16. *Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure.*

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1. *Introduction.*

After several investigations since last year we have qualitatively ascertained the nature of energy dissipation in seismic vibrations of a structure. Since in spite of its importance on the earthquake-proof construction the problem hardly received the attention of investigators until recently and furthermore the treatment of the problem was not too easy, we took a somewhat approximate method of solution, particularly in the form of dissipating waves, with a view to obtaining qualitative results as simply as possible. As a matter of fact, the scattered waves that dissipate from the structure into the ground are longitudinal as well as transverse waves, associated with surface waves, the consideration of all kinds of dissipating waves in an exact way being therefore practically impossible. We made consequently such a rough approximation with respect to the form of scattered longitudinal waves as taking $\varDelta=a\cos\theta\,\exp.i(pt-hr)/r$ instead of its more accurate form $\varDelta=a\cos\theta\,\exp.i(pt-hr)\{1/r+1/i\,hr^2\}$, under the assumed condition that the surface waves are temporarily omitted. The next step was to use a rather general case, $y_{x=0}=u_{0x=0}+u_{r=\varepsilon,\,\theta=0}$ and $\partial u_0/\partial x_{x=0}=a$ *finite*, in lieu of its more convenient condition, $y_{x=0}=u_{0\,x=0}+u'_{0\,x=0}+u_{r=\varepsilon,\,\theta=0}$ and $\partial(u_0+u'_0)/\partial x_{x=0}=0$, where u'_0 is the displacement of normally reflected waves. Another important assumption was to put $\lambda=14\mu$ in place of taking arbitrary λ and μ. Furthermore, we neglected higher orders of such apparently small quantities as ε, the result of which is sometimes contradictory to the assumed condition of the problem. Generally speaking, however, these assumptions are not likely to affect the main charater of the problem resulting from its solution, owing probably to the reason that the assumed conditions deviate little, physically, from the actual ones.[1] Since, on the other hand, our study is being

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 682~683, 687, 690; 918~919.

understood more or less by many investigators quite recently and the assumptions above cited are liable to be criticized by them, we are now in position to improve the problem in a stricter way. Our new study attempted from this intention, shows that the calculation under the improved assumptions is not too much difficult and furthermore gives rise to such new results as incapable of being found in the previous study. The theoretical part of the improved problem will be shown in this paper.

2. *A simplest structure subjected to incident longitudinal waves.*[2)]

Let the incident and reflected longitudinal waves with their displacements orientated vertically be

$$u_0 = e^{i(pt+hx)}, \qquad u_0' = e^{i(pt-hx)}, \tag{1}$$

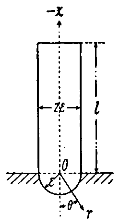

Fig. 1.

where $h^2 = \rho p^2/(\lambda+2\mu)$. The scattered longitudinal as well as transverse waves and the vibration of the structure are expressed by

$$\left.\begin{aligned} \Delta &= a\cos\theta\, e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \\ \varpi &= -\beta\sin\theta\, e^{i(pt-kr)}\left(\frac{1}{r}+\frac{1}{ikr^2}\right), \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} u_1 &= -\frac{a}{h^2}\cos\theta\frac{d}{dr}e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \\ v_1 &= \frac{a}{h^2}\sin\theta\frac{1}{r}e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} u_2 &= -\frac{4\beta}{k^2}\cos\theta\frac{1}{r}e^{i(pt-kr)}\left(\frac{1}{r}+\frac{1}{ikr^2}\right), \\ v_2 &= \frac{2\beta}{k^2}\sin\theta\frac{1}{r}\frac{d}{dr}e^{i(pt-kr)}\left(1+\frac{1}{ikr}\right), \end{aligned}\right\} \tag{4}$$

$$u' = Be^{i(pt+h'x)} + Ce^{i(pt-h'x)}, \tag{5}$$

$$h^2 = \frac{\rho p^2}{\lambda+2\mu}, \qquad k^2 = \frac{\rho p^2}{\mu}, \qquad h'^2 = \frac{\rho' p^2}{E}, \tag{6}$$

where u_1, v_1 and u_2, v_2 satisfy accurately the equations of motion for both longitudinal and transverse bodily waves and u' that for the vibration of the structure; ρ, λ, μ; ρ', E being densities and elastic con-

2) *Bull. Earthq. Res. Inst.*, **13** (1935), 682.

stants of the earth and the structure respectively.

Let us assume that the structure is a uniform circular cylinder of radius ε and of length l, then the boundary conditions are such that, at the upper end of the structure, $x=-l$,

$$\frac{\partial u'}{\partial x}=0; \tag{7}$$

and at its lower end, $x=0$, $r\to\varepsilon$,

$$(u_1+u_2)_{\max.}=-(v_1+v_2)_{\max.}, \tag{8}$$

$$u'=u_0+u_0'+(u_1+u_2)_{\theta=0}, \tag{9}$$

$$E\pi\varepsilon^2\frac{\partial u'}{\partial x}=\int_{\varphi=0}^{\varphi=2\pi}\int_{\theta=0}^{\theta=\frac{\pi}{2}}\mu\left\{\frac{\partial(v_1+v_2)}{\partial r}-\frac{v_1+v_2}{r}+\frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\theta}\right\}r_2\sin^2\theta\, d\varphi d\theta\,,$$

$$+\int_{\varphi=0}^{\varphi=2\pi}\int_{\theta=0}^{\theta=\frac{\pi}{2}}\left\{\lambda\Delta+2\mu\frac{\partial(u_1+u_2)}{\partial r}\right\}r^2\sin\theta\cos\theta\, d\varphi d\theta\,. \tag{10}$$

Substituting the solutions in these conditions and taking the real part only, we get

$$u_0=\cos(pt+hx)\,, \tag{1'}$$

$$u'=4\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cosh'(x+l)\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{11}$$

where

$$\left.\begin{aligned}
&P=2\Gamma_1\cos h'l+\frac{3E\varepsilon}{\mu l}\Lambda_1 h'l\sinh'l\,,\\
&Q=2\Gamma_2\cos h'l+\frac{3E\varepsilon}{\mu l}\Lambda_2 h'l\sinh'l\,,\\
&\Gamma_1=3+\nu(h'l)^2\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right),\\
&\Gamma_2=\sqrt{\nu}\,(h'l)\left\{3\left(\sqrt{\frac{\lambda}{\mu}+2}+1\right)+\nu(h'l)^2\left(\frac{\lambda}{\mu}+2-2\sqrt{\frac{\lambda}{\mu}+2}\right)\right\},\\
&\Lambda_1=\frac{\mu}{\lambda+2\mu}+2-\nu(h'l)^2\,,\\
&\Lambda_2=\sqrt{\nu}\,(h'l)\left(\sqrt{\frac{\mu}{\lambda+2\mu}}+2\right),\\
&\qquad\nu=\frac{\rho E\varepsilon^2}{\rho'(\lambda+2\mu)l^2}\,.
\end{aligned}\right\} \tag{12}$$

The right-hand term under the root sign of the denominator of (11) represents the effect of the dissipation of energy scattered as seismic waves. The larger the absolute value of the term under consideration, the larger will be the decrease in the amplitudes of vibration at the period corresponding to the resonance condition, $2\Gamma_1 \cosh h'l + (3E\varepsilon/\mu l)\cdot \Lambda_1 h'l \sinh h'l = 0$, of the case without dissipation of energy. It should be remembered that the equation showing the resonance condition in the present accurate calculation is somewhat modified from the previous one. Numerical examples of the result of the present as well as following sections will be left to next occasion.

3. *A tall structure with rigid floors subjected to incident transverse waves.*[3]

Let $\rho, \lambda, \mu; \rho'(=m/al_1)$, $G(=12\cdot 4Ej^2/l_1^2)$ be the density and elastic constants of the earth, and the effective density and the effective rigidity of the structure, where E, j, l_1, a are Young's modulus, radius of gyration of section, length and total sum of the sectional areas of the columns between two adjacent floors respectively. Provided the incident transverse waves in the earth are transmitted vertically upwards, the solutions of the incidednt as well as reflected waves, and those of the structural vibrations are as follows:

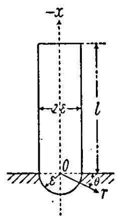

Fig. 2.

$$u_0 = e^{i(pt+kx)}, \qquad u_0' = e^{i(pt-kx)}, \tag{13}$$

$$u' = Be^{i(pt+k'x)} + Ce^{i(pt-k'x)}, \tag{14}$$

where $k^2 = \rho p^2/\mu$, $k'^2 = \rho' p^2/G$, the forms of scattered waves being the same as those in (3), (4). The boundary conditions are expressed by

$$\frac{\partial u'}{\partial x} = 0 \tag{15}$$

at $x = -l$, and

$$(u_1+u_2)_{\max.} = -(v_1+v_2)_{\max.}, \tag{16}$$

$$u' = u_0 + u_0' + (u_1+u_2)_{\theta=0}, \tag{17}$$

$$G\pi\varepsilon^2 \frac{\partial u'}{\partial x} = \int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi} \mu\left\{\frac{\partial(v_1+v_2)}{\partial r} - \frac{v_1+v_2}{r} + \frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\theta}\right\} r^2 \sin^2\theta\, d\varphi d\theta$$

$$+ \int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi} \left\{\lambda\Delta + 2\mu\frac{\partial(u_1+u_2)}{\partial r}\right\} r^2 \sin\theta \cos\theta\, d\varphi d\theta, \tag{18}$$

3) *Bull. Earthq. Res. Inst.*, **13** (1935), 687.

at $x=0$, $r=\varepsilon$. Substituting the solutions in these conditions and taking real parts, we obtain

$$u_0=\cos(pt+kx), \tag{13'}$$

$$u'=4\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos k'(x+l)\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{19}$$

where

$$\left.\begin{aligned}
P&=2\Gamma_1\cos k'l+\frac{3G\varepsilon}{\mu l}\Lambda_1 k'l\sin k'l,\\
Q&=2\Gamma_2\cos k'l+\frac{3G\varepsilon}{\mu l}\Lambda_2 k'l\sin k'l,\\
\Gamma_1&=3\left(\frac{\lambda}{\mu}+2\right)+\nu(k'l)^2\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right),\\
\Gamma_2&=\sqrt{\nu}\,(k'l)\left\{3\left(\frac{\lambda}{\mu}+2+\sqrt{\frac{\lambda}{\mu}+2}\right)+\nu(k'l)^2\left(\sqrt{\frac{\lambda}{\mu}+2}-2\right)\right\},\\
\Lambda_1&=\left(\frac{2\lambda}{\mu}+5\right)-\nu(k'l)^2,\\
\Lambda_2&=\sqrt{\nu}\,(k'l)\left(2\sqrt{\frac{\lambda}{\mu}+2}+1\right),\\
\nu&=\frac{\rho G\varepsilon^2}{\rho'\mu l^2}.
\end{aligned}\right\} \tag{20}$$

The solutions show certain features of vibrations similar to those in the previous case.

4. *A tall structure with flexible floors subjected to incident transverse waves.*[4)]

Let ρ, λ, μ; $\rho'(=m/al_1)$, j be the density and elastic constants of the earth, the effective density of the structure and the radius of gyration of a section of a column, where m, l_1, a are mass concentrated on every floor, and the length and total sum of the sectional areas of columns between two adjacent floors respectively. From the equation of motion of the structure

$$\frac{\partial^2 y}{\partial t^2}+\frac{Ej^2}{\rho'}\frac{\partial^4 y}{\partial x^4}=0, \tag{21}$$

4) *Bull. Earthq. Res. Inst.*, **13** (1935), 690.

we find

$$y=e^{ipt}\left[Ae^{i\sqrt{p}cx}+Be^{-i\sqrt{p}cx}+Ce^{\sqrt{p}cx}+De^{-\sqrt{p}cx}\right], \tag{22}$$

in which $c=(\rho'/Ej^2)^{\frac{1}{4}}$. The forms of incident waves, u_0, plane reflected waves u_0', and scattered waves, u_1, v_1; u_2, v_2 are respectively the same as those in (13), (3), (4). Using the same figure as Fig. 2, the boundary conditions are such that, at $x=-l$,

$$\frac{\partial^2 y}{\partial x^2}=0, \qquad \frac{\partial^3 y}{\partial x^3}=0; \tag{23), (24}$$

and, at $x=0$, $r=\varepsilon$,

$$(u_1+u_2)_{\max.}=-(v_1+v_2)_{\max.}, \tag{25}$$

$$y=u_0+u_0'+(u_1+u_2)_{\theta=0}, \tag{26}$$

$$\frac{\partial y}{\partial x}=0, \tag{27}$$

$$-E\pi\varepsilon^2 j^2\frac{\partial^3 y}{\partial x^3}=\int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi}\mu\left\{\frac{\partial(v_1+v_2)}{\partial r}-\frac{(v_1+v_2)}{r}+\frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\theta}\right\}r^2\sin^2\theta\, d\varphi d\theta$$

$$+\int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi}\left\{\lambda\Delta+2\mu\frac{\partial(u_1+u_2)}{\partial r}\right\}r^2\sin\theta\cos\theta\, d\varphi d\theta. \tag{28}$$

The final solutions in real forms are as follows:

$$u_0=\cos(pt+kx), \tag{13''}$$

$$y=M\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left\{pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right\}, \tag{29}$$

where

$$\left.\begin{aligned}
M=&\cos\sqrt{p}\,cl\cosh\sqrt{p}\,c(x+l)+\cosh\sqrt{p}\,cl\cos\sqrt{p}\,c(x+l)\\
&+\sin\sqrt{p}\,cl\sinh\sqrt{p}\,c(x+l)-\sinh\sqrt{p}\,cl\sin\sqrt{p}\,c(x+l)\\
&+\cos\sqrt{p}\,cx+\cosh\sqrt{p}\,cx,\\
P=&2(\cos\sqrt{p}\,cl\cosh\sqrt{p}\,cl+1)\Gamma_1+\frac{3E\varepsilon j^2}{\mu l^3}(\sqrt{p}\,cl)^3\\
&.(\cos\sqrt{p}\,cl\sinh\sqrt{p}\,cl+\sin\sqrt{p}\,cl\cosh\sqrt{p}\,cl)\Delta_1,\\
Q=&2(\cos\sqrt{p}\,cl\cosh\sqrt{p}\,cl+1)\Gamma_2+\frac{3E\varepsilon j^2}{\mu l^3}(\sqrt{p}\,cl)^3\\
&.(\cos\sqrt{p}\,cl\sinh\sqrt{p}\,cl+\sin\sqrt{p}\,cl\cosh\sqrt{p}\,cl)\Delta_2,
\end{aligned}\right\} \tag{30}$$

$$\left.\begin{aligned}
&\Gamma_1=3\left(\frac{\lambda}{\mu}+2\right)+\nu(\sqrt{p}\,cl)^4\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right),\\
&\Gamma_2=\sqrt{\nu}\,(\sqrt{p}\,cl)^2\left\{3\left(\frac{\lambda}{\mu}+2+\sqrt{\frac{\lambda}{\mu}+2}\right)+\nu(\sqrt{p}\,cl)^4\left(\sqrt{\frac{\lambda}{\mu}+2}-2\right),\right.\\
&\varDelta_1=\left(\frac{2\lambda}{\mu}+5\right)-\nu(\sqrt{p}\,cl)^4,\\
&\varDelta_2=\sqrt{\nu}\,(\sqrt{p}\,cl)^2\left(2\sqrt{\frac{\lambda}{\mu}+2}+1\right),\\
&\qquad\nu=\frac{\rho E j^2\varepsilon^2}{\rho'\mu l^4}.
\end{aligned}\right\}\tag{31}$$

The resonance condition is specified by $P=0$, the displacement of the structure then assuming the form

$$y=\frac{M\sqrt{\Gamma_1^2+\Gamma_2^2}}{Q}\cos\left\{pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}\pm\frac{\pi}{2}\right\}.$$

5. *General theory in the case of a framed structure.*

Incident, reflected, and scattered waves are assumed to have the same forms as those in the preceding two sections. The solution of the vibratory motion of columns between $(s-1)$th and sth floors is determined from the differential equation of the type, $\partial^4 y_s/\partial x_s^4=0$, as follows:

$$y_s=(A_s+B_s x_s+C_s x_s^2+D_s x_s^3)e^{ipt}\qquad(s=1,\ 2,\ \cdots,\ n),\tag{32}$$

where x_s is the coordinate of a point of the columns measured negatively from their respective lower ends, y_s the horizontal deflection of point x_s. Let l_s, E_s, I_s be the length, Young's modulus, and the sum of the moments of inertia of cross sections of all the columns between $(s-1)$th and the sth floors, and m_s mass concentrated at sth floor. Then, imagining a figure similar to Fig. 2, the boundary conditions are as follows.

$$x_n=-l_n;\qquad E_n I_n\frac{\partial^3 y_n}{\partial x_n^3}+m_n\frac{\partial^2 y_n}{\partial t^2}=0,\tag{33}$$

$$\frac{\partial y_n}{\partial x_n}=0\ \text{(rigid floors)},\quad\text{or}\quad\frac{\partial^2 y_n}{\partial x_n^2}=0\ \text{(flexible floors)},\tag{34}$$

$$x_s=-l_s,\ x_{s+1}=0;\ E_{s+1}I_{s+1}\frac{\partial^3 y_{s+1}}{\partial x_{s+1}^3}=E_s I_s\frac{\partial^3 y_s}{\partial x_s^3}+m_s\frac{\partial^2 y_s}{\partial t^2},\tag{35}$$

$$y_s = y_{s+1}\,, \tag{36}$$

$$\frac{\partial y_s}{\partial x_s}=0\,,\quad \frac{\partial y_{s+1}}{\partial x_{s+1}}=0 \text{ (rigid floors),}$$

$$\text{or } \frac{\partial y_s}{\partial x_s}=\frac{\partial y_{s+1}}{\partial x_{s+1}}\,,\quad E_{s+1}I_{s+1}\frac{\partial^2 y_{s+1}}{\partial x_{s+1}^2}=E_sI_s\frac{\partial^2 y_s}{\partial x_s^2} \text{ (flexible floors),} \tag{37}$$

$$(s=1,\ 2,\ \cdots,\ n-1)$$

$$x_1=0\,,\quad r=\varepsilon;\quad (u_1+u_2)_{\max.}=-(v_1+v_2)_{\max.}\,, \tag{38}$$

$$y_1=u_0+u_0'+(u_1+u_2)_{r=\varepsilon,\ \theta=0}\,, \tag{39}$$

$$\frac{\partial y_1}{\partial x_1}=0 \text{ (clamped base), or } \frac{\partial^2 y_1}{\partial x_1^2}=0 \text{ (hinged base),} \tag{40}$$

$$-E\pi\varepsilon^2 j^2\frac{\partial^3 y_1}{\partial x_1^3}=\int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi}\mu\left\{\frac{\partial(v_1+v_2)}{\partial r}-\frac{(v_1+v_2)}{r}+\frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\theta}\right\}r^2\sin^2\theta\,d\varphi d\theta$$
$$+\int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=\pi}\left\{\lambda\Delta+2\mu\frac{\partial(u_1+u_2)}{\partial r}\right\}r^2\sin\theta\cos\theta\,d\varphi d\theta\,. \tag{41}$$

Substituting the solutions for incident, reflected, and scattered waves, as well as for the vibratory motion of all the columns in these boundary conditions in respective cases it is possible to obtain the deflections or bending moments of columns.

From the relations (38), (39), (41), which are connected with the nature of dissipation, we obtain

$$\Gamma A_1+\frac{9E_1 j_1^2\varepsilon}{\mu}\Lambda D_1=2\Gamma, \tag{42}$$

where

$$\left.\begin{aligned}
&\Gamma=\Gamma_1+i\Gamma_2\,,\quad \Lambda=\Lambda_1+i\Lambda_2\,,\\
&\Gamma_1=\frac{3(\lambda+2\mu)}{\mu}+\nu\gamma_1\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda+2\mu}{\mu}}\right),\\
&\Gamma_2=\sqrt{\nu\gamma_1}\left\{3\left(\frac{\lambda+2\mu}{\mu}+\sqrt{\frac{\lambda+2\mu}{\mu}}\right)+\nu\gamma_1\left(\sqrt{\frac{\lambda+2\mu}{\mu}}-2\right)\right\},\\
&\Lambda_1=\frac{2\lambda+5\mu}{\mu}-\nu\gamma_1\,,\\
&\Lambda_2=\sqrt{\nu\gamma_1}\left(2\sqrt{\frac{\lambda+2\mu}{\mu}}+1\right),\\
&\qquad\nu=\frac{E_1\rho I_1\varepsilon^2}{\mu m_1 l_1^3}\,.
\end{aligned}\right\} \tag{43}$$

These relations hold in any case of a framed structure irrespective of the conditions of columns at the floors as well as at the base. It should be borne in mind that the symbols

$$\left.\begin{array}{c} \gamma_s=\dfrac{m_s p^2 l_s^3}{E_s I_s}\ (s=1,2,\cdots,n)\,, \qquad \eta_s=\dfrac{E_{s+1}I_{s+1}l_s^3}{E_s I_s l_{s+1}^3}\,, \\ \xi_s=\dfrac{l_{s+1}}{l_s}\ (s=1,2,\cdots,n-1) \end{array}\right\} \tag{44}$$

are commonly used in (43) and in treatments which will appear hereafter. Again, in the subsequent sections, we shall mainly show the general expressions of A_s, B_s, C_s, D_s, by means of which it is possible to write the formulae for the deflections (or bending moments) of columns. The expressions of bending moments will also be added, though for simplest cases, with a view to getting a quick application of the result.

6. *A structure with rigid floors and clamped base.*[5]

From (33), (34), (35), (36), (37), (39) we find the relations such that[6]

$$\left.\begin{array}{l} B_s=0\,,\quad C_s=\dfrac{3}{2}l_s D_s\,,\quad (s=1,2,3,\cdots,n) \\ A_{s+1}=\dfrac{1}{2}(2A_s+l_s^3D_s),\ (s=1,2,3,\cdots,n-1) \\ D_{s+1}=\dfrac{E_sI_s}{12E_{s+1}I_{s+1}l_s^3}\left\{-2\gamma_sA_s+(12-\gamma_s)l_s^3D_s\right\}, \\ \qquad\qquad\qquad\qquad (s=1,2,3,\cdots,n-1) \\ A_n=\dfrac{12-\gamma_n}{2\gamma_n}l_n^3D_n\,. \end{array}\right\} \tag{45}$$

By means of these relations and (42), we obtain the absolute values of A_s, B_s, C_s, D_s, the final result being shown below. The incident waves are assumed to be of the type $e^{i(pt+kx)}$ in determining the coefficients under consideration, whereas they should take the form $\cos(pt+kx)$ in the solution of bending moments described here.

(i) $n=1$;

$$\left.\begin{array}{c} A_1=\dfrac{2(12-\gamma_1)\Gamma}{\Phi}\,,\quad B_1=0\,,\quad D_1=\dfrac{4\gamma_1\Gamma}{\Phi l_1^3}\,,\quad C_1=\dfrac{3}{2}l_1D_1\,, \\ \Phi=(12-\gamma_1)\Gamma+\dfrac{18E_1j_1^2\varepsilon}{\mu l_1^3}\gamma_1\Delta\,, \end{array}\right\} \tag{46}$$

5) *Bull. Earthq. Res. Inst.*, **13** (1935), 700.
6) *Ditto*, 919.

$$EI\frac{\partial^2 y}{\partial x^2}\Big/ 2\,p^2 ml=\frac{6\sqrt{\Gamma_1^2+\Gamma_2^2}}{\sqrt{P^2+Q^2}}\left(1+\frac{2x}{l}\right)\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \quad (47)$$

where $E=E_1$, $I=I_1$, $m=m_1$, $l=l_1$, and

$$\left.\begin{aligned} P&=(12-\gamma)\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_1,\\ Q&=(12-\gamma)\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_2.\end{aligned}\right\} \quad (48)$$

(ii) $n=2$;

$$\left.\begin{aligned} &A_1=2\left\{12\gamma_2\eta_1-(12-\gamma_1)(12-\gamma_2)\right\}\frac{\Gamma}{\Phi},\\ &D_1=-4\left\{12\gamma_2\eta_1+\gamma_1(12-\gamma_2)\right\}\frac{\Gamma}{l_1^3\Phi},\\ &A_2=-2.12(12-\gamma_2)\frac{\Gamma}{\Phi}, \quad D_2=-4.12\gamma_2\frac{\Gamma}{l_2^3\Phi},\\ &B_1=B_2=0, \quad C_1=\frac{3}{2}l_1D_1, \quad C_2=\frac{3}{2}l_2D_2,\\ &\Phi=\left\{12\gamma_2\eta_1-(12-\gamma_1)(12-\gamma_2)\right\}\Gamma-\frac{18E_1j_1^2\varepsilon}{\mu l_1^3}\left\{12\gamma_2\eta_1+\gamma_1(12-\gamma_2)\right\}\Lambda.\end{aligned}\right\} \quad (49)$$

$$\left.\begin{aligned} EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/ 2\,p^2ml&=6(\gamma-24)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left(1+\frac{2x_1}{l}\right)\\ &\quad .\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\ EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/ 2\,p^2ml&=6.12\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left(1+\frac{2x_2}{l}\right)\\ &\quad .\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\end{aligned}\right\} \quad (50)$$

in which $E=E_1=E_2$, $I=I_1=I_2$, $m=m_1=m_2$, $l=l_1=l_2$, and

$$\left.\begin{aligned} P&=\left\{(12-\gamma)^2-12\gamma\right\}\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma(24-\gamma)\Lambda_1,\\ Q&=\left\{(12-\gamma)^2-12\gamma\right\}\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma(24-\gamma)\Lambda_2.\end{aligned}\right\} \quad (51)$$

(iii) $n=3$;

$$
\left.\begin{aligned}
A_1&=2\Big[12\eta_2\gamma_3\Big\{12\eta_1+(12-\gamma_1)\Big\}\\
&\qquad\qquad+(12-\gamma_3)\Big\{12\eta_1\gamma_2-(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\frac{\Gamma}{\Phi},\\
D_1&=-4\Big[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\frac{\Gamma}{l_1^3\Phi},\\
A_2&=2.12\Big\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\Big\}\frac{\Gamma}{\Phi},\\
D_2&=-4.12\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\frac{\Gamma}{l_2^3\Phi},\\
A_3&=-2.12^2(12-\gamma_3)\frac{\Gamma}{\Phi},\qquad D_3=-4.12^2\gamma_3\frac{\Gamma}{\Phi},\\
B_1&=B_2=B_3=0,\quad C_1=\frac{3}{2}l_1D_1,\quad C_2=\frac{3}{2}l_2D_2,\quad C_3=\frac{3}{2}l_3D_3,
\end{aligned}\right\}\tag{52}
$$

$$
\begin{aligned}
\Phi=&\Big[12\eta_2\gamma_3\Big\{12\eta_1+(12-\gamma_1)\Big\}+(12-\gamma_3)\Big\{12\eta_1\gamma_2-(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\Gamma\\
&-\frac{18E_1j_1^2\varepsilon}{\mu l_1^3}\Big[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\Lambda.
\end{aligned}\tag{53}
$$

$$
\left.\begin{aligned}
EI\frac{\partial^2y_1}{\partial x_1^2}\Big/2\,p^2ml&=6(12-\gamma)(36-\gamma)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left(1+\frac{2x_1}{l}\right)\\
&\qquad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\
EI\frac{\partial^2y_2}{\partial x_2^2}\Big/2\,p^2ml&=6.12(24-\gamma)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left(1+\frac{2x_2}{l}\right)\\
&\qquad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\
EI\frac{\partial^2y_3}{\partial x_3^2}\Big/2\,p^2ml&=6.12^2\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\left(1+\frac{2x_3}{l}\right)\\
&\qquad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),
\end{aligned}\right\}\tag{54}
$$

in which $E=E_1=E_2=E_3$, $I=I_1=I_2=I_3$, $m=m_1=m_2=m_3$, $l=l_1=l_2=l_3$, and

$$\left.\begin{aligned} P&=(1728-864\gamma+60\gamma^2-\gamma^3)\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma(12-\gamma)(36-\gamma)\Lambda_1\,, \\ Q&=(1728-864\gamma+60\gamma^2-\gamma^3)\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\gamma(12-\gamma)(36-\gamma)\Lambda_2\,. \end{aligned}\right\} \quad (55)$$

(iv) The cases, for which n is greater than 4, will be dealt with separately.

7. *A structure with flexible floors and clamped base.*[7]

In this case too we assume the incident waves, $e^{i(pt+kx)}$, in the determination of A_s, B_s, C_s, D_s, and the ones, $\cos(pt+kx)$, in the solution of bending moments.

(i) $n=1$;

$$\left.\begin{aligned} &A_1=4(3-\gamma_1)\frac{\Gamma}{\Phi}\,,\quad D_1=2\gamma_1\frac{\Gamma}{l_1^3\Phi}\,,\quad B_1=0\,,\quad C_1=3l_1D_1\,, \\ &\Phi=2(3-\gamma_1)\Gamma+\frac{9E_1j_1^2\varepsilon}{\mu l_1^3}\gamma_1\Lambda\,. \end{aligned}\right\} \quad (56)$$

$$EI\frac{\partial^2 y}{\partial x^2}\bigg/2\,p^2ml=6\left(1+\frac{x}{l}\right)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \quad (57)$$

where $E=E_1$, $I=I_1$, $\cdots$, and

$$\left.\begin{aligned} P&=2(3-\gamma)\Gamma_1+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_1\,, \\ Q&=2(3-\gamma)\Gamma_2+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_2\,. \end{aligned}\right\} \quad (58)$$

(ii) $n=2$;

$$\left.\begin{aligned} &A_1=2\left[3\eta\gamma_2\left\{\xi^2(12-\gamma_1)+4(1+3\xi)\right\}-4(3-\gamma_1)(3-\gamma_2)\right]\frac{\Gamma}{\Phi}\,, \\ &\qquad B_1=0\,, \\ &C_1=-6\left\{3\eta\gamma_2(2+2\xi-\xi^2\gamma_1)+2\gamma_1(3-\gamma_2)\right\}\frac{\Gamma}{l_1^2\Phi}\,, \\ &D_1=-4\left\{3\eta\gamma_2(1-\xi^2\gamma_1)+\gamma_1(3-\gamma_2)\right\}\frac{\Gamma}{l_1^3\Phi}\,, \\ &A_2=12\left\{3\xi\eta\gamma_2(1+2\xi)-2(3-\gamma_2)\right\}\frac{\Gamma}{\Phi}\,, \\ &B_2=12\xi\left\{3\eta\gamma_2(1+2\xi)+\gamma_1(3-\gamma_2)\right\}\frac{\Gamma}{l_2\Phi}\,, \\ &C_2=-18\gamma_2(2+\xi\gamma_1)\frac{\Gamma}{l_2^2\Phi}\,,\qquad D_2=-6\gamma_2(2+\xi\gamma_1)\frac{\Gamma}{l_2^3\Phi}\,, \end{aligned}\right\} \quad (59)$$

7) *Bull. Earthq. Res. Inst.*, **13** (1935), 704.

$$\Phi=\left[3\eta\gamma_2\left\{\xi^2(12-\gamma_1)+4(1+3\xi)\right\}-4(3-\gamma_1)(3-\gamma_2)\right]\Gamma$$
$$-\frac{18E_1j_1^2\varepsilon}{\mu l_1^3}\left\{3\eta\gamma_2(1-\xi^2\gamma_1)+\gamma_1(3-\gamma_2)\right\}\varDelta, \tag{60}$$

where $\xi=\xi_1$, $\eta=\eta_1$,

$$\left.\begin{aligned} EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/2p^2ml&=6\left\{(18-5\gamma)+4(3-2\gamma)\frac{x_1}{l}\right\}\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\\ &\quad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\ EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/2p^2ml&=18(2+\gamma)\left(1+\frac{x_2}{l}\right)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\\ &\quad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \end{aligned}\right\} \tag{61}$$

in which $E=E_1=E_2$, $I=I_1=I_2$, ... ,

$$\left.\begin{aligned} P&=(36-108\gamma+7\gamma^2)\Gamma_1+\frac{36Ej^2\varepsilon}{\mu l^3}(3-2\gamma)\varDelta_1,\\ Q&=(36-108\gamma+7\gamma^2)\Gamma_2+\frac{36Ej^2\varepsilon}{\mu l^3}(3-2\gamma)\varDelta_2. \end{aligned}\right\} \tag{62}$$

(iii) $n=3$; the general case being omitted, it is assumed that $E=E_1=E_2=E_3$, $I=I_1=I_2=I_3$, ...

$$\left.\begin{aligned} A_1&=4(108-1296\gamma+393\gamma^2-13\gamma^3)\frac{\Gamma}{\Phi}, & B_1&=0,\\ C_1&=6\gamma(216-294\gamma+19\gamma^2)\frac{\Gamma}{l^2\Phi}, & D_1&=2\gamma(108-372\gamma+31\gamma^2)\frac{\Gamma}{l^3\Phi},\\ A_2&=24(18-171\gamma+23\gamma^2)\frac{\Gamma}{\Phi}, & B_2&=-6\gamma(324-216\gamma+7\gamma^2)\frac{\Gamma}{l\Phi},\\ C_2&=36\gamma(18+13\gamma-2\gamma^2)\frac{\Gamma}{l^2\Phi}, & D_2&=6\gamma(24-10\gamma-5\gamma^2)\frac{\Gamma}{l^3\Phi},\\ A_3&=72(6-23\gamma-3\gamma^2)\frac{\Gamma}{\Phi}, & B_3&=-12\gamma(234-15\gamma-\gamma^2)\frac{\Gamma}{l\Phi},\\ C_3&=18\gamma(12+36\gamma+\gamma^2)\frac{\Gamma}{l^2\Phi}, & D_3&=6\gamma(12+36\gamma+\gamma^2)\frac{\Gamma}{l^3\Phi}, \end{aligned}\right\} \tag{63}$$

$$\Phi=2(108-1296\gamma+393\gamma^2-13\gamma^3)\Gamma+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma(108-372\gamma+31\gamma^2)\varDelta, \tag{63}$$

$$\left.\begin{aligned}EI\frac{\partial^2 y_1}{\partial x_1^2}\bigg/2\,p^2ml&=6\left\{(216-294\gamma+19\gamma^2)+(108-372\gamma+31\gamma^2)\frac{x_1}{l}\right\}\\&\qquad\cdot\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\EI\frac{\partial^2 y_2}{\partial x_2^2}\bigg/2\,p^2ml&=18\left\{2(18+13\gamma-2\gamma^2)+(24-10\gamma-5\gamma^2)\frac{x_2}{l}\right\}\\&\qquad\cdot\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\EI\frac{\partial^2 y_3}{\partial x_3^2}\bigg/2\,p^2ml&=18(12+36\gamma+\gamma^2)\left(1+\frac{x_3}{l}\right)\\&\qquad\cdot\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\end{aligned}\right\}\quad(65)$$

where

$$\left.\begin{aligned}P&=2(108-1296\gamma+393\gamma^2-13\gamma^3)\Gamma_1+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma(108-372\gamma+31\gamma^2)\Lambda_1,\\Q&=2(108-1296\gamma+393\gamma^2-13\gamma^3)\Gamma_2+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma(108-372\gamma+31\gamma^2)\Lambda_2.\end{aligned}\right\}\quad(66)$$

8. *A structure with rigid floors and hinged base.*[8]

The assumptions with respect to the form of incident waves are the same as those in the preceding two sections.

(i) $n=1$;

$$\left.\begin{aligned}&A_1=4(3-\gamma_1)\frac{\Gamma}{\Phi},\quad B_1=-6\gamma_1\frac{\Gamma}{l_1\Phi},\quad C_1=0,\quad D_1=2\gamma_1\frac{\Gamma}{l_1^3\Phi},\\&\qquad\Phi=2(3-\gamma_1)\Gamma+\frac{9E_1j_1^2\varepsilon}{\mu l_1^3}\gamma_1\Lambda.\end{aligned}\right\}\quad(67)$$

$$EI\frac{\partial^2 y}{\partial x^2}\bigg/2\,p^2ml=\frac{6x}{l}\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\quad(68)$$

where $E=E_1$, $I=I_1$; $\cdots$, and

$$\left.\begin{aligned}P&=2(3-\gamma)\Gamma_1+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_1,\\Q&=2(3-\gamma)\Gamma_2+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma\Lambda_2,\end{aligned}\right\}\quad(69)$$

---

8) *Bull. Earthq. Res. Inst.*, **13** (1935), 708.

(ii) $n=2$;

$$\left.\begin{aligned}
&A_1=4\left\{12\eta\gamma_2-(3-\gamma_1)(12-\gamma_2)\right\}\frac{\Gamma}{\Phi}, \quad B_1=6\left\{12\eta\gamma_2+\gamma_1(12-\gamma_2)\right\}\frac{\Gamma}{l_1\Phi},\\
&C_1=0, \qquad\qquad D_1=-2\left\{12\eta\gamma_2+\gamma_1(12-\gamma_2)\right\}\frac{\Gamma}{l_1^3\Phi},\\
&A_2=-12(12-\gamma_2)\frac{\Gamma}{\Phi}, \qquad B_2=0,\\
&C_2=-36\gamma_2\frac{\Gamma}{l_2^2\Phi}, \qquad D_2=-24\gamma_2\frac{\Gamma}{l_2^3\Phi},
\end{aligned}\right\} \tag{70}$$

$$\Phi=2\left\{12\eta\gamma_2-(3-\gamma_1)(12-\gamma_2)\right\}\Gamma-\frac{9E_1j_1^2\varepsilon}{\mu l_1^3}\left\{12\eta\gamma_2+\gamma_1(12-\gamma_2)\right\}\Lambda, \tag{71}$$

where $\eta=\eta_1$;

$$\left.\begin{aligned}
EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/2\,p^2ml&=6(24-\gamma)\frac{x_1}{l}\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\\
&\quad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\
EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/2\,p^2ml&=36\left(1+\frac{2x_2}{l}\right)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\\
&\quad.\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),
\end{aligned}\right\} \tag{72}$$

where $E=E_1=E_2, \cdots\cdots,$ and

$$\left.\begin{aligned}
P&=2(36-27\gamma+\gamma^2)\Gamma_1+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma(24-\gamma)\Lambda_1,\\
Q&=2(36-27\gamma+\gamma^2)\Gamma_2+\frac{9Ej^2\varepsilon}{\mu l^3}\gamma(24-\gamma)\Lambda_2.
\end{aligned}\right\} \tag{76}$$

(iii) $n=3$;

$$\begin{aligned}
A_1&=4\Big[12\eta_1\left\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\right\}\\
&\qquad+(3-\gamma_1)\left\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\right\}\Big]\frac{\Gamma}{\Phi},\\
B_1&=6\Big[12\eta_1\left\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\right\}\\
&\qquad-\gamma_1\left\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\right\}\Big]\frac{\Gamma}{l_1\Phi},\\
C_1&=0,
\end{aligned}$$

$$\left.\begin{aligned}
D_1 &= -2\Big[12\eta_1\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\} \\
&\qquad -\gamma_1\Big\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\Big\}\Big]\frac{\Gamma}{l_1^3\Phi}, \\
A_2 &= 12\Big\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\Big\}\frac{\Gamma}{\Phi}, \qquad B_2=0, \\
C_2 &= -36\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\frac{\Gamma}{l_2^2\Phi}, \\
D_2 &= -24\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\frac{\Gamma}{l_2^3\Phi}, \\
A_3 &= -144(12-\gamma_3)\frac{\Gamma}{\Phi}, \qquad B_3=0, \\
C_3 &= -432\gamma_3\frac{\Gamma}{l_3^2\Phi}, \qquad D_3=-288\gamma_3\frac{\Gamma}{l_3^3\Phi},
\end{aligned}\right\} \tag{74}$$

$$\begin{aligned}
\Phi &= 2\Big[12\eta_1\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}+(3-\gamma_1)\Big\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\Big\}\Big]\Gamma \\
&\quad -\frac{9E_1j_1^{2\varepsilon}}{\mu l_1^3}\Big[12\eta_1\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}-\gamma_1\Big\{12\eta_2\gamma_3-(12-\gamma_2)(12-\gamma_3)\Big\}\Big]\Lambda.
\end{aligned} \tag{75}$$

$$\left.\begin{aligned}
EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/2p^2ml &= 6(12-\gamma)(36-\gamma)\frac{x_1}{l}\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}} \\
&\qquad \cdot\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \\
EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/2p^2ml &= 36(24-\gamma)\left(1+\frac{2x_2}{l}\right)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}} \\
&\qquad \cdot\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \\
EI\frac{\partial^2 y_3}{\partial x_3^2}\Big/2p^2ml &= 432\left(1+\frac{2x_3}{l}\right)\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}} \\
&\qquad \cdot\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),
\end{aligned}\right\} \tag{76}$$

in which $E=E_1=E_2=E_3, \cdots\cdots,$ and

$$\left.\begin{aligned}
P &= 2(432-540\gamma+51\gamma^2-\gamma^3)\Gamma_1+\frac{9Ej^{2\varepsilon}}{\mu l^3}\gamma(12-\gamma)(36-\gamma)\Lambda_1, \\
Q &= 2(432-540\gamma+51\gamma^2-\gamma^3)\Gamma_2+\frac{9Ej^{2\varepsilon}}{\mu l^3}\gamma(12-\gamma)(36-\gamma)\Lambda_2.
\end{aligned}\right\} \tag{77}$$

9. *A four-storied structure with rigid floors and clamped base.*[9]

$$
\left.\begin{aligned}
A_1 &= 2\Big[-12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\frac{\Gamma}{\Phi}, \\
D_1 &= 4\Big[12\eta_2(12\eta_1-\gamma_1)\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\frac{\Gamma}{l_1^3\Phi}, \\
A_2 &= 2.12\Big[-12\eta_2\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+(12-\gamma_2)\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big]\frac{\Gamma}{\Phi}, \\
D_2 &= 4.12\Big[12\eta_2\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+\gamma_2\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big]\frac{\Gamma}{l_2^3\Phi}, \\
A_3 &= 2.12^2\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\frac{\Gamma}{\Phi}, \\
D_3 &= 4.12^2\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\}\frac{\Gamma}{l_3^3\Phi}, \\
A_4 &= 2.12^3(12-\gamma_4)\frac{\Gamma}{\Phi}, \qquad D_4=4.12^3\gamma_4\frac{\Gamma}{l_4^3\Phi}, \\
B_s &= 0, \qquad C_s=\frac{3}{2}l_sD_s. \quad (s=1,\ 2,\ 3,\ 4)
\end{aligned}\right\} \quad (78)
$$

$$
\begin{aligned}
\Phi &= \Gamma\Big[-12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big] \\
&\quad+\frac{18E_1j_1^2\varepsilon}{\mu l_1^3}\Delta\Big[12\eta_2(12\eta_1-\gamma_1)\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\} \\
&\quad+\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]. \quad (79)
\end{aligned}
$$

9) *Bull. Earthq. Res. Inst.*, **13** (1935), 919.

$$
\left.
\begin{aligned}
&EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/ 2\,p^2 ml = 6(24-\gamma)\Big\{24(6-\gamma)+(12-\gamma)^2\Big\}\Big(1+\frac{2x_1}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/ 2\,p^2 ml = 6.12(12-\gamma)(36-\gamma)\Big(1+\frac{2x_2}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_3}{\partial x_3^2}\Big/ 2\,p^2 ml = 6.12^2(24-\gamma)\Big(1+\frac{2x_3}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_4}{\partial x_4^2}\Big/ 2\,p^2 ml = 6.12^3\Big(1+\frac{2x_4}{l}\Big)M\,,
\end{aligned}
\right\}\qquad (80)
$$

where $E=E_1=E_2=E_3=E_4,\ \cdots\cdot$, and

$$
\left.
\begin{aligned}
&M=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\Big(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\Big)\,,\\
&P=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi A_1\,,\quad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi A_2\,,\\
&\phi=-12\gamma(24-\gamma)^2+\Big\{-12\gamma+(12-\gamma)^2\Big\}^2\,,\\
&\psi=\gamma(24-\gamma)\Big\{24(6-\gamma)+(12-\gamma)^2\Big\}\,.
\end{aligned}
\right\}\qquad (81)
$$

10. *A five-storied structure with rigid floors and clamped base.*[10]

$$
\begin{aligned}
A_1=2\Bigg\{&12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\Big[12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}+\Big\{-12\eta_1\gamma_2\\
&+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]-\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big[-12\eta_2\gamma_3\Big\{12\eta_1\\
&+(12-\gamma_1)\Big\}+(12-\gamma_3)\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\Bigg\}\frac{\Gamma}{\Phi}\,,
\end{aligned}
$$

$$
\begin{aligned}
D_1=-4\Bigg\{&12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\Big[12\eta_2(12\eta_1-\gamma_1)-\Big\{12\eta_1\gamma_2\\
&+\gamma_1(12-\gamma_2)\Big\}\Big]+\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big[12\eta_2\gamma_3(12\eta_1-\gamma_1)\\
&+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\Bigg\}\frac{\Gamma}{l_1^3\Phi}\,,
\end{aligned}
$$

10) *Bull. Earthq. Res. Inst.*, **13** (1935), 920.

$$A_2=2.12\Big[12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\Big\{12\eta_2+(12-\gamma_2)\Big\}$$
$$-\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big\{-12\eta_2\gamma_3+(12-\gamma_2)(12-\gamma_3)\Big\}\Big]\frac{\Gamma}{\Phi},$$

$$D_2=-4.12\Big[12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}(12\eta_2-\gamma_2)$$
$$+\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\Big]\frac{\Gamma}{l_2^3\Phi},$$

$$A_3=2.12^2\Big[12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}$$
$$-(12-\gamma_3)\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big]\frac{\Gamma}{\Phi},$$

$$D_3=-4.12^2\Big[12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}$$
$$+\gamma_3\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big]\frac{\Gamma}{l_3^3\Phi},$$

$$A_4=-2.12^3\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\frac{\Gamma}{\Phi},$$

$$D_4=-4.12^3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\frac{\Gamma}{l_4^3\Phi},$$

$$A_5=-2.12^4(12-\gamma_5)\frac{\Gamma}{\Phi},\qquad D_5=-4.12^4\gamma_5\frac{\Gamma}{l_5^3\Phi},$$

$$B_s=0,\qquad C_s=\frac{3}{2}l_sD_s,\quad (s=1,\ 2,\ \cdots,\ 5) \tag{82}$$

$$\Phi=\Gamma\Bigg\{12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\Big[12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}+\Big\{-12\eta_1\gamma_2+(12$$
$$-\gamma_1)(12-\gamma_2)\Big\}\Big]-\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big[-12\eta_2\gamma_3\Big\{12\eta_1$$
$$+(12-\gamma_1)\Big\}+(12-\gamma_3)\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\Bigg\}$$
$$-\frac{18E_1\gamma_1^2\varepsilon}{\mu l_1^3}\Delta\Bigg\{12\eta_3\Big\{12\eta_4\gamma_5+\gamma_4(12-\gamma_5)\Big\}\Big[12\eta_2(12\eta_1-\gamma_1)-\Big\{12\eta_1\gamma_2$$
$$+\gamma_1(12-\gamma_2)\Big\}\Big]+\Big\{-12\eta_4\gamma_5+(12-\gamma_4)(12-\gamma_5)\Big\}\Big[12\eta_2\gamma_3(12\eta_1$$
$$-\gamma_1)+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\Bigg\}. \tag{83}$$

$$\left.\begin{aligned}
&EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/2\,p^2ml=6\Big\{(12-\gamma)^2-12\gamma\Big\}\\
&\qquad\qquad\qquad\cdot\Big\{(12-\gamma)(48-\gamma)+144\Big\}\Big(1+\frac{2x_1}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/2\,p^2ml=6.12(24-\gamma)\Big\{(12-\gamma)(24-\gamma)-12\gamma\Big\}\Big(1+\frac{2x_2}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_3}{\partial x_3^2}\Big/2\,p^2ml=6.12^2(12-\gamma)(36-\gamma)\Big(1+\frac{2x_3}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_4}{\partial x_4^2}\Big/2\,p^2ml=6.12^3(24-\gamma)\Big(1+\frac{2x_4}{l}\Big)M\,,\\
&EI\frac{\partial^2 y_5}{\partial x_5^2}\Big/2\,p^2ml=6.12^4\Big(1+\frac{2x_5}{l}\Big)M\,,
\end{aligned}\right\}\qquad(84)$$

in which $E=E_1=E_2=\cdots,\ \cdots,$ and

$$\left.\begin{aligned}
&M=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\Big(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\Big)\,,\\
&P=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_1\,,\qquad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_2\,,\\
&\phi=-12\gamma(12-\gamma)(24-\gamma)(36-\gamma)\\
&\qquad+\Big\{(12-\gamma)^2-12\gamma\Big\}\Big[(12-\gamma)\Big\{(12-\gamma)^2-24\gamma\Big\}-144\gamma\Big]\,,\\
&\psi=\gamma\Big\{(12-\gamma)^2-12\gamma\Big\}\Big\{(12-\gamma)(48-\gamma)+144\Big\}\,.
\end{aligned}\right\}\qquad(85)$$

11. *A six-storied structure with rigid floors and clamped base.*[11]

$$\begin{aligned}
A_1=2\Bigg[&R\Bigg\{-12\eta_3\gamma_4\Big[12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}+\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\\
&+(12-\gamma_4)\Big[-12\eta_2\gamma_3\Big\{12\eta_1+(12-\gamma_1)\Big\}+(12-\gamma_3)\Big\{-12\eta_1\gamma_2\\
&+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\Bigg\}+12S\eta_4\Bigg\{12\eta_3\Big[12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}\\
&+\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]+\Big[-12\eta_2\gamma_3\Big\{12\eta_1+(12-\gamma_1)\Big\}\\
&\qquad+(12-\gamma_3)\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\Bigg\}\Bigg]\frac{\Gamma}{\Phi}\,,
\end{aligned}$$

11) *Bull. Earthq. Res. Inst.*, **14** (1936),

$$
\begin{aligned}
D_1=-4\Bigg[&-R\bigg\{12\eta_3\gamma_4\Big[12\eta_2(12\eta_1-\gamma_1)-\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\\
&+(12-\gamma_4)\Big[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\bigg\}\\
&+12S\eta_4\bigg\{12\eta_3\Big[12\eta_2(12\eta_1-\gamma_1)-\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\\
&\qquad-\Big[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\Big\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\Big\}\Big]\bigg\}\Bigg]\frac{\Gamma}{l_1^3\Phi},
\end{aligned}
$$

$$
\begin{aligned}
A_2=2.12\bigg\{&R\Big[-12\eta_3\gamma_4\Big\{12\eta_2+(12-\gamma_2)\Big\}+(12-\gamma_4)\Big\{-12\eta_2\gamma_3\\
&+(12-\gamma_2)(12-\gamma_3)\Big\}\Big]+12S\eta_4\Big[12\eta_3\Big\{12\eta_2+(12-\gamma_2)\Big\}\\
&\qquad+\Big\{-12\eta_2\gamma_3+(12-\gamma_2)(12-\gamma_3)\Big\}\Big]\bigg\}\frac{\Gamma}{\Phi},
\end{aligned}
$$

$$
\begin{aligned}
D_2=4.12\bigg\{&R\Big[12\eta_3\gamma_4(12\eta_2-\gamma_2)+(12-\gamma_4)\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\Big]\\
&+12S\eta_4\Big[-12\eta_3(12\eta_2-\gamma_2)+\Big\{12\eta_2\gamma_3+\gamma_2(12-\gamma_3)\Big\}\Big]\bigg\}\frac{\Gamma}{l_2^3\Phi},
\end{aligned}
$$

$$
A_3=2.12^2\Big[R\Big\{-12\eta_3\gamma_4+(12-\gamma_3)(12-\gamma_4)\Big\}+12S\eta_4\Big\{12\eta_3+(12-\gamma_3)\Big\}\Big]\frac{\Gamma}{\Phi},
$$

$$
D_3=4.12^2\Big[R\Big\{12\eta_3\gamma_4+\gamma_3(12-\gamma_4)\Big\}+12S\eta_4(-12\eta_3+\gamma_3)\Big]\frac{\Gamma}{l_3^3\Phi},
$$

$$
A_4=2.12^3\Big\{R(12-\gamma_4)+12S\eta_4\Big\}\frac{\Gamma}{\Phi},\qquad D_4=4.12^3(R\gamma_4-12S\eta_4)\frac{\Gamma}{l_4^3\Phi},
$$

$$
A_5=2.12^4R\frac{\Gamma}{\Phi},\qquad D_5=-4.12^4S\frac{\Gamma}{l_5^3\Phi},
$$

$$
A_6=-2.12^5(12-\gamma_6)\frac{\Gamma}{\Phi},\qquad D_6=-4.12^5\gamma_6\frac{\Gamma}{l_6^3\Phi},
$$

$$
B_s=0,\qquad C_s=\frac{3}{2}l_sD_s,\quad (s=1,\ 2,\ \cdots,\ 6) \tag{86}
$$

$$
\begin{aligned}
\Phi=\Gamma\Bigg[&R\bigg\{-12\eta_3\gamma_4\Big[12\eta_2\Big\{12\eta_1+(12-\gamma_1)\Big\}+\Big\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\Big\}\Big]\\
&+(12-\gamma_4)\Big[-12\eta_2\gamma_3\Big\{12\eta_1+(12-\gamma_1)\Big\}+(12-\gamma_3)\Big\{-12\eta_1\gamma_2
\end{aligned}
$$

$$+(12-\gamma_1)(12-\gamma_2)\}]\}+12S\eta_4\{12\eta_3[12\eta_2\{12\eta_1+(12-\gamma_1)\}$$

$$+\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\}]+[-12\eta_2\gamma_3\{12\eta_1+(12-\gamma_1)\}$$

$$+(12-\gamma_3)\{-12\eta_1\gamma_2+(12-\gamma_1)(12-\gamma_2)\}]]\}]$$

$$-\frac{18E_1j_1^2\varepsilon}{\mu l_1^3}\Lambda[-R\{12\eta_3\gamma_4[12\eta_2(12\eta_1-\gamma_1)-\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\}]$$

$$+(12-\gamma_4)[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\}]\}$$

$$+12S\eta_4\{12\eta_3[12\eta_2(12\eta_1-\gamma_1)-\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\}]$$

$$-[12\eta_2\gamma_3(12\eta_1-\gamma_1)+(12-\gamma_3)\{12\eta_1\gamma_2+\gamma_1(12-\gamma_2)\}]\}],$$

$$R=12\eta_5\gamma_6-(12-\gamma_5)(12-\gamma_6)\,,\qquad S=12\eta_5\gamma_6-\gamma_5(12-\gamma_6)\,. \tag{87}$$

$$\left.\begin{aligned}
EI\frac{\partial^2 y_1}{\partial x_1^2}\Big/2\,p^2ml&=6(12-\gamma)(24-\gamma)(36-\gamma)\\
&\qquad\cdot\left\{(24-\gamma)^2-432\right\}\left(1+\frac{2x_1}{l}\right)M\,,\\
EI\frac{\partial^2 y_2}{\partial x_2^2}\Big/2\,p^2ml&=6.12\left\{(18-\gamma)^2-180\right\}\left\{(30-\gamma)^2-180\right\}\left(1+\frac{2x_2}{l}\right)M\,,\\
EI\frac{\partial^2 y_3}{\partial x_3^2}\Big/2\,p^2ml&=6.12^2(24-\gamma)\left\{(24-\gamma)^2-288\right\}\left(1+\frac{2x_3}{l}\right)M\,,\\
EI\frac{\partial^2 y_4}{\partial x_4^2}\Big/2\,p^2ml&=6.12^3(12-\gamma)(36-\gamma)\left(1+\frac{2x_4}{l}\right)M\,,\\
EI\frac{\partial^2 y_5}{\partial x_5^2}\Big/2\,p^2ml&=6.12^4(24-\gamma)\left(1+\frac{2x_5}{l}\right)M\,,\\
EI\frac{\partial^2 y_6}{\partial x_6^2}\Big/2\,p^2ml&=6.12^5\left(1+\frac{2x_6}{l}\right)M\,,
\end{aligned}\right\} \tag{88}$$

in which $E=E_1=E_2=\cdots,\ \cdots$, and

$$\left.\begin{aligned}
&M=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\
&P=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_1, \qquad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_2,\\
&\phi=\left\{\gamma(24-\gamma)^2-12(12-\gamma)^2\right\}^2-12\gamma(12-\gamma)^2(36-\gamma)^2,\\
&\psi=\gamma(12-\gamma)(24-\gamma)(36-\gamma)\left\{(24-\gamma)^2-432\right\}.
\end{aligned}\right\} \tag{89}$$

12. *General law in and comparison of the results of the present as well as the original problems.*

The deflections, y, or bending moments, $EI(\partial^2y/\partial x^2)$ of columns of the structure in any case have the types:

$$y=f_1(x)M, \qquad EI\frac{\partial^2y}{\partial x^2}=f_2(x)M, \tag{90}$$

where

$$M=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \tag{91}$$

$$P=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_1, \quad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_2, \text{ (or similar forms)} \tag{92}$$

and Γ_1, Γ_2, Λ_1, Λ_2 are functions of λ, μ, $m_1p^2l_1^3/E_1I_1$, $E_1\rho I_1\varepsilon^2/\mu m_1l_1^3$, but independent of the number of stories and the conditions with respect to the stiffness of floors, etc.; whereas ϕ, ψ, which were shown in the respective preceding sections, are functions of the structural conditions, but not affected by the dissipation nature of the ground. $\phi=0$ gives the resonance condition of a structure in the case that the ground, on which the structure is standing, is so rigid as does not participate in dissipation, whereas $\psi=0$ indicates the corresonance condition under the criterion shown in our foregoing papers. Actually, $\psi=0$ is the resonance condition of the structure in the case that the ground is so soft as does not partake in dissipation. $f_1(x)$, $f_2(x)$ are quite of the same forms as those also shown in the same papers.

If there were such a condition as $Ej^2\varepsilon/\mu l^3=0$, y or $EI(\partial^2y/\partial x^2)$ would have ϕ as its denominator. The phenomena of perfect resonance of the structural vibration — in usual sense — would then result in the condition under consideration.

It may seem that our present and original theories are different each other particularly with respect to the forms of P as well as Q.

In our original theory P and Q had the terms similar to $\phi\Gamma_1$ and $(18Ej^2\varepsilon/\mu l^3)\psi A_2$ respectively, and the terms corresponding to the second one in P and the first one in Q in (92) did not exist. If however the higher orders in $h\varepsilon$ and $k\varepsilon$ were not neglected in the original theory, M would be of the form

$$M=\sqrt{\frac{(\Gamma_1-3\sqrt{(\lambda+2\mu)/\mu}\,A_2)^2+(\Gamma_2+3\sqrt{(\lambda+2\mu)/\mu}\,A_1)^2}{P^2+Q^2}}$$

$$\cdot\cos\left\{pt+\tan^{-1}\frac{\Gamma_2+3\sqrt{(\lambda+2\mu)/\mu}\,A_1}{\Gamma_1-3\sqrt{(\lambda+2\mu)/\mu}\,A_2}-\tan^{-1}\frac{Q}{P}\right\}, \tag{93}$$

$$\left.\begin{aligned}
&P=\phi\Gamma_1+\frac{36}{a}\sqrt{\frac{\lambda+2\mu}{\mu}}\sqrt{\frac{m_1E_1I_1}{\rho\mu l_1^3}}\frac{1}{\sqrt{\gamma_1}}\psi A_1,\\
&Q=\phi\Gamma_2+\frac{36}{a}\sqrt{\frac{\lambda+2\mu}{\mu}}\sqrt{\frac{m_1E_1I_1}{\rho\mu l_1^3}}\frac{1}{\sqrt{\gamma_1}}\psi A_2,\\
&\Gamma_1=2\sqrt{\gamma_1\nu}\left[4\left\{\sqrt{\frac{(\lambda+\mu)^2}{(\lambda+2\mu)\mu}}+1-2\sqrt{\frac{\lambda+2\mu}{\mu}}\right\}+\gamma_1\nu\left(1-2\sqrt{\frac{\mu}{\lambda+2\mu}}\right)\right],\\
&\Gamma_2=4\left(1-\sqrt{\frac{\lambda+2\mu}{\mu}}\right)(\gamma_1\nu+4),\\
&A_1=-\left(4+\sqrt{\frac{\mu}{\lambda+2\mu}}\gamma_1\nu\right),\quad A_2=\sqrt{\gamma_1\nu}\left\{2\left(\sqrt{\frac{\mu}{\lambda+2\mu}}+1\right)+\frac{\mu}{\lambda+2\mu}\gamma_1\nu\right\},\\
&\qquad\nu=\frac{E_1\rho I_1\varepsilon^2}{\mu m_1 l_1^3}.
\end{aligned}\right\} \tag{94}$$

When $h\varepsilon\to 0$, $k\varepsilon\to 0$, M assumes the form

$$M=\frac{1}{\sqrt{\phi^2+\left\{\frac{12}{a}\sqrt{\frac{m_1E_1I_1}{\rho\mu l_1^3}}\frac{\psi}{\sqrt{\gamma_1}}\right\}^2}}\cos\left(pt+\tan^{-1}\frac{12E_1j_1^2\psi}{\mu l_1^3 k\phi}\right). \tag{95}$$

It will therefore be seen that the old and new theories are not essentially different with each other in their qualitative natures.

13. *The forms of $f_1(x)$, $f_2(x)$.*

As already stated, $f_1(x)$ and $f_2(x)$ shown in (90) are quite of the same forms for the present as well as for the original theories. It is somewhat difficult at present to describe the nature of $f_1(x)$, whereas $f_2(x)$'s of various cases are in obedience with a very simple law such that

$f_2(x_s)$ for columns between sth and $(s-1)$th floors of an n-storied structure
$=12f_2(x_{s-1})$ for columns between $(s-1)$th and $(s-2)$th floors of an $(n-1)$-storied structure
$=12^2f_2(x_{s-2})$ for columns between $(s-2)$th and $(s-3)$th floors of an $(n-2)$-storied structure
$= \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$ (96)

These relations hold only for the structure with extremely rigid floors and clamped base.

In conclusion we wish to express our thanks to the Council of the Foundation for the Promotion of Scientific and Industrial Research of Japan, with whose aid the study of present series was begun.

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16. 構造物に於ける震動逸散理論の吟味

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これまで種々の場合について論じた構造物震動の逸散問題では，計算を容易にする爲に二三の點を簡單化してあつたが，問題が最近多くの人に眞實に注意される傾向となつて來たから，問題のできるだけ嚴格な取扱をなすことにして，今迄の理論全體を吟味して見たのである．その結果は數理の點に於て部分的に所々變つて來たが，全體としては大した變化がないことがわかつた．又，計算結果による逸散の性質も大體に於て同じであるばかりでなく，今迄多少都合の惡かつた點は寧ろ除外されるやうになつたのである．例へば，建物が極端に小さい場合に逸散が少いことや，建物の層數によつて最大屈曲モーメントの起る柱の違ひなどが數理的によくわかるやうになるのである．但し何れにしても計算が如何にも複雜であつて，實際問題に應用する場合に多大の困難を感じる次第である．しかしこの應用も只今着々として進行中である．