30. *Theoretical and Experimental Study of Initial Motion of Seismographs and the Quantitative Study of First Impulsion of Earthquake.*

Part II.—An Experimental Study of the Initial Motion of Seismographs caused by Motions of Short Duration.

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A proposal was made by the Subcommittee on Seismology of the National Research Council of Japan to the forthcoming Congress of the International Union of Geodesy and Geophysics at Edinburgh in view of the unofficial circular letter of Prof. M. Ishimoto addressed to seismological stations throughout the world for the international co-operation with respect to quantitative studies of the initial motion of earthquakes. With the object of supporting the idea, Prof. Ishimoto advised us to study experimentally the initial motion of seismographs. At the same time one[1] of the authors showed that simple method, first suggested by Dr. Tsuboi[2], of reducing the first ground impulsion by observing the magnitude of the initial deflection and time of first zero from the beginning of motion on seismograms, may be applied with tolerable accuracy regardless of the form of the ground motion. This conclusion was derived by comparing the results of theoretical calculations for some types of ground motion of regular mathematical form. But the result of our present shaking table experiments proved the applicability of the above simple method to cases of somewhat irregular motions imparted to the shaking table by the unaided hand irrespective of the type of external motion. It also proved the applicability of the usual equation of motion

$$\ddot{a} + 2\varepsilon\dot{a} + n^2 a = -V(\ddot{\sigma} - gi) \tag{1}$$

either in the statical or dynamical state including the beginning of motion.

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1) H. KAWASUMI, Part I of the present study, *Bull. Earthq. Res. Inst.*, **14** (1936), 319～338.

2) C. TSUBOI, *Bull. Earthq. Res. Inst.*, **12** (1934), 426～445.

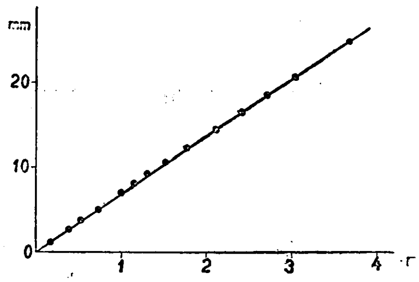
The seismographs used were an acceleration-seismograph designed by Prof. Ishimoto and the velocity- and displacement-seismographs designed by Mr. T. Hagiwara. All the seismographs are of the inverted pendulum type with air damper and mechanical recording, the construction details and the results of applications of which as well as comparative studies of them have already been reported by the respective designers on a number of occasions.[3]

1. Standardisation of Instruments, I.

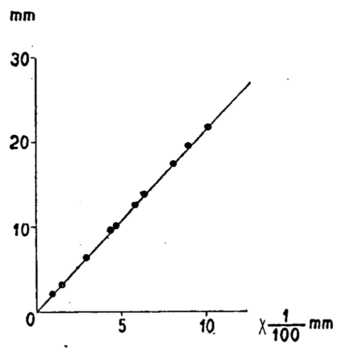
As it behove us to know all the instrumental constants exactly in each experiment, special calibration were made as far as possible.

(i) *Geometrical Magnifications.*

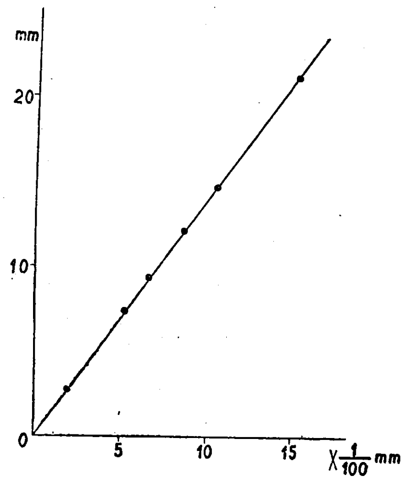
Geometrical magnification V_G as defined by Prof. Ishimoto [4] is the



a. Displacement-seismograph.



c. Acceleration-seismograph.



b. Velocity-seismograph.

Fig. 1. Determination of geometrical magnification.
Ordinate: Deflection of indicator on the record.
Abscissa: Displacement of the pendulum bob.

3) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **9** (1931), 316～332, 473～484; **10** (1932), 171～187; **12** (1934), 234～248; **13** (1935), 592～607; **14** (1936) 240～247. T. HAGIWARA, *Bull. Earthq. Res. Inst.*, **12** (1934), 776～787; **13** (1935), 138～145.

4) See M. Ishimoto, *loc. cit.* 3) and "Vibration Experiments and their Measurements", (Japanese.), *Kyôritusya Experimental Engineering Tracts*, Tokyo (1934).

magnification on the record of the displacement of the centre of gravity of the pendulum bob, and differs from the dynamical magnification for the infinitely rapid vibration V.

The difference between V_G and V increases with magnification, the number of magnifying levers, and their moments of inertia.

V_G was obtained by comparing the deflection of the indicator and the displacement of the centre of gravity of the bob as measured by means of a dial guage. (See Fig. 1, a, b, and c.)

From these diagrams we have, $V_G=$(a) 6·99 for the displacement-seismograph, which however was reduced in all the later experiments to 6/16·7 of this value, that is, 2·51. The values for the velocity- and acceleration seismographs are (b) 139 and (c) 214 respectively. A fairly linear relation between the deflection of the indicator and the displacement of the bob is noticable.

(ii) *Proper Periods of Free Oscillations and Soild Frictions.*

For determining these quantities the damping was minimised, and the free oscillations were recorded on a quick revolving drum by synchronous motor (9·3 mm per sec). The periods thus determined were (a) 6·87, (b) 0·502 and (c) 0·120 sec for the displacement-, velocity- and acceleration-seismogaphs respectively. And the width of solid friction ρ were obtained from the relation[5]

$$w_{k+1}=vw_k+\frac{\rho}{2(1+v)}, \tag{2}$$

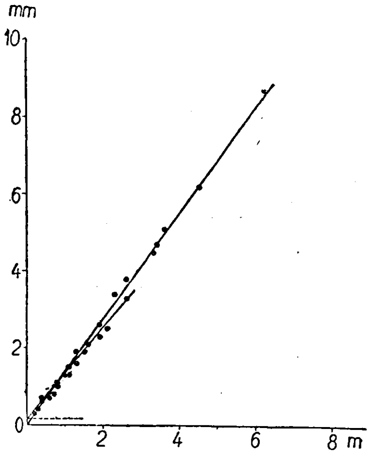
where w_k is the k-th double amplitude and v is the damping ratio. By the graphical method (Fig. 2, a, b, c), we obtaine (a) 0·02, (b) 0·04 and (c) 0·04 mm as being at the most for the displacement-, velocity-, and acceleration-seismographs respectively. These small quantities are altogether negligible even in the present study. As will also be seen from the figures the damping ratios v are (a) 1·3, (b) 1·06 and (c) 1·02, so that the period above obtained may be adopted as T_0 without any corrections.

(iii) *Dynamical Magnification and Sensibility as determined from Statical Inclination Experiments.*

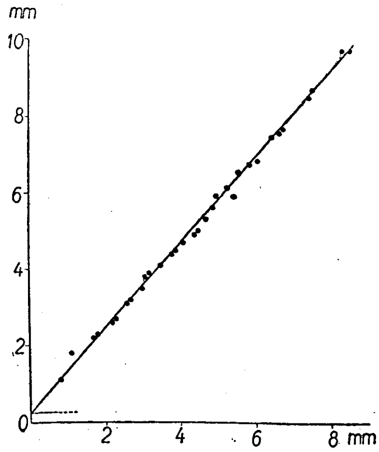
The equation of motion of seismograph (1) reduces for the present statical case to

$$n^2a=Vgi,$$

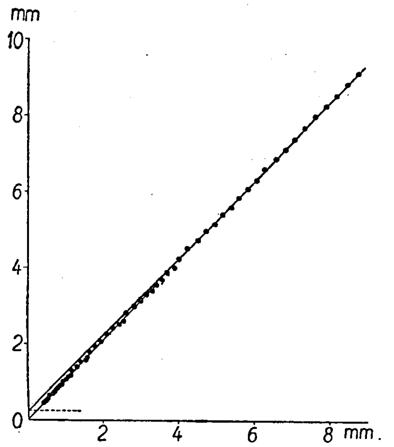
5) B. GALITZIN, *Comptes Rendus, Comm. Sis. Perm.*, St., Petersburg, 5 (1912), 35~83, or Vorlesungen über Seismometrie, (1914), 491.



a. Displacement-seismograph.



b. Velocity-seismograph.



c. Acceleration-seismograph.

Fig. 2. Determination of solid friction ρ.
Ordinate: w_k,
Abscissa: w_{k+1}.

from which we have

$$V=\frac{n^2}{g}\frac{a}{i}=\frac{n^2}{g}\frac{da}{di}, \tag{3}$$

and we also have as the sensibility of the acceleration seismograph

$$S_c=\frac{V}{n^2}=\frac{a}{gi}=\frac{1}{g}\frac{da}{di}. \tag{4}$$

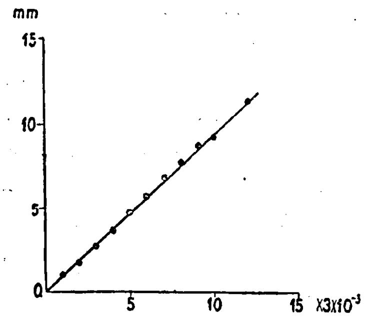
The values of $\frac{da}{di}$ were determined on an experimental platform whose inclination was measured by telescope and scale, or on the platform designed for an Ishimoto tiltometer, in which minute change in level may be made by means of a tangent screw.

$$\frac{da}{di}=\text{(a) } 3020, \text{ (b) } 624, \text{ (c) } 55{\cdot}6 \text{ in cm/radian,}$$

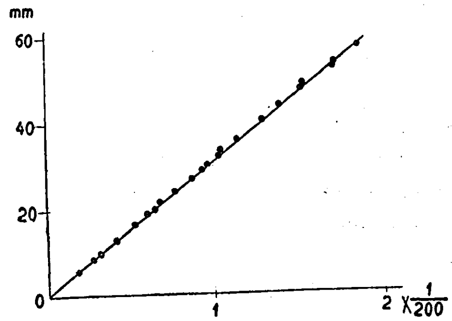
whence we have

$$V=\text{(a) } 2{\cdot}58, \quad \text{(b) } 100, \quad \text{(c) } 156,$$

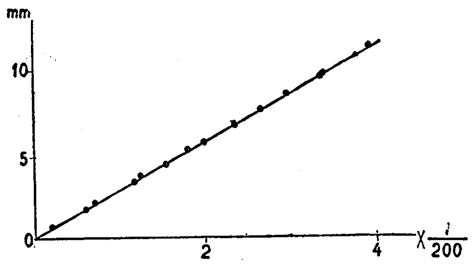
respectively, while the sensibility of the acceleration-seismograph works out to $S_c=0{\cdot}57$ mm/gal.



a. Displacement-seismograph.



b. Velocity seismograph.



ç. Acceleration-seismograph.

Fig. 3. Determination of *da/di*.
Ordinate: *a*, deflection of indicator on the record.
Abscissa: *i*, inclination of experimental table in radian.

2. Standardisation of the Instruments, II.

Althogh we have already determined some almost invariable instrumental constants, we have still to determine the damping factors as well as the dynamical magnifications under actual working conditions, since the former is liable to change with atmospheric conditions, etc., while the latter is subjected to certain factors that differ according as to whether it is in the dynamical state or in the statical, for example the effect of bending of the magnifying lever.

We therefore determined, as far as possible dynamically, the instrumental constants in the working state in each shaking table experiment. This was conveniently done by using the shaking table constructed by Prof. Ishimoto, full details of which were described by him in his paper[6] "Dètermination de la limite perceptible des secousses" in the Bulletin of this Institute.

(i) *Shaking Table.*

The shaking table, which is made of wood of dimensions 200 cm × 93 cm, is suspended by four ropes from hooks in the ceiling. The ropes are crossed obliquely so that the motion of the table is confined to one direction only. The free vibration of the table is heavily damped by dampers of metallic plates immersed in a reservoirs of viscous oil. Periodic vibration of the table is attained by means of power generated by a 1/2 horse power motor, which drives an eccentric rotor with long rods by which the horizontal component of the motion is communicated to the shaking table through a junction by means of a brass helical spring. The period is easily changed from about 0·1 to 10 sec by regulating the electric current and the gear ratios between the motor and eccentric rotor, while the amplitude is easily altered by changing the eccentricities of the rotors, or by chaing the leverage at the connecting device of the horizontal rods as well as altering the stiffness of the helical spring.

The foregoing three seismographs were mounted on the shaking table and their motions recorded on a single drum, which also recorded the motion of the table with magnification 2·38. The drum was rotated by a synchronous motor, the time being marked by lifting the indicating pointer by means of electro-magnets.

The experimental arrangements will be seen from the photographs, Fig. 4 and 5, Plate XXII. The instrumental constants were determined by observing resonance curves, that is the actual magnification curves for harmonic motions in the stationary state obtained by comparing the recorded table motions with the seismographic records. By means of the formula

$$\mathfrak{B}=\frac{Vp^2}{\sqrt{(n^2-p^2)^2+4n^2\varepsilon^2}}=\frac{V}{\sqrt{(1-u^2)^2+4h^2u^2}}, \qquad u=\frac{n}{p}, \qquad (5)$$

we can generally determine V, $T_0=\frac{2\pi}{n}$, and ε or $h=\frac{\varepsilon}{n}$, from observations of $\mathfrak{B}$ and p.

6) M. ISHIMOTO et M. OTUKA, *Bull. Earthq, Res. Inst.*, **11** (1933), 113~121.

(ii) *Dynamical Magnification for Infinitely Rapid Vibration V.*

The value of V was estimated by the magnifications for motions with shortest periods[7] in the case of the displacement- and velocity-seismographs, while that in the case of acceleration-seismographs was determined by examining the sensibility for long periods, which is actually constant for motions with periods above 0·2 sec, namely,

$$V= \quad \text{a)} \quad 1{\cdot}90, \qquad \text{b)} \quad 110,$$

$$S_c=\frac{a}{\sigma_m p^2}=0{\cdot}60 \text{ mm/gal}, \tag{6}$$

where σ_m and p are amplitude and frequency of the table motin $\sigma=\sigma_m \sin pt$. We can therefore have the dynamical magnification by the formula (4). We are not certain whether the differences in the S_c or in the V as already determined from the statical inclination experiment from the one here obtained is due to the bending of the arms or to other dynamical causes or to a change in the proper period of the seismograph, but if we were to attribute it to the latter cause, the proper period for the acceleration-seismograph would work out to $T_0=0{\cdot}117$ sec, which, compared with the former determination $T_0=0{\cdot}120$ sec, is within the range of observational accuracy.

(iii) *Damping Constant and Proper Period.*

From

$$\left(\frac{\sigma_m}{a_m}\right)^2\left(\frac{V}{\mathfrak{V}}\right)^2=(1-u^2)^2+4h^2u^2=U^2 \tag{7}$$

we determined by the method of least squares the proper period T_0 and the damping constant h by the formula

$$U^2-U_0^2=\frac{\partial U_0^2}{\partial T_0}\delta T_0+\frac{\partial U_0^2}{\partial h_0}\delta h_0 \tag{8}$$

for the displacement-seismograph, but for the velocity- and acceleration-seismographs we determind h only, because our observations of $\frac{\sigma_m}{a_m}$ for short period vibrations were not sufficient. The results as determined from tests at each initial motion experiments were

7) Since quick vibrations, such as would cause resonance of the indicator arms were of course avoided, magnification for the accelaration-seismograph could not be determined in this way.

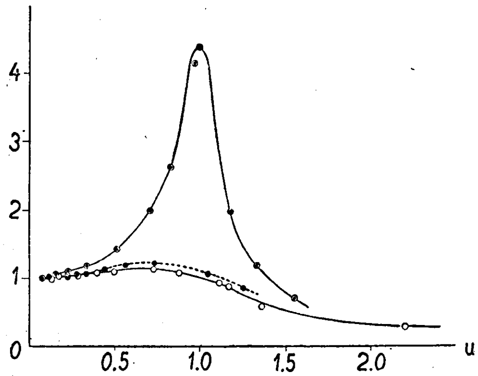


Fig. 6 a. Resonance curves of displacement-seismograph.

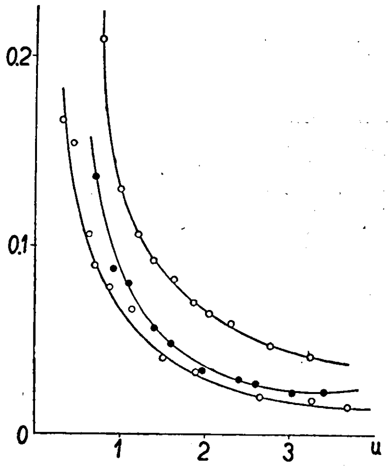


Fig. 6 b. Resonance curves of velocity-seismograph.

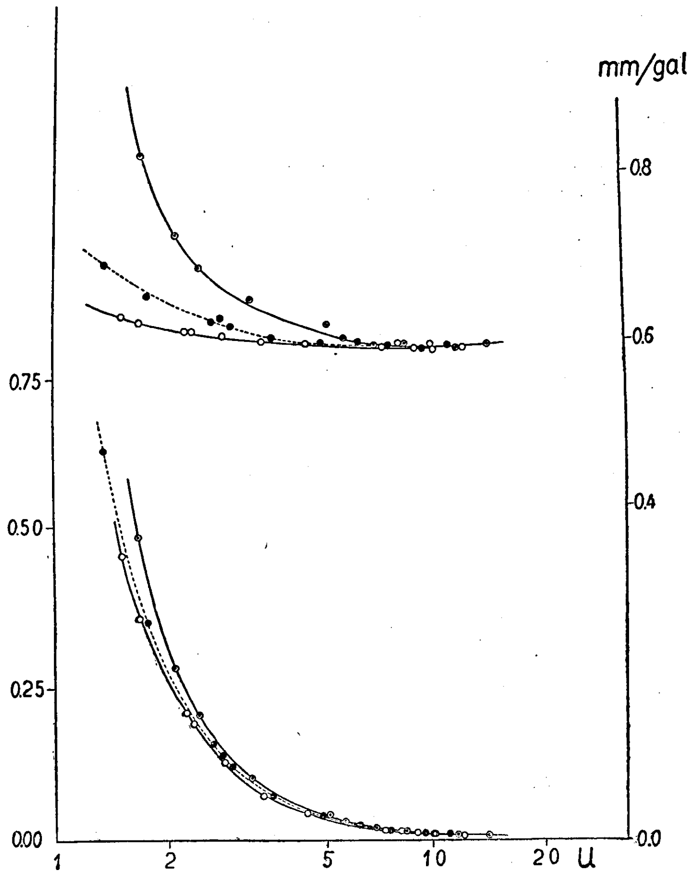


Fig. 6 c. Resonance curves (below) and sensibility curves (above) of acceleration-seismograph.

(a) (1) $T_0=6{\cdot}56$ sec, $h=0{\cdot}513$,
(2) $T_0=6{\cdot}57$, $h=0{\cdot}134$,
(3) $T_0=(6{\cdot}22)$, $h=0{\cdot}490$.
(b) (1) $h=7{\cdot}99$,
(2) $h=6{\cdot}25$,
(3) $h=3{\cdot}69$.
(c) (1) $h=0{\cdot}555$,
(2) $h=0{\cdot}599$,
(3) $h=0{\cdot}301$.

The mean sensibility of the velocity-seismograph was also determined, namely,

$$S_v=\frac{a_m}{\sigma_m p}=12{\cdot}7\,\text{cm/kin}. \qquad (9)$$

Having now obtained all the necessary instrumental constants, we shall summaries them here in tabular form.

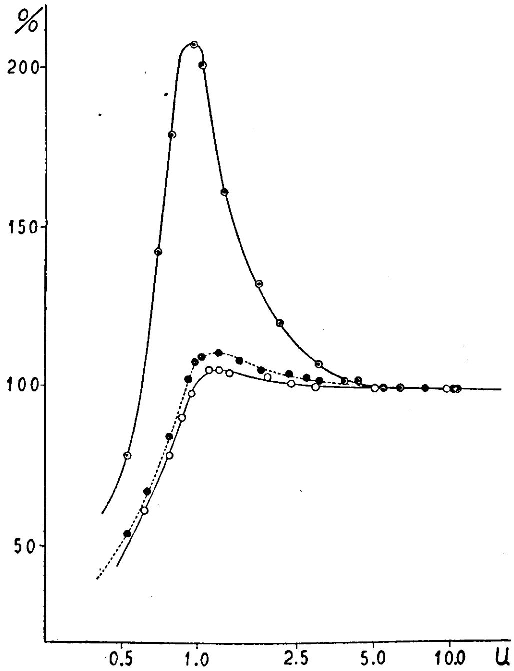


Fig. 7. Sensibility curves of acceleration-seismographs.

The sensibility variation with period for the acceleration-seismograph was also examined by applying a periodic force by means of an elastic rubber string connected to a small hook at the bob of the pendulum at one end and pulled periodically by an eccentric

Table I.
Instrumental constans for the seismographs used.

	Displacement-seismograph	Velocity-seismograph	Acceleration-seimograph
Weight of bob (kg)	6	15	15
V_G	2·51	139	214
V (stat.)	2·58	100	164
V (dyn.)	1·90	110	164
V (intial motion)	1·83	112	15
T_0 (sec)	6·565	0·502	0·118
h	0·513	7·99	0·555
h	0·134	6·25	0·599
h	0·490	3·69	0·301
ρ (mm)	0·02	0·04	0·04
S		12·7 cm/kin	0·60 mm/gal

rotor at the other end with constant amplitude and variable frequency. In this way the sensibility for short period was found (Fig. 7).

3. Transient Motion of Seismographs caused by Motions of Short Duration.

As to the experimental study of the initial motion of seismographs, which is our principal aim, the motions, for which the curve to be applied in reducing the magnitude of the first given impulsion have been prepared, are $\sigma = \sin pt$ and $2 \sin pt - \sin 2pt$, both due to Dr. C. Tsuboi, and $3 \sin pt - \sin 3pt$ for the ordinary seismographs and all the three cases for the Galitzin seismographs by one of the writers. But, in addition to the fact that motion of type e^{-ct^2} or $3 \sin pt - \sin 3pt$ is most frequently observed in actual earthquakes, it was found that the magnification, considered as a function of the time of

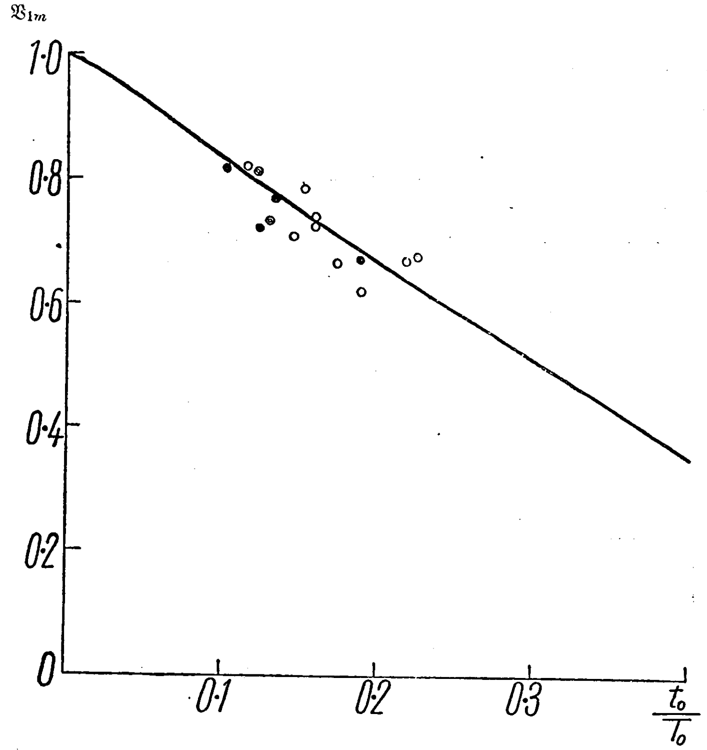


Fig. 13. Comparison of the magnification for the first maximum experimentally observed (h=0·51 and 0·49 inclusive) with the theoretical curve for h=0·5.
○ for table motion of nearly one-sided impulsion,
● for single vibration,
◎ for one and a half vibration.

first zero from the beginning measured in fractions of the proper period of seismograph for $\sigma=2\sin pt-\sin 2pt$ or $3\sin pt-\sin 3pt$, which is nearly equal to each other, may be applied as an intermediate magnification for reducing the various forms of ground motion with tolerable accuracy. We shall therefore verify this point by experiments with the shaking table, using somewhat irregular motions generated by the anaided hands and of types resembling the mathematical forms considered theoretically, as well as other forms not so considered. A part of the experimental results are reproduced in the annexed plates. The motions of the shaking table and of the displacement-, velocity- and acceleration-seismographs are arranged from the top in each photograph. The instrumental constants determined at the same time are given below the photographs. (Fig. 8, 9, 10, 11, 12, in Plate XXIII～XXVI.)

The measured $\mathfrak{B}_{1m}$, $\mathfrak{B}_{2m}$, $\frac{t_0}{T_0}$, and $\frac{\bar{a}_{1m}}{a_{2m}}$ for the displacement-seismograph are given in the follwing table and is compared in Fig. 13 with the theoretical curve. It will be seen that the theoretical curve is applicable with fair accuracy to motions attained experimentally.

Table II.

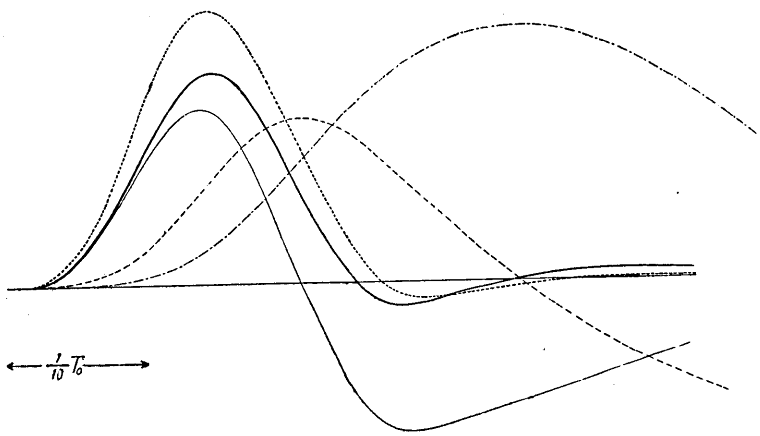
The measured $\mathfrak{B}_{1m}$, $\mathfrak{B}_{2m}$, t_0/T_0 and a_{1m}/a_{2m}. (Displacement-seismograph), for table motion of nearly one-sided impulsion (○), of single vibration ● and of one and a half vibration (⊙).

h	$\mathfrak{B}_{1m}$	$\mathfrak{B}_{2m}$	t_0/T_0	type of σ
0·490	0·77	1·09	0·13	●
	0·68	0·57	0·23	○
	0·74	0·49	0·16	○
0·134	0·83	0·84	0·25	●
	0·88	0.44	0.15	○
	0·62	0.38	0.19	○
0·513	0·82	1·08	0·10	●
	0·73	0·48	0·16	○
	0·67	0·60	0·22	○
	0·81	1·36	0.12	⊙
	0·53	0.24	0·17	○
	0·58	0·55	0·12	○
	0·54	0.85	0·19	●
	0·57	0·41	0·15	○
	0·66	0·33	0·12	○
	0·59	0·96	0·13	⊙

We shall next examine the theoretical basis on which some authors may ask whether the simple equation of motion of the seismograph is applicable or not to such a transient state as the beginning of a motion. On the other hand we shall at the same time examine the applicability of Poincaré-Lippmann's[8] method of integrating term by term of

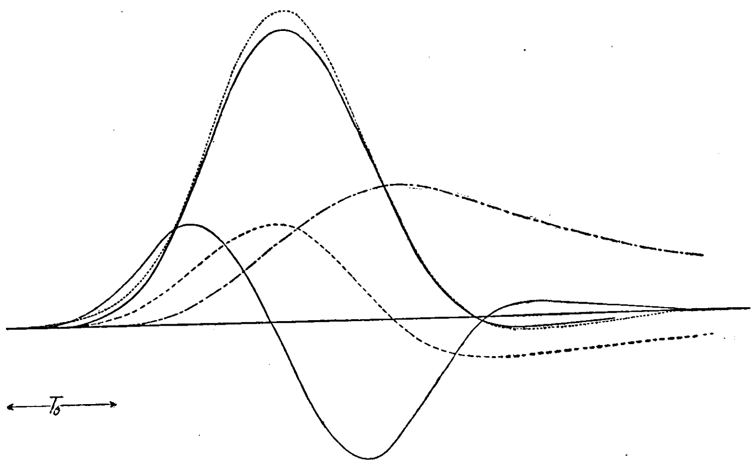
$$a+2\varepsilon\int_0^t adt+n^2\int_0^t dt\int_0^t adt=-V\sigma. \tag{10}$$

Readings of a were made from enlarged photographs, and corrections applied for the arcual movements of the indicator. Althogh the amplitude correction to reduce $a=L\sin\theta$ or $L\tan\theta$ to $L\theta$ was not necessary in the present cases, the correction for time $L(1\text{-}\cos\theta)$, where L is the length of the last indicator arm, was effective even in amplitudes of about 1 cm in the casc of $L=13$ or 15 cm. The motions were plotted on section paper, and the first and second integrals obtained by means of Coradi's intergraph. For the kind loan of this instrument the writers take this opportunity of offering their thanks to Prof. Takahasi of the Institute of Naval Architecture of this University. The resulting displacements thus worked out after applying whenever necessary little corrections for the zero line assuming the form $\alpha+\beta t+\gamma t^2$ are compared with the table motion in Fig. 14, (a), (b), (c). To prevent confusion, we have indicated the resulting $V\sigma$ curve

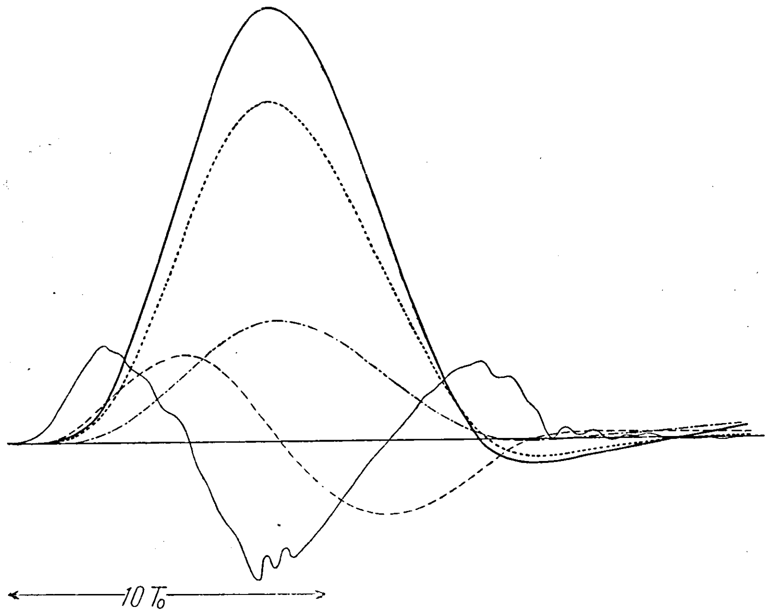


a. Displacement-seismograph.

8) F. FOUQÉ, *Trembrements de terre*, (1889), 46~47.
G. LIPPMANN, *Comptes Rendus*, **110** (1890), 440~446.



b. Velocity-seismograph.



c. Acceleration-seismograph.

Fig. 14. Reduction of table motion by Poincaré-Lippmann's method. Thin full line: seismographic record a, broken line: $\int a dt$, chain line: $\int dt \int a dt$, thick full line: $V\sigma$ and dotted line: table motion σ.

on a smaller scale instead of σ itself. This proves the correctness of the usual equation of motion, as well as the accuracy of the instrumental constants n and ε that we determined as mentioned above. The magnifications V worked out in this way, are (a) 1·84, (b) 113 and

(c) 153 respectively, agreeing well with the former dynamical determination within the limits of observational accuracy.

The reader will have already noticed the resemblance of records of the velocity- or acceleration-seimograph to the actual velocity or acceleration of the motion of the shaking table σ. This is also confirmed by the fact that the first integral of a due to the velocity seismograph and the double integral of the same due to the acceleration-seismograph, which are shown in Fig. 14 b and c, closely resemble the motion of the shaking table. A quantitative verification will be made from the relative magnitude of the factor to be multiplied to the first integral of the a curve for the velocity-seismograph and that to the double integral for the acceleration-seismograph in order to obtain $V\sigma$ by formula (10), i. e., the factors to the first, second, and third terms of (10) are

(b) 1, 115·68, 52·41,
(c) 1, 21·12, 361·90,

for the velocity- and acceleration-seismographs respectively. This fact has a very important practical application because a comparison of the records of the displacement-seismograph and the velocity- or acceleration-seismograph will readily indicate the form of ground motion, which is very convenient in reducing the magnitude of the first impulsion. In this sense, acceleration- and velocity-seismographs with still higher magnifications so as to enable the registration of distant earthquakes would be very convenient, although, unless carefully examined, the presence of waves of smaller period totally mask those of slower motions.

Conclusion.

An Ishimoto acceleration-seismograph and Hagiwara velocity- and displacement-seismographs were used for the shaking table experiment.

(i) The instrumental constants were determined as far as possible by statical as well as dynamical methods. The results are summarised in Table I.

(ii) The transient motion of seismographs caused by some types of somewhat irregular motions, resembling the forms already investigated theoretically as well as other types of motions, were studied experimentally, and the applicability of the simple method of reduction of the first ground impulsion described in Part I of the present paper was verified. The applicability of the usual equation of motion was

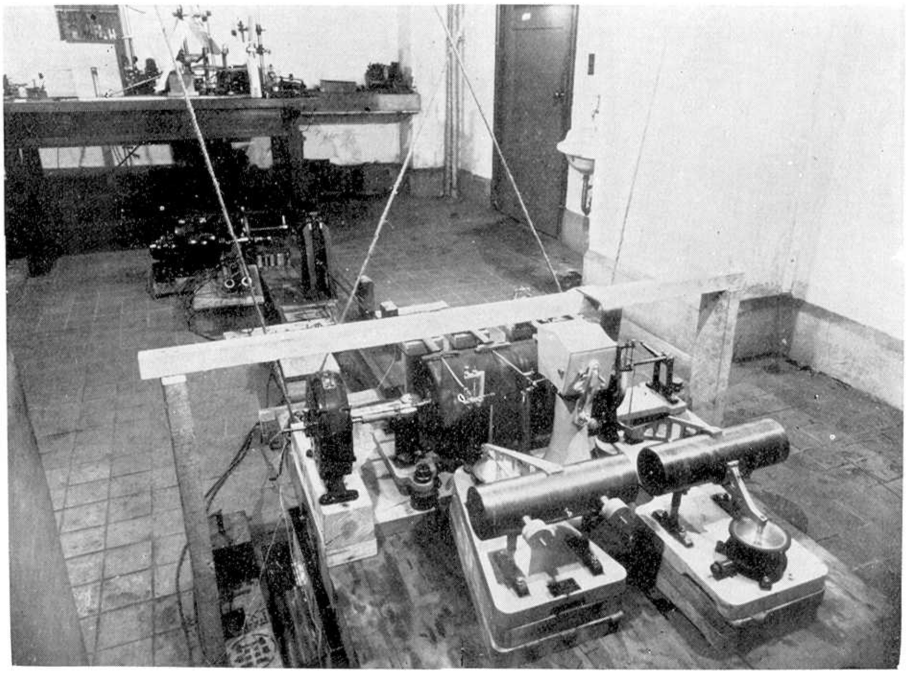


Fig. 4. Experimental arrangement of shaking table.

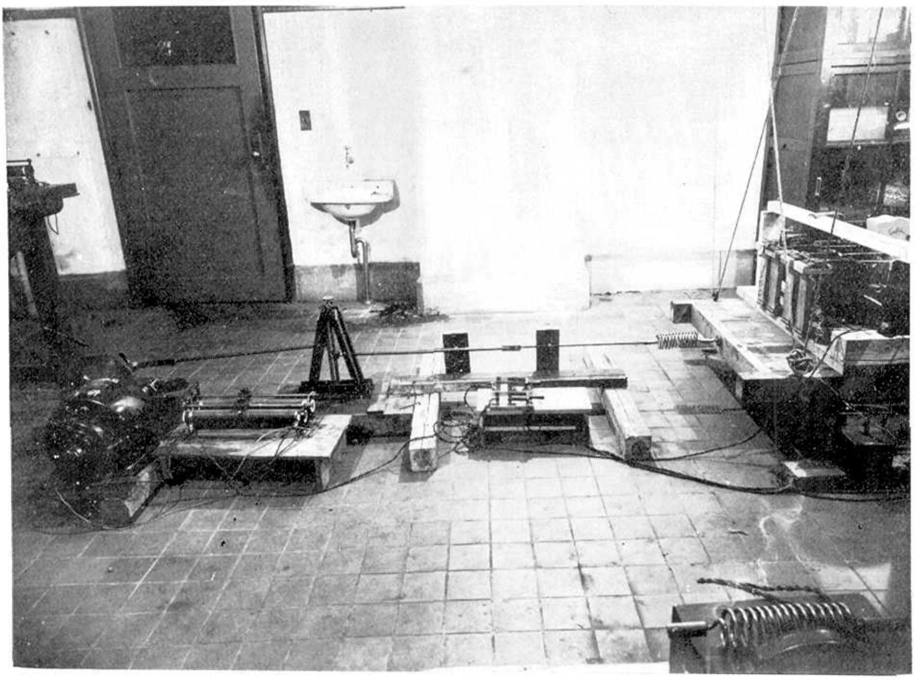


Fig. 5. Connection between motor and shaking table.

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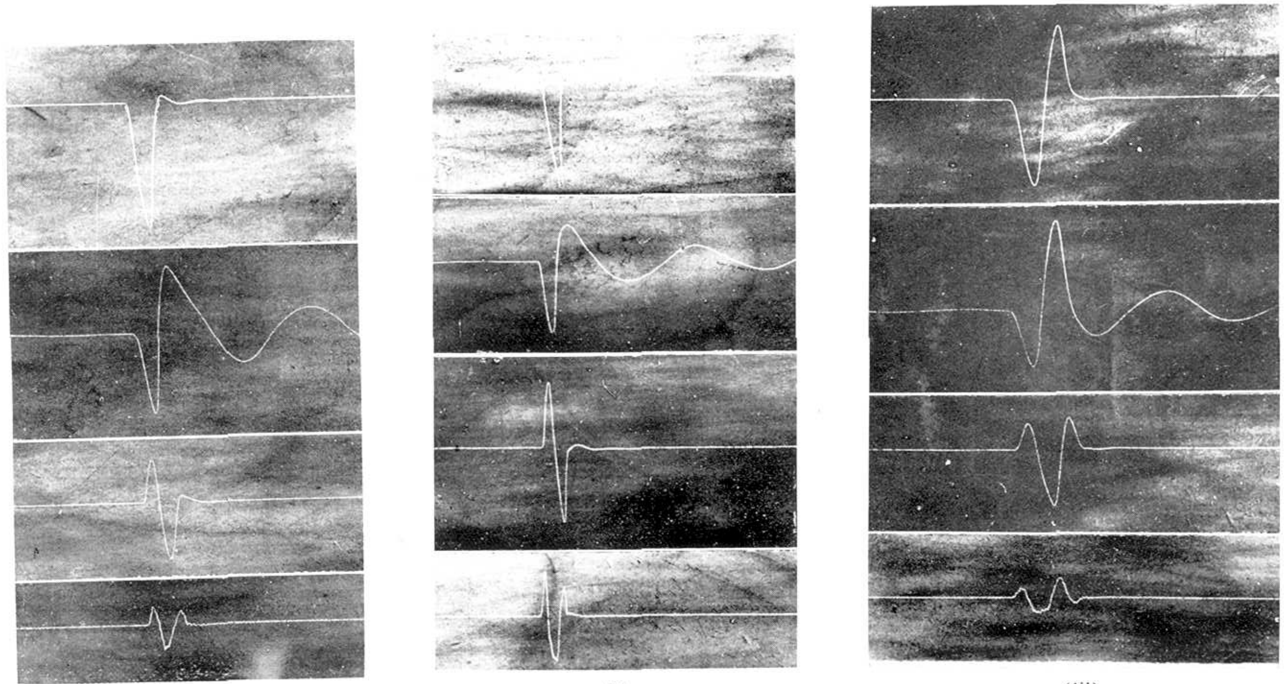

(i) (ii) (iii)

Fig. 8. Comparison of the motion of shaking table with those of displacement-, velocity- and acceleration-seismographs (from the top to below).
(a) $h=0{\cdot}134$, (b) $h=7{\cdot}99$, (c) $h=0{\cdot}555$.

[Bull. Earthq. Res. Inst., Vol. XIV, Pl. XXIV.]

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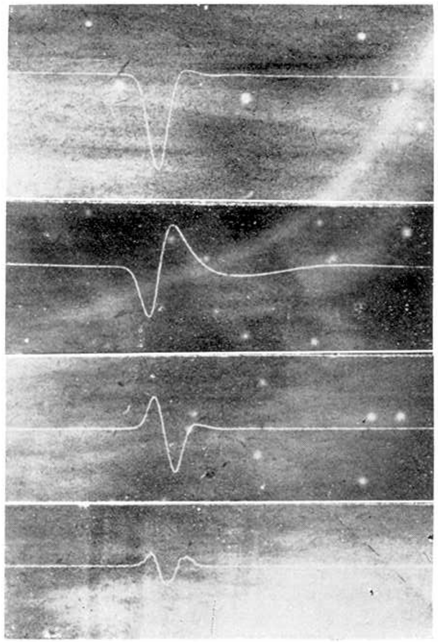
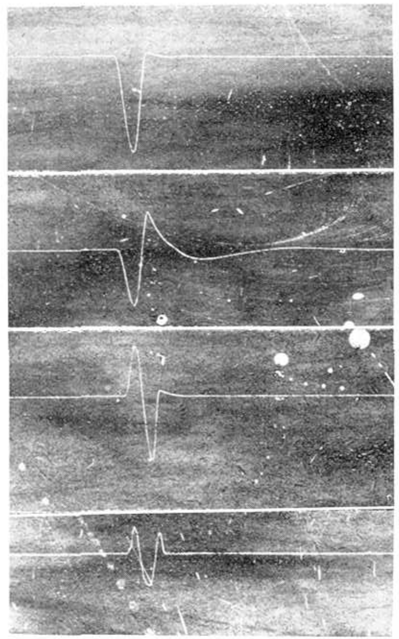


(i)



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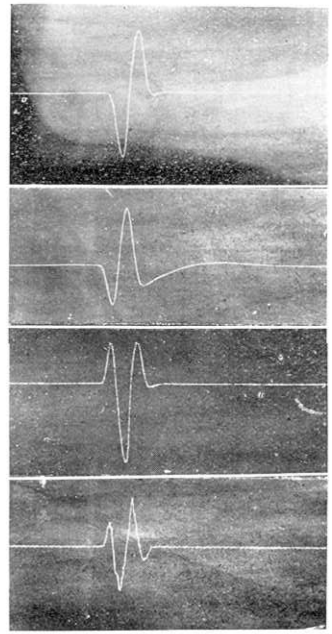
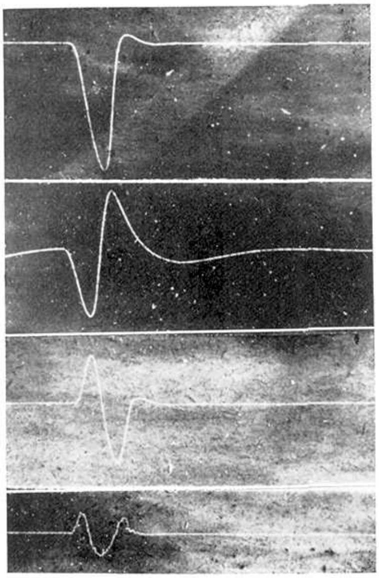


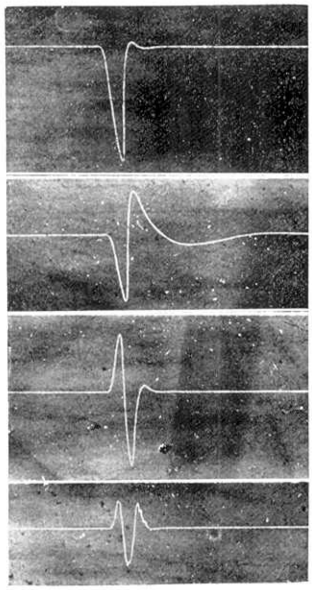
(iii)

Fig. 9.

(a) $h=0{\cdot}489$, (b) $h=7{\cdot}99$, (c) $h=0{\cdot}555$.

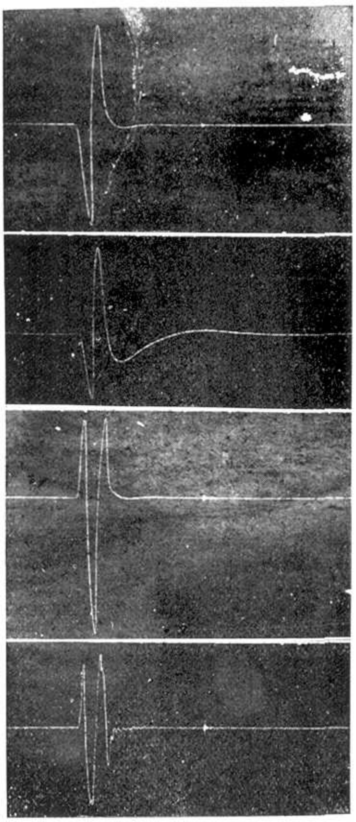


(i)

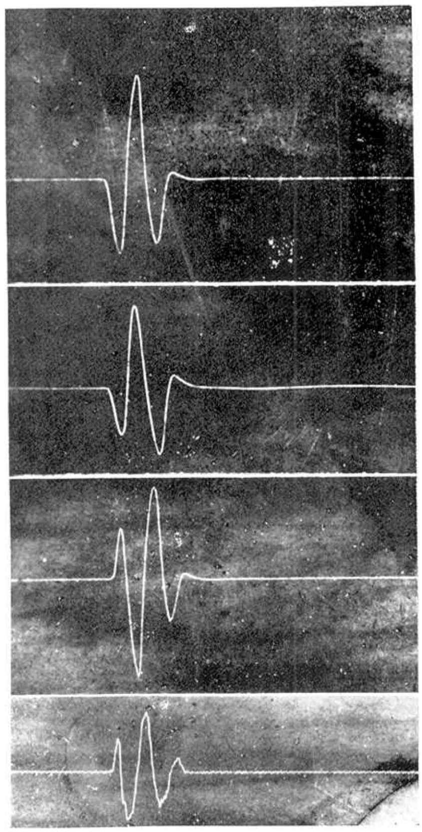


(ii)

（震研彙報 第十四號 圖版 河角・木下）



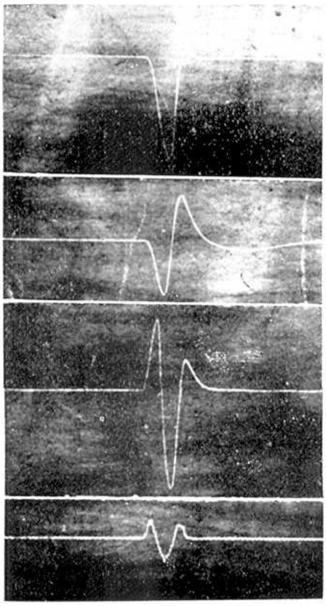
(iii)



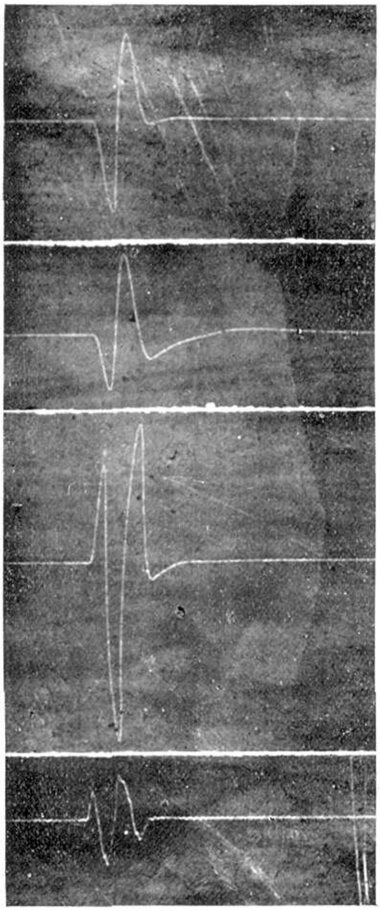
(iv)

Fig. 10.

(a) $h=0{\cdot}513$, (b) $h=7{\cdot}99$, (c) $h=0{\cdot}555$.



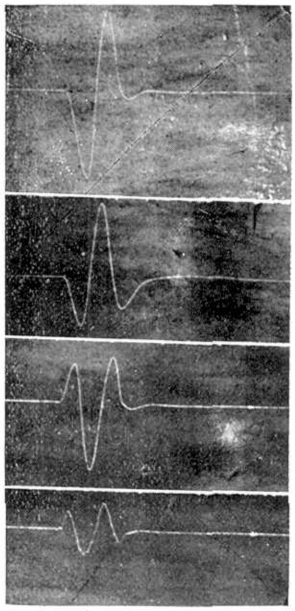
(i)



(ii)

Fig. 11.

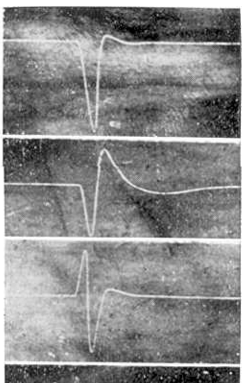
(a) $h=0{\cdot}513$, (b) $h=3{\cdot}69$, (c) $h=0{\cdot}301$.



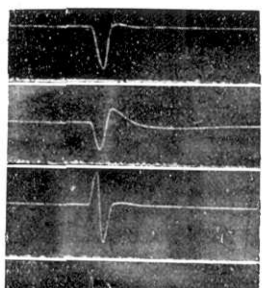
(i)

Fig. 12.

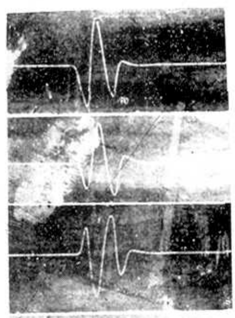
(震研彙報 第十四號 圖版 河角・木下)



(ii)



(iii)



(iv)

Fig. 12.

(a) $h=0{\cdot}513$, (b) $h=6{\cdot}25$, (c) $h=0{\cdot}599$.

also proved even for the start of motion of the pendulum with air damper so powerful that the damping constant h was 7·99. The accuracy of the method of integrating term by term due to Poincaré and Lippmann was also examined.

In conclusion, the writers wish to acknowledge their heartiest thanks to Prof. Ishimoto for his kind advice and encouragement throughout the course of our present study. They also wish to express their thanks to the Council of the Foundation for the Promotion of Scientific and Industrial Research of Japan, whose aid greatly assisted the progress of our work.

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30. 地震計初動の理論的及び實驗的研究並びに地震初動の定量的研究

（其の2）地震計初動の實驗的研究

地震研究所 {河角廣, 木下潤一}

本論文第1部に於て得たる所を確める爲に振動臺を用ひて地震計の初動實驗を行つた．此れに用ひた地震計は石本式加速度地震計，萩原式速度及び變位地震計である．靜力學的，動力學的方法により出來るだけ器械常數を測定し，各實驗毎に動力學的檢定を行つた．其の結果理論的に研究されて居る樣な形に近い運動を手にて振動臺に與へた場合にも，又其の他の運動をさせた場合にも第1部に於て得た第1動に對する倍率曲線 $\mathfrak{V}_{1m}\ (t_0/T_0)$ が相當な精度に於て利用出來る事を確め，更に項別積分の方法により各種地震計よりその運動を求め極めて滿足なる結果が得られた．かくて地震計の運動方程式

$$\ddot{a}+2\varepsilon\dot{a}+n^2a=-V(\ddot{\sigma}-gi)$$

が靜力學的にも動力學的にも，ここに初動の場合にもあてはまる事を知つた．

尙本研究に對する不斷の教示，激勵に對し石本先生に深甚の感謝の意を表し，合はせて本研究に對し補助を與へられた日本學術振興會に對し感謝する次第である．
