29. *Theoretical and Experimental Study of Initial Motion of Seismographs and the Quantitative Study of First Impulsion of Earthquake.*

Part I.—Initial Motion of Seismographs caused by Ground Motion of Shock Type, with Special Reference to a Simple Method of Reducing the First Impulsion of Earthquake Motion.

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Thirty years have elapsed since Omori[1] discovered that the initial motion of earthquake is directed toward or away from the epicentre, and that the geographical distribution of its direction as well as different types of seismograms may be correlated with the mechanism of earthquake occurrence. When Galitzin[2] pointed out that the azimuth of the epicentre of a distant earthquake could be derived from the direction of the first impulsion of P, he was careful enough to work out a method of reducing the amplitude of the first ground motion, though he assumed it to be of sinusoidal form with a sudden beginning. But as his treatment was restricted to the case of his own seismograph with electromagnetic recording, H. Benndorf[3] invented another method of reduction though his result was vitiated by certain unfortunate errors. While Galitzin[4] was endeavouring to investigate the earth's internal structure from the emergence angle deduced from the observed magnitude of the first impulsion of P, a new field was opened by T. Shida, who showed that it is possible to infer the mechanism of earthquake occurrence from a study of the geographical distribution of initial dilatational or compressional waves. The prob-

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1) F. OMORI, *Publ. Earthq. Invest. Commit.*, **21** (1905), esp. 58~61. *Bull. Earthq. Invest. Comm.*, **1** (1907), 145~154; *Report Earthq. Invest. Comm.*, **68** A (1910), 3~19.

2) B. GALITZIN, *Bull. d. Acad. Imp. d. Sci. St. Petersbourg*, (1909), or *Verhandl. d. Int. Seismol. Assoc. Zermatt*, (1909), 132~141, and *Vorlesungen über Seismometrie*, (1914).

3) H. BENNDORF, *Mitt. d. Erdbeben Komm. Wien*, N. F., **46** (1913). 122~169.

4) B. GALITZIN, *Comptes Rendus Comm. Sis. Perm. Petrograd*, **7** (1919), 185~334. (His posthumous work.)

lem was extended by S. T. Nakamura, T. Matuzawa, K. Suda, and others to near earthquakes, and by E. Gherzi, O. Somville, P. Byerly and others to distant earthquakes. The more recent advances made on the same lines in its quantitative study scarcely need comment[5].

But since for such quantitative studies suitable corrections to the recorded magnitude of initial motion are indispensable, Berlage[6], Nakamura[7], Matuzawa[8] and Meisser[9] took these effect into consideration, and worked out methods of reducing the first ground impulsion. There still remained however persons who questioned the validity of the initial conditions for the solution of the equation of motion of seismometers, $\ddot{a}+2\varepsilon\dot{a}+n^2a=-V\ddot{\sigma}$. Though Wiechert[10], Galitzin[11], and Nakamura rightly put $a_0=0$ and $\dot{a}_0=-V\dot{\sigma}_0$ at $t=0$ when the ground begins to move from rest, S. Kusakabe[12] and O. Meisser put $a_0=\dot{a}_0=0$, notwithstanding that $\sigma_0\gtrless 0$. The present writer[13], who took up the problem in 1931, justified the first condition and also were able to confirm the applicability of the harmonic factor with tolerable accuracy to ground motion with a continuously varying period and amplitude after about one cycle from the beginning.

For reduction of the transient part the writer suggested the use of the reduction introduced by Galitzin and Nakamura. Then, by comparing records obtained from an ordinary displacement-seismograph and an Ishimoto acceleration-seimograph, T. Suzuki[14] found the existence of a ground motion of shock type that began gradually and enumerated the motions of some specified seismographs under the influence of ground motion of the type e^{-ct^2}, while C. Tsuboi[15] fully investigated the transient motion of a pendulum caused by an external vibration that began either suddenly or gradually, ($\sin pt$ or $2\sin pt-\sin 2pt$). But later ground motions of approximately one

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5) For a fuller account of this problem, the reader is referred to the writer's paper in the *Bull. Earthq. Res. Inst.*, **11** (1933), 403~453; **12** (1934), 43~705, and his "A Historical Review of the Problem of the Initial Motion of an Earthquake" which will be published in the near future.

6) H. P. BERLAGE, *Doktordissertation, Buchdrukerei, J. Waltman, Delft*, (1934).

7) S. T. NAKAMURA, *Proc. Imp. Acad.*, **3** (1927), 32.

8) T. MATUZAWA, *Proc. Imp. Acad.*, **3** (1927), 68~71.

9) O. MEISSER, *Veröff. Reichsanstalt f. Erdbebenforschung, Jena*, **9** (1929).

10) E. WIECHERT, *Abhandl. K. Ges. Göttingen, Math.-Phy. Kl.*, N.F., **2** (1903), 1~125.

11) B. GALITZIN, *for ex. Vorlesungen über Seismometrie*, (1914).

12) S. KUSAKABE, *General Seismology*, (1927), 90, (Japanese).

13) H. KAWASUMI, *Disin*, **4** (1931), 71~94, and 167~169.

14) T. SUZUKI, *Bull. Earthq. Res. Inst.*, **12** (1934), 15~18, and 155~162.

15) C. TSUBOI, *Bull. Earthq. Res. Inst.*, **12** (1934), 426~445.

sided impulsion were found on several occasions[16], with the result that studies[17] on the motion of seismographs caused by certain specified ground motions have also been published. But there are no studies covering instruments with so wide a range of constants as in the cases investigated by Tsuboi, neither has any inquiry been made relating to the selection of the appropriate ground motion in a practical study nor any examination into the suitabiltiy of each assumption when applied to other cases. In order to construct a complete table of reducing ground motions of shock type referred to above, the writer in 1934 intended to calculate as an exercise in seismometry at the Seismological Institute the case for ground motion of the type $3\sin pt-\sin 3pt$ in the time interval $0\leqq pt\leqq\pi$ only, the motion being a symmetrical one sided impulsion easy to calculate. He was greatly encouraged by the generous loan of Dr. Tsuboi of the results of his calcutations of the sinusoidal ground motion ($\sin pt$) during $0\leqq pt\leqq 300°$. The table was completed with the colaboration of R. Satô and the kind assistance of students, S. Kunori, T. Yamamura, Y. Hattori, S. Hukunaga, K. Kimura and K. Satô, after about a year's interruption, although a part of the results was read on April 3, 1935, at the annual meeting of the Physico-mathematical Society of Japan. A case exactly the same has also been treated by H. Martin[18] of Jena, both theoretically and experimentally, but not in so complete a manner as we would have wished. He has again erred in putting $a_0=\dot{a}_0=0$ as the initial condition for the acceleration-seismographs in opposition to Tsuboi.

In the meanwhile, M. Ishimoto, the director of the Earthquake Research Institute proposed in an unofficial circular letter to include the magnitudes and directions of the first motions P as well as other important phases in the bulletins of all seismological stations. In view of the encouraging response that met Ishimoto's proposal, the Subcommittee on Seismology of the National Research Council of Japan proposed to the forthcomming International Congress to be held at Edinburgh, that Seismological bulletins published by all seismological stations shall contain

1) The components of amplitude of first impulsion of P as well as its period in addition to the usual data on its direction as hitherto

16) K. WADATI, *Kensin Zihô*, 8 (1934), 21~31.
H. KAWASUMI, *Read at the meeting of Phys.-math. Soc. Japan*, on April 3, 1935.
J. A. SHARPE, *Bull. Seismol. Soc. Amer.*, 25 (1935), 199~222.

17) M. KIZIMA, *Kensid Zihô*, 7 (1935), 97~107.
G. NISHIMURA and T. TAKAYAMA, *Disin*, 7 (1935), 100~117.

18) H. MARTIN, *Veröff. Reichsanstalt f. Erdbebenforschuag, Jena*, 26 (1935).

given by certain stations.

2) Similar data relating to the first impulsion of any other particular phase whenever it is clearly recognised.

It is explained in the proposal that the components of amplitude shall be seismographic readings divided by their respective magnifications. The magnification for the present purpose should not, from the ideal point of view, be the constant instrumental magnification, nor the usual one for the harmonic motion in stationary condition, but should be some variable magnification for the transient state of the beginning of the respective ground motion. To put this proposal into rigorous practice necessitates the use of numerical integration, which would be very laborious. Moreover there is the uncertainty as to which of the various methods just described of dealing with the proposal could be used satisfactorily. These were the main points of the objection raised by some to Ishimoto's proposal. To obtain the approval of the International Congress, some simple and convenient method should be offered, or an examination made the accuracy of the methods now in use and the result made known.

For this purpose the results of our calculation for the case of single ground impulsion will be described below and compared with other cases with the view of finding a simple but workable method of obtaining the first ground impulsion.

We shall now deduce the motion of a pendulum caused by a sinusoidal ground motion of finite length using the proof given in the writer's previous paper, with certain generalisations in favour of the initial condition $a_0=0$ and $\dot{a}_0=-V\dot{\sigma}_0$.

The elementary solution of the equation of motion

$$\ddot{a}+2\varepsilon\dot{a}+n^2a=-V\ddot{\sigma} \tag{1}$$

corresponding to $\sigma=Ae^{i\omega t}+Be^{-i\omega t}$ is

$$a=A\frac{V\omega^2}{n^2-\omega^2+2\varepsilon\omega i}e^{i\omega t}+B\frac{V\omega^2}{n^2-\omega^2-2\varepsilon\omega i}e^{-i\omega t}.$$

If we put $A=f(\omega)d\omega$, $B=g(\omega)d\omega$ and integrate from 0 to ∞, the result

$$a=\int_0^\infty\frac{V\omega^2f(\omega)}{n^2-\omega^2+2\varepsilon\omega i}e^{i\omega t}d\omega+\int_0^\infty\frac{V\omega^2g(\omega)}{n^2-\omega^2-2\varepsilon\omega i}e^{-i\omega t}d\omega$$

is also the solution of (1) corresponding to

$$\sigma(t)=\int_0^\infty f(\omega)\,e^{i\omega t}d\omega+\int_0^\infty g(\omega)\,e^{i\omega t}d\omega\,,$$

so long as $f(\omega)$ and $g(\omega)$ are independent of time.

Comparing it with Fourier's double integral theorem

$$\sigma(t)=\frac{1}{2\pi}\int_0^\infty d\omega\int_{-\infty}^\infty \sigma(\lambda)\, e^{i\omega(t-\lambda)}d\lambda+\frac{1}{2\pi}\int_0^\infty d\omega\int_{-\infty}^\infty \sigma(\lambda) e^{-i\omega(t-\lambda)}d\lambda\,, \tag{2}$$

we have the general solution

$$a=\frac{V}{2\pi}\int_{-\infty}^\infty \frac{\omega^2 d\omega}{n^2-\omega^2+2\varepsilon\omega i}\int_{-\infty}^\infty \sigma(\lambda)\, e^{i\omega(t-\lambda)}d\lambda\,, \tag{3}$$

corresponding to the arbitrary ground motion $\sigma(t)$.

In the special case

$$\sigma(t)=\begin{cases} 0 & \text{for} \quad t<0\,, \\ \sin pt & \quad 0\leqq t\leqq t_1\,, \\ 0 & \quad t_1<t\,, \end{cases} \tag{4}$$

with which we are concerned, we have

$$a=\frac{V}{2\pi}\int_{-\infty}^\infty \frac{\omega^2 d\omega}{n^2-\omega^2+2\omega\varepsilon i}\int_0^{t_1} \sin p\lambda\, \sigma^{i\omega(t-\lambda)}d\lambda$$

$$=\frac{V}{4\pi}\left[\left\{\int_{-\infty}^\infty \frac{\omega^2 e^{i\omega t}d\omega}{(\omega-\alpha)(\omega-\beta)(\omega-p)}-\int_{-\infty}^\infty \frac{\omega^2 e^{i\omega t}d\omega}{(\omega-\alpha)(\omega-\beta)(\omega+p)}\right\}\right.$$

$$\left.-\left\{e^{ipt_1}\int_{-\infty}^\infty \frac{\omega^2 e^{i\omega(t-t_1)}d\omega}{(\omega-\alpha)(\omega-\beta)(\omega-p)}-e^{-ipt_1}\int_{-\infty}^\infty \frac{\omega^2 e^{i\omega(t-t_1)}d\omega}{(\omega-\alpha)(\omega-\beta)(\omega+p)}\right\}\right], \tag{5}$$

where α and β are the roots of $\omega^2-2\varepsilon\omega i-n^2=0$,

$$\alpha=\sqrt{n^2-\varepsilon^2}+\varepsilon i\,,\qquad \beta=-\sqrt{n^2-\varepsilon^2}+\varepsilon i\,. \tag{6}$$

The infinite integral

$$I_\pm=\int_{-\infty}^\infty \frac{\omega^2 e^{i\omega\tau}d\omega}{(\omega-\alpha)(\omega-\beta)(\omega\pm p)} \tag{7}$$

is easily evaluated by contour integral along the real axis with a small indentation at $\omega=-p$ or $+p$ and infinite semi-circle on the upper or lower half of the complex plane according as τ is positive or negative. The result is

$$I_{\pm}=\left\{\begin{array}{ll} -\pi i\dfrac{p^2}{p^2+(\alpha+\beta)p+\alpha\beta}e^{\mp ip\tau}, & \text{for} \quad \tau<0, \\ \pi i\dfrac{p^2}{p^2\pm(\alpha+\beta)p+\alpha\beta}e^{\mp ip\tau}+\dfrac{2\pi i}{\alpha-\beta}\left\{\dfrac{\alpha^2 e^{i\alpha\tau}}{\alpha\pm p}-\dfrac{\beta^2 e^{i\beta\tau}}{\beta\pm p}\right\}, & \text{for} \quad \tau>0. \end{array}\right\} \tag{8}$$

and a little arithmetic shows that

(i) for $t<0$, $t-t_1<0$,

$$a=0,$$

(ii) for $0\leq t\leq t_1$,

$$a=\frac{V\sigma_m i}{2}\left[p^2\left\{\frac{e^{-ipt}}{p^2+(\alpha+\beta)p+\alpha\beta}-\frac{e^{ipt}}{p^2-(\alpha+\beta)p+\alpha\beta}\right\}\right.$$
$$\left.+\frac{2p}{(\alpha-\beta)}\left\{\frac{\alpha^2 e^{i\alpha t}}{\alpha^2-p^2}-\frac{\beta^2 e^{i\beta t}}{\beta^2-p^2}\right\}\right]$$

$$=\frac{V\sigma_m}{\sqrt{(1-u^2)^2+4h^2u^2}}\left\{\sin(pt-\Delta)-\frac{u}{\mu}e^{-\varepsilon t}\sin(\gamma t-\delta)\right\}, \tag{9}$$

where $u=n/p$, $\gamma=\sqrt{n^2-\varepsilon^2}=n\mu$,

$$\tan\Delta=\frac{2hu}{u^2-1} \quad \text{and} \quad \tan\delta=\frac{2h\mu}{u^2-1+2h^2},$$

(iii) for $t_1<t$,

$$a=\frac{V\sigma_m i}{2(\alpha-\beta)}\left[2p\left\{\frac{\alpha^2 e^{i\alpha t}}{\alpha^2-p^2}-\frac{\beta^2 e^{i\beta\tau}}{\beta^2-p^2}\right\}\right.$$
$$\left.-e^{ipt_1}\left\{\frac{\alpha^2 e^{i\alpha(t-t_1)}}{\alpha-p}-\frac{\beta^2 e^{i\beta(t-t_1)}}{\beta-p}\right\}+e^{-ipt_1}\left\{\frac{\alpha^2 e^{i\alpha(t-t_1)}}{\alpha+\beta}-\frac{\beta^2 e^{i\beta(t-t_1)}}{\beta+p}\right\}\right], \tag{10}$$

and in the special case with which we are concerned, $pt=(2k+1)\pi$, k being an integer, it reduces to

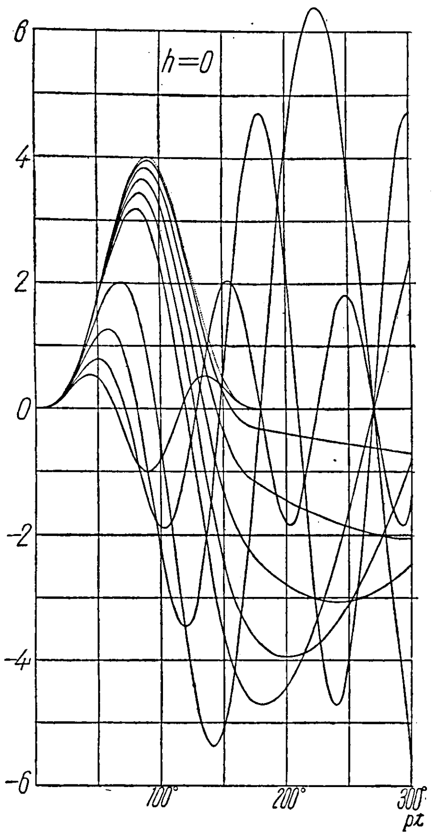
$$=\frac{-V\sigma_m}{\sqrt{(1-u^2)^2+4h^2u^2}}\frac{u}{\mu}\left\{e^{-\varepsilon t}\sin(\gamma t-\delta)+e^{-\varepsilon(t-t_1)}\sin\gamma(t-t_1-\delta)\right\}. \tag{11}$$

From the above solution we can see that the pendulum begins to move at $t=0$ with initial velocity $\dot{a}_0=-Vp\sigma_m=-V\dot{\sigma}_0$, while at $t=t_1$ the forced oscillations with frequency p come to an end, generating anew the proper oscillation of the pendulum, besides the one generated at $t=0$. In the case $pt=(2k+1)\pi$, the proper motion there generated is exactly equal to that generated at the beginning. The result is evident from the law of superposition characteristic of the linear differential equation.

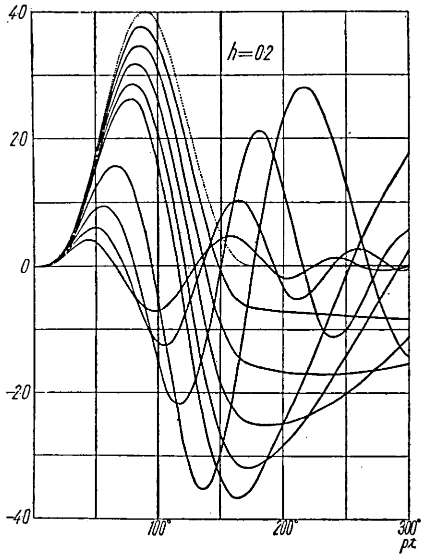
By means of the above solution the motion of the pendulum caused by

$$\sigma(t)=\left\{\begin{array}{ll} 0\,, & t<0\,, \\ 3\sin pt-\sin 3pt\,, & 0<pt<\pi\,, \\ 0\,, & \pi<pt\,, \end{array}\right\} \tag{12}$$

was obtained with the aid of Dr. Tsuboi's calculations. The results are shown in the following figures and tables. The instrumental magnification V is assumed to be unity.



(a)



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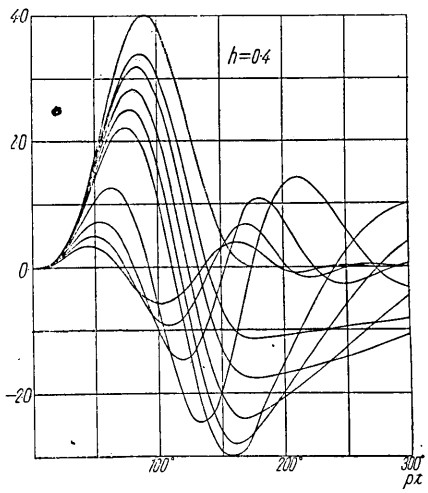
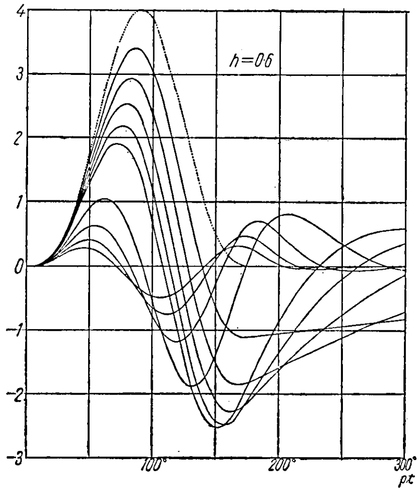
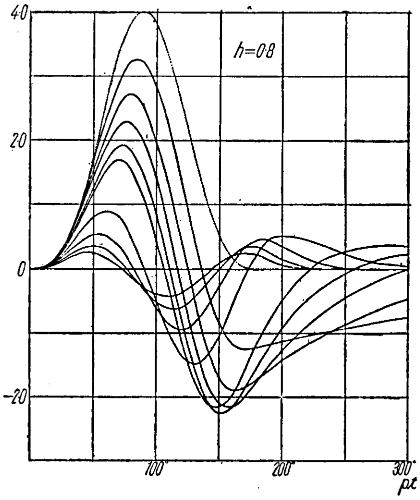


(c)

Fig. 1. Motion of ordinary seismographs (full line curve) caused by ground motion $\sigma=3\sin pt-\sin 3pt$ (dotted line). Respective curves correspond to $u=n/p=0{\cdot}2$, $0{\cdot}4$, $0{\cdot}6$, $0{\cdot}8$, $1{\cdot}0$, $2{\cdot}0$, $3{\cdot}0$, $4{\cdot}0$ and $5{\cdot}0$.



(d)



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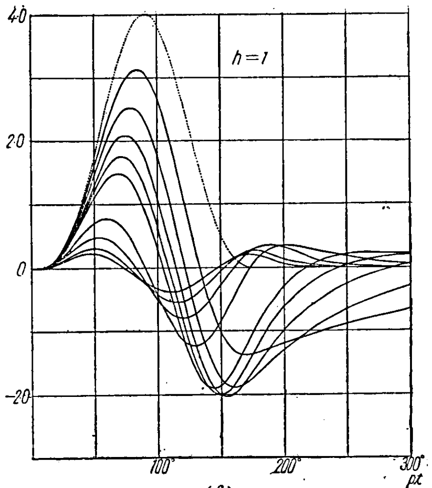

(f)

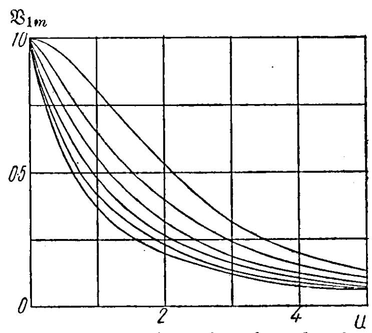

Fig. 2. Magnification for the first maximum ($\mathfrak{B}_{1m}$).

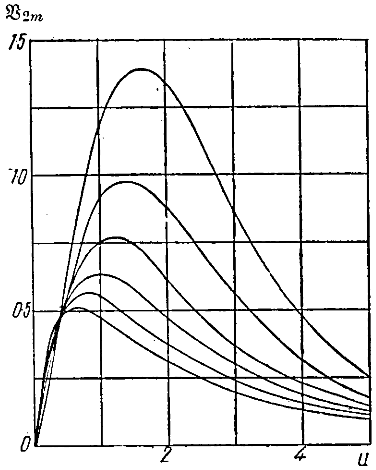

Fig. 3. Magnification for the second maximum.

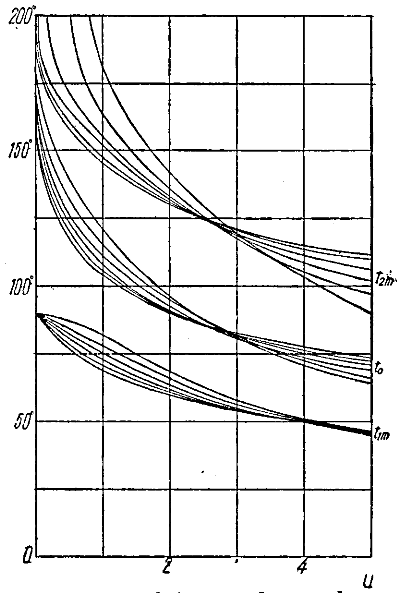

Fig. 4. Times of first and second maxima (pt_{1m}), (pt_{2m}), and time of first zero (pt_0).

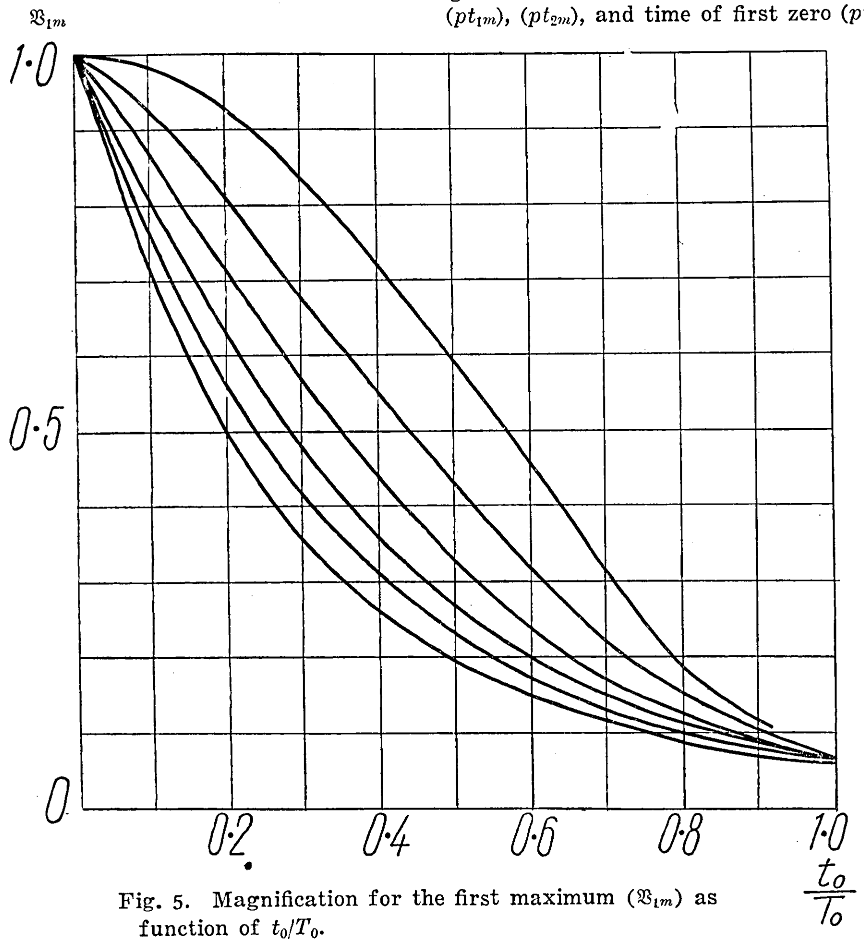

Fig. 5. Magnification for the first maximum $(\mathfrak{V}_{1m})$ as function of t_0/T_0.

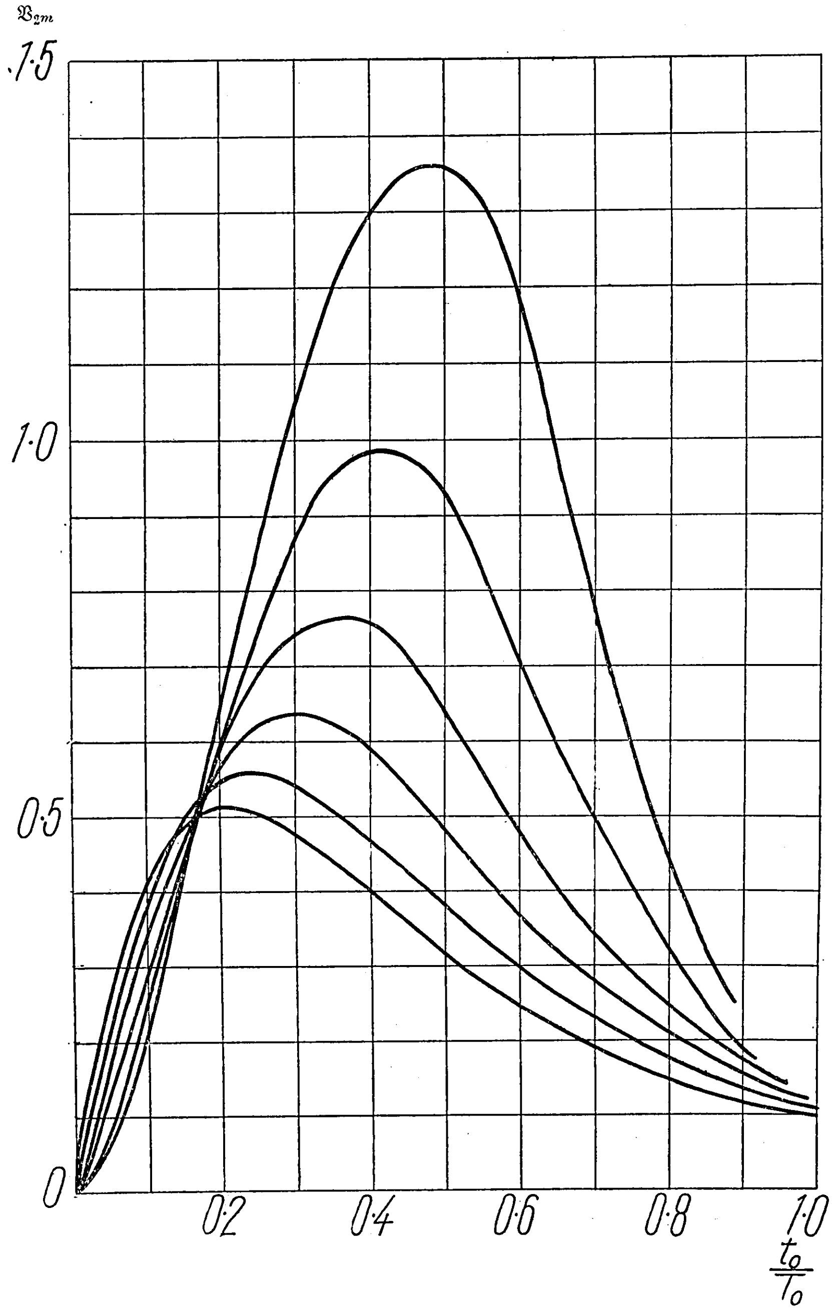

Fig. 6. Magnification for second maximum ($\mathfrak{V}_{2m}$) as function of t_0/T_0.

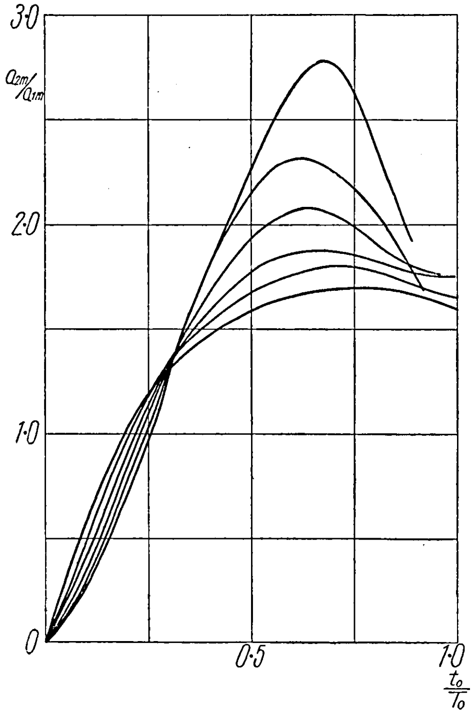

Fig. 7. Ratio of the first and second maxima.

Table I.

Magnification for the First Maximum ($\mathfrak{V}_{1m}$).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0·2	0·988	0·943	0·896	0·850	0·813	0·783
0·4	0·960	0·866	0·796	0·731	0·678	0·630
0·6	0·915	0·795	0·705	0·634	0·570	0·520
0·8	0·860	0·715	0·623	0·545	0·478	0·435
1·0	0·798	0·656	0·551	0·475	0·420	0·370
2·0	0·530	0·395	0·314	0·263	0·223	0·195
3·0	0·313	0·233	0·179	0·158	0·133	0·118
4·0	0·195	0·152	0·123	0·104	0·090	0·078
5·0	0·130	0·104	0·083	0·070	0·065	0·060

Table II.

Magnification for the Second Maximum ($\mathfrak{V}_{2m}$).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0·2			0·225	0·279	0·315	0·345
0·4	0·465	0·429	0·470	0·465	0·475	0·475
0·6	0·765	0·630	0·603	0·575	0·540	0·510
0·8	0·985	0·799	0·704	0·621	0·566	0·503
1·0	1·178	0·918	0·750	0·635	0·553	0·475
2·0	1·343	0·881	0·615	0·470	0·375	0·310
3·0	0·868	0·533	0·370	0·295	0·239	0·198
4·0	0·475	0·311	0·231	0·188	0·156	0·130
5·0	0·250	0·175	0·145	0·123	0·108	0·095

Table III.

Time of First Maximum (pt_{1m}).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0.2	89°	88°	87°	86°	85°	84°
0·4	88	85	84	83	80	79
0·6	87	83	81	79	77	75
0·8	84	80	78	76	74	72
1·0	82	78	75	72	71	70
2·0	69	66	64	63	61	60
3 0	58	56	54	55	55	54
4·0	51	51	50	50	51	51
5·0	45	45	44	46	46	46

Table IV.

Time of Second Maximum (pt_{2m}).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0·2			175°	175°	170°	170°
0·4	295°	237°	176	169	164	161
0·6	242	188	168	162	159	155
0·8	202	172	162	158	152	150
1·0	181	164	157	152	148	145
2·0	142	136	133	132	131	130
3·0	119	118	119	119	120	121
4·0	103	105	108	112	115	115
5·0	90	97	102	106	110	111

Table V.

Time of First Zero (pt_0).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0·2	156·5°	148°	143°	139°	136°	133°
0·4	144·5	137	131	127·5	124·5	121·5
0·6	135	128	122·5	120	116·5	114
0·8	127·5	121	116·5	113	110	108
1·0	120·5	115	111	108	106	104
2·0	96	94	92·5	91·5	90·5	90
3·0	81	81	80·5	81·5	82	83
4·0	71	72·5	73·5	75·5	76·5	77·5
5·0	64	66·5	68·5	71	72·5	73·5

Table VI.

Time of First Zero Measured in Fraction of Proper Period of Pendulum (t_0/T_0).

u \ h	0	0·2	0·4	0·6	0·8	1·0
0·2	0·087	0·082	0·080	0·077	0·076	0·074
0·4	0·161	0·152	0·147	0·142	0·139	0·135
0·6	0·225	0·214	0·204	0·200	0·195	0·190
0·8	0·284	0·269	0·259	0·251	0.244	0·240
1·0	0·335	0·320	0·309	0·300	0·295	0·289
2·0	0·533	0·522	0·513	0·508	0·503	0·500
3·0	0·675	0·674	0·671	0·679	0·683	0·691
4·0	0·789	0·806	0·817	0·839	0·850	0·861
5·0	0·889	0·923	0·952	0·986	1·008	1·020

Table VII.

Ratio of the First and Second Maxima a_{1m}/a_{2m}.

u \ h	0	0·2	0·4	0·6	0·8	1·0
0.2			0·251	0·328	0·388	0·442
0·4	0·485	0·495	0·590	0·635	0·702	0·753
0·6	0·835	0·793	0·852	0·909	0·950	0·980
0·8	1·148	1·120	1·135	1·141	1·185	1·156
1·0	1·480	1·400	1·355	1·331	1·317	1·286
2·0	2·440	2·230	1·962	1·790	1·682	1·590
3·0	2·780	2·280	2·070	1·875	1·800	1·685
4·0	2·439	2·050	1·885	1·809	1·740	1·681
5·0	1·925	1·680	1·762	1·754	1·652	1·581

We next calculated the motion of the Galitzin seismograph ($T_0=T_1$, $\mu=0$) caused by

(i) $\sin pt$ for $0\leqq t$,

(ii) $3\sin pt-\sin 3pt$ for $0\leqq pt\leqq\pi$,

(iii) $2\sin pt-\sin 2pt$ for $0\leqq t$,

by mean of Galitzin's own solution

$$a_1=\frac{kA_1T_0\sigma_m}{\pi l}u\left\{e^{-u\xi}\left[a_0+a_1\xi+a_2\xi^2+a_3\xi^3\right]+g\cos\xi+h\sin\xi\right\}, \qquad (13)$$

where $\xi=pt$ and

$$a_0=\frac{1-6u^2+u^4}{(1+u^2)^4}\,, \qquad a_3=-\frac{1}{6}\frac{u^3}{1+u^2}\,,$$

$$a_1=\frac{u(3-u^2)}{(1+u^2)^3}\,, \qquad g=-a_0\,,$$

$$a_2=\frac{1}{2}\frac{u^2(3+u^2)}{(1+u^2)^2}\,, \qquad h=\frac{4u(1-u^2)}{(1+u^2)^4}\,.$$

The results are shown in the following table and figures.

The fator $\frac{kA_1T_0}{\pi l}$ is omitted in the following calculation. Therefore to obtain the first ground impulsion we have to divide the recorded first maximum (a_{1m}) by $\mathfrak{V}_{1m}\frac{kA_1T_0}{\pi l}$ or second maximum (a_{2m}) by $\mathfrak{V}_{2m}\frac{kA_1T_0}{\pi l}$.

Table VIII.

Magnification for the First and Second Maxima ($\mathfrak{V}_{1m}$, $\mathfrak{V}_{2m}$), and their Raito ($\mathfrak{V}_{2m}/\mathfrak{V}_{1m}=a_{2m}/a_{1m}$) and Epochs ($pt_{1m}$, pt_{2m}), as well as Time of First Zero pt_0, and t_0/T_0.

Galitzin Seismograph ($T_0=T_1$, $\mu=0$).

(i) $\sigma(t)=\sin pt$, (ii) $3\sin pt-\sin 3pt$, (iii) $2\sin pt-\sin 2pt$.

(i)

u	$\mathfrak{V}_{1m}$	$\mathfrak{V}_{2m}$	a_{2m}/a_{1m}	pt_{1m}	pt_{2m}	pt_0	t_0/T_0
0·2	0·150	0·235	1·57	127°	294°	207	0·115
0·4	0·154	0·308	2·00	99	268	165	0·183
0·6	0·137	0·284	2·07	83	233	140	0·233
0·8	0·121	0·239	1·98	72	210	121	0·269
1·0	0·105	0·196	1·87	58	191	107	0·297
1·5	0·077	0·122	1·58	45	152	84	0·350
2·0	0·060	0·079	1·32	35	125	69	0·384
3·0	0·044	0·039	0·84	25	100	54	0·450
4·0	0·033	0·017	0·52	21	75	45	0·500
5·0							

(ii)

u	$\mathfrak{V}_{1m}$	$\mathfrak{V}_{2m}$	a_{2m}/a_{1m}	pt_{1m}	pt_{2m}	pt_0	t_0/T_0
0·2	0·131	0·063	0·48	121		214	0·119
0·4	0·153	0·121	0·79	105	233	159	0·177
0·6	0·148	0·164	1·11	95	193	139	0·232

(to be continued.)

(ii) (*continued.*)

u	$\mathfrak{B}_{1m}$	$\mathfrak{B}_{2m}$	a_{2m}/a_{1m}	pt_{1m}	pt_{2m}	pt_0	t_0/T_0
0·8	0·136	0·188	1·38	89°	174°	127°	0·254
1·0	0·121	0·195	1·61	82·5	163·5	117	0·328
1·5	0·091	0·176	1·93	71·5	145	102·5	0·428
2·0	0·066	0·140	2·13	65	133	92·5	0·513
3·0	0·041	0·088	2·15	53·5	119	80	0·667
4·0	0·028	0·051	1·34	46	108	75·5	0·839
5·0	0·022	0·033	1·53	41	102	69	0·958

(iii)

u	$\mathfrak{B}_{1m}$	$\mathfrak{B}_{2m}$	a_{2m}/a_{1m}	pt_{1m}	pt_{2m}	pt_0	t_0/T_0
0·2							
0·4	0·151	0·296	1·96	133°	257°	184°	0·204
0·6							
0·8	0·117	0·300	2·56	111	221	153	0·340
1·0	0·102	0·273	2·67	102	219	142	0·394
1·5							
2·0	0·045	0·126	2·80	78	173	112	0·622
3·0	0·027	0·068	2·49	93	152	96	0·801
4·0	0·019	0·037	1·92	55	140	90	1·000
5·0							

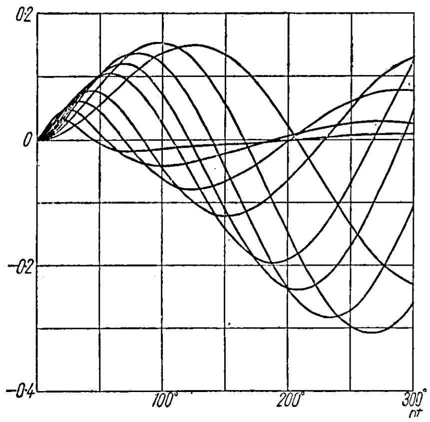

Fig. 8. Motion of the galvanometer of Galitzin seismograph caused by $\sigma=\sin pt$. Respective curves correspond to $u=n/p$ $=0{\cdot}2$, 0·4, 0·6, 0·8, 1·0, 1·5, 2·0, 3·0 and 4·0.

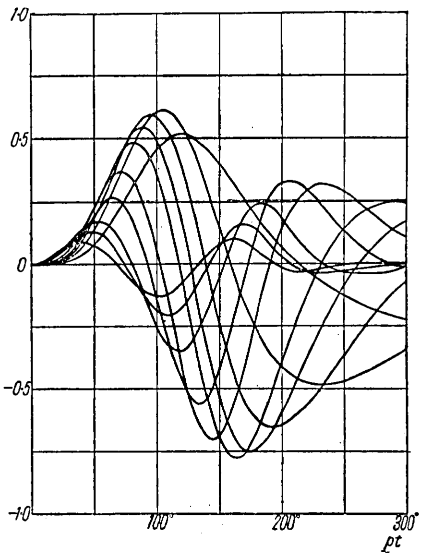

Fig. 9. Motion of the galvanometer of Galitzin seismograph caused by $\sigma=3\sin pt-\sin 3pt$. $0\leqq pt\leqq\pi$. Respective curves correspond to $u=n/p=$ 0·2, 0·4, 0·6, 0·8, 1·0, 1·5, 2·0, 3·0, 4·0 and 5·0.

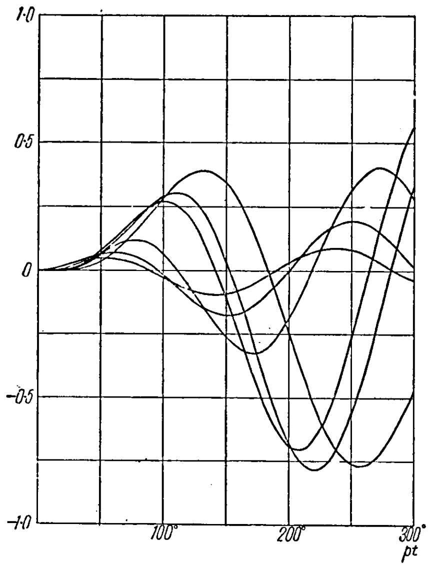

Fig. 10. Motion of galvanometer of Galitzin seismograph caused by $\sigma = 2\sin pt - \sin 2pt$, $0 \leqq pt$. Respective curves correspond to $u = n/p = 0{\cdot}4$, $0{\cdot}8$, $1{\cdot}0$, $2{\cdot}0$ and $4{\cdot}0$.

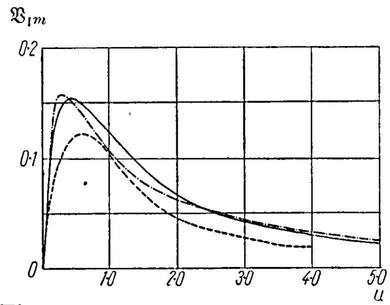

Fig. 11. Magnification for the first maximum caused by $\sigma = \sin pt$ (chain line), $3\sin pt - \sin 3pt$ (full line), or $2\sin pt - \sin 2pt$ (broken line). (Galitzin seismograph.)

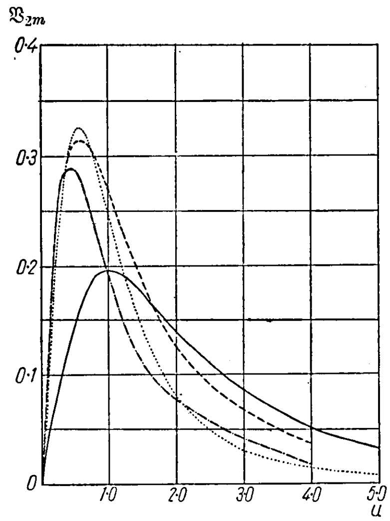

Fig. 12. Magnification for the second maximum. Broken, full and chain line correspond to the same as in Fig. 11. Dotted line represent the magnification for $\sin pt$ in stationary state. (Galitzin seismograph.)

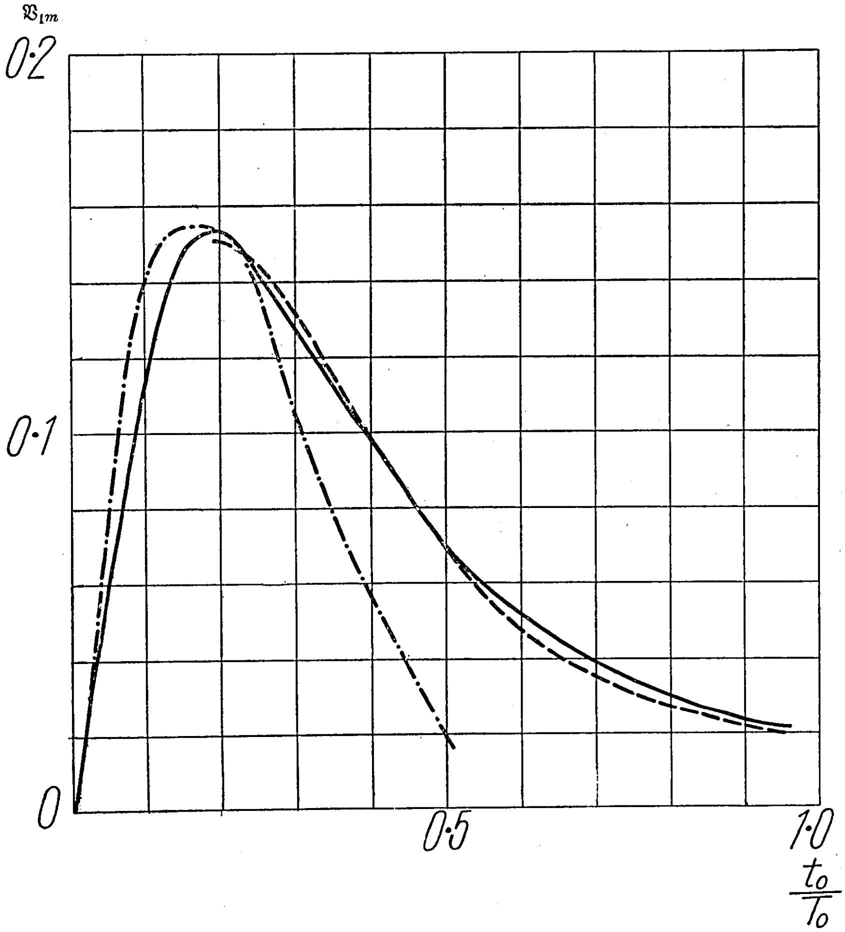

Fig. 13. Magnification for the first maximum, chain, full and broken lines correspond to the ground motion $\sigma = \sin pt$, $3\sin pt - \sin 3pt$, and $2\sin pt - \sin 2pt$ respectively. (Galitzin seismograph.)

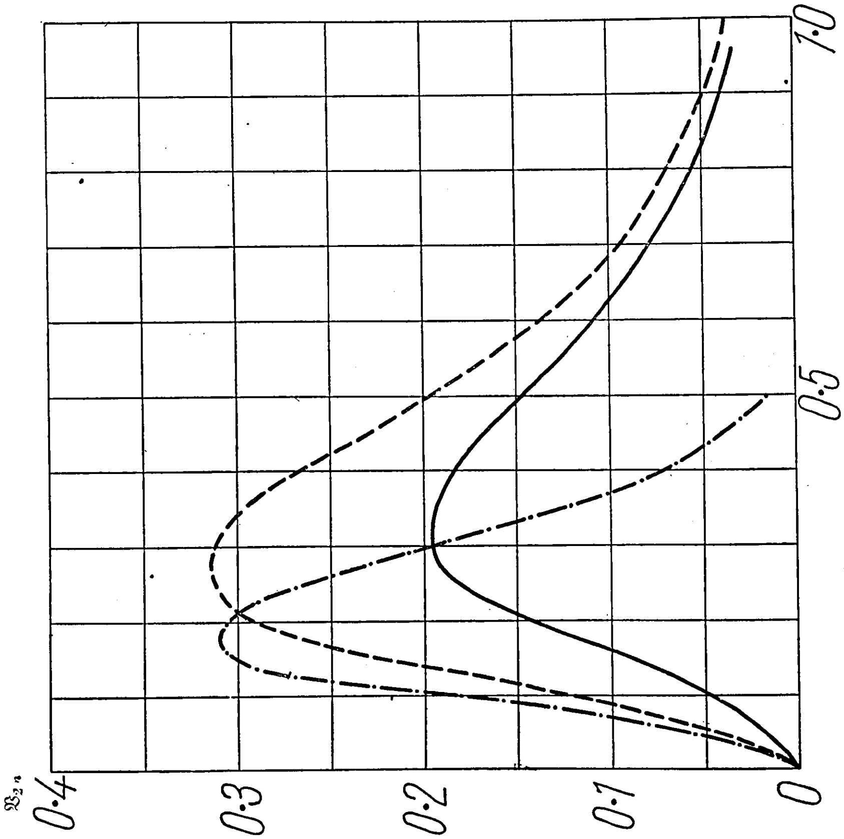

Fig. 14. Magnification for the second maximum. (Galitzin seismograph.)

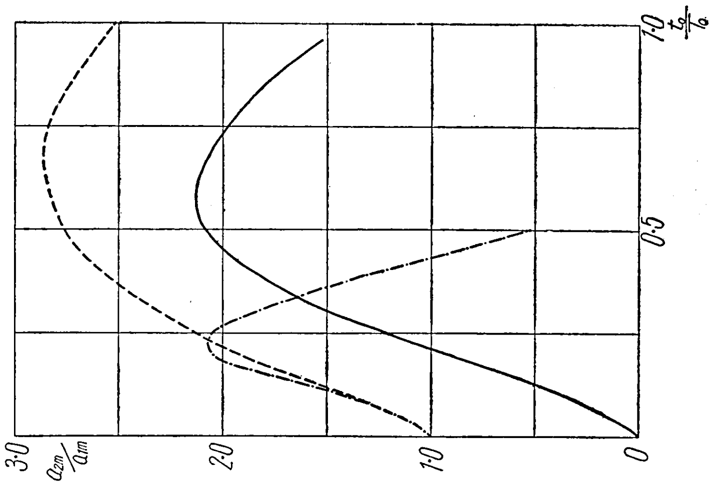

Fig. 15. Ratio of first and second maxima. (Galitzin seismograph.)

We shall now proceed to compare the present results with the calculations previously mentioned authors and examine the accuracies of each result when other formulae are used. After some study and trial, tabulation of the magnification for the first maximum as the

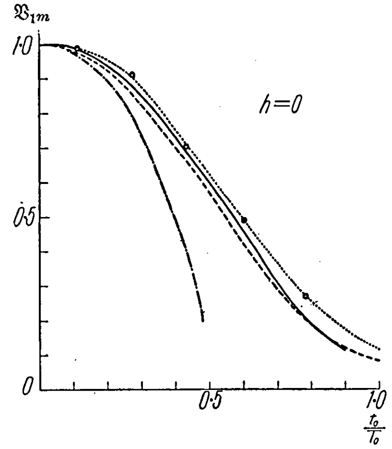

Fig. 16. Comparison of the $\mathfrak{B}_{1m}(t_0/T_0)$ for $\sigma=e^{-ct^2}$ (dotted line), $3\sin pt-\sin 3pt$ (full line), $2\sin pt-\sin 2pt$ (broken line), and $\sin pt$ (chain line). (Ordinary Seismograph.)

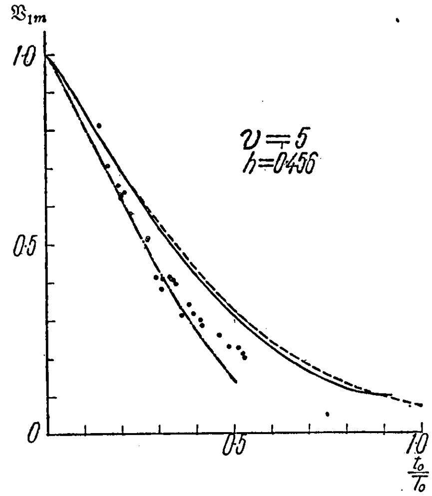

Fig. 17. Comparison of the $\mathfrak{B}_{1m}(t_0/T_0)$ for $\sigma=3\sin pt-\sin 3pt$ (full line), $2\sin pt-\sin 2pt$ (broken line), $\sin pt$ (chain line) and $te^{\beta t}\sin pt$ (points, different values of β inclusive). (Ordinary seismograph.)

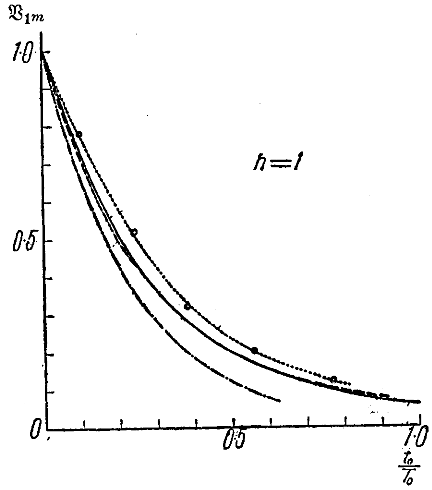

Fig. 18. Comparison of the $\mathfrak{B}_{1m}(t_0/T_0)$ for $\sigma=e^{-ct^2}$ (dotted line), $3\sin pt-\sin 3pt$ (full line), $2\sin pt-\sin 2pt$ (broken line) and $\sin pt$ (chain line). (Ordinary seismograph.)

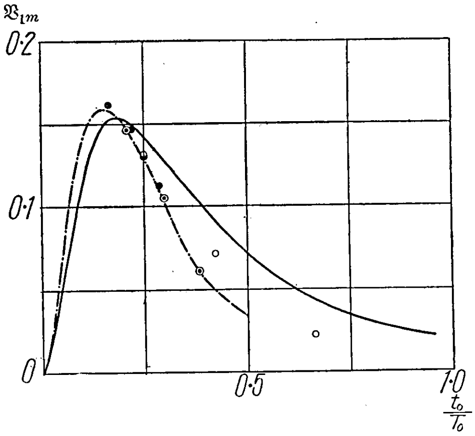

Fig. 19. Comparison of the $\mathfrak{B}_{1m}(t_0/T_0)$ for $\sigma=3\sin pt-\sin 3pt$ (full line), $\sin pt$ (chain line) and $e^{\beta t}\sin pt$ (points, different kind of points correspond to different value of β). (Galitzin seismograph.)

function of the time of first zero from the beginning measured in fractions of the proper period of the pendulum t_0/T_0, was found very convenient in practice for the reduction of the first impulsion, as already anticipated by Tsuboi although he used $2t_0/T_0$ instead of t_0/T_0. The determination of the time of the first zero from the beginning of the ground motion far surpasses in accuracy those of the times of maximum and minimum. The magnification of the first maximum is almost fairly invariable irrespective of the form of ground motion (see Figs. 16, 17, 18 and 19), while the other intervals, such as the first and second zero, are liable to be disturbed by other waves arriving later.

But closer examination shows that the magnitude of the first seismographic reading for the given t_0/T_0 is the larger the more gradual the beginning of the ground motion. The case of $\sigma(t)=e^{-ct^2}$ calculated dy Suzuki (which the writer was able to examine on the original diagams kindly loaned by the author) is the largest, the next being the case for $\sigma(t)=2\sin pt-\sin 2pt$, or $3\sin pt-\sin 3pt$, which begins with $\dot{\sigma}_0=\ddot{\sigma}_0=0$, while the case $\sigma(t)=te^{\alpha t}\sin pt$ calculated by Berlage and Kizima is intermediate between the above two cases and that for $\sigma(t)=\sin pt$ or the $e^{\beta t}\sin pt$ of J. Rybner[19]. Besides the improbability of the existence of an initial motion with finite velocity as believed in by some authorities, and the actual frequent appearance of the almost one-sided impulsion with the gradual beginning of an earthquake, the curves for $\sigma(t)=2\sin pt-\sin 2pt$ or $3\sin pt-\sin 3pt$ may be used in practice as the intermediate one. Yet still higher accuracy could be obtained if we could judge the ground motion more accurately by a comparison of the second maximum and other quantities by referring to the respective cases or by interpolation. Discrimination of the particular type of ground motion may be conveniently made by comparing the ratios of amplitudes of the recorded first with that of the second maxima. A still better way would be to install seismographs of various constants, like the acceleration- and velocity-seismometers.

Concluding Remarks.

In view of the frequent appearance of first impulsions of the shock type, the motion of ordinary seismographs as well as those of the Galitzin type caused by ground motion $\sigma=3\sin pt-\sin 3pt$ were theoretically calculated. Then as to actual application, the fair ac-

19) J. RYBNER, *Gerlands Bertr. z. Geophys.*, **31** (1931), 259~281.

curacy of this assumption though applied to other forms of ground motions was examined, and a simple method of reducing the first impulsion of earthquake with tolerable accuracy was pointed out. That is by reducing by means of magnification of the first maximum for the ground motion $2\sin pt - \sin 2pt$ or $3\sin pt - \sin 3pt$, taking the time of first zero from the beginning measured in fractions of the proper period of the pendulum as parameter.

In conclusion the writer offers his heartiest thanks to Prof. Ishimoto for his kind encouragement and to Dr. C. Tsuboi for the generous loan of his results of calculation. Thanks are due also to R. Satô and others who assisted him in the numerical computations. He also wish to express his thanks to the Council of the Foundation for the Promotion of Scientific and Industrial Research of Japan, whose aid greatly assisted the preparation of this paper.

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29. 地震計初動の理論的及實驗的研究並びに地震初動の定量的研究

（共の1）衝動性地動による各種地震計の運動及び地震初動を求むる簡便法に就て

地震研究所　河　角　廣

地震初動の定量的研究の國際的協力に關する日本學術研究會議地震分科會よりの萬國測地學地球物理學協會の國際會議への提案に關聯して，筆者は實際地震の際に屢々觀測される衝擊性地動に對する各種地震計の運動を計算し，これと他の地動に對する場合と比較し，地震記象上に於ける初動の倍率 $\mathfrak{B}_{1m}$ を t_0/T_0（t_0 は動き始めから最初の零點までの時間，T_0 は地震計の週期）の函數と見る時は $\mathfrak{B}_{1m}(t_0/T_0)$ 曲線はかなり種々の地動に對し實用上差支へない程度しか各の間で違がない事を知つた．然も坪井氏の計算した地動 $\sigma=2\sin pt-\sin 2pt$ 或は本論文に用ひた $\sigma=3\sin pt-\sin 3pt$ に對する $\mathfrak{B}_{1m}(t_0/T_0)$ 曲線は殆んど一致するものであるが今迄考へられて居る地動の形に對する倍率曲線 $\mathfrak{B}_{1m}(t_0/T_0)$ のほぼ中庸の値を示す故，兩者の中何れかを用ふれば現在必要とする程度に於て十分な程度に地震初動の大きさを知る事が出來る．

尙本研究に際し計算の勞を分たれた佐藤柳造氏及び地震學科學生，九里偷一，山村常正，服部保正，早川正己，福永三郎，木村耕三，佐藤光之助の諸氏に厚く感謝の意を表し，不斷の激勵を賜つた石本先生に深謝すると共に，本研究に對する補助に對し日本學術振興會に感謝する次第である．
