnetised by a coil $M.C$ carrying a direct current from a series of batteries. Another coil

10) John. M. IDE, has been experimented on the rock specimens by means of the same idea as adopted by us. *Rev. Sic. Instruments*, **6** (1935), 296. *Proc. Nat. Acad. Sci.*, **22** (1936), 81.

C, which is also fixed to the same iron stick, is intended to carry an alternating current produced in the generator D_1. The vibrating plate is set in forced vertical vibration of the same frequency as the alternating electric current.

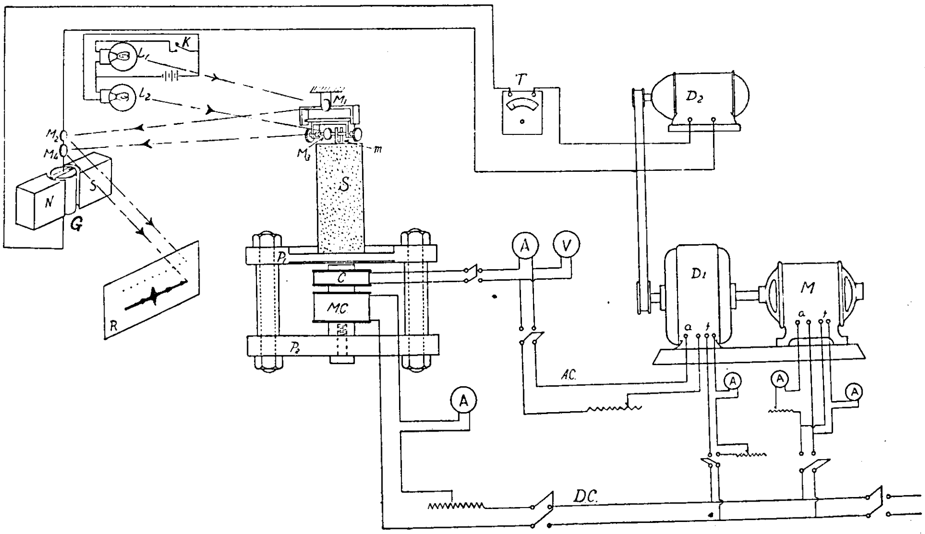


Fig. 8. Sckematic diagram of arrangement of the experimental apparatus for producing longitudinal vibrations of soils by vibration method.

a: Armature terminal.
A: Ampere-meter.
A.C: Alternating current circuit.
C: *A.C.* coil.
D.C: Direct current circuit.
D_1: Alternating generator.
D_2: Dynamo of the Weston tachometer.
f: Field terminal.
G: Galvanometer.
K: Key.
L_1: Light source.
L_2: 〃 〃
m: Magnet.

M: *D.C.* motor.
M_1: Lens mirror.
M_3: 〃
M_2: Plane mirror.
M_4: 〃
M.C: *D.C.* coil.
P_1: Circular iron disc for vibration.
P_2: Circular iron disc with electro-magnet.
R: Bromide paper for record.
S: Soil (specimen).
T: Voltmeter of the Weston tachometer.

Magnification of the amplitude of the top of soil specimen. When the vibrating plate is set in vibration, the longitudinal waves produced by this are propagated from the bottom of the soil specimen *S*. When the frequency of this vibrating plate reaches a condition suitable to provoke a stationary wave in the soil specimen, the amplitude of its top attains to the maximum. The vibration of its top should be perceived even by the tip of a finger, but we must magnify by some device to record continuously the variation of its vibration amplitude. We have, therefore, adopted a pair of pivot and small magnet m to get a sufficient magnification of it. A part of this small magnet m, then, was inserted into the top of the soil specimen. This small magnet will not disturb the vibration of the soil specimen as its weight is very insignificant. The pivot, of 1·99 mm in diameter, is in contact with the upper part of this magnet m. On this pivot we have attached a small lens mirror M_3 having the focal length about 50 cm. The rotation of this pivot was manifested by the deflection of a light beam reflected by this lens mirror. In order to diminish the friction to the smallest possible degree at the contact point between the magnet and the pivot, we have made the frame of the pivot moved easily at the centre of another pivot, which is shown clearly in Fig. 8. On that frame we have attached moreover another plane mirror M_1.

When the soil specimen was set in longitudinal vibration, the pivot takes a rotational motion, which is shown by the mechanism above mentioned. In this way the amplidude of the top of the soil specimen is magnified as much as about 2000 times, and the amount of this amplitude is to be detected by a recording system that will be explained later.

Motor-generator. We have employed a motor-generator driven by a direct current fed by secondary cells. The number of rotation of the generator D_1 of 20 pole pieces can be changed continuously with the increase of resistances in the speed regulation circuit. We have, therefore, the output of the alternating current, having a preferred frequency within the limit of 700 per second.

Tachometer. In order to determine the number of revolutions of the generator D_1, an electric Weston tachometer was employed. This tachometer consisted of two elements, that is, a small dynamo D_2 and a voltmeter T. This dynamo is connected with the generater D_1 by means of a cloth belt. We have, however, employed two pulleys of different diameters, one diameter being twice the other, so as to reduce the number of revolutions to half. The number of revolutions of this small dynamo is, of course, read off from the index of the voltmeter which corresponds to the frequency of the vibrating plate, which frequency we wish to know.

Galvanometer. In order to get a continuous reading of the variable frequency of the vibrating plate, we employed a galvanometer G; and to obtain a suitable sensibility of this galvanometer, we have regulated the amount of direct current which passes through the field of exciting coils. The moving coil of this galvanometer was inserted in the tachometer circuit. The amount of deflection of this moving coil is, therefore, proportional to the deflection of the voltmeter T. Consequently we can determine the frequency of the vibrating plate from the amount of this deflection.

We have attached two plane mirrors M_2 and M_4 on this moving coil, one of these serves as the detector of the amplitude variation of the soil specimen, and the other serves as the scale for the number of frequency of resonance.

Recording device. In order to obtain the record of the amplitude of vibration of the soil specimen, we employed a system containing a light source L_2, the lens mirror M_3 on the pivot, and the plane mirror M_4 on the moving coil of the galvanometer. As is demonstrated in Fig. 8, there is a beam of light radiated from a slit of the light source L_2, which at first falls on the lens mirror M_3 is reflected to the plane mirror M_4 and then re-reflected back again, focussing on a photographic bromide paper R. This bromide paper was set at about one meter's distance from the lens mirror M_3 In this way we have succeeded in obtaining the record on the bromide paper R; it is needless to say that the amplitude of vibration is given in ordinate and the frequency of vibration in abscissa on the records thus obtained. In order to ascertain the accurate frequency at the resonance position, we have further employed another system which contains a light sourece L_1, the lens mirror M_1 on the frame of the pivot, and the plane mirror M_2 on the moving coil of the galvanometer. A beam of light radiated from a slit of the light source L_1, formerly reflected by the fixed lens mirror M_1, is now reflected by the plane mirror M_2. The formation of the slit image of this light source L_1 is also made on the same bromide paper R near the image representing the amplitude of vibration of the soil specimen.

When we close the circuit of light L_1 by a key K, corresponding to any frequency read off from the voltmeter T, we can get a point on the same bromide paper R. At any rate, we have closed the circuit every 200 revolutions per minute of the generator D_1, so that a series of points was obtained in the record such as is shown in Fig. 8.

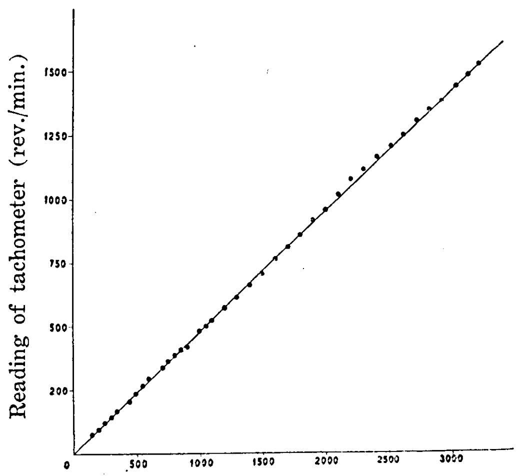
In this way we can determine precisely the frequency of the resonance position.

(b) Testing of the apparatus.

Proper period of the vibrating plate. It seems to us that it is one of the most important matters of investigation to ascertain whether or not the effects of the proper period of the vibrating plate predominate in the photographic record, where we hope there is no vibrational effect, but that of the soil specimen itself. To diminish the effect of the proper period of the vibrating plate in our experiment, we employed three degrees of plate thickness one by one and reached, at last, the thickness of 0·5 cm. With the last-obtained thickness, we proved by passing the alternating current in the coil C, that the proper period of this vibrating plate was beyond our use. Naturally, in this case we have no soil specimen on the vibrating plate.

We have found, anyhow, that the fundamental frequency of the vibrating plate was about 400 per second. In order to find whether or not this period should be varied by a dead weight of metal loaded on the vibrating plate, we repeated the same experiment with several weights up to the limits of 2 kg. From the results of these experiments, we have ascertained that the proper period is scarecely affected by these loaded weights. We may, therefore, conclude in our results that there are no effects to be taken into consideration, but the characteristics of the soil specimens.

Calibration of the tachometer. To obtain the real reading of the Weston tachometer, we have executed a calibration on this tachometer in comparing it to another standard tachometer and then an experiment with the stroboscopic method. We compared, at any rate, the readings of the voltmeter with the number of revolutions of the generator measured by these above two methods. The relation between them thus obtained is shown in Fig. 9. From this relation, we can find the correction to the readings of the Weston tachometer.



Reading of the standard tachometer (rev./min.).

Fig. 9. Relation between the reading of the Weston tachometer and that of standard one.

(c) Methods of the experiments.

The methods of the experiments are the following:

1) Setting the soil specimen on the vibrating plate, we gave it the vibration up to the limit. Naturally, during this process its amplitude was continuously recorded by the photographic method. We repeated

the same manipulation on the identical material with variable heights so as to get a reducing curve, from which we could calculate the constant of its elasticity.

2) We have experimented with the soils in the natural state and sometimes in the recomposed state. At any rate, the specimens were taken from several places in Tôkyô by means of the boring process. The soils of Maru-no-uti, Hongô, and Komatugawa are taken out from depths of 8 m, 2 m, and 20 m repectively beneath the ground surface.

3) The soils of Maru-no-uti and Hongô were tested in both the natural and recomposed states, and other soils in the recomposed state only. The specimens in the natural state were made into the form of a rectangular prism, about 4 cm $\times$ 5 cm in cross section, and the ranges of 20 cm$\sim$40 cm in initial height, which was dinimished by and by with a cutting knife. The specimens in the recomposed state were also made into the form of the rectangular prism, the dimensions of which were about the same as those used in the natural state. To make the form of these specimens we have put the soil into a mould made of a wooden box. Consequently it seems to us that the degree of packing is in the state of the maximum.

By the way, we took substances, such as agar-agar and a kind of commercial rubber, for the purpose of comparing their elastic constants with those of soils. We have made a solution of agar-agar, the weight of which is 125 gr in 5·5 l of water, and poured into a hollow rectangular box of zinc. This solution was thereafter solidified. We have cut this block of agar-agar in the form of a rectangular prism to make a testing specimen. We have also experimented on the rubber specimen in the form of a cylinder, the diameter of which was 4·83 cm, and the height of which was changed in the range of 2 cm$\sim$20 cm.

4) We determined the apparent density of the soil specimen from the measuring of its mass and the volume of the specimen just after every experiment.

5) In order to know the properties of the specimen in relation to moisture content, we have executed the same manipulation on the same material with the variable moisture content. We have made the change in moisture content by drying the specimen from the initial wet state leaving it in the ordinary room conditions. The moisture content of the specimen was calculated from the ratio of the weight of water in the specimen to that of its original wet state.

When the soil specimen becomes hard like a solid body by the diminishing of its moisture content, it is rather removable owing to its jumping on the vibrating plate during its vibration; however, we have succeeded in carrying out the experiments by fixing the lower part of the soil specimen to the vibrating plate by means of oil clay.

5. Results of the Experiments.

(a) Records.

Some examples of actual photographs[11] thus obtained are shown in

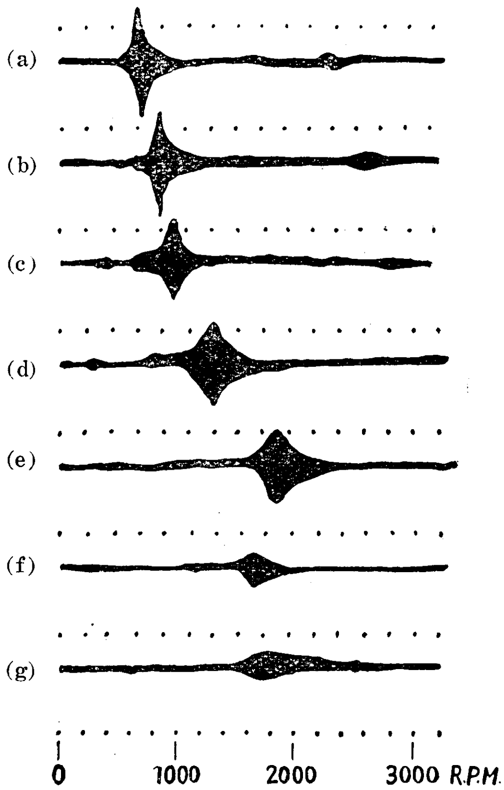


Fig. 11. Resonance curves of soil taken from Maru-no-uti. Moisture content 48·6%.

(a) h=24·3 cm, N=110 sec^{-1}.
(b) h=19·7 , N=134 .
(c) h=14·9 , N=151 .
(d) h=10·0 , N=201 .
(e) h= 5·9 , N=297 .
(f) h= 3·1 , N=255 .
(g) h= 1·6 , N=300 .

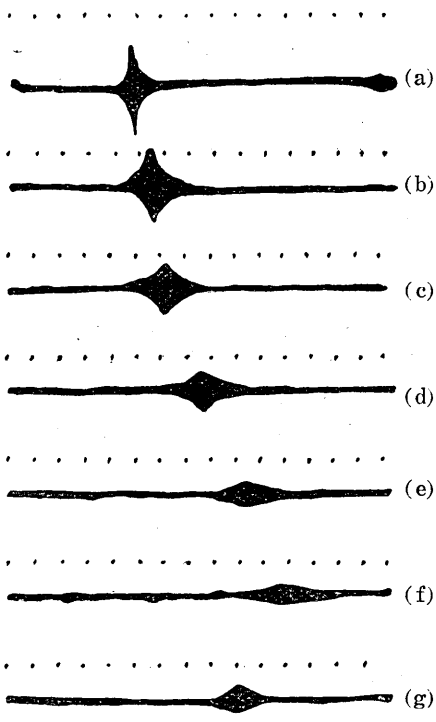


Fig. 12. Resonance curves of soil taken from Maru-no-uti. Moisture content 42·3%.

(a) h=24·8 cm, N=161 sec^{-1}.
(b) h=20·2 , N=177 .
(c) h=16·0 , N=208 .
(d) h=11·1 , N=250 .
(e) h= 6·5 , N=318 .
(f) h= 3·5 , N=365 .
(g) h= 2·0 , N=310 .

11) Some of the curves are deviated from horizontal. The reason for this departure from horizontal is due to a shrinkage of the soil specimen during its vibration.

Figs. 10, 13 and 15, and some of hand reproductions from actual photographs are shown in Figs. 11, 12, 14, and 16~19. These show the difference according to the kinds, the heights, and the moisture contents of the soil specimens.

As will be seen in these figures, the characteristics of resonance are almost alike; but we observe the fact that the resonance curves tend to flatten when their heights or their moisture contents are diminished. As we have already stated in the theoretical part in Section 3 that the sharpness of the curves shows the degree of the solid viscosity in the specimen, we can, therefore, determine its value from the flatness of these curves. It will be confirmed from these curves that the apparent effect of solid viscosity becomes great in the following two cases;

1) When the height of the specimen is lowered.

2) When the moisture content of the specimen is diminished.

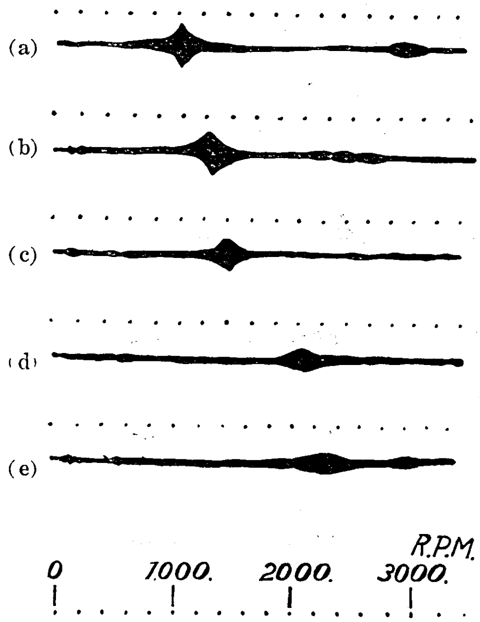


Fig. 14. Resonance curves of soil taken from Hongô. Moisture content 52·3%.

(a) $h=27{\cdot}5$ cm, $N=173\ \text{sec}^{-1}$.
(b) $h=20{\cdot}5$,, $N=212$,,
(c) $h=14{\cdot}4$,, $N=242$,,
(d) $h=9{\cdot}5$,, $N=347$,,
(e) $h=6{\cdot}0$,, $N=392$,,

However, in this effect, any other disturbances caused by the friction between the pivot and the magnet, etc., seem to be included; therefore this flatness of the curve may not be suitable for determining the coefficient of solid viscosity of the soil specimens. For determining its value we have, therefore, noticed the curved portions near the origin in the reducing curves. We have succeeded in obtaining its values.

(b) Determination of velocity of the elastic waves and the solid viscosity coefficient of soil specimen.

As is already known from the theoretical treatments of the longitudinal vibration in a prismatic bar of various heights, which are shown graphically in the figures 3 and 4, we can determine the velocity and the solid viscosity coefficient. The theoretical relation between the resonance period T and the hight h of the specimen is given in equation (16).

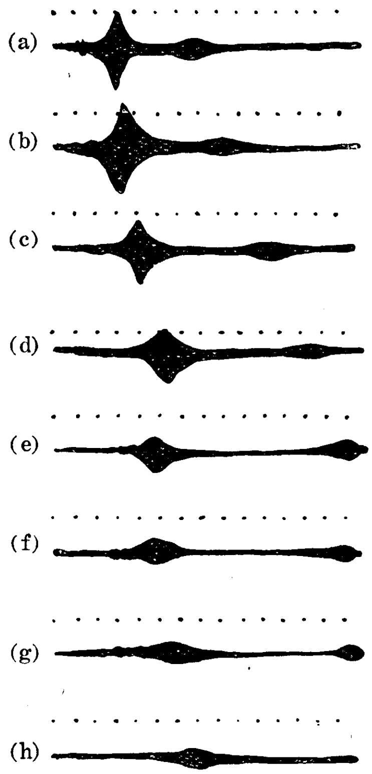


Fig. 16. Resonance curves of loam.
(a)~(d): moisture content 17·3%, $\rho=2{\cdot}02$.
(e)~(f): " " 22·1%, $\rho=2{\cdot}15$.

(a) $h=14{\cdot}1$ cm, $N=$ 99 sec^{-1}.
(b) $h=11{\cdot}0$, $N=110$.
(c) $h=$ 8·0 , $N=139$.
(d) $h=$ 5·0 , $N=161$.
(e) $h=13{\cdot}5$, $N=139$.
(f) $h=11{\cdot}0$, $N=162$.
(g) $h=$ 8·0 , $N=183$.
(h) $h=$ 5·0 , $N=231$.

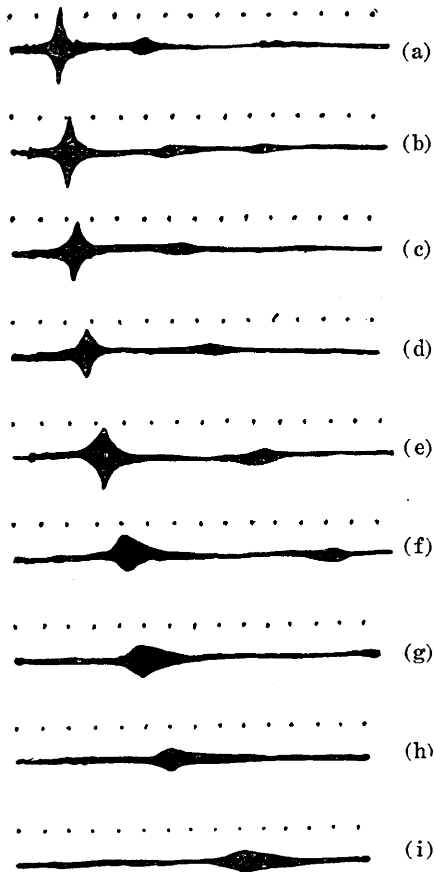


Fig. 17. Resonance curves of clay. Moisture content 30·7%, $\rho=1{\cdot}98$.

(a) $h=45{\cdot}5$ cm, $N=$ 68 sec^{-1}.
(b) $h=40{\cdot}0$, $N=$ 78 .
(c) $h=34{\cdot}8$, $N=$ 91 .
(d) $h=30{\cdot}0$, $N=$ 99 .
(e) $h=24{\cdot}9$, $N=113$.
(f) $h=20{\cdot}2$, $N=142$.
(g) $h=14{\cdot}9$, $N=175$.
(h) $h=10{\cdot}0$, $N=216$.
(i) $h=$ 5·1 , $N=328$.

We have anyhow employed this equation to determine these two quantities.

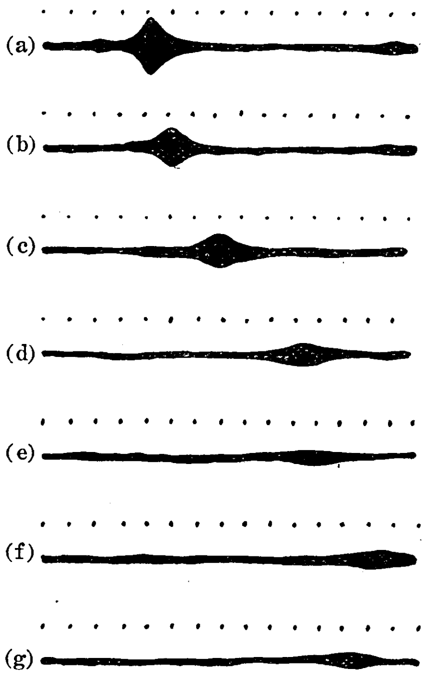


Fig. 18. Resonance curves of rubber.

(a)	h=20·0 cm,	N=142 sec^{-1}.
(b)	h=14·6 ,	N=170 .
(c)	h=10·0 ,	N=242 .
(d)	h= 5·0 ,	N=367 .
(e)	h= 2·8 ,	N=342 .
(f)	h= 2·05 ,	N=488 .
(g)	h= 1·15 ,	N=407 .

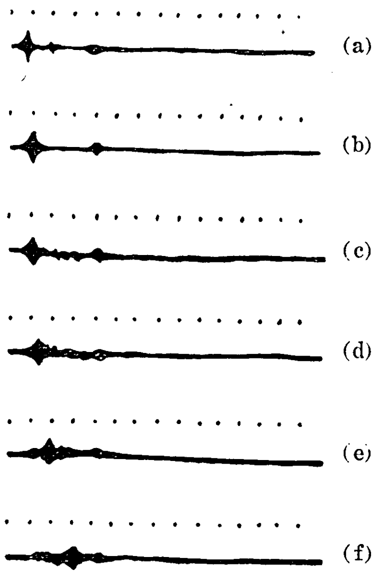


Fig. 19. Resonance curves of agar-agar.

(a)	h=12·8 cm,	N=28 sec^{-1}.
(b)	h=10·0 ,	N=33 .
(c)	h= 8·0 ,	N=39 .
(d)	h= 6.0 ,	N=50 .
(e)	h= 4·3 ,	N=59 .
(f)	h= 2·0 ,	N=97 .

(*I*) *Velocity of longitudinal wave in soils.* By means of the fundamental resonance period T and the corresponding height h of the soil specimen, we have traced a diagram, in which the period T is taken as ordinate and the height h as abscissa. Figs. 20∼26 are the diagrams showing these relations, the values of which are tabulated in Tables II∼VII. It is believed that the mode of vibration of the soil specimen corresponds to that of the

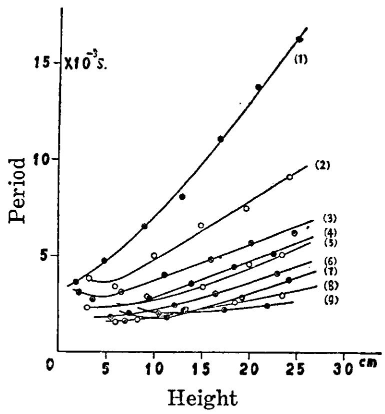


Fig. 20. Relation between the fundamental resonance period T and the height h. (Soil of Maru-no-uti in the natural state.)
(1) Moisture content 50%. (2 48·6%. (3) 42·3%. (4) 36·7%. (5) 43·6%. (6) 40·8%. (7) 36·7%. (8) 30·5%. (9) 32·7%.

clamped- and free-end bar, the loop of which is at its top and the node is at its bottom; the reasons for which will be explained later. We can see

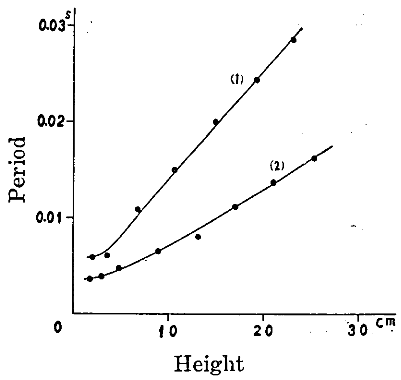


Fig. 21. Relation between the fundamental resonance period T and the height h. (Soil of Maru-no-uti.) Moisture content 50%.
(1) Recomposed state.
(2) Natural state.

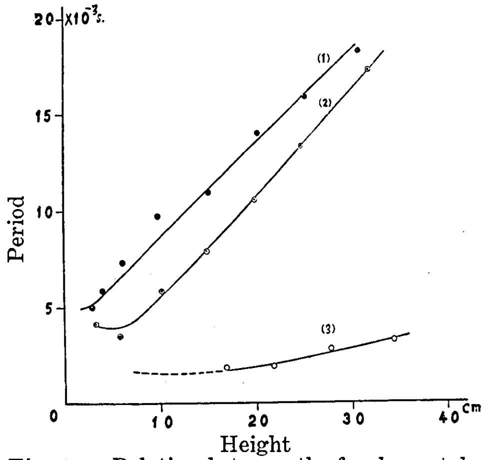


Fig. 22. Relation between the fundamental resonance period T and the height h. (Silt of Komatugawa.)
(1) Moisture content 49·5%. (2) 29·3%. (3) 15·9%.

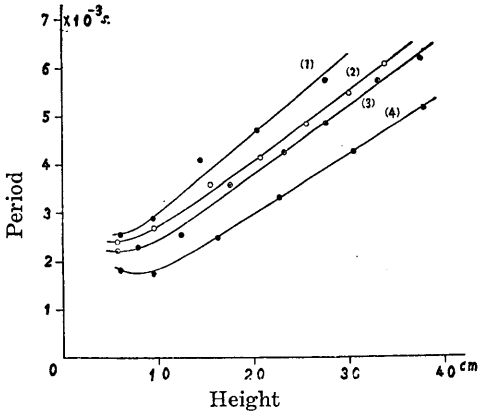


Fig. 23. Relation between the fundamental period T of resonance and the height h. (Loam of Hongô.)
(1) Moisture content 52·3%. (2) 49·7%. (3) 46·8%. (4) 41·5%.

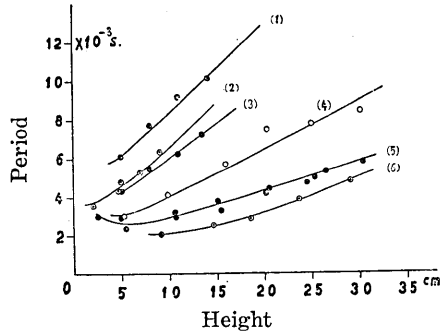


Fig. 24. Relation between the fundamental period T of resonance and the height h. (Loam of No 1.)
(1) Moisture content 17·3%. (2) 16·7%. (3) 22·1%. (4) 19%. (5) 21%. (6) 15·5%.

that the characteristics of these curves are similar to those obtained by the theoretical treatment shown in the figures 3 and 4. It is almost impossible to find the height corresponding to the period that becomes infinitely great. According to the relation represented by the equation (16), we have succeeded in finding out the values of the longitudinal wave velocity v and the solid viscosity coefficient γ of the soil specimen from these graphs.

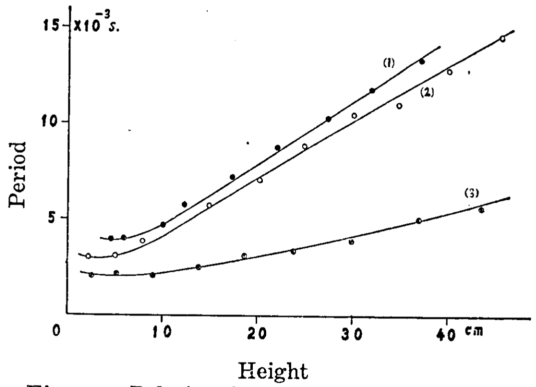


Fig. 25. Relation between the fundamental resonance period T and the height h. (Clay of No. 2.)
(1) Moisture content 30·7%. (2) 26·6%. (3) 22·6%.

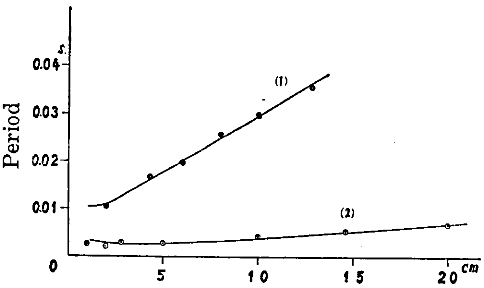


Fig. 26. Relation between the fundamental period T of resonance and the height h.
(1) Agar-agar. (2) Rubber.

In the process of determination of the velocity of elastic waves in the soil specimen, we must, at any rate, trace a straight line, which is to be an asymptote to the reducing curve passing through the origin. This straight line represents the relation between the fundamental resonance period T and the height h, such as $T=4h\Big/\sqrt{\dfrac{E}{\rho}}$.

(2) *Solid viscosity coefficient.* In order to determine the solid viscosity coefficient γ, we have compared a series of the theoretical curves of different value of γ, which is represented by equation $T=4h\Big/\sqrt{\dfrac{E}{\rho}-\dfrac{\pi^2\gamma^2}{16\rho^2h^2}}$, and picked up one of these which coincides with the reducing curve obtained experimentally. We can thus decided the value of γ. These values of v and γ thus obtained are shown in Table VIII. Young's moduli E of these soil specimens were then computed from the relation $E=v^2\rho$, in which ρ is the density and v is the velocity. These values are also shown in Table VIII.

All of Young's moduli and the solid viscosity coefficients of the soil specimen in the natural state have larger value compared with those in the recomposed state. This phenomenon can be seen, for example, in the soils taken from Maru-no-uti shown in Fig. 10 or in the reducing curves in Fig. 22.

Table II. Height, Fundamental Frequency of Resonance Vibration, and Density of Silty-clay of Maru-no-uti.

No. 1	Height (cm)	25·2	21·0	17·0	13·1	8·9	4·8	2·9	1·7
	Density	1·44	1·44	1·45	1·46	1·47	1·48	1·47	1·49
	Freq. (sec^{-1})	61	73	90	124	153	210	256	277
No. 2	Height (cm)	24·8	20·2	16·0	11·1	6·5	3·5	2·0	
	Density	1·49	1·45	1·44	1·44	1·44	1·49	1·50	
	Freq. (sec^{-1})	161	177	208	250	318	365	310	
No. 3	Height (cm)	24·3	19·7	14·9	10·0	5·9	3·1	1·6	
	Density	1·34	1·36	1·37	1·35	1·40	1·38	1·38	
	Freq. (sec^{-1})	110	134	151	201	297	255	300	
No. 4	Height (cm)	23·5	20·0	15·1	9·2	5·5	3·0		
	Density	1·61	1·61	1·61	1·70	1·75	1·78		
	Freq. (sec^{-1})	197	217	319	318	558	419		
No. 5	Height (cm)	22·0	17·3	14·2	10·5	6·9	3·8		
	Density	1·50	1·66	1·62	1·70	1·84	1·72		
	Freq. (sec^{-1})	417	451	465	487				
No. 6	Height (cm)	22·6	18·4	13·9	9·5	5·1			
	Density	1·62	1·64	1·66	1·60	1·59			
	Freq. (sec^{-1})	195	223	280	457				
No. 7	Height (cm)	23·0	16·5	11·2	7·0				
	Density	1·58	1·59	1·56	1·50				
	Freq. (sec^{-1})	247	328	416	624				
No. 8	Height (cm)	24·2	19·3	13·2	11·3	7·4	4·6		
	Density	1·64	1·63	1·64	1·64	1·60	1·58		
	Freq. (sec^{-1})	263	355	470	554	490			
No. 9	Heignt (cm)	23·5	18·5	13·3	8·3	6·0			
	Density	1·45	1·46	1·48	1·49	1·50			
	Freq. (sec^{-1})	332	341	451	590	658			
No. 10 recomp.	Height (cm)	23·0	19·1	14·9	10·6	6·8	3·8	2·1	
	Density	1·44	1·44	1·43	1·49	1·48	1·49	1·49	
	Freq. (sec^{-1})	35	41	49	67	87	168	173	

Table III. Height, Frequency, and Density of Silt of Komatugawa.

No. 1	Height (cm)	31·0	25·4	20·3	15·2	9·9	6·1	4·0	3·0
	Density	1·69	1·69	1·72	1·72	1·81	1·98	1·92	1·70
	Freq. (sec^{-1})	55	63	71	97	103	137	198	170

(to be continued.)

Table III. (*continued.*)

No. 2	Height (cm) Density Freq. (sec^{-1})	32·0 1·80 57	24·9 1·88 75	20·0 1·95 96	15·0 2·01 129	10·1 2·13 172	5·80 2·14 288	3·3 2·03 247	2·1 2·00 207
No. 3	Height (cm) Density Freq. (sec^{-1})	34·4 1·70 196	27·9 1·75 365	21·9 1·76 532	16·9 1·76 583	11·1 1·78	6·0 1·86		

Table IV. Height, Frequency, and Density of Loam of Hongô.

No. 1	Height (cm) Density Freq. (sec^{-1})	27·5 1·43 173	20·5 1·46 212	14·4 1·42 242	9·5 1·43 347	6·0 1·48 392			
No. 2	Height (cm) Density Freq. (sec^{-1})	33·8 1·28 163	30·0 1·28 182	25·6 1·28 207	20·6 1·29 239	15·4 1·29 277	9·7 1·29 384	5·8 1·29 417	
No. 3	Height (cm) Density Freq. (sec^{-1})	37·4 1·21 160	33·0 1·24 173	27·5 1·23 205	23·2 1·23 235	17·5 1·19 277	12·4 1·18 392	7·8 1·22 435	5·8 1·26 450

Table V. Height, Frequency, and Density of Loam (No. 1).

No. 1	Height (cm) Density Freq. (sec^{-1})	9·0 2·41 158	7·0 2·43 190	5·0 2·40 208	4·7 2·30 231	3·0 2·41 260	2·0 2·27 284	
No. 2	Height (cm) Density Freq. (sec^{-1})	14·1 2·03 99	11·0 2·15 110	8·0 2·25 130	5·0 2·26 161			
No. 3	Height (cm) Density Freq. (sec^{-1})	13·5 1·90 139	11·0 1·96 162	8·0 1·90 183	5·0 1·92 231			
No. 4	Heigot (cm) Density Freq. (sec^{-1})	30·5 2·05 173	25·4 2·09 202	20·6 2·10 228	15·5 2·15 305	10·7 2·20 338	5·5 2·26 430	3·1 2·26 263
No. 5	Height (cm) Density Freq. (sec^{-1})	26·5 2·06 190	24·5 2·10 212	20·3 2·10 245	15·1 2·13 260	10·6 2·12 309	5·0 2·25 343	2·5 2·21 333

(*to be continued.*)

Table V. (*continued.*)

No. 6	Height (cm)	30·2	25·0	20·3	16·0	9·9	5·3	3·0
	Density	2·08	2·09	2·09	2·15	2·14	2·16	2·23
	Freq. (sec^{-1})	119	130	133	177	238	333	470
No. 7	Height (cm)	29·1	23·6	18·6	14·6	9·0	4·7	
	Density	2·11	2·12	2·12	2·14	2·15	2·29	
	Freq. (sec^{-1})	212	257	353	400	485		

Table VI. Height, Frequency, and Density of Clay (No. 2).

No. 1	Height (cm)	45·5	40·0	34·8	30·0	24·9	20·2	14·9	10·0	5·1	2·2
	Density	1·86	1·87	1·88	1·87	1·86	1·88	1·90	1·95	2·02	2·03
	Freq. (sec^{-1})	68	78	91	99	113	142	175	216	328	333
No. 2	Height (cm)	37·1	31·9	27·3	22·0	17·3	12·3	7·9	3·0		
	Density	1·73	1·77	1·78	1·78	1·83	1·89	1·79	1·97		
	Freq. (sec^{-1})	75	85	97	115	139	173	260	254		
No. 3	Height (cm)	43·5	36·9	29·9	23·8	18·7	13·8	9·0	5·2	2·6	
	Density	1·92	1·92	1·95	1·99	1·99	1·99	2·04	2·01	2·08	
	Freq. (sec^{-1})	177	197	257	295	321	399	490	462	454	

Table VII. Height, Frequency, and Density of Agar-agar and Rubber.

Agar-agar	Height (cm)	12·8	10·0	8·0	6·0	4·3	2·0	
	Density	1·02	0·99	1·01	1·03	1·06	1·05	
	Freq. (sec^{-1})	28	33	39	50	59	97	
Rubber	Height (cm)	20·0	14·6	10·0	5·0	2·8	2·05	1·15
	Density	1·76	1·73	1·79	1·79	1·73	1·78	1·73
	Freq. (sec^{-1})	142	170	242	367	342	488	407

(c) Effect of moisture content on the constants of elasticity and solid viscosity.

In order to observe the effect of moisture content on the physical properties of soils, we have arranged the computed values from the experiments (Table VIII) in several diagrams. Always the moisture content is considered abscissa; and the velocity v, Young's modulus E, and the solid viscosity coefficient γ as ordinate in each case. These curves are shown in Figs. 27～29.

Table VIII. Young's Modulus, Velocity, Coefficient of Solid Viscosity, and Moisture Content of Soils.

Kinds of Soils	No.	Density ρ	Moisture content %	Velocity m/sec	Young's modulus E (c.g.s.)	Coef. of solid viscosity γ (c.g.s.)
Silty-clay (Maru-no-uti) (No. 1~9: natural state. No. 10: recomposed state.)	1	1·46	50·0	62	$5{\cdot}61\times10^{7}$	$4{\cdot}05\times10^{4}$
	2	1·46	42·3	166	$4{\cdot}05\times10^{8}$	$2{\cdot}32\times10^{5}$
	3	1·40	48·6	110·7	$1{\cdot}72\times10^{8}$	1·27 ″
	4	1·68	43·6	189	$6{\cdot}00\times10^{8}$	2·51 ″
	5	1·67	32·7	437	$3{\cdot}19\times10^{9}$	$1{\cdot}11\times10^{6}$
	6	1·63	42·9	181	$5{\cdot}34\times10^{8}$	$2{\cdot}48\times10^{5}$
	7	1·56	40·8	240	$8{\cdot}99\times10^{8}$	3·77 ″
	8	1·62	36·7	267	$1{\cdot}16\times10^{9}$	2·95 ″
	9	1·47	30·5	344	$1{\cdot}74\times10^{9}$	6·30 ″
	10	1·48	50·0	31	$1{\cdot}42\times10^{7}$	$2{\cdot}26\times10^{4}$
Silt (Komatugawa) (Recomposed state)	1	1·78	49·5	69	$8{\cdot}47\times10^{7}$	$1{\cdot}28\times10^{5}$
	2	1·99	29·3	75	$1{\cdot}12\times10^{8}$	$6{\cdot}18\times10^{4}$
	3	1·77	15·9	426	$3{\cdot}21\times10^{9}$	$7{\cdot}61\times10^{5}$
Loam (Hongô) (Natural state)	1	1·44	52·3	200	$5{\cdot}76\times10^{8}$	$2{\cdot}86\times10^{5}$
	2	1·29	49·7	221	6·30 ″	2·52 ″
	3	1·22	46·8	245	7·32 ″	2·50 ″
	4	1·15	41·5	300	$1{\cdot}04\times10^{9}$	2·84 ″
Loam (No. 1) (Recomposed state)	1	2·37	16·7	76	$1{\cdot}37\times10^{8}$	$1{\cdot}40\times10^{5}$
	2	2·11	17·3	64	0·86 ″	1·17 ″
	3	1·92	22·1	94	1·70 ″	1·74 ″
	4	2·16	19·0	219	$1{\cdot}04\times10^{9}$	4·86 ″
	5	2·14	21·0	150	$4{\cdot}82\times10^{8}$	3·63 ″
	6	2·13	15·5	250	$1{\cdot}51\times10^{9}$	4·97 ″
Clay (No. 2) (Recomposed state)	1	1·91	30·7	112	$2{\cdot}40\times10^{8}$	$1{\cdot}81\times10^{5}$
	2	1·82	26·6	126	$2{\cdot}89\times10^{8}$	1·85 ″
	3	1·99	22·5	309	$1{\cdot}90\times10^{9}$	6·28 ″
Rubber		1·77		114·5	$2{\cdot}35\times10^{8}$	$1{\cdot}41\times10^{5}$
Agar-agar		1·03		14·8	$2{\cdot}25\times10^{6}$	$6{\cdot}53\times10^{3}$

We can conclude that the velocity or Young's modulus of soils decreases somewhat rapidly with the increase of the moisture content, though the manner of decreasing differs according to each soil specimen. The solid viscosity coefficient of the soils at Maru-no-uti decreases with the increase of the moisture content, while in the case of the loam at

Hongô, this coefficient is somewhat constant, even with the increase of the moisture content to a certain limit.

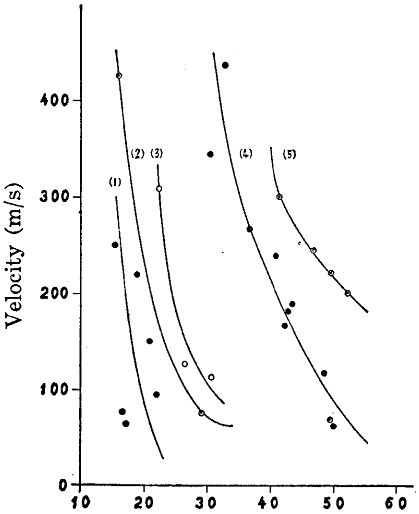


Fig. 27. Relation between the velocity and the moisture content.
(1) Loam of No. 1. (2) Silt of Komatugawa. (3) Clay of No. 2. (4) Silty-clay of Maru-no-uti. (5) Loam of Hongô.

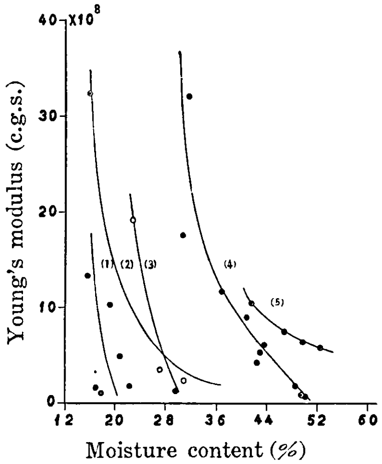


Fig. 28. Relation between Young's modulus and the moisture content.
(1) Loam of No. 1. (2) Silt of Komatugawa. (3) Clay of No. 2. (4) Silty-clay of Maru-no-uti. (5) Loam of Hongô.

(d) Partials.

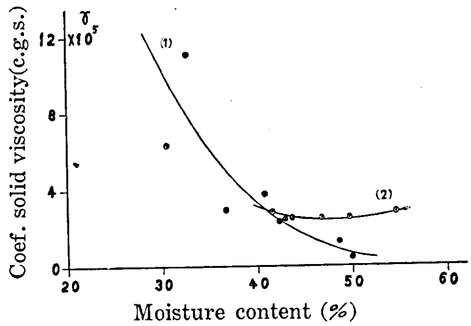


Fig. 29. Relation between the coefficient of solid viscosity and the moisture content.
(1) Silty-clay of Maru-no-uti.
(2) Loam of Hongô.

As the determination of the mode of vibration of soil specimen is very important in our experiments, we have studied especially the resonance frequencies of the partials in every case. Three examples of the relation between the frequency of resonance and the order of partials are shown in Fig. 30. In this figure we take the frequency as ordinate and the order of successive maximum amplitude in the record as abscissa. We find that these points are not in a straight line which corresponds to the result obtained from the theoretical treatment in Section 3. We believe that this is due to the effect of solid viscosity in the soil specimen. As will be seen in this figure, the frequency of the partial of the second maximum amplitude attiains about three times that of the first. In general, the frequency of the partials is about odd number of times that of the first maximum amplitude which represented in the figures 10 and 11.

We ascertain, therefore, that the observed vibration of the soil specimen can be explained as the longitudinal vibration in a clamped- and free-end bar.

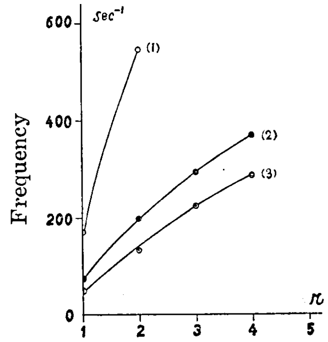


Fig. 30. Relation between the resonance frequency and the order of partials, r, of soils.
(1) Loam of Hongô, $h=27\cdot5$ cm. (2) Silty-clay of Maru-no-uti, $h=25\cdot2$cm. (3) Silt of Komatugawa, $h=31\cdot0$ cm.

From the curvature of these curves represented in Fig. 30, we were able to determine the solid viscosity coefficient by comparing these with the theoretical values shown in Fig. 5. However, in this case we have not adopted this method for determining the solid viscosity coefficient γ.

6. Summary and Conclusion.

We may now briefly summarize the results of the present study.

1) We studied the elastic properties of sub-surface soils of several regions in Tôkyô. The soils were taken out in the natural state from underground in Hongô, Maru-no-uti, and Komatugawa by means of the boring process. All of soil specimens were made into the form of a rectangular prism, and tested in both the natural and recomposed states.

2) We obtained the longitudinal wave velocity v in these soil specimens by means of the vibration method, and then computed their Young's moduli E from the relation $E=\rho v^2$.

3) The apparent effect of solid viscosity becomes great according to the variable height and variable moisture content of the soil specimen. The smaller the height and the degree of moisture content of the specimen, the more the resonance curves tend to flatten.

4) We determined the solid viscosity coefficient γ by comparing the reducing curves obtained experimentally, with the theoretical curves which are represented by the equation

$$T = \frac{4h}{\sqrt{\frac{E}{\rho} - \frac{\pi^2 \gamma^2}{16\rho^2 h^2}}},$$

in which the symbols have the same meaning as before.

5) The longitudinal wave velocity, Young's modulus, and the solid viscosity coefficient of the soils in the natural state are greater than those obtained in the recomposed state.

6) The wave velocity, Young's modulus, and the solid viscosity coefficient decrease somewhat rapidly with the increase of the moisture content. The coefficient of solid viscosity of these soils is of the order of $10^4 \sim 10^6$ and the velocity varies from about 60 m sec$\sim$400 m sec at moisture content ranges of about 50%$\sim$20%.

7) The frequency of the partials of the second maximum amplitude attains about three times that of the first. The observed vibration of the soil specimen can be, therefore, explained as the longitudinal vibration in a clamped- and free-end bar.

In conclusion, our sincerest thanks are due to Dr. T. Watanabe, the member of Geotechnical Committee, Government Railways of Japan, who kindly gave us the soils and the results of their mechanical analysis, which we used in these experiments.

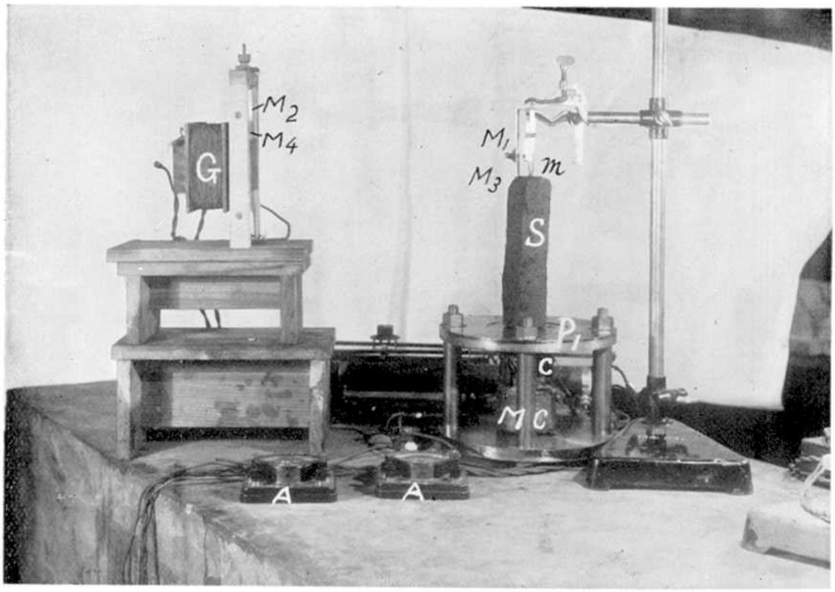


Fig. 6. Experimental Apparatus (1).
(The symbols in this figure have the same meaning as in Fig. 8.)

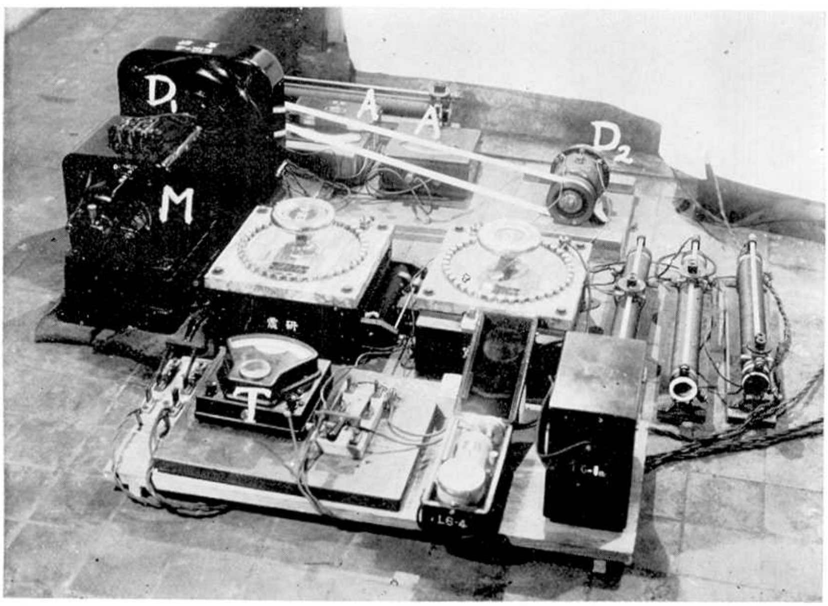


Fig. 7. Experimental Apparatus (2).
(The symbols in this figure have the same meaning as in Fig. 8.)

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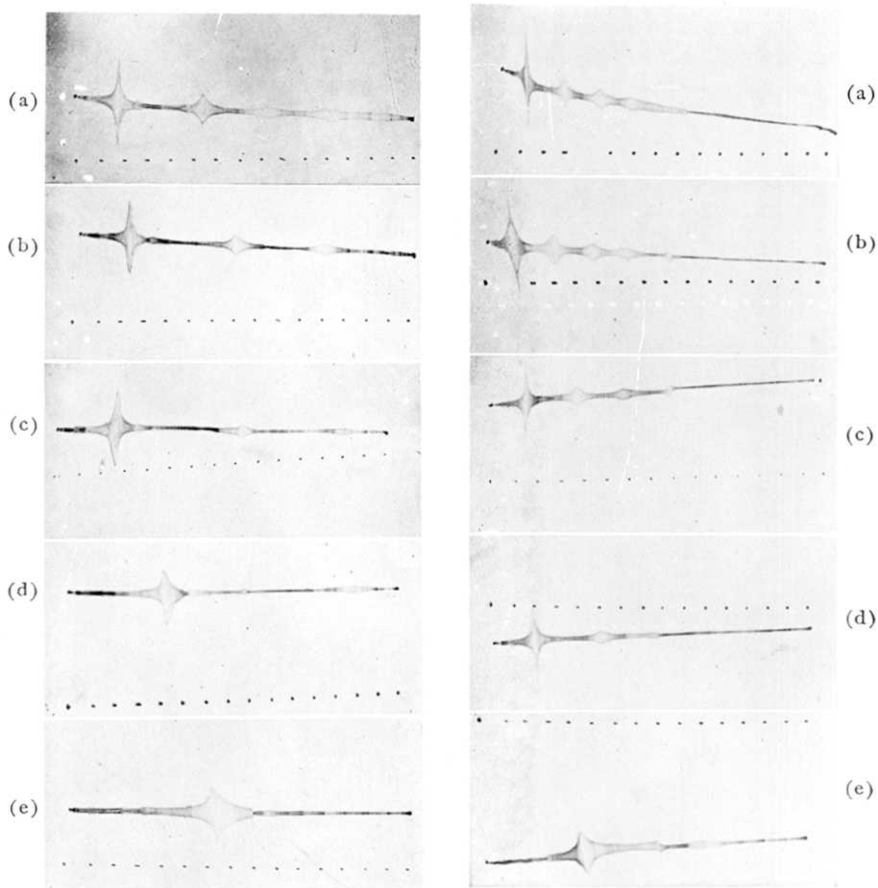

(i) Natural state. (ii) Recomposed state.

Fig. 10. Resonance curves of soils taken from Maru-no-uti.
Moisture content 50%, $\rho=1\cdot46$, h=height, N=frequeney.

(i)
- (a) $h=25\cdot2$ cm, $N=61$ $\sec^{-1}$.
- (b) $h=21\cdot0$, $N=73$.
- (c) $h=17\cdot0$, $N=90$.
- (d) $h=8\cdot9$, $N=153$.
- (e) $h=4\cdot8$, $N=210$.

(ii)
- (a) $h=23\cdot0$ cm, $N=35$ $\sec^{-1}$.
- (b) $h=19\cdot1$, $N=41$.
- (c) $h=14\cdot9$, $N=49$.
- (d) $h=10\cdot6$, $N=67$.
- (e) $h=3\cdot8$, $N=168$.

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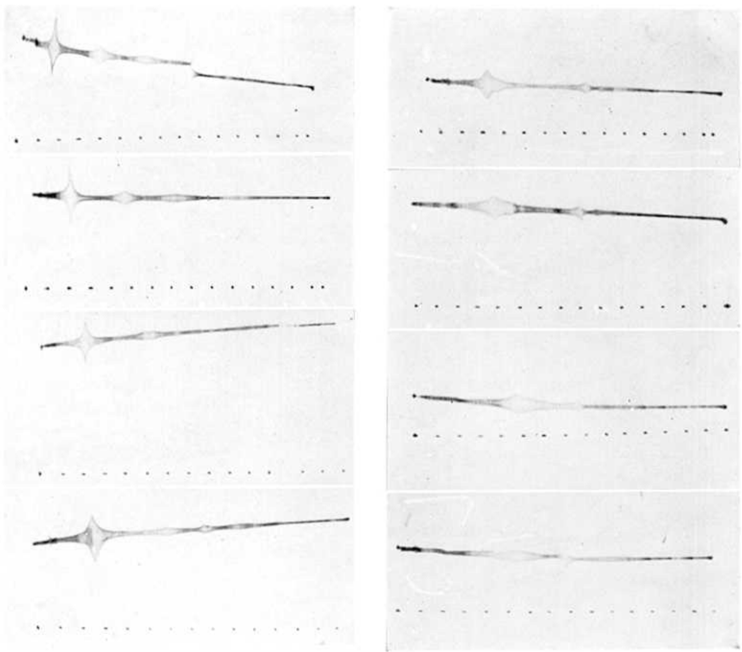

Fig. 13. Resonance curves of soils taken from Komatugawa. Moisture content 49·5%, $\rho=1{\cdot}78$.

(a)	$h=31{\cdot}0$ cm,	$N=55$ sec^{-1}.	(b)	$h=25{\cdot}4$ cm,	$N=63$ sec^{-1}.
(c)	$h=20{\cdot}3$,,	$N=71$,,	(d)	$h=15{\cdot}2$,,	$N=97$,,
(e)	$h=9{\cdot}9$,,	$N=103$,,	(f)	$h=6{\cdot}1$,,	$N=137$,,
(g)	$h=4{\cdot}0$,,	$N=198$,,	(h)	$h=3{\cdot}0$,,	$N=170$,,

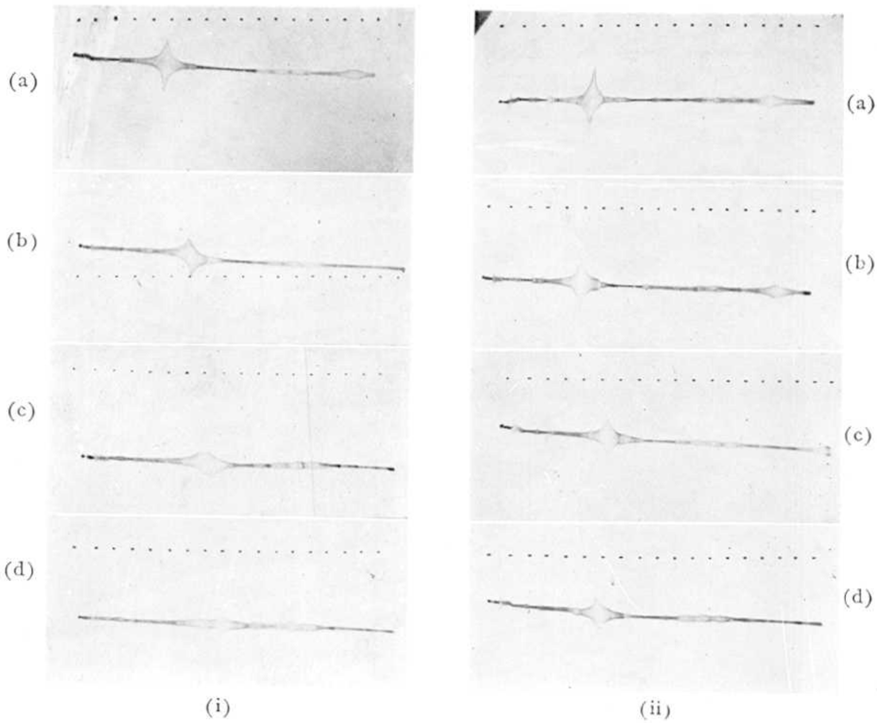

Fig. 15. Resonance curves of soils taken from Hongô, Imperial University.

(i) Moisture content 49·7%, $\rho=1\cdot29$. (ii) Moisture content 46·8%, $\rho=1\cdot22$.

(i)
- (a) $h=33\cdot8$ cm, $N=163$ sec^{-1}.
- (b) $h=25\cdot6$,, $N=207$,,
- (c) $h=20\cdot6$,, $N=239$,,
- (d) $h=15\cdot4$,, $N=277$,,

(ii)
- (a) $h=37\cdot4$ cm, $N=160$ sec^{-1}.
- (b) $h=33\cdot0$,, $N=173$,,
- (c) $h=27\cdot5$,, $N=205$,,
- (d) $h=23\cdot2$,, $N=235$,,

58. 振動方法による土の彈性測定

第1報 ヤング率の測定

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地震動の性質の各地に於て異る事は大森博士等の注目した事であるが，最近加速度地震計等の觀測により各地に卓越振動の存在する事が明かにされ，地震動の解釋に就て一段の發展を示した如く思はれる．

地震の發生により地震波が震源より放射され，地殼内を傳播して地表に到達すれば各土地に存する固有振動の二次的に誘發され，共れが地震動中，特に加速度の大なる卓越振動として觀測される．各土地に斯樣の卓越振動の存する所以は，地表に比較的彈性の小なる所謂表面土層の存在する結果であり，此の層の固有振動が卓越振動を生ぜしめると考へられるに至つた．從つて地震動の性質を闡明するに當つては，先づ表面層を形成する物質の彈性の研究が重要であると信じられる．

此の論文は要するに表面土の彈性を研究したものであり，一般の自然土に於ては多くの因子に左右されて其の本性を取り出す事が比較的困難であるために，各地に於ける自然の堆積狀態の土を成る可く害さぬ樣にして實驗室に持ち來し，共れを適當の形狀を具へた土柱となし，其の彈性を測定する事とした．土の彈性に就ては從來靜力學的の研究はあるが，元來土は粘彈性物質であつて以上の方法を以てしては粘性，彈性の兩性質を分けて測定する事が殆ど不可能である．從つて今回の實驗には毎秒 0～700 の範圍に連續的に振動數の變化し得る振動盤を採用し，其の上に土柱を乘せ其の共鳴する振動數を求めて結局土中に於ける縱波の速度 v を測定し，次いで

$v=\sqrt{\dfrac{E}{\rho}}$ の式より共の物質のヤング率を求めたのである．

試驗土の種類は沈泥，赤土，粘土等であるが，この外に此れらと比較するために寒天及び彈性ゴムをも取扱つた．自然の堆積狀態に於て試驗した土は丸の内の第一生命保險相互株式會社建築敷地の地表面下約 8 m の表面層中より採取せる沈泥及び本鄕帝國大學構内第二食堂わきの地表下 2 m の表面層より採取せる赤土（ローム層土）である．此の丸の內の土に於ては 50% の含水率で 62 m/sec の速度及び 5.61×10^7 c.g.s.（比重 1.46）のヤング率を得，本鄕の土に於ては 52.3% の含水率に於て 200 m/sec の速度及び 5.76×10^8 c.g.s.（比重 1.44）のヤング率を測定した．上述の速度及びヤング率は含水量と共に變化する，即ち含水量の減少に伴ひ增加し，含水量の增加と共に減少する事が判明した．丸の內の土では速度に於て 60 m/sec より 400 m/sec の程度に迄變化し，ヤング率に於て 10^7 より 10^9 (c.g.s.) の程度に迄變化する．但し含水率の變化の範圍は 50% より 30% 迄の程度である．本鄕の赤土では 50% より 40% 迄の含水率の變化に對し，速度及びヤング率の變化は夫々 200 m/sec より 300 m/sec 迄及び 10^8 より 10^9 (c.g.s.) 迄の程度である．又固體粘度は大略 10^5 の程度であるが，含水量と共に變化する．例へば丸の內の土に於て含水量の 50% より 30% 迄の範圍の變化に對し，固體粘度の變化は 10^4 より 10^5 (c.g.s.) の程度の範圍である．

自然の堆積狀態の土とその組織を破碎してねり直した狀態の土とではその速度及び彈性率は著しく異り，ねり直しの狀態のものは小さな値を示す事が判明した．