14. *On the Relation between Seismic Origins and Radiated Waves.*

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1. Several attempts have been made mathematically and experimentally with a view to ascertaining the mechanism of a seismic origin from the nature of the waves radiated therefrom. But, the shock-generating deformation of the earth's crust differs somewhat from the one assumed from the theory of elasticity, while the prevalent ideas on the nature of radiated waves from the same origin, even under the theory of elasticity, can not yet be said to be satisfactory. It is therefore of pressing importance to investigate the problem in the most rational manner possible, notwithstanding that a number of inherent ambiguities exist in the problem.

In a previous paper[1] we arrived at the conclusion that wave phenomena are independent of initial stress, and that the total wave energy is radiated from the origin itself at which a shock or stress is applied.

The absolute amount of wave energy is the same whether a given stress is applied at the origin or released from it. It was also shown that a given additive statical energy is also radiated (in the form of waves) from the same origin in both conditions of the problem. The same conclusion may also be obtained from the discussion in Love's text book[2], a brief summary of which is shown in the next section.

2. In the first place, the correspondence of the movement at the origin and the radiated waves will be discussed. The work done by the surface force at the origin, namely the applied surface energy, is

$$\int dt \iint \left(X_\nu \frac{\partial u}{\partial t} + Y_\nu \frac{\partial v}{\partial t} + Z_\nu \frac{\partial w}{\partial t} \right) dS, \qquad (1)$$

where X_ν, Y_ν, Z_ν, u, v, w are components of force and displacement

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[1] K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **14** (1936), 10~17.

[2] A. E. H. LOVE, *Mathematical Theory of Elasticity*, §§ 61~62.

at the surface in x, y, z directions respectively. Using the relations

$$X_\nu = X_x \cos(x,\ \nu) + X_y \cos(y,\ \nu) + X_z \cos(z,\ \nu),\ \cdots\cdots,$$
$$Y_z = Z_y,\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,$$

and applying Green's theorem on the solid bounded by the surface at the origin and one at a sufficiently large distance from that origin, the integral (1) reduces to

$$\int dt \left\{ \iiint \left(\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} \right) \frac{\partial u}{\partial t} + \left(\frac{\partial X_y}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} \right) \frac{\partial v}{\partial t} \right.$$
$$\left. + \left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} + \frac{\partial Z_z}{\partial z} \right) \frac{\partial w}{\partial t} \right\} dxdydz$$
$$+ \iiint \left[X_x \frac{\partial e_{xx}}{\partial t} + \cdots + \cdots + Y_z \frac{\partial e_{yz}}{\partial t} + \cdots + \cdots \right] dxdydz \qquad (2)$$

The increase of kinetic energy in the body is roughly

$$\int dt \iiint \rho \left(\frac{\partial^2 u}{\partial t^2} \frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial t^2} \frac{\partial v}{\partial t} + \frac{\partial^2 w}{\partial t^2} \frac{\partial w}{\partial t} \right) dxdydz\ . \qquad (3)$$

When the differential equations of wave motion are used, (3) can be replaced by

$$\int dt \iiint \left[\left(\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} \right) \frac{\partial u}{\partial t} + \cdots + \cdots \right] dxdydz\ . \qquad (4)$$

Thus, from (2) and (4) it is possible to express the excess work done by the surface force beyond the increase of the kinetic energy as follows.

$$\int dt \iiint \left[X_x \frac{\partial e_{xx}}{\partial t} + Y_{yy} \frac{\partial e_{yy}}{\partial t} + Z_z \frac{\partial e_{zz}}{\partial t} + Y_z \frac{\partial e_{yz}}{\partial t} + X_z \frac{\partial e_{xz}}{\partial t} + X_y \frac{\partial e_{xy}}{\partial t} \right] dxdydz,$$

or

$$\int dt \iiint [X_x \delta e_{xx} + \cdots + \cdots + Y_z \delta e_{yz} + \cdots +]\, dxdydz\ , \qquad (5)$$

which is possible for any change of stress conditions, $\delta e_{xx}, \cdots\cdots$ being increments of strain components in δt.

Let δT_1 and δU be the respective increments of kinetic energy and the intrinsic energy, both in δt per unit volume. Again, let δW_1 and δQ be the work done by the surface force and the mechanical value of heat supplied in δt. Then, the first law of thermodynamics gives

$$\int_t \iiint (\delta T_1 + \delta U)\, dx dy dz = \int_t (\delta W_1 + \delta Q). \tag{6}$$

By means of (5) we have

$$\begin{aligned} &\int_t \delta W_1 - \int_t \iiint \delta T_1 dx dy dz \\ &\qquad = \int_t \iiint (X_x \delta e_{xx} + \cdots + \cdots + Y_z \delta e_{yz} + \cdots + \cdots) dx dy dz\,. \end{aligned} \tag{7}$$

Thus

$$\int_t \iiint \delta U dx dy dz = \int_t \delta Q + \int_t \iiint (X_x \delta e_{xx} + \cdots + \cdots)\, dx dy dz\,. \tag{8}$$

In the shock problem, which belongs to adiabatic phenomena, it is possible to put

$$\delta Q = 0\,.$$

Hence

$$\delta U = X_x \delta e_{xx} + \cdots + \cdots\,.$$

Since the right-hand side is an exact differential, it is possible to replace U by the potential energy function W showing the energy stored per unit volume of the body by the strain, so that

$$X_x = \frac{\partial W}{\partial e_{xn}}\,,\ \cdots\cdots,\ Y_z = \frac{\partial W}{\partial e_{yz}}\,,\ \cdots\cdots \tag{9}$$

It will be seen from (8) and (9) that our previous conclusions[3] with respect to the energy of elastic waves as well as that of the statical strain is valid.

Since, as shown in the foregoing and in our paper cited there, the wave phenomena are independent of the initial stress, namely they are reckoned from the standard state of a body, it is possible to conclude that, even if a large volume of the earth's crust were uniformly compressed by earth pressure, the type of the radiated waves will always correspond with that of deformation at the origin merely during wave generation without being under any influence of the initial stress. Thus, for longitudinal waves, the rarefactional waves and the compressional waves correspond respectively with waves of pull type and of push type. A similar conclusion naturally holds for transverse waves.

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[3] K. SEZAWA, *Bull. Earthq. Res. Inst.*, **13** (1935), 729~739; K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **14** (1936), 10~17.

3. The simplest possible mechanism of radiation of seismic waves will next be dealt with. It has already been shown that the surface stress at the origin forms an important link in the evolution of seismic waves, so that postulation of the phenomena of break of materials obviously becomes necessary.

We have quite a number of theories on the break or failure of materials, among which the plastic theories due to Mohr, Mises, etc., may be accepted as reliable. From investigations made by Bridgman[4] and Taylor[5], Mises's theory and a few others in the same field appear most adapted to experimental results, but, owing to the simplicity in the form of its expression and to the fact that there is but little difference in their essential characters, Mohr's maximum shear stress theory may temporarily be used. The earth's pressure

$$p_0 = \frac{1}{2}\rho_0 g \frac{a^2-r^2}{r}, \tag{10}$$

where a, ρ_0 are radius and mean density of the Earth, may be transformed to

$$p_0 = \rho g z \tag{11}$$

in the case $r \to a$, where ρ is mean density of the earth in the vicinity of the Earth's surface and z the depth from the same surface. Thus, the vertical component of stress in the earth's crust is

$$Z_z = \rho g z\,. \tag{12}$$

If we use the parameters of elastic deformation (which may not be valid in the actual case but which's may be used for approximately determining its nature), the transverse components of stress are expressed by

$$X_x = Y_y = \frac{\lambda}{\lambda+2\mu}\rho g z\,. \tag{13}$$

This relation exists under the conception that no displacement is possible in horizontal directions in the earth's crust, whence it follows that from Mohr's theory, for instance, a maximum shear stress of amount

$$\tau_{\max} = \frac{Z_z - X_x}{2} = \frac{Z_z - Y_y}{2} = \frac{2\mu}{\lambda+2\mu}\rho g z \tag{14}$$

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[4] P. W. BRIDGMAN, *Proc. Amer. Acad.*, **58** (1927). 166; etc.

[5] G. I. TAYLOR, *Proc. Roy. Soc.*, **145** (1934), 1~17.

acts in a direction 45–degrees to the horizontal plane. From this expression it will be seen that for a great depth, τ_{max} is exceptionally large.

It is however obscure whether or not the earth's crust at a great depth is subjected to such a large shearing stress. My idea at present is that large shear stress in the crust at such a depth under usual conditions is rather improbable. One reason for it is that μ should be very small compared with λ under high pressure, particularly for statical deformation, with the result that τ_{max} is not too large at a great depth. This explanation however may not be convincing, seeing that for μ to be so very small compared with λ is virtually outside the scope of usual experiments. An alternative explanation is that, since the earth's crust at great depth is not liable to slide or to displace, such plastic deformation as is usually known cannot occur.

As a matter of fact nearly every authority on plastic deformation assumes that displacements of material points are permissible even under high hydrostatic pressure, so that the stresses are always accompanied by deformation (or deformation velocity) of material points. Break of materials may rather be accounted for as the result of relative displacements of its configuration points as will be recognized from the experiments under high hydrostatic pressure due to Bridgman, etc. Thus, even should a material, after subjection to hydrostatic pressure without relative displacements of points in it, be released from that pressure, the material would not experience the state of break. It follows then that, since the sliding or displacement of the crust at a great depth is not likely to happen, any actual break of materials under the assumed τ_{max} seems improbable.

In order that τ_{max} which is assumed from the theory of plasticity, may be active in the break of materials in the case of the earth's crust, the crust should become deformable. For that purpose the neighbourhood of the crust under consideration should be semi-fluid or should have faults. The faults themselves therefore do not radiate seismic energy unless they have a certain form of friction, but they are important in the sense that the crustal part in the neighbourhood of fault regions make the shearing stress τ_{max} active. It is evident from the explanation in Section 2 that the relative movements of the crust at a frictionless fault[6] contribute very little to radiation of the seismic energy.

The activity of earth blocks in seismology would be almost mean-

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6) Fault having large friction partake somewhat of the nature of a continuous medium.

ingless were the condition already cited to be disregarded. Thus, if an earth block were completely surrounded by frictionless faults as well as by the land surface, any movement of the block merely with respect to the faults under consideration, would be incapable of radiating sensible seismic energy. If, on the other hand, the block were bounded by faults at some part of its boundary, and were continuous elastically or plastically with outer medium at the remaining part of that boundary, the latter part is liable to be in the actual stressed condition owing to easy deformation of the crust, thus evolving a large amount of seismic energy in the case of break or sliding. It should be borne in mind that local fusion of the earth's crust in the case without faults, has the same effect as that in the case of partly existing faults. Although the mechanism of deep-seated earthquakes may be of this nature, I shall not discuss at present the problem in relation to that matter. My explanation here, at any rate, may appear to differ but little from the prevalent belief. But, the fact that the radiation of seismic waves requires stress change at the origin, is a totally different matter that was arrived at through my theoretical investigations.

It should be borne in mind that, since plastic theories due to Mises, etc., fairly conventionally take volume integrals, the belief, that critical plastic energy is radiated directly from the volume of the body, is liable to be accepted. Actually, however, stresses concentrated locally in the body due to critical plastic deformation cause break of the body at some surfaces (internally) and the surface energy or work given at these surfaces is then radiated in forms of seismic waves, etc. Thus, it is by no means necessary to apply pressure at a free surface to get surface energy or work.

The present study shows that the mechanism of seismic origin is probably of the nature of plastic deformation as the result of a relatively sudden movement and mainly of shearing type. The radiated waves would then be mainly distortional. In the usual earthquakes of relatively shallow origins, transverse waves are predominant rather than the longitudinal. My result seems then to be more in line with Jeffreys's particular conclusion(7) that P waves are merely generated from S waves by reflection at discontinuity surfaces than with other theories.

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7) *Publ. Bureau Centr. Séis., Int.*, A, No. 11 (1935).

14. 震源と震波との關係に就て

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1. 震源さ震波さの關係さいふやうな大きな問題は一朝一夕に決定出來るものではないけれごも，少くさも種々の權威ある力學的研究に基づく機構を作つて見てそれさ抵觸しなければ，これが一つの Particular solution さ考へても差支がないであらう．地震現象そのものを一種の力學的實驗さ見てそれから結論を出すことはむしろ望ましいのであるけれごも，それは單に Boundary surface の問題さも考へられるから，假りにそれが事實を決定するさしても，科學的價値は少いものさいはればならぬ．

2. 最初に，地震波動の勢力が如何なる場所から出るものであるかを調べる爲に，Love の敎科書にある關係さ我々が前に提出した論文さから次の如き結論に達した．地震波の勢力は固體中のある面に應力の或る速さの變化（正負の符號に關係せず）さそれに伴ふ變位が起るさきにその面から發生する．從て單に固體中全體から勢力が出ることはなく，强ひて固體全體を震源さする爲には固體中全體に Disturbances を加へるか取去るかしなければならぬ．固體の一部に Disturbance を作用せしめた結果固體中から勢力が出るさ考へるのは誤りであつて，それは波動が一度傳播してからの結果に過ぎないのである．上の結論の結果を具體的に考へるさ，地震の發生には應力變化を伴ふ斷層のやうなものが必要さなる譯である．從て，斷層面に於て破壞力や固體摩擦が何等働かぬやうな場合は地震發生に殆ご貢獻しない譯になる．多くの人の研究によるさ，かくの如き斷層もあるらしく思はれるけれごも，斯る斷層は震波を大して發散しないものさ考へられる．倘序でに附加へたいことは，地殼に如何に壓力が加はつてをつても，地震縱波の押しさ引きさは夫々密波さ粗波さに相當することである．これは震源の面に力を加へることゝ力を取除くことゝは單に應力の符號の問題に過ぎぬさいふ理論さ密接の關係がある．

3. 次に如何にして震源に應力變化が起り得るかさいふ問題を吟味して見る．これには種々の事柄が考へられるけれごも，前述の如く力學上に根據のない說明法では問題の決定性が少い．たさひ地球化學の考へを用ひても，地震波の出る直接の機構には一度尤もらしい力學を通さなければ，彈性波の如き地震波の勢力が作られないのは明かである．最も容易に不衡力及び應力變化を考へ易いのはプラスチック性の理論（粘性液體の問題さ混同せざる）である．これには種々有名な人の研究があり，現在ではその解き得る極限に達した感があるが，さもかくそれによることにする．しかしこれ等の理論は物質點が互に距離の變化をなし得る場合についてゞある．然るに一方には高氣壓に於けるプラスチック性の實驗がある．之等の結果を綜合して見るさ，地殼內のある面で適當の速度の應力變化さそれに從て起る勢力發散には，物質の變位性が必要になるのである．この爲には地殼の一部に既存の斷層があり，その附近の地殼が破壞（主さしてプラスチックに）すればよいのである．しかしその置換さして地殼の一部が熱の關係で熔ければ物質の變位性が可能になり得る譯である．但しそれには熱力學及物性論から多少の難點が存在する．只今のところでは斷層のみを考へて置くことにして，さて地殼のブロックの一部分が既存斷層に境され，殘りの部分が外部さ固體的に續いてゐるさすれば，斯るブロックは，外さ連つてゐる部分の破壞に依つて大きな震波勢力を發散し得ることになる．しかしそのブロックが斷層により完全に外部さ境されてをれば，斯る斷層が大した勢力を出し得ないのは明かである．筆者は七八年前に一里四方もある豆腐の如き剛性の物體が方形のまゝ傾く爲には，その周圍の固體摩擦だけにしても如

何に小さいかといふことを以て他の人のブロック理論（筆者のブロック理論は關東地震の翌年に出してある）を吟味したことがあるが，上述の既存斷層とは斯る極端なるブロックを形成する斷層を指すのである．

茲に注意したいことは，Mises 等のプラスチック論で，或容積内のプラスチック的の勢力が一定値に達したときに，プラスチックの辷りが起る形となつてゐる爲に，この辷りが起る場合に上述の容積中の勢力がその儘で彈性波として送られるものと誤解されることである．Mises 等の容積積分形が既に相當に Conventional のものであるから問題にならぬことは明であるが，たとひ之が本當に容積積分であるとしても，波動の起るのは，實際はプラスチックの或極限に達したときに，固體に大なる剪力ができ，その結果特定の場所に應力の集中を生じ，こゝが破壞を起すのである．而して波の勢力はこの破壞面（理想的にいへば）から發散するのである．Surface energy 又は Surface work とはこの事であつて，何も自由面を壓力で押す必要はないのである．

以上述べた考方は，所謂 Elements を取つたものであつて，このやうな Elements を數多く結合して所謂 Coupled action となすことは複雜なだけであつて考方としては容易である．そのやうにしても特に新しい事柄がわかるとは考へられないから，こゝでは Elements だけを取扱つておいた次第である．

尙，地震波に橫波及び同種の波の多いことはプラスチック性の說明に一層好都合な譯である．深發地震についてはその說明に多少困難な點もあるやうであるが，その問題には餘り觸れないことゝして置いた．

終りに附加へたいことは，この論文では地震波の出る機構が固體の破壞の如く固體の中の面に關係することを力說してあるけれども，彈性波が固體の破壞のみに原因するといふ說ではないことである．波動が出る爲には必ず物質の慣性力が直接又は間接に誘起しなければならぬことは力學の敎へる所である．その爲には或容積が急にふくれるとか，物が衝突するとか，固體面に急に力が加はるとかいふことがあればよい．しかし地殼の場合に最も考へ易いのは固體の破壞又は類似の事，卽ち固體の中の面に應力の急變化が起るといふことである．そのやうな意味に於て破壞といふ言葉を委しい說明もなく用ひたのである．