37. *An Experimental Method to solve the Equation of Motion of the Seismograph*[*].

By Takato FUCHIDA.

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1. Introduction.

It goes without saying that it is one of the most important problems in the earthquake research to solve the equation of motion of the seismograph,

$$\ddot{x}+2\varepsilon\dot{x}+n^2x=f''(t). \tag{1}$$

The method of numerical calculation which has been adopted for the use of solving the above equation cannot be employed in the case in which the form of $f''(t)$ is very complicated. The writer therefore tried to solve the equation experimentally. By this method, the solution of (1) can easily be obtained in the cases when the form of $f''(t)$ is complex as well as simple. By this experimental method, however, the solution can be obtained only for the initial conditions,

$$x=0,\ \dot{x}=0 \text{ at } t=0. \tag{2}$$

2. Principle.

The sensitive galvanometer of moving coil type is used for the purpose of measuring the weak direct current or the ballistic current. The transient motion in the oscillation of the suspension of the galvanometer was used in this experiment.

The equation of motion of the galvanometer is

$$K\ddot{\theta}+a\dot{\theta}+c\theta=Gi, \tag{3}$$

in which K, a, c and G are respectively the moment of inertia of the moving coil, the coefficient of damping of air, the restitutive force of a string and the torque by unit current. Therefore in case of the connection as shown in Fig. 1, the equation of motion is expressed in a form

$$K\ddot{\theta}+\left(a+\frac{G^2}{S}\right)\dot{\theta}+c\theta=GI'\cdot \tag{4}$$

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[*] Communicated by N. Miyabe.

Taking into account the next relation,

$$I' = I\frac{R'}{R+R'}$$

and substituting

$$2\varepsilon = \frac{\left(a+\frac{G^2}{S}\right)}{K}, \qquad n^2 = \frac{c}{K}, \qquad A = \frac{G}{K}\,\frac{R'}{R+R'},$$

the equation (4) becomes

$$\ddot{\theta} + 2\varepsilon\dot{\theta} + n^2\theta = AI.^{1)} \tag{5}$$

Although the last equation is of the same type as the equation (1), the right hand side in it corresponding to $f''(t)$ in (1) is proportional to the current.

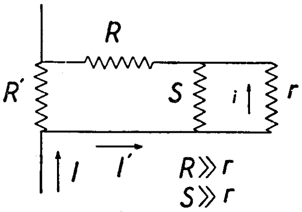

Fig. 1.

Now, as (5) is the equation of motion of the galvanometer, it is possible to know θ experimentally which represents the variation in I. Hence when I is proportional to $f'(t)$, the record of θ shows the quantity proportional to x in (1). But as the moving coil of the galvanometer is at rest at the zero position till the current I begins to flow, the initial conditions must be

$$\theta = 0 \text{ and } \dot{\theta} = 0 \text{ at } t = 0$$

without regard to the form of I. The solution thus obtained cannot always stand for the solution of the equation (1) of motion of the seismograph, for the initial conditions by above mentioned solution cannot always be satisfied in the actual case of the motion of the seimograph when it is subjected to earthquake shocks.

3. Experimental Arrangement.

The experimental arrangements are shown in Fig. 2. A photoelectric cell was used to supply the current I proportional to $f''(t)$ in (1). In Fig. 2 P is a glass plate in which is printed the curve of $f''(t)$ just like a talky film of variable area system. A light source L_1 has a linear filament the image of which is adjusted to focus to P. The flux of rays passing through P is condensed by a lense and projected on the photoelectric cell P. T.. P is driven by a small series-motor the speed of which can be changed at will and a drum

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1) When the self-inductance L of the circuit is an important factor, the term $\frac{LK}{R}\dddot{\theta}$ must be added, though it is neglected in the present discussion because of its smallness in comparison with the other terms.

of a camera is also rotated by the same motor therefore the driving speed of the film is proportional to that of P. In the writer's experiment, it was felt most convenient to make the speed of the film proportional to that of P without making it constant, so that the speed of P can be changed within a range, the ratio of the maximum to the minimum speed being about one hundred. Time marks were recorded on the film by means of a lamp L_3 to check the time axis of records.

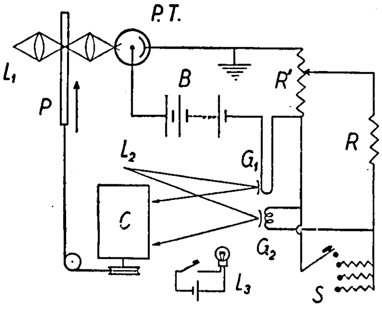

Fig. 2.

The characteristic curve of the photoelectric cell P. T. (P. T.—50—G) was made almost linear by taking care of connections, in order to avoid the effects due to sparks in the motor and other disturbances that may happen in weak current experiments.

To measure the current I, a torsion-string galvanometer G_1 was used, the instrumetal constants of which being;

Period	1/50 sec.
Resistance of string	10 Ω.
Current sensitivity	$4\cdot10^{-7}$ Amp.

The proper period of the torsional oscillation of this galvanometer suspension is 1/50 sec, very short compared with the variation of $f''(t)$ in the present experiment, so that its deflection is approximately proportional to $f''(t)$.

The galvanometer G_2 is of a type of moving coil which plays the most important rôle in this experiment, its constants being as follows:

Period	7 sec.
Resistance of coil	420 Ω.
Current sensitivity	$7\cdot10^{-10}$ Amp.
Resistance for critical damping	14000 Ω.

The shunt resistance S was placed to change the damping ratio in the galvanometer G_2.

Experiments are made for four cases with different shunt resistances: e. i.,

$S=\infty$ . . . . $b=0\cdot05$
$S=36000\ \Omega$ . . . . $b=0\cdot4$
$S=20000\ \Omega$ . . . . $b=0\cdot7$
$S=14000\ \Omega$ . . . . $b=1\cdot0$
$(b=\varepsilon/n)$

As shown in Fig. 1, the larger the resistance R compared with that of the galvanometer G_2, the simpler the relations between I and I'. A resistance of 1 meg was therefore used for R. The current which flows in G_2 can be so adjusted by a potentiometer R' that the deflection of G_2 is recorded with proper amplitudes.

4. Results of Experiments.

In solving the equation of motion of the seismograph (1), various cases with different combinations of n, b and $f''(t)$ must be dealt with. The values of b can be changed at will by changing the shunt resistance S, whereas n cannot be changed, as it is an instrumental constant of the galvanometer G_2 used in this experiment. The same results as if n is changed, however, can be obtained by changing the driving speed of P. When P is driven at the speed N times as large, the equation of motion (1) becomes

$$\ddot{x}+2\varepsilon\dot{x}+n^2x+f''(Nt). \tag{6}$$

Putting $Nt=\tau$, (6) becomes

$$\frac{d^2x}{d\tau^2}+2b\left(\frac{n}{N}\right)\frac{dx}{d\tau}+\left(\frac{n}{N}\right)^2 x=\frac{1}{N^2}f''(\tau). \tag{7}$$

In the last equation $\frac{n}{N}$ and $\frac{1}{N^2}f''(\tau)$ correspond to n and $f''(t)$ in (1) respectively. The solution of the equation,

$$\ddot{x}+2b\left(\frac{n}{N}\right)\dot{x}+\left(\frac{n}{N}\right)^2x=f''(t),$$

can therefore be found by reading records through the N^2 times larger scale.

The constancy in dynamical magnifying power of the apparatus, corresponding to the accelerograph, can be tested by the observation of the deflection of G_2 when P is driven at different speed.

For the test of the apparatus, in the present experiment, $f''(t)$ is given in square form which is expressed analytically by

$$f''(t)=\frac{4}{\pi}\left(\sin t+\frac{1}{3}\sin 3t+\frac{1}{5}\sin 5t+\cdots\cdots\cdots\right). \tag{8}$$

Several examples of the results are shown in Fig. 3. From the results as shown in Fig. 3, it can be said that the apparatus, the period of which is about one third of that of the square wave and is critically damped, can record practically the squre form of the wave. From the results of theoretical calculation of the amplitude charactristics, it is concluded that the condition, $b \fallingdotseq 0{\cdot}7$, under which the apparatus is slightly oscillatory, is needed in order to extend the range of use of it

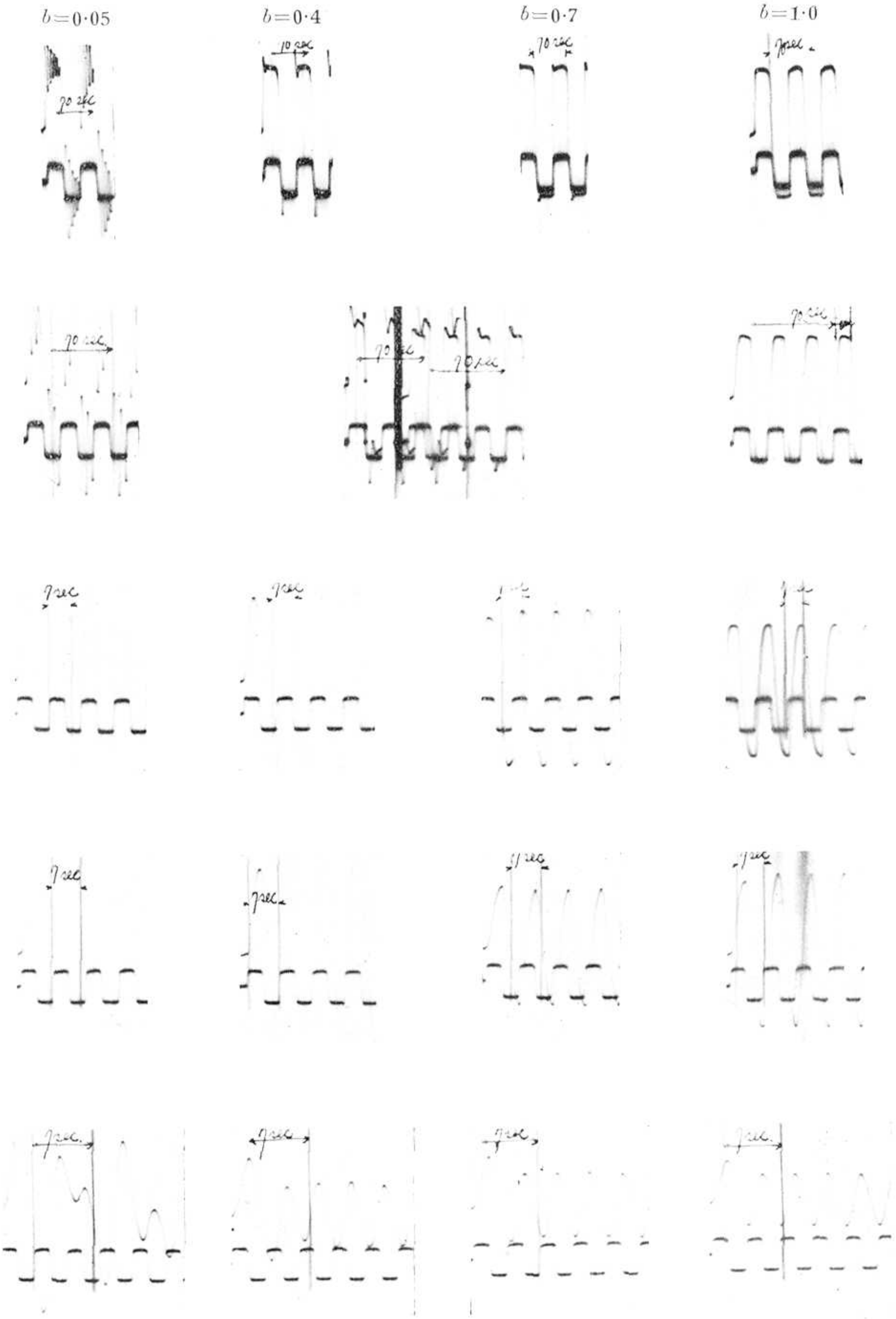

Fig. 3. Time marks of 7 sec are recorded in this experiment, which is equal to the period of G_2.

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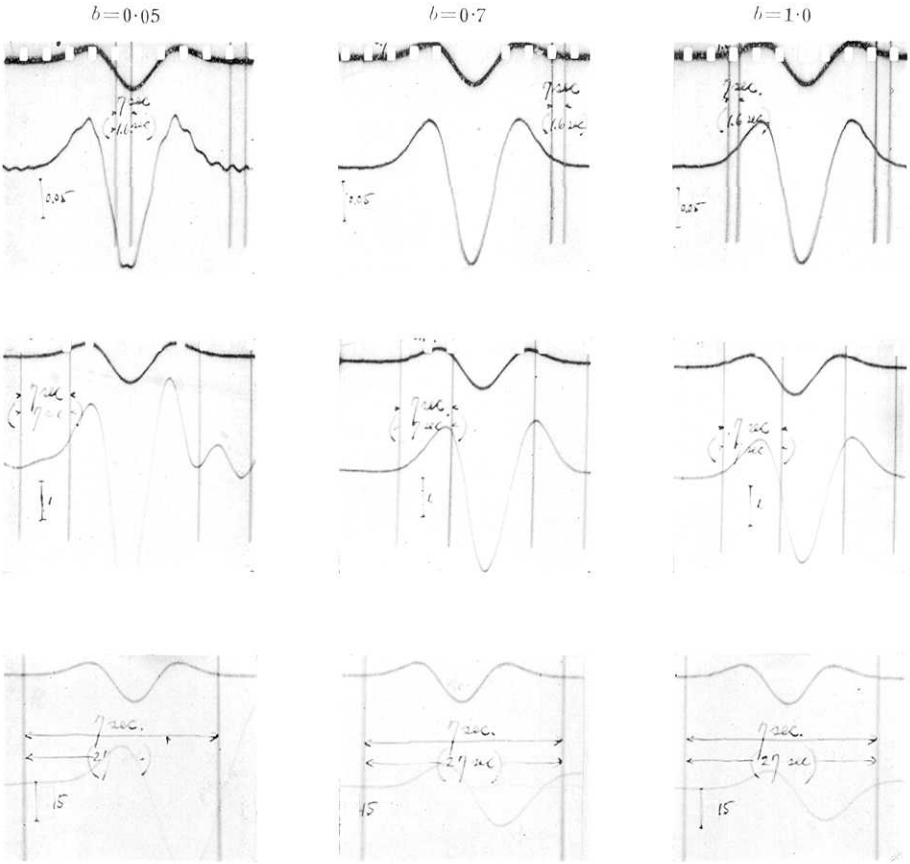
$b=0{\cdot}05$ $b=0{\cdot}7$ $b=1{\cdot}0$

Fig. 4. Time marks in brackets and scales at sides show periods and scales corresponding to n/N.

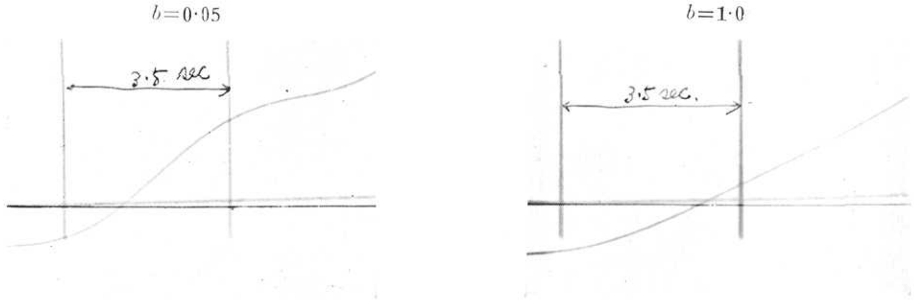
$b=0{\cdot}05$ $b=1{\cdot}0$

Fig. 6.

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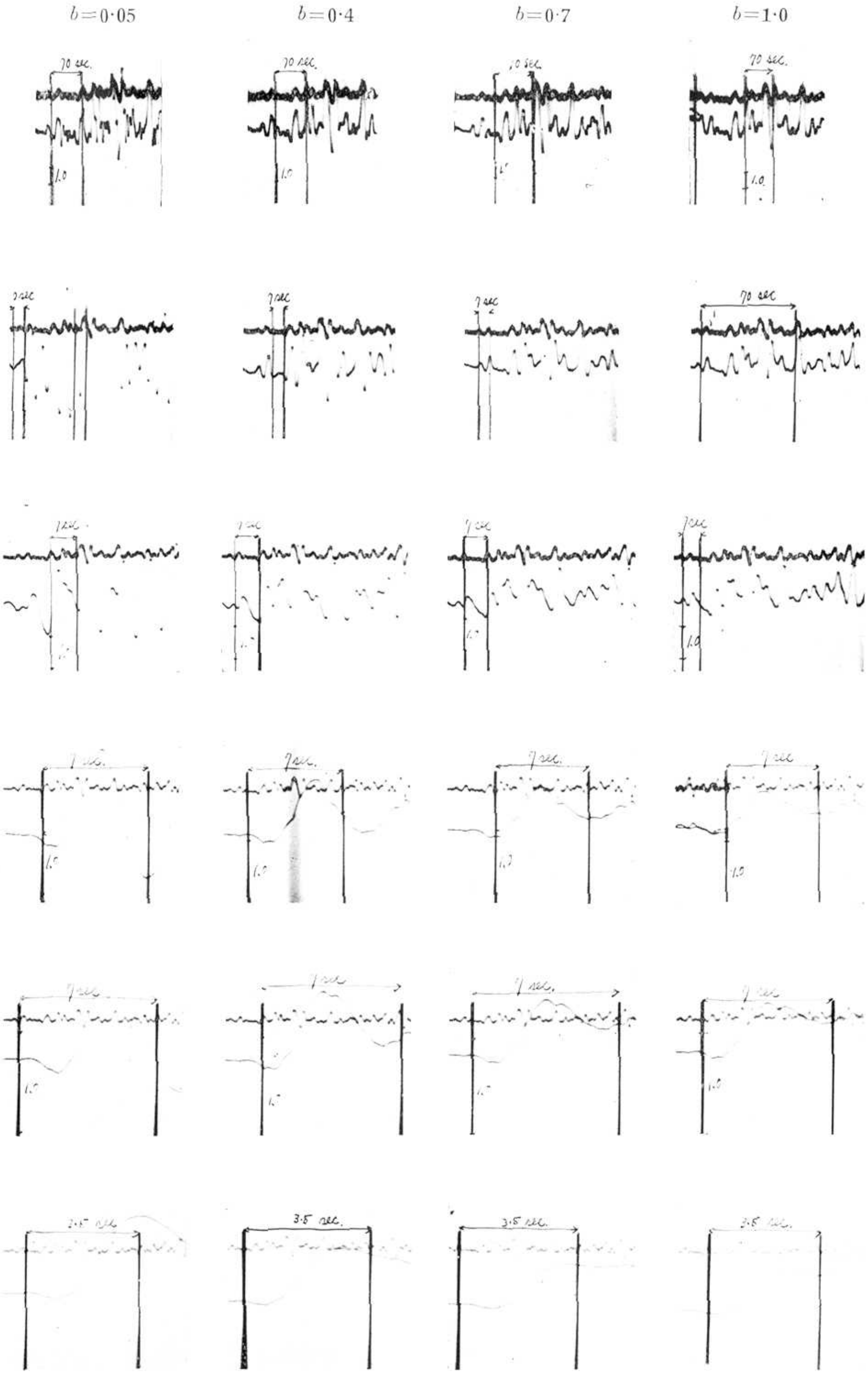

Fig. 5.

up to the resonance. But this conclusion is of cource correct only in the case of pure sine wave and is nothing but the reference to the case of complex waves.

Fig. 4 shows results obtained when $f''(t)=2(2t^2-1)e^{-t^2}$. This example has been already numerically calculated by Mr. S. Suzuki[2] which the present writer took for the purpose of comparing the experiment with the calculation. Comparing the both results, We notice that they are very similar in forms.

In Fig. 5 several examples are shown when $f''(t)$ is given in very complicated form. The preliminary portion in a record of Ishimoto's accelerograph was taken as an example of complex waves. It is also found that the more fast P is driven, the more distinctly the deflection of the galvanometer tend to show the quantity of $\int\int f''(t)\,dtdt$. If therefore $f''(t)$ shows the acceleration of the earthquake shocks, the deflection of the galvanometer when P is driven very fast, is almost proportional to the displacement of the ground. In Fig. 5, the deflection of the galvanometer seems to show the permanent displacement. The fact that the image of the mirror deflects to one side only, does not seem to have come from errors due to the nonlinear photoelectric characteristics, the slight inclination of the axis of the curve of $f''(t)$ which affects, as shown in Fig. 6, and so on. But it cannot be concluded that it is the true permanent displacement, since the adopted record of the accelerograph does not always show the true acceleration of the ground due to the earthquake shocks and the problem concerning the initial conditions and the continuity of the true displacement have not yet been discussed thouroughly.

In conclusion the writer wishes to express his best thanks to Dr. N. Miyabe for his kind advices and guidances.

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37. 振動方程式を實驗的に解く一方法

淵　田　隆　門

$t=0$ に於いて $x=\dot{x}=0$ なる初期條件に對する $\ddot{x}+2\epsilon\dot{x}+n^2x=f''(t)$ の解を實驗的に求むる方法である．卽ち $f''(t)$ に比例する電流を檢流計に流して其の運動を記錄せしめるのである．$f''(t)$ の簡單な場合や複雜な場合等を二三試みた．本法は數値計算によりて出來ないやうな複雜な場合も容易に解を得る事の出來るのを利點とする．

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2) S. Suzuki, *Bull. Earthq. Res. Inst.*, **12** (1934), 15.