34. *Energy Dissipation in Seismic Vibrations of Actual Buildings Predicted by means of an Improved Theory.*

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(Read April 21, 1936.—Received June 20, 1936.)

1. *Formulae for the problem.*

In the previous paper[1] we obtained improved formulae for the energy dissipation in seismic vibrations of a structure as the result of more accurate treatment of the points that had been somewhat approximate in the original problem. As suggested in that paper, the improved theory, when applied to actual structures, gives rise to new results that cannot be found in the original study.

The solution of the vibratory motion of the columns between the $s-1$th and sth floors is

$$y_s=(A_s+B_s x_s+C_s x_s^2+D_s x_s^3)e^{ipt}. \quad (s=1,\ 2,\cdots\cdots,\ n) \tag{1}$$

The boundary conditions in the case of a clamped base and extremely rigid floors are as follows.

$$x_n=-l_n; \qquad E_n I_n\frac{\partial^3 y_n}{\partial x_n^3}+m_n\frac{\partial^2 y_n}{\partial t^2}=0, \qquad \frac{\partial y_n}{\partial x_n}=0, \tag{2), (3}$$

$$x_s=-l_s, \quad x_{s+1}=0; \quad E_{s+1}I_{s+1}\frac{\partial^3 y_{s+1}}{\partial x_{s+1}^3}=E_s I_s\frac{\partial^3 y_s}{\partial x_s^3}+m_s\frac{\partial^2 y_s}{\partial t^2}, \tag{4}$$

$$\frac{\partial y_s}{\partial x_s}=0, \quad \frac{\partial y_{s+1}}{\partial x_{s+1}}=0, \quad y_s=y_{s+1}, \quad (s=1,\ 2,\cdots\cdots,\ n-1) \tag{5), (6), (7}$$

$$x_1=0, \quad r=\varepsilon; \qquad (u_1+u_2)_{max.}=-(v_1+v_2)_{max.}, \tag{8}$$

$$\frac{\partial y_1}{\partial x_1}=0, \quad y_1=u_0+u_0'+(u_1+u_2)_{r=\varepsilon,\ \theta=0}, \tag{9), (10}$$

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1) K. SEZAWA and K. KANAI, "Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure", *Bull. Earthq. Res. Inst.*, 14 (1936), 164~188. The significance of each symbol used in the present paper is shown in the preceding paper under consideration.

$$-E\pi\varepsilon^2 j^2\frac{\partial^3 y_1}{\partial x_1^3}=\int_{\varphi=0}^{\pi}\int_{\theta=0}^{\pi}\Big[\mu\Big\{\frac{\partial(v_1+v_2)}{\partial r}-\frac{v_1+v_2}{r}+\frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\theta}\Big\}\sin^2\theta$$

$$+\Big\{\lambda\Delta+2\mu\frac{\partial(u_1+u_2)}{\partial r}\Big\}\sin\theta\cos\theta\Big]r^2 d\varphi d\theta. \qquad (11)$$

In the present paper the cases of a 3-storied and 7-storied structures will be studied. Since the solution of a 3-storied structure has been formulated in pp. 174∼175 in one[2] of our previous papers, and that of a 7-storied structure in pp. 189∼192 of another paper,[3] details of the mathematical expressions ascertained by means of the improved theory will not be given. It is however rather important to note that the deflections as well as the bending moments on columns for any case have a common factor of type such that

$$M=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\Big(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\Big), \qquad (12)$$

where

$$\left.\begin{aligned}
&P=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_1, \quad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_2,\\
&\Gamma_1=\frac{3(\lambda+2\mu)}{\mu}+\nu\gamma_1\Big(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda+2\mu}{\mu}}\Big),\\
&\Gamma_2=\sqrt{\nu\gamma_1}\Big\{3\sqrt{\frac{\lambda+2\mu}{\mu}}\Big(1+\sqrt{\frac{\lambda+2\mu}{\mu}}\Big)+\nu\gamma_1\Big(\sqrt{\frac{\lambda+2\mu}{\mu}}-2\Big)\Big\},\\
&\Lambda_1=\frac{2\lambda+5\mu}{\mu}-\nu\gamma_1, \quad \Lambda_2=\sqrt{\nu\gamma_1}\Big(2\sqrt{\frac{\lambda+2\mu}{\mu}}-1\Big),\\
&\nu=\frac{E_1\rho I_1\varepsilon^2}{\mu m_1 l_1^3},
\end{aligned}\right\} \qquad (13)$$

in which

$$\left.\begin{aligned}
&\phi=1728-864\gamma+60\gamma^2-\gamma^3,\\
&\psi=\gamma(12-\gamma)(36-\gamma)
\end{aligned}\right\} \qquad (14)$$

in the case $n=3$; and

2) *loc. cit.* 1).

3) K. SEZAWA and K. KANAI, "Energy Dissipation in Seismic Vibrations of a Seven-storied Structure. Nature of Corresonance", *Bull. Earthq. Res. Inst.*, **14** (1936), 189∼200.

$$\left.\begin{aligned}\phi&=(12-\gamma)\Big\{(12-\gamma)(24-\gamma)-144\Big\}\Big[-36\gamma(24-\gamma)(36-\gamma)\\&\qquad\qquad\qquad\qquad+\{(12-\gamma)(24-\gamma)-144\}^2\Big].\\\psi&=\gamma\Big\{(12-\gamma)^2(36-\gamma)-144(24-\gamma\ \Big\}\Big\{(12-\gamma)(36-\gamma)^2-144(24-\gamma)\Big\}\end{aligned}\right\}\quad(15)$$

in the case $n=7$.

If $(Ej^2\varepsilon)/(\mu l^3)=0$, the denominator of M has a factor ϕ, so that, were γ to satisfy $\phi=0$, M would assume an infinitely large value. If, on the other hand, $(Ej^2\varepsilon)/(\mu l^3)=\infty$, the denominator of M has a factor ψ, so that, were γ to satisfy $\psi=0$, M would again assume an infinitely large value. $\phi=0$, $\psi=0$ correspond respectively to the resonance and corresonance conditions in our previous conception That the two conditions $(Ej^2\varepsilon)/(\mu l^3)=0$ and $(Ej^2\varepsilon)/(\mu l^3)=\infty$ indicate critical cases in both of which there is very little dissipation, appears probable even from the ordinary physical standpoint.

In the case where $(Ej^2\varepsilon)/(\mu l^3)$ is neither zero nor infinity, the value of M becomes maximum within every range, the lower and the upper criticals of which are conditioned by every root of $\phi=0$ and that of $\psi=0$. Generally speaking, the value of γ that gives each of the maximum bending moments, increases from a value satisfying $\phi=0$ to that satisfying $\psi=0$ in accordance with increase in $(Ej^2\varepsilon)/(\mu l^3)$ from 0 to ∞.

2. *A three-storied structure.*

In the case of a 3-storied structure γ satisfies $\phi=0$ when

$$\gamma=2{\cdot}3765,\qquad 18{\cdot}6630,\qquad 38{\cdot}9605;$$

and γ satisfies $\psi=0$ when

$$\gamma=12,\qquad 36.$$

Thus, the two maxima of the bending moments in the resonance curves lie successively between $\gamma=2{\cdot}3765$ and 12, and between 18·6630 and 36.

The greatest value of the maximum bending moment is usually that in the range between $\gamma=2{\cdot}3765$ and 12 in the resonance curves. If the greatest bending moment were to be at γ near $\gamma=2{\cdot}3765$, that moment would be induced in the columns below the first floor, whereas, if the greatest moment were to be at γ near $\gamma=12$, that moment would be induced in the columns between the first floor and the roof, the greatest moments in the columns of both two floors then being the same. If, on the other hand, the greatest bending moment were to arise at an intermediate γ in the range under consideration, that moment in the columns between the first and the second floors would be the greatest.

The calculated result for the case of Mitsubishi Naka-13 gô-kan is shown in Figs. 1, 2. The data of structure for the calculation are the same as those in an earlier paper[4] of ours, whereas E and ρ being $E=2{\cdot}1.10^9$ kg/m², $\rho=2{\cdot}0.10^3$ kg *mass/m*³, the value of $\mu(\lambda=\mu)$ is assumed to be (i) $\mu=8.10^7$ kg *mass/ms*² (corresponding to $\sqrt{\mu/\rho}=200$m/s), and (ii) $\mu=2.10^7$ kg *mass/ms*² (corresponding to $\sqrt{\mu/\rho}=100\,m/s$), the two cases belonging to Figs. 1, 2 respectively. The vertical strips of full and broken lines represent the conditions $\phi=0$, $\psi=0$ respectively.

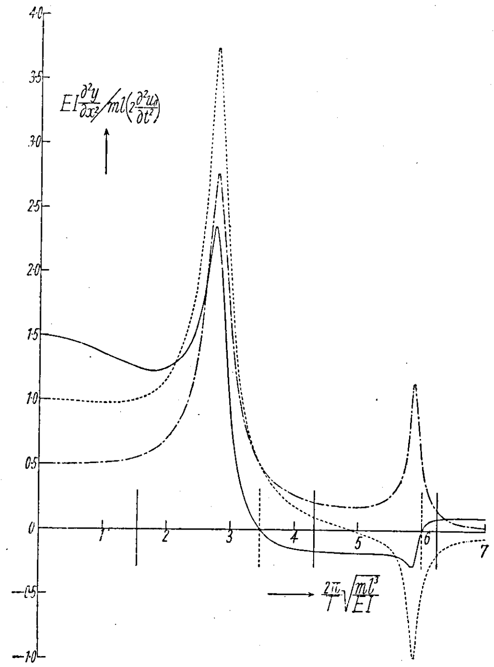

Fig. 1. Mitsubishi Naka-13gô-kan. $\sqrt{\mu/\rho}=200$ m/sec. Full, broken, and chain lines represent moments in columns of ground, first, and second floors respectively.

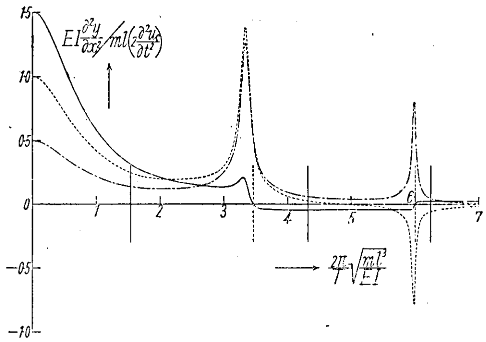

Fig. 2. Mitsubishi Naka-13gô-kan. $\sqrt{\mu/\rho}=100$m/sec. Full, broken, and chain lines represent moments in columns of ground, first, and second floors respectively.

Upon comparing the results in Figs. 1, 2 it will be seen that dissipation is more pronounced in the case of soft ground than in rigid ground.

The greatest bending moment in the present case lies between $\gamma=2{\cdot}3765$ and 12, namely $\sqrt{\gamma}=2{\cdot}78$ in the first case, $\sqrt{\mu/\rho}=200\,m/s$, and $\sqrt{\gamma}=3{\cdot}34$ in the second, $\sqrt{\mu/\rho}=100\,m/s$, in

4) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 930.

both of which the greatest moments are induced in columns between the first and the second floors.

With a view to getting the solution of a problem such that the calculated natural frequency would coincide with the actual probable one (0·2 sec), we assume a rather small value of EI instead of calculating for the case in which the floor is sensibly flexible.[5] If we were to take 1/4 of the value of EI just mentioned so as to obtain a principal natural period of about 0·2 sec, it would be necessary to assume $\sqrt{\mu/\rho}=100\,m/s$ and $50\,m/s$ respectively to make the result in Figs. 1, 2 available. Since such conditions as that $I=I'/4$ (I' being the calculated value) and $\sqrt{\mu/\rho}=200\,m/s\sim100\,m/s$ are rather probable ones in the present structure, it seems that the results in Figs. 1, 2 are likely to represent dissipation somewhat in excess of what we should expect in the same structure.

3. *A seven-storied structure.*

In the case of a 7-storied structure, γ satisfies $\phi=0$ when

$$\gamma=0{\cdot}525,\quad 4{\cdot}584,\quad 12,\quad 21{\cdot}49,\quad 31{\cdot}42,\quad 40{\cdot}03,\quad 45{\cdot}95;$$

and γ satisfies $\psi=0$ when

$$\gamma=2{\cdot}37,\quad 9{\cdot}04,\quad 18{\cdot}66,\quad 29{\cdot}35,\quad 38{\cdot}99,\quad 45{\cdot}60.$$

The maxima of the bending moments in the resonance curves lie successively between $\gamma=0{\cdot}525\sim2{\cdot}37$, $4{\cdot}584\sim9{\cdot}04$, $12\sim18{\cdot}66$, $21{\cdot}49\sim29{\cdot}35$, $31{\cdot}42\sim38{\cdot}99$, $40{\cdot}03\sim45{\cdot}60$. The position of every maximum moment changes from the lowest critical to the upper in each of the above ranges in accordance with changes in $(Ej^2\varepsilon)/(\mu l^3)$ from 0 to ∞.

The greatest value of the maximum bending moments is usually that in the range $\gamma=0{\cdot}525\sim2{\cdot}37$. If the greatest bending moment were to be near $\gamma=0{\cdot}525$ in the resonance curves, that moment would be induced in the columns below the first floor, whereas, if the greatest bending moment were to be near $\gamma=2{\cdot}37$, that moment would be induced in columns between the 3rd and 5th floors, the greatest moment in the columns of the two floors then being the same. In accordance with the change in the position of the greatest bending moment in the resonance curves, namely from $\gamma=0{\cdot}525$ to $2{\cdot}37$, the greatest bending moment is induced in columns of different stories, namely from

5) K. SEZAWA and K. KANAI, "The Effect of the Stiffness of Floors on the Horizontal Vibrations of a Framed Structure", *Bull, Earthq. Res. Inst.*, **14** (1936), 367~376.

the columns below the first floor to those between the 3rd and 4th floors.

Finally, it will be seen that the gaps between the successive ranges, within which the resonance frequencies lie, become narrower, excepting the lowest gap, with increase in the order of resonance. In the present case the values of the gaps under consideration are such that 2·21, 2·96, 2·83, 2·07, 1·04, 0·35. It should be remembered that the final value of γ for which $\phi=0$ is not the critical of any range.

The calculation as applied to the case of the Old Kaizyô Building is shown in Figs. 3, 4. The value of I in the calculation is the same as that for the columns only in Table I in one of our papers,[6] whereas E, ρ, λ, μ being the same as those in the preceding section, cases $\sqrt{\mu/\rho}=200\,m/s$ and $\sqrt{\mu/\rho}=100\,m/s$ correspond to those in Figs. 3, 4.

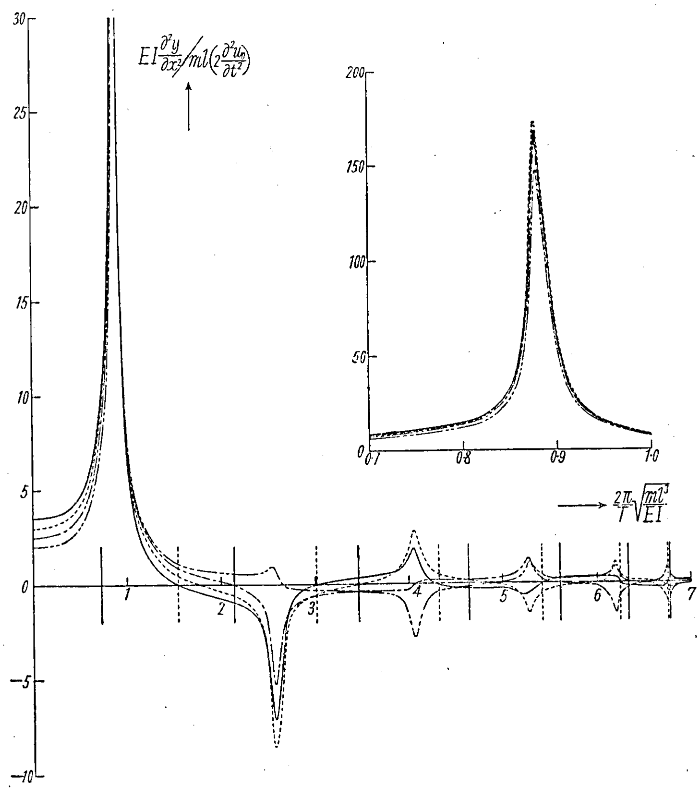

Fig. 3a. Old Kaizyô Building, $\sqrt{\mu/\rho}=200$ m/sec. Full, broken, chain, and double chain lines represent moments in columns of ground, first, second, and third floors respectively.

6) K. SEZAWA and K. KANAI, *loc. cit.* 3).

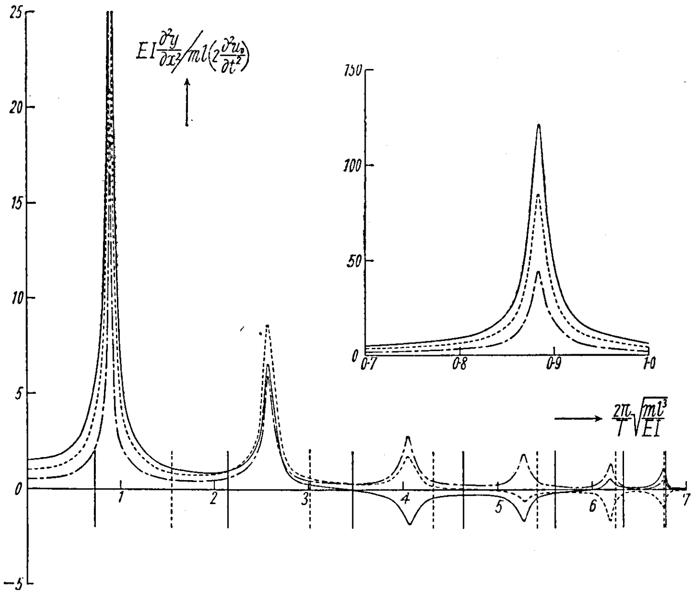

Fig. 3b. Old Kaizyô Building. $\sqrt{\mu/\rho}=200$ m/sec. Full, broken, and chain lines represent mome..ts in columns of fourth, fifth, and sixth floors respectively.

The vertical strips of full and broken line types again represent the conditions $\phi=0$, $\psi=0$ respectively.

The effect of rigidity of ground on dissipation is more marked in the present case than in the preceding one. In the case of $\sqrt{\mu/\rho}=$ 200 m/s, dissipation is extremely small.[7)]

The greatest bending moment in the present case lies in the range between 0·525 and 2·37, namely $\sqrt{\gamma}=0{\cdot}884$ in the first case, $\sqrt{\mu/\rho}=$ 200 m/s, and $\sqrt{\gamma}=1{\cdot}30$ in the second, $\sqrt{\mu/\rho}=100$ m/s. While the greatest moment in the first case is induced in columns between the first and second floors, the same moment in the second case is induced in columns between the third and fourth floors.

Since the value of I just mentioned is incidentally the same as one quarter of the moments of inertia of all the vertical members shown in the same table, the value of I here adopted may alternatively be interpreted as its value that has been modified in lieu of calculating the case in which the floor is sensibly flexible. As a matter of fact the

7) The effect of such a deeply piled foundation as in this building is rather to greatly increase the dissipation owing to the enormously increased area of contact surface between the piles and the ground. But, we are here discussing the problem of a pileless structure wherein the dissipation is not very marked.

present calculation appears to show a real probable case, for the reason that the principal natural period of the structure is about 0·46 sec.. However, to us at present, it is not certain whether the value of $\sqrt{\mu/\rho}$ is near 100 m/s or near 200 m/s. If nevertheless $\sqrt{\mu/\rho}$ were

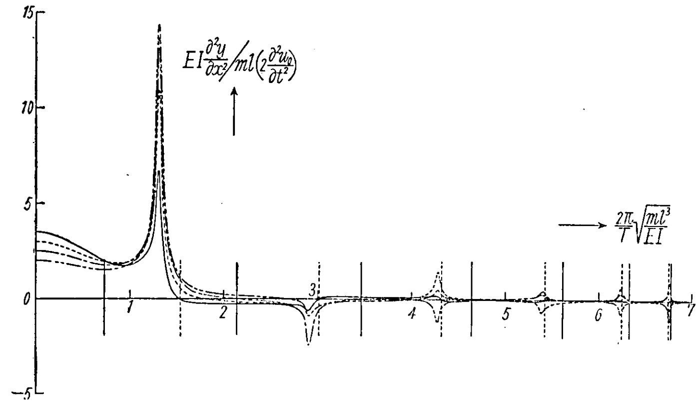

Fig. 4a. Old Kaizyô Building. $\sqrt{\mu/\rho}$=100 m/sec. Full, broken, chain, and double chain lines represent moments in columns of ground, first, second, and third floors respectively.

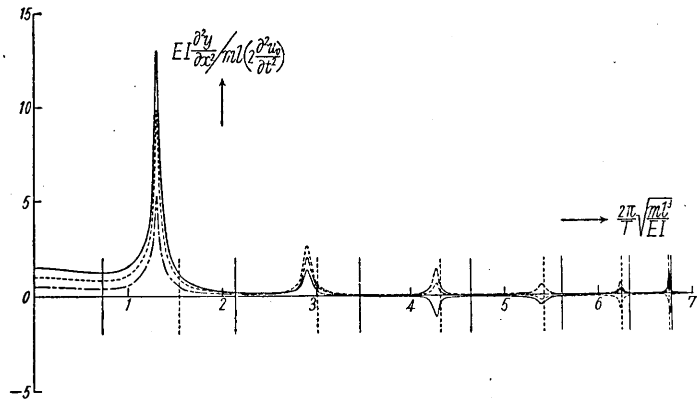

Fig. 4b. Old Kaizyô Building. $\sqrt{\mu/\rho}$=100 m/sec. Full, broken, and chain lines represent moments in columns of fourth, fifth, and sixth floors respectively.

to be near 100 m/s, it would be possible to conclude that the damping of the seismic vibration is mainly due to dissipation of the vibrational energy into the ground, in the present case at all events.

4. *Concluding remarks.*

Every maximum of the bending moments induced in the columns lies within a certain specified range of resonance curves, the lower and the upper criticals of the range under consideration indicating the resonance of cases satisfying the condition $(Ej^2\varepsilon)/(\mu l^3)=0$ and the condition $(Ej^2\varepsilon)/(\mu l^3)=\infty$ respectively, namely the resonance and cor-resonance conditions implied in our previous conception. The vibrational frequencies at which the bending moments become maximum depend on the ratio of $(Ej^2\varepsilon)/(\mu l^3)$, their values increasing from the lower criticals to the upper with increase in $(Ej^2\varepsilon)/(\mu l^3)$ from 0 to ∞. When $(Ej^2\varepsilon)/(\mu l^3)=0$ or ∞, the vibration energy hardly dissipates into the ground, the bending moments under resonances in such a condition assuming infinitely large values. If, on the other hand, $(Ej^2\varepsilon)/(\mu l^3)$ were neither 0 or ∞, the energy dissipated into the ground would be of such an amount as shall conform with the difference in the value of $(Ej^2\varepsilon)/(\mu l^3)$.

The greatest maximum bending moment is within the lowest range just described. The columns in which the greatest bending moment is induced differ with difference in the value of $(Ej^2\varepsilon)/(\mu l^3)$. When $(Ej^2\varepsilon)/(\mu l^3)=0$, the columns in which the greatest bending moment is induced are those below the first floor. The larger the value of $(Ej^2\varepsilon)/(\mu l^3)$, the more the columns of floors still further up partake of the greatest bending moment. In the limiting case where $(Ej^2\varepsilon)/(\mu l^3)=\infty$, the floor under consideration is that which is immediately below the roof or that a few floors below the roof. The fact that the columns that were damaged most in the case of great earthquakes were those on floors a few stories above the ground floor, appears now to be well explained by the present study on the dissipation of vibrational energy.

In conclusion we wish to express our thanks to the Council of the Foundation for the Promotion of Scientific and Industrial Research in Japan, with whose aid the present series of investigations was begun, and also to the members of the staff of the Mitsubishi Co. and those of the Tokyo Fire and Marine Insurance Co., who kindly allowed us the use of their valuable data on these structures.

34. 改良せる理論から見た實在建物の震動勢力逸散性

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前回の論文で震動逸散の問題をできるだけ嚴格に取扱つて理想的結果を出して置いたが，この論文ではその理論を應用して，三菱仲十三號館（3 階）と海上ビルデイング（7 階）の場合の震動逸散性とを推定して見たのである．その結果から之等の建物の震動逸散性が一層よくわかつたのみでなく，今迄どうしてもはつきりしなかつた構造物の震動の性質が明瞭になつたのである．

何れの場合にも床の變位と柱の屈曲モーメントとは，强制振動がある振動數（1 個でなく順次高次のものがある）で極大値を取る．この振動數は土地の剛度が極端に剛い場合の建物の各次の固有振動數と土地の剛度が極端に柔い場合のその各次の固有振動數との間の値であり，その値は建物の剛度と土地の剛度との比，即ち $(Ej^2\varepsilon)/(\mu l^3)$ の如き數によつて定まるものである．且つ又，$(Ej^2\varepsilon)/(\mu l^3)=0$ の場合と $(Ej^2\varepsilon)/(\mu l^3)=\infty$ の場合とには，上記の變位や屈曲モーメントが無限大となり逸散性が無くなるけれども，その中間の場合には何れも有限値となつて逸散性のあることがわかる．換言すれば前の論文で示したところの共振と餘共振との間の振動數で現れ，且つその振動數は $(Ej^2\varepsilon)/(\mu l^3)$ の値によつて變化することが知られるのである．

柱の屈曲モーメントは上記の極大値中で最低次に相當するものが最大となるが，その場合に何れの床の柱のモーメントが一番大きくなるかといふと，その極大値を取るべき振動數の値，いひ換へれば $(Ej^2\varepsilon)/(\mu l^3)$ の値によつて變るものである．$(Ej^2\varepsilon)/(\mu l^3)=0$ の場合には最下階の柱の屈曲モーメントが最も大きくなり，$(Ej^2\varepsilon)/(\mu l^3)$ の增加とともに少しづつ上の層の柱の屈曲モーメントが最大値を取るやうになる．極論な場合即ち $(Ej^2\varepsilon)/(\mu l^3)=\infty$ になると屋根から 1 階か 2 階下の柱の屈曲モーメントが最大になるものである．このやうな性質は土地の震動數が公平にあらゆる値を取る場合についていはれるから非常に都合がよいものといふことができる．大地震の時に高層建物で地上數階目かに建物の被害が多かつたといはれてゐるが，只今の逸散理論を應用すれば簡單に說明がつくのである．關東地震直後に筆者や其他の先輩學者方が建物の自己振動を添附したり，高次の共振を入れたり，或は他の種類の振動型を入れたりして，無理に說明を試みようとしたことなどもあるけれども，何れの場合にもディスターバンスの種々のものゝ機會均等性を考へると不都合な點から免れ得なかつたものである．