32. *Dissipation Waves Accompanying Forced Seiches in a Bay.*

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1. In the preceding papers[1),2)] we studied the problem of the decay of seiches in an epicontinental sea as well as in straits. Although the same problem in the case of a bay is of more practical importance, it was not possible for us to solve that problem until now owing to certain difficulties in mathematical calculation. As however the determination of the decay factor of seiches still remains a difficulty, we shall now merely ascertain the nature of the waves dissipated from a bay under the condition of forced seiches.

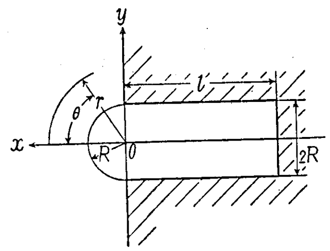

Fig. 1.

2. We shall suppose for simplicity that the bay is of rectangular shape and uniform depth, its breadth, length, and depth being $2R$, l, ξ respectively. The outer sea is also assumed to be of uniform depth ξ, while we shall take the axis of x directed outward with its origin at the mouth of the bay.

The incident waves (u_1, w_1) and the reflected waves (u_2, w_2) in the outer sea are

$$\left.\begin{aligned} u_1 &= ie^{i(\sigma t+fx)}, & w_1 &= -f\xi e^{i(\sigma t+fx)}, \\ u_2 &= -iAe^{i(\sigma t-fx)}, & w_2 &= -f\xi Ae^{i(\sigma t-fx)}, \end{aligned}\right\} \qquad (1)$$

where u, w signify the horizontal displacements and the surface elevations of waves, and $2\pi/f$ is the wave length corresponding to the period $2\pi/\sigma$.

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1) K. SEZAWA, "Growth and Decay of Seiches in an Epicontinental Sea," *Bull. Earthq. Res. Inst.*, **13** (1935), 476~483.

2) K. SEZAWA and K. KANAI, "Damped Free Osillation and Amplitudes in Resonance, with Special Reference to Decay of Seiches in Straits," *Bull. Earthq. Res. Inst.*, **14** (1936), 1~9.

The boundary condition at $x=0$ corresponding to the coast line is $u_1+u_2=0$, so that $A=1$. Thus we get

$$\left.\begin{aligned} u_1+u_2&=-2e^{i\sigma t}\sin fx, \\ w_1+w_2&=-2f\xi e^{i\sigma t}\cos fx. \end{aligned}\right\} \tag{1'}$$

This condition is approximate in the sense that the general reflection of the disturbance at the coast line is not affected by the existence of the bay.

The forms of the forced seiches in the bay are written $(u_1'+u_2', w_1'+w_2')$, where

$$\left.\begin{aligned} u_1'&=iCe^{i(\sigma t+fx)}, & w_1'&=-f\xi Ce^{i(\sigma t+fx)}, \\ u_2'&=-iBe^{i(\sigma t-fx)}, & w_2'&=-f\xi Be^{i(\sigma t-fx)}. \end{aligned}\right\} \tag{2}$$

At the end of the bay, namely at $x=-l$, we have $u_1'+u_2'=0$, so that

$$C=Be^{2ifl}, \tag{3}$$

the expressions for forced seiches then being

$$\left.\begin{aligned} u_1'+u_2'&=-2Be^{ifl}\sin f(l+x)e^{i\sigma t}, \\ w_1'+w_2'&=-2f\xi Be^{ifl}\cos f(l+x)e^{i\sigma t}. \end{aligned}\right\} \tag{4}$$

The expressions for the dissipation waves in the outer sea, which are radiated from the mouth of the bay, may be put in the forms

$$\left.\begin{aligned} U_2&=-\sum_{n=0}^{\infty}a_n\frac{\partial H_n^{(2)}(fr)}{\partial(fr)}\cos n\theta e^{i\sigma t}, \\ W_2&=-f\xi\sum_{n=0}^{\infty}a_nH_n^{(2)}(fr)\cos n\theta e^{i\sigma t}, \\ V_2&=\frac{1}{fr}\sum_{n=0}^{\infty}a_nnH_n^{(2)}(fr)\sin n\theta e^{i\sigma t}. \end{aligned}\right\} \tag{5}$$

where U_2, W_2, V_2 are the radial, vertical, and transverse components of displacement.

We now write

$$\left.\begin{aligned} U_1&=(u_1+u_2)\cos\theta, & U'&=(u_1'+u_2')\cos\theta, \\ W_1&=w_1+w_2, & W'&=w_1'+w_2', \\ V_1&=(u_1+u_2)\sin\theta, & V'&=(u_1'+u_2')\sin\theta. \end{aligned}\right\} \tag{6}$$

Were the length of the bay relatively long compared with its breadth, the boundary conditions at the mouth of the bay would be put approximately

$$\left.\begin{aligned}&[W_1+W_2]_{r=R,\ y=y}=[W']_{x=0,\ y=y},\\&\left[\int_{\theta=-\pi/2}^{\theta=\pi/2}U_2Rd\theta\right]_{r=R}=\left[\int_{y=-R}^{y=R}U'dy\right]_{x=0}.\end{aligned}\right\}\tag{7}$$

In these conditions the conception that the vertical elevation of the water at $r=R$ is the same as that at $x=0$ corresponding to the same value of y, and that the instantaneous mean flux of water through the boundary $r=R$ is the same as that at the mouth $x=0$, is implied. It should be borne in mind that, while U_2 takes important part in the flux of water, V_2 hardly contributes to the same flux.

Substituting from (5), (6) in (7), we obtain

$$2\cos fx-B\{e^{-ifx}+e^{2ifl+ifx}\}+\sum_{n=0}^{\infty}a_nH_n^{(2)}(fr)\cos n\theta=0,$$

$$(x=0,\quad r=R)\tag{8}$$

$$B\left\{\sin 2f'l+i(1-\cos 2f'l)\right\}=\left[-\sum_{n=-}^{\infty}a_n\frac{\sin\frac{n\pi}{2}}{n}\frac{\partial H_n^{(2)}(fr)}{\partial(fr)}\right].\quad(r=R)\tag{9}$$

It is well known that

$$\left.\begin{aligned}&e^{ifx}=J_0(fr)+2iJ_1(fr)\cos\theta+\cdots+2i^nJ_n(fr)\cos n\theta+\cdots,\\&e^{-ifx}=J_0(fr)-2iJ_1(fr)\cos\theta+\cdots+(-1)^n2i^nJ_n(fr)\cos n\theta+\cdots.\end{aligned}\right\}\tag{10}$$

Thus (8) is replaced by[3)]

$$2e^{ifl}B\left\{J_0(fR)\cos fl+2\sum_{n=1}^{\infty}J_n(fR)\cos n\theta\cos\left(\frac{n\pi}{2}+fl\right)\right\}$$
$$-\sum_{n=0}^{\infty}a_nH_n^{(2)}(fR)\cos n\theta-2\left\{J_0(fR)+2\sum_{n=1}^{\infty}(-1)^nJ_{2n}(fR)\cos 2n\theta\right\}=0.\tag{8'}$$

Equating the coefficient of each $\cos m\theta$ in (8′) to zero, we obtain

3) We used a similar method of treatment in the problem of seismic waves. K. SEZAWA, "Scattering of Elastic Waves and Some Allied Problems," *Bull. Earthq. Res. Inst.*, **3** (1927), 19~41; *ditto*, "The Reflection of Elastic Waves generated from an Internal Point of a Sphere", *ibid.*, **4** (1928), 123~130; T. MATUZAWA also used such a treatment in his recent paper, "Über Schattenwellen und Kernwellen", *ibid.*, **13** (1935), 18~38.

$$\left.\begin{aligned} a_0 &= \frac{2J_0(fR)\,(Be^{ifl}\cos fl-1)}{H_o^{(2)}(fR)}, \\ a_{2n+1} &= \frac{-4(-1)^n Be^{ifl} J_{2n+1}(fR)\sin fl}{H_{2n+1}^{(2)}(fR)}, \\ a_{2n+2} &= \frac{-(-1)^n J_{2n+2}(fR)\,(4Be^{ifl}\cos fl-1)}{H_{2n+2}^{(2)}(fR)}. \end{aligned}\right\} \tag{11}$$

Substituting (11) in (8′) it is possible to determine B. When the values of B thus determined are again substituted in (11), we obtain the values of a_0, a_1, a_2, $\cdots$, a_{2n+1}, a_{2n+2}, $\cdots$. We have calculated two cases, namely (i) $fR=1$, $fl=3$; (ii) $fR=24$, $fl=72$.[4]

3. In the case $fR=1$, $fl=3$; B, C, a_0, $\cdots$, a_{2n+1}, $\cdots$ assume values such that

$$\left.\begin{aligned} &B=0{\cdot}950-i0{\cdot}1421, && C=0{\cdot}8728-i0{\cdot}4019, \\ &a_0=-0{\cdot}1135-i0{\cdot}1016, && a_1=0{\cdot}059-i0{\cdot}2597, \\ &a_2=0{\cdot}06268+i0{\cdot}1890, && a_3=0{\cdot}000515+i0{\cdot}001748, \\ &a_4=0{\cdot}0000812+i0{\cdot}0003466, && \cdots\cdots\cdots\cdots, \end{aligned}\right\} \tag{12}$$

so that the equations for the dissipated waves are obtained by substituting these constants in (5), whereas the expression of the forced seiches is as follows.

$$W'=-2f\xi e^{3i}(0{\cdot}950-i0{\cdot}1421)\cos 3\left(1+\frac{x}{l}\right)e^{i\sigma t}. \tag{13}$$

The distribution of $W_2/f\xi$ for $fr=40$ and the different values of θ is shown in Fig. 2. Since, from the nature of the problem, resonance of the seiches is defined by $fl=\pi/2$, $3\pi/2$, $\cdots$ and corresonance of the same seiches by $fl=\pi$, 2π, $\cdots$, it is possible to assume that the present condition of the problem, namely $\cos fl=-0{\cdot}99$, is very close to first corresonance.

The maximum vertical elevations of the seiches at $x=-l$ as well as at $x=0$ assume the values

$$W'_{x=-l}=1{\cdot}921\,f\xi, \qquad W'_{x=0}=1{\cdot}900\,f\xi \tag{14}$$

respectively.

4. In the case of $fR=24$, $fl=72$; B, C, a_0, $\cdots$, a_{2n+1}, $\cdots$ assume values such that

4) There are a number of bays in Japan that have the dimensional ratio of $l/R=3$. Turuga, Oohunato, Hutami, and Hakodate are the bays, that may be mentioned as examples of the present case.

$$\left.\begin{array}{ll}
B=0{\cdot}079+i0{\cdot}2913, & C=0{\cdot}2119+i0{\cdot}2151, \\
a_0=-0{\cdot}3675-i0{\cdot}4934, & a_1=0{\cdot}2187+i0{\cdot}1903, \\
a_2=0{\cdot}2895+i0{\cdot}0350, & a_3=-0{\cdot}1918-i0{\cdot}2342, \\
a_4=-0{\cdot}01916-i0{\cdot}00746, & a_5=0{\cdot}10413+i0{\cdot}2843, \\
a_6=-0{\cdot}3010-i0{\cdot}3050, & a_7=0{\cdot}03658-i0{\cdot}2365, \\
a_8=0{\cdot}1711+i0{\cdot}9029, & a_9=-0{\cdot}04892+i0{\cdot}04454, \\
a_{10}=0{\cdot}5909-i0{\cdot}9015, & a_{11}=-0{\cdot}1818-i0{\cdot}02189, \\
a_{12}=-0{\cdot}4567+i0{\cdot}0182, & a_{13}=0{\cdot}1201+i0{\cdot}2802, \\
a_{14}=-0{\cdot}3226-i0{\cdot}6390, & a_{15}=0{\cdot}04784-i0{\cdot}04301, \\
a_{16}=-0{\cdot}7467+i0{\cdot}6170, & a_{17}=0{\cdot}2179+i0{\cdot}1921, \\
a_{18}=-0{\cdot}3277-i0{\cdot}3933, & a_{19}=0{\cdot}04777-i0{\cdot}0429, \\
a_{20}=-0{\cdot}7356+i0{\cdot}3574, & a_{21}=0{\cdot}10464+i0{\cdot}2838, \\
a_{22}=0{\cdot}0759-i1{\cdot}0402, & a_{23}=-0{\cdot}2177-i0{\cdot}07372, \\
a_{24}=0{\cdot}3353+i0{\cdot}4378, & a_{25}=0{\cdot}08646-i0{\cdot}02003, \\
a_{26}=-0{\cdot}1350-i0{\cdot}0797, & a_{27}=-0{\cdot}01618+i0{\cdot}00806, \\
a_{28}=0{\cdot}02111+i0{\cdot}00914, & a_{29}=0{\cdot}001726-i0{\cdot}0009759, \\
a_{30}=-0{\cdot}001772-i0{\cdot}000731, & a_{31}=-0{\cdot}0001113+i0.0000639, \\
a_{32}=0{\cdot}0000914+i0{\cdot}00003753, & \cdots\cdots\cdots\cdots\cdots\cdots,
\end{array}\right\} \qquad (15)$$

While the equations for the dissipated waves are obtained by substituting these constants in (5) the expression of the forced seiches is written by

$$W'=-2f\xi e^{72i}(0{\cdot}079+i0{\cdot}2913)\cos 72\left(1+\frac{x}{l}\right)e^{i\sigma t}. \qquad (16)$$

The distribution of $W_2/f\xi$ for $fr=960$, that is for the same r as in the preceding case, and the different values of θ is also plotted in Fig. 2. Since, in the present case, $\cos fr=-0{\cdot}96726$, the condition of the problem is again very close to corresonance. It appears then that, from the results of the present as well as previous sections, the difference in dissipation waves with the difference in the frequency of disturbing waves is confirmable owing to the fact that the condition of the problem for both cases is capable of being assumed as corresonance.

The maximum vertical elevations of the seiches at $x=-l$ as well as at $x=0$ assume the values

$$W'_{x=-l}=0{\cdot}604f\xi, \qquad W'_{x=0}=0{\cdot}584f\xi. \qquad (17)$$

5. Comparison of the results of the two cases, namely (i) $fR=1$, $fl=3$, (ii) $fR=24$, $fl=72$, shows that, the higher the frequency of incident waves, the more is the energy of the dissipation waves in the outer sea polarised in the neighbourhood of the azimuth parallel to the lengthwise direction of the bay. The ratio of the vetical elevation of the dissipation waves at the radial distance $r/2R=20$ to that of incident waves is shown below.

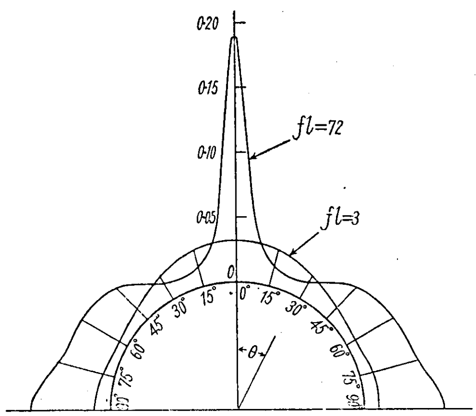

Fig. 2.

θ	0°	5°	15°	30°	45°	60°	80°	90°
$fR=1$	0·0312	——	0·0284	0·0209	0·01151	0·00645	0·01101	0·01279
$fR=24$	0·1899	0·0580	0·01547	0·01424	0·0346	0·0416	——	0·0627

These values are also assumed by the similar ratio for the horizontal displacements.

Another imporatant fact is that, while the total energy of the dissipation waves for the case of longer incident waves is much less than the one for the case of shorter incident waves as will be seen from the above table, the general amplitudes of the seiches in the bay for the former case is much greater than those for the latter as shown by (14) and (17). This is because of the energy of seiches more dissipated for higher frequency disturbance.

Although in the present paper the nature of the difference of the dissipation waves resulting from different vibrational condition of the bay, say, resonance or corresonance or intermediate condition, has not been studied, it appears however that the difference under consideration is very similar to the one in the case of the epicontinental sea or of the straits. For examples, in the condition close to resonance the vertical elevation at the mouth of the bay is very small, whereas in the condition close to corresonance the horizontal displacement at the same mouth is very small. The equation (4) in the present paper also suggests this fact.

32. 灣のセイシに伴ふ逸散性

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以前の研究で陸棚や海峡に現れるセイシが外海への逸散波のために減衰する狀態を考へたが，灣についてはまだ試みてなかつたのである．この問題を一般的に取扱ふことは相當困難であるから，玆にはこの強制セイシの場合を取り且つディスターバンスの週期が遲いものと速いものとを考へてそれらの比較研究を試みた譯である．共振のときに灣口での波高が零になり，餘共振のときに灣口での水平流れが零になることや，その外之等の場合の振幅比などの問題は大體わかつてゐるから只今は特に吟味しないことにした．

ディスターバンスの波長が長い場合と極く短い場合とでは逸散波やセイシの性質に著しい相異がある．波長が長い場合には，外海のすべての方向へ餘り極端な差異がなく逸散波が傳播するけれども，極く短くなると，灣口の向きの方向のみへ大きな勢力の逸散波が傳播し，所謂勢力傳播の方向性が現れるものである．

今一つ大切なことは，波長の長いときと短いときを比較して見ると，短いときの方が，逸散波の全勢力が遙かに大きくなり，セイシの振幅は逆に小さくなることである．振動勢力逸散の根本的性質であるとはいへ，セイシの場合にもよく現れてをるのは面白いともいひ得る．