18. On a Problem Concerning the Internal Structure of the Earth as Discussed from the Time-distance Curve of the Formosa Earthquake of April 20, 1935.

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The genuine discontinuity or so called first order discontinuity in the velocity distribution of seismic waves within the earth which is now under discussion is that supposed at the depth of 300 or 400 km from the earth's surface. The well known discontinuities near the earth's surface and at the boundary of the inner core are beyond the scope of present study. If we leave the earliest qualitative suggestions of Milne,[1] Omori,[2] Imamura,[3] Laska,[4] Benndorf[5] and Knott[7] out of account, the first qualitative discussion was that of S. Mohorovičić,[7] who remarked in 1916 that the velocity may suddenly decrease at the depth of about 400 km. On the other hand Byerly[8] showed in 1926 that the time-distance curve of the Montana earthquake showed abrupt change of slope at about $\Delta=20°$, which might correspond to sudden increase of velocity at the depth of 400 km. In a study of a deep-focus earthquake one[9] of the writers also considered the existence of such discontinuous increase of velocity at about 450 km. In 1931 Jeffreys[10] noticed from the statistical investigation of time-distance curves that the time curves show abrupt bending at about 20° from the epicentre, and worked out the discontinuity of velocity of about 25% at the depth of 273 km. Reviewing again Jeffreys data one of the writers reached to the conclusion that the negligence of the discontinuity is permissible so long as the observational accuracy at that time is concerned.

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1) J. MILNE, *Brit. Assoc. Rep.*, 1902.
2) F. OMORI, *Publ. Earthq. Inv. Comm.*, **5** (1901), **13** (1903), **21** (1905).
3) A. IMAMURA, *Publ. Earthq. Inv. Comm.*, **16** (1904), 1~117.
4) W. LASKA, *Mitt. Erdbebenkomm., Wien.* [N.F.], **23** (1904).
5) H. BENNDORF, *do.*, **29** (1905); **31** (1904).
6) C. G. KNOTT, *Proc. Roy. Soc. Ed.* **28** (1908), 217~230.
7) S. MOHOROVIČIĆ, *Gerlands Beitr. z. Geophys.*, **14** (1916), 187.
8) P. BYERLY, *Bll. Seis. Soc. Amer.*, **16** (1626), 209.
9) H. KAWASUMI, *Journ. Met. Soc. Japan*, [ii], **5** (1927), 232~240.
10) H. JEFFREYS, *M. N. R. A. S., Geophys. Suppl.*, **2** (1931), 329~348.
11) H. KAWASUMI, *Jap. Journ. Astro. Geophys.*, **9** (1931), 15~22.

And Jeffreys[12] himself withdrew his statement in a later investigations, and Gutenberg and Richter[13] also presented negative opinion.

But recently the problem regained eminent supporters as Neumann[14] Byerly,[15] Lehmann[16] and Jeffreys and Bullen[17]. These authorities investigated independently time-distance curves, and found abrupt bending of P- and S-curves at the epicentral distance of about 16° or 19°. Moreover Lehmann presented conspicuous seismograms which show the appearence of late P- and S-phases beyond the epicentral distance of about 20°, and this will decide whether the time-distance curves in this region are double or merely simple curves with strong curvature, provided the appearance of the first smaller P- and S-phases at these distances is independent to the mechanism of the earthquake occurrence. The depths of the discontinuity surface estimated by Neumann, Byerly, Lehmann and Jeffreys and Bullen were 270, 216~315, and about 400 km respectively.

On the other hand, Matuzawa[18] also found strong bend in the P-curve of the Sanriku Earthquake of March 2, 1933, at $\Delta=21°$, but he stated that the change of the slope seems to be gradual.

Thus special studies of this important problem with any available data are urgent task of present seismology. The disastrous Formosa Earthquake of April 20, 1935 provided us with a good opportunity for this purpose, having numerous observations in the range of epicentral distances in question. And Sagisaka and Miura[19] have already elucidated upon this data a discontinuity surface of the second order at the depth of about 300 km from the earth's surface. But we independently took the opportunity to examine quantitative basis of these opinions by the analysis of the time-distance curve of this earthquake to throw some light on this problem.

But as the earthquake was not strong enough to be distinctly observed at the stations in Kwantô and Tôhoku districts which lie in the ranges of epicentral distances concerned, that is in the vicinity of $\Delta=20°$, where microseisms were prevailing at that time. And the time observation alone does not afford conclusive results, but closer

12) H. JEFFREYS, *M. N. R. A. S. Geophys. Suppl.*, **2** (1931), 399~407.

13) B. GUTENBERG and C. F. RICHTER, *Bull. Seism. Soc. Amer.*, **21** (1931), 216.

14) F. NEUMANN, *Trans. Amer. Geophys. Union,* (1933), 329.

15) P. BYERLY, *Bull. Seism. Soc. Amer.*, **24** (1934), 81~99.

16) I. LEHMANN, *Meddelelse Geod. Inst.*, **5** (1934), 1~45.

17) H. JEFFREYS and K. E. BULLEN, *Publ. du Bureau Centr. Seismol. Int.* (A), **11** (1935).

18) T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **13** (1935), 171~193.

19) K. SAGISAKA and T. MIURA, *Kensin Zihô*, **9** (1935), 37~42.

re-examinations of seismograms, which on the writers' request are now in progress by the members of the Central Meteorological Observatory, will give some clue on this problem[20].

1. Determination of the Epicentre.

For such quantitative study as mentioned above we must determine the position of the epicentre as accurately as possible. We, therefore, gathered all the available data from Kisyô Yôran and bulletins of foreign observatories. Following Jeffreys[21] and Matuzawa[22] we assumed cubic equation for the time-distance curve ($t=t_0+\alpha\Delta-\beta\Delta^3$) and analysed the time of arrival of P-phase to determine the position of epicentre (φ_0, λ_0) and constants of time distance curve (t_0, α and β). In this analysis, we first adopted the position determined by the Central Meteorological Observatory[23]

$$\varphi_0 = 24°\,21'\text{ N},$$

$$\lambda_0 = 120°49'\text{ E},$$

as the primary epicentre, and the distance to each station was calculated by using geocentric latitude. Borrowing then the cubic equation

$$t_{\text{cal}} = 1{\cdot}6 + 14{\cdot}55\Delta - 0{\cdot}0022\Delta^3,$$

determined by Matuzawa from the Sanriku Earthquake, the travel time t_{cal} was calculated. The correction δt_r for the difference of distance from the centre of the earth due to the ellipticity was calculated by the simple formula $\delta t_r = (r - r_0)/10$ sec.

Table I.

Station	Δ	P	δt_r	ε_1	$\delta\Delta$	δt	ε_2
	° ′	h m s	s	s	′	s	s
Taityû	0 07	7 2, 03·8	0·0	0·0	42	18·1	0·0
Arisan	48	2, 14·0	0·0	+0·1	42	18·1	+0·8
Kwarenkô	58	2, 14·8	0·0	−1·3	42	18·0	−0·7
Taihoku	1 03	2, 16·8	0·0	−0·8	42	18·0	−1·5

(*to be continued.*)

20) After the present paper has been written, the writers received detailed discussion of Gutenberg and Richter on this problem, which confirms their previous opinion. *Gerlands Beitr. z. Geophys.*, **45** (1935), 280~360.

21) H. JEFFREYS, *M. N. R. A. S Geophys. Suppl.*, **1** (1928), 500~521, etc..

22) T. MATUZAWA, *loc. cit.*, 17.

23) "Report of the Severe Formosa Earthquake", *Kensin Zihô*, **9** (1935), 1~63.

Table I. (*continue*'.)

Station	Δ		P	δt_r	ε_1	$\delta\Delta$	δt	ε_2
	°	′	h m s	s	s	′	s	s
Tainan	1	22	7 2, 20·0	0·0	−1·9	42	18·0	−1·6
Hôkotô		33	2, 22·0	0·0	−2·6	42	18·0	−1·8
Taitô		38	2, 26·3	+0·1	+0·4	42	18·0	+1·2
Takao		43	2, 31·9	+0·1	+4·6	42	18·0	+5·6
Kôsyun	2	18	2, 36·5	+0·1	+0·8	41	17·9	+2·1
Isigakizima	3	14	2, 49·6	0·0	+0·9	41	17·8	+1·9
Hongkong	6	36	3, 30	+0·1	−1·6	40	17·5	−3
Naha	6	37	3, 42·5	0·0	+6·3	40	17·6	+7·0
Zikawei	6	53	3, 37	−0·2	−3·0	40	17·5	−2
Naze	8	56	4, 08·9	−0·1	−0·1	39	17·3	+0·6
Manila	9	46	4, 19	−0·2	−0·7	38	17·1	0
Tomie	10	00	4, 37·3	−0·4	+9·3	38	17·1	+14·8
Kagosima	11	22	4, 45·9	−0·2	+1·8	37	16·8	+2·2
Nagasaki		38	4, 45·7	−0·3	+0·2	38	17·1	+0·5
Unzendake		52	4, 51·5	−0·3	+2·9	36	16·7	+3·2
Miyasaki	12	11	4, 54·0	−0 2	+1·3	36	16·6	+1·4
Kumamoto		12	4, 54·4	−0·3	+1·7	36	16·6	+1·9
Saga		17	4, 59·9	−0·3	+6·0	36	16·6	+5·5
Ituhara		24	5, 50·5	−0 3	+55·1	36	16·6	+55·7
Hukuoka		35	4, 57·7	−0·3	−0·5	36	16·6	−0·3
Husan		59	5, 14·5	−0·3	+11·0	35	16·5	+11·0
Ooita	13	05	5, 04·9	−0·3	+0·6	35	16·5	−0·0
Simonoseki		09	5, 16·0	−0·3	+10·2	35	16·5	+10·2
Taikyû		24	5, 12·5	−0·4	+3·3	35	16·4	+3·5
Uwazima		42	5, 24·9	−0·3	+11·7	34	16·3	+11·6
Simidu		43	5, 12·6	−0·3	−0·8	34	16·3	−0·9
Phulien		44	5, 10	+0·1	+0·8	34	16·3	−4·7
Zinsen	14	06	5, 18·1	−0·4	+0·0	34	16·2	+12·5
Matuyama		11	5, 19·8	−0·3	+0·6	34	16·2	+0·1
Keizyô		17	5, 20·5	−0·4	−0·2	34	16·2	−0·4
Kure		20	5, 51·4	−0·3	+30·1	34	16·1	+2·0
Hiroshima		22	5, 10·0	−0·3	−12·0	34	16·1	−12·1
Hamada		29	5, 21·5	−0·3	−1·8	34	16·1	−2·1
Kôti		33	5, 20·0	−0·3	−4·3	33	16·1	−4·7
Dairen		34	5, 34·1	−0·5	+9·8	33	16·1	+9·4
Muroto		50	5, 35·8	−0·3	+7·7	33	16·0	+7·4
Tadotu	15	08	5, 44·4	−0·3	−7·4	33	16·0	+12·2
Heizyô		16	5, 36·0	−0·5	+2·7	33	16·0	+2·0
Okoyama		29	5, 32·0	−0·3	−4·5	33	16·0	−2·7
Sakai		37	5, 44·8	−0·4	+6·1	32	15·7	+6·0

(*to be continued.*)

Table I. (*continued.*)

Station	Δ	P	δt_r	ε_1	δΔ	δt	ε_2
	° ′	h m s	s	s	′	s	s
Tokusima		7 5, 32·9	−0·4				
Sumoto	15 43	5, 41·0	−0·3	+1·7	32	15·9	+0·7
Wakayama	16 04	5, 41·0	−0·3	−2·6	32	15·8	−3·8
Siomisaki	04	5, 44·0	−0·3	+0·3	32	15·8	−1·9
Kôbe	20	5, 47·6	−0·3	+0·5	32	15·7	−0·7
Eikô	21	5, 46·5	−0·5	−0·4	32	15·7	−1·8
Oosaka	33	5, 47·0	−0·3	−2·7	32	15·7	−4·1
Toyooka	36	5, 51·0	−0·4	+0·7	32	15·7	−0·7
Yagi	39	5, 51·6	−0·3	+0·6	32	15·7	−3·1
Miyadu	50	5, 47·6	−0·4	−5·4	31	15·7	−7·4
Kyôto	54	5, 54·2	−0·3	+0·1	32	15·7	−1·8
Kameyama	17 16	5, 59·5	−0·3	+0·4	31	15·6	−1·1
Tu	16	6, 04·3	−0·3	+5·5	31	15·6	−3·7
Hikone	23	6, 01·7	−0·4	−0·3	31	15·5	−0·2
Ibukisan	34	6, 02·9	−0·4	−1·4	31	15·5	−3·7
Gihu	48	6, 06·1	−0·4	+0·8	31	15·4	−0·8
Nagoya	49	6, 08·1	−0·3	+2·5	31	15·4	+0·5
Hukui	51	5, 56·2	−0·4	−9·9	31	15·4	−11·7
Hamamatu	18 06	6, 04·8	−0·3	−4·3	30	15·4	+3·3
Kanazawa	23	5, 20·0	−0·4	+7·1	30	15·3	+5·7
Omaesaki	24	5, 20·7	−0·3	+7·6	30	15·3	+5·2
Takayama	32	6, 57·5	−0·4	+42·8	30	15·3	+40·6
Husiki	47	6, 20·7	−0·4	+2·9	30	15·3	+0·2
Toyama	49	6, 18·3	−0·4	+0·2	30	15·2	−2·1
Hatidyôzima	19 02	6, 20·4	−0·3	−0·3	29	15·2	−3·1
Iida	05	6, 19·0	−0·4	−2·1	29	15·2	−5·1
Matumoto	05	6, 30·9	−0·4	+9·8	29	15·2	+6·9
Numadu	06	6, 21·9	−0·3	+0·7	29	15·2	−3·9
Kôhu	10	6, 25·9	−0·4	+4·0	29	15·2	+0·8
Misima	11	6, 19·4	−0·3	+0·8	29	15·2	−6·0
Hunatu	14	6, 21·1	−0·4	−1·5	29	15·2	−5·0
Nagano	28	6, 36·2	−0·4	+10·6	29	15·1	+7·5
Oiwake	31	6, 32·0	−0·4	+5·7	29	15·1	+3·0
Titizima	37	6, 23·7	−0·1	−3·7	29	15·1	−13·4
Tomisaki	43	6, 32·0	−0·3	+3·4	28	14·9	−1·9
Yokosuka	45	6, 55·8	−0·3	+26·7	29	15·1	+23·0
Yokohama	50	6, 36·0	−0·4	+6·1	28	15·0	+2·3
Maebasi	55	6, 32·0	−0·4	+1·2	28	15·0	−2·7
Kumagaya	20 00	6, 38·8	−0·4	+6·9	28	15·0	+3·1
Tôkyô	02	6. 38·8	−0·4	+4·6	28	15·0	+2·7

(*to be continued.*)

Table I. (*continued.*)

Station	Δ	P	δt_r	ε_1	δΔ	δt	ε_2
	° ′	h m s	s	s	′	s	s
Wazima	20 32	7 6, 23·1	−0·4	−15·0	28	14·9	−19·2
Tukubasan	32	6, 32·0	−0·4	−6·1	28	14·9	−10·4
Utunomiya	34	6, 44·8	−0·4	+6·4	28	14·9	+2·0
Kakioka	36	6, 45·0	−0·4	+6·1	28	14·9	+1·8
Niigata	46	6, 24·5	−0·5	−15·3	27	14·9	−25·1
Tyôsi	51	6, 56·9	−0·4	+15·1	27	14·8	+10·4
Vladivostock		6, 41	−0·6				
Mito	52	6, 55·0	−0·4	+13·0	26	14·7	+8·2
Aidu	21 14	7, 06·6	−0·4	+20·4	27	14.8	+15·3
Hukusima	44	6, 49·1	−0·5	−2·4	26	14·7	−8·6
Yamagata	46	7, 07·8	−0·5	+15·6	26	14·7	+9·9
Sendai	22 09	7. 24·5	−0·5	+28·1	26	14·6	+22·4
Midusawa	47	6, 58·0	−0·5	−5·5	26	14·5	−12·3
Morioka	23 04	7, 07·7	−0·5	+1·1	25	14·5	+47·3

Using the formula

$$t_{ob}-t_{cal}=\delta t_0+\frac{\partial t}{\partial\varphi_0}\delta\varphi_0+\frac{\partial t}{\partial\lambda_0}\delta\lambda_0+\delta\alpha\Delta+\Delta\beta\Delta^3\,, \tag{1}$$

the corrections to the assumed values (δt_0, $\delta\varphi_0$, $\delta\lambda_0$, $\delta\alpha$ and $\delta\beta$) were determined by the method of least squares.[24] Discarding the observations subjected to distinct errors, 46 stations in the range $\Delta=0{\cdot}8°$ and 18° were used in the analysis.

The results were

$$\delta t_0=0\overset{s}{\cdot}86\pm0\overset{s}{\cdot}91\ \text{(P. E.)},$$
$$\delta\varphi_0=-1\overset{\prime}{\cdot}4\pm2\overset{\prime}{\cdot}5\,,$$
$$\delta\lambda_0=-11\overset{\prime}{\cdot}4\pm2\overset{\prime}{\cdot}6\,,$$
$$\delta\alpha=-0{\cdot}21\pm0{\cdot}13\,,$$
$$\delta\beta=0{\cdot}00003\pm0{\cdot}00033.$$

24) As is usually the case with large earthquake causing damage, the hypocentral depth of this earthquake seems to be about 10 km from the examination of the P-curve near the epicentre. This depth was worked out by Y. Oka ("Preliminary Report of the Severe Middle Formosa Earthquake", published by the Taihoku Observatory) as well as by members of the Central Meteorological Observatory (*Kensin Zihô*, **9** (1935), 8). This is also confirmed by the examination of reflected waves. And we can well neglect the consideration of the effect of this depth in the present analysis.

$$\therefore \quad \left. \begin{aligned} \varphi_0 &= 24^\circ\, 19'.6 \pm 2'.5 \text{ N}, \\ \lambda_0 &= 120^\circ\, 37'.6 \pm 2'.6 \text{ E}, \end{aligned} \right\} \qquad (2)$$

and

$$\left. \begin{aligned} t = 7^{h}\, 2^{m}\, 2^{s}.5 &+ 14.337\Delta - 0.00217\Delta^3, \\ \pm 0.9 \ &\pm 0.13 \quad \pm 0.00033. \end{aligned} \right\} \qquad (3)$$

The probable error of individual observation were $\rho = 1.5$ sec ($\sigma = 2.2$ sec). The residue ε_1 of each observation were tabulated in Table I. The epicentre determined lies near the River Taikwo-kei in the southern vicinity of a town called Taikwo.

2. Examination of the Assumption of the Second Order Discontinuity.

Now that the epicentre and the time distance curve up to $\Delta = 18^\circ$ have been determined, we have only to examine the fitness of the curve with the observations above that distance. But as we have already stated, the observations show such a considerable scatter that we cannot get definite conclusion from the sole examination of these data. We are therefore compelled to compare it with the existing time-distance curves. The comparisons were made with two curves, the one was that of Jeffreys-Bullen table[25] of 1935 and the other was the mean[26] of the P-curve of the Sanriku Earthquake and reduced P-curve of the deep-focus earthquake of Feb. 20, 1932. (Table II.)

Table II. Mean P-curve (M) and its derivative.

Δ	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°
t (M) sec	278·6	288·9	299·3	309·3	319·4	329·3	339·0	348·5	357·9	367·2	376·3
$\frac{dt}{d\Delta}$ sec/degree	10·53	10·34	10·20	10·03	10·025	9·80	9·59	9·44	9·36	9·175	9·20

25) H. JEFFREYS and K. E. BULLEN, *Publ. Bur. Centr. Seismol. Int.*, (A), **11** (1935).

26) It was derived by the present writers by the revision of the data of the Sanriku Earthquake through the kindness of Prof. Matuzawa, and making use of the data in a paper by H. Kawasumi and R. Yosiyama (*Disin*, **6** (1934), 415). On comparing the two component curves we obtained that the delay in starting defined by Jeffreys was 7·9 sec, which is exactly the same with the value obtained by Jeffreys and Bullen. And this value was subtracted from the mean value in the above comparison. The analysis of the mean curve will be reported in near future.

In each case the extrapolated time curve of the Formosa Earthquake becomes nearly continuous with the time curve compared at about $\Delta=20°$, though the slope suddenly increases by about 10% in passing the joint.

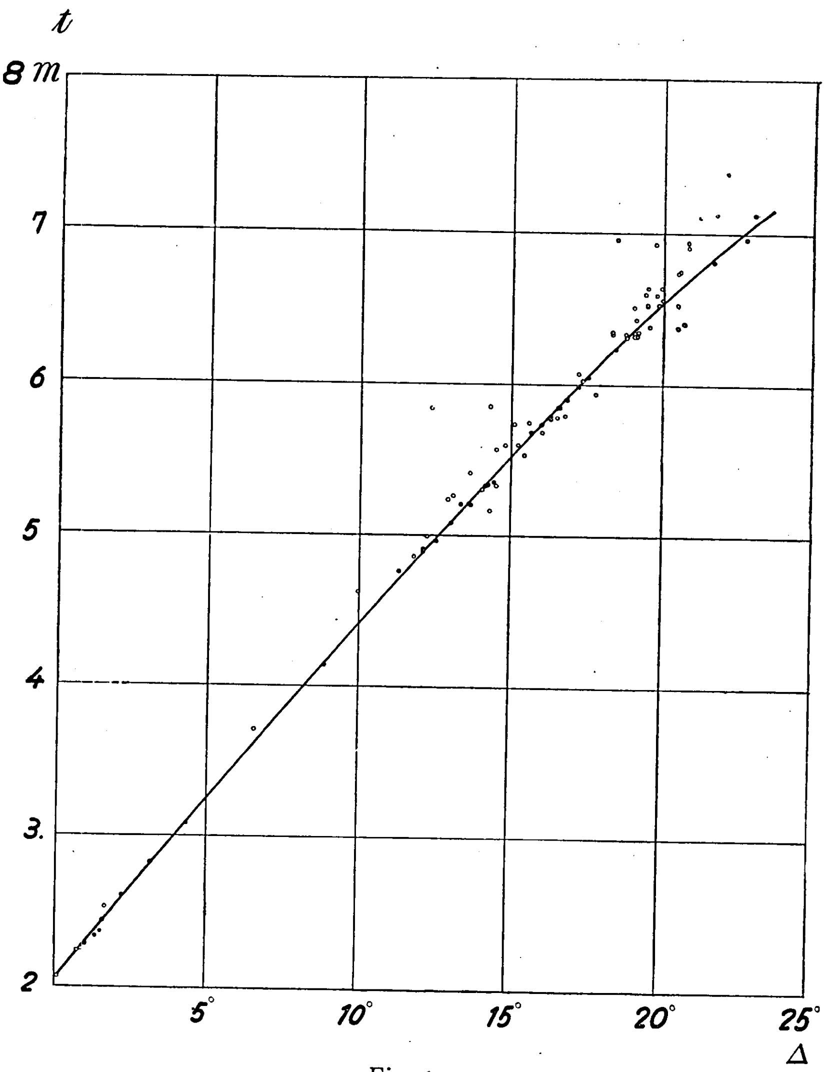

Fig. 1.

If we were to smooth these discontinuities, the necessary corrections derived, for example, by Comrie's method are, as we see in the raws of $-\frac{3}{35}\Delta_4$ in Table III, amount only to small fractions of a second. And such correction may be quite permissible from the present observational accuracy. But if this is actually the case, the consequence contradicts to Lehmann's discovery of the sudden decrease of amplitude at the epicentral distance here concerned. Moreover the scatter of the observational points of the time-distance curve of the present earthquake at the very distances seems also disadvantageous to the above consequence. (See Fig. 1.)

Table III. Smoothed P-curves and the Correction applied by Comrie's method.

Δ	17	18	19	20	21	22	23	24	25	26	27	28
J. B.	233·1	245·4	257·6	269·8	280·4	290·6	300·6	310·4	319·8	329·1	338·2	347·2
M	233·1	245·4	257·5	269·5	280·8	291·5	301·5	311·5	321·4	331·1	340·6	350·0
$-\frac{3}{35}\Delta_4$(J.B)	−0·03	−0·01	+0·14	−0·21	+0·08	0	+0·02	0	−0·05	+0·07	−0·03	0
$-\frac{3}{35}\Delta_4$(M)	−0·03	+0·01	+0·01	0·07	−0·15	0·06	+0·02	0·01	−0·01	−0·01	+0·01	+0·02

Though the time observation of the present earthquake is insufficient to be decisive without support of the amplitude observations, there is an investigation by Sagisaka and Miura on the present as-

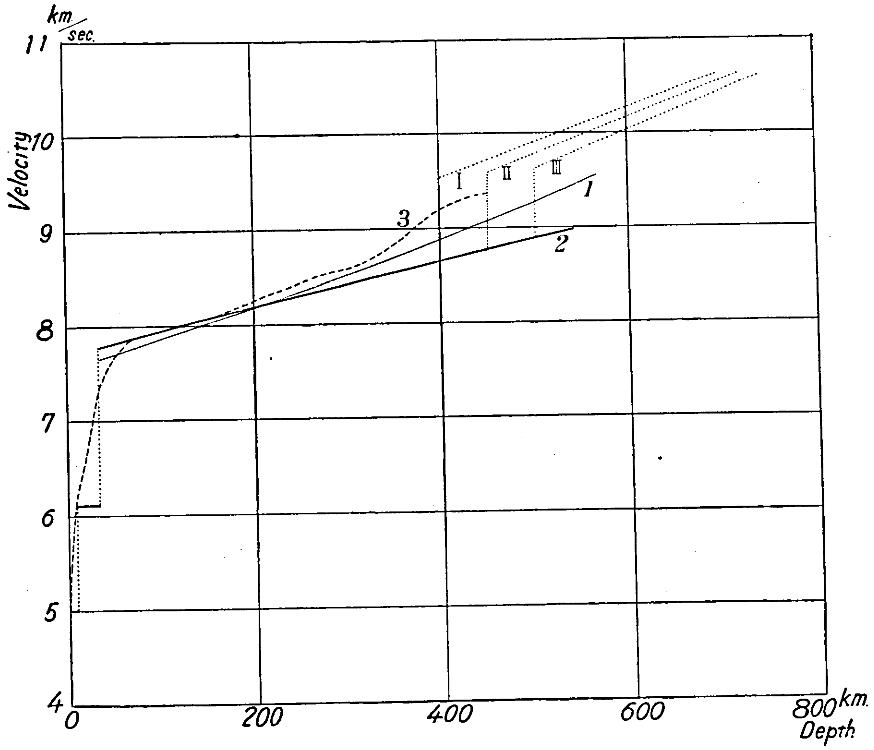

Fig. 2. Velocity distribution of P-wave. 1: The result of chapter 2. 2 (I, II, III): The results of chapter 3. 3: The results of Sagisaka and Miura.

sumption, so we shall here cite their results (curve 3 in Figs. 2 and 3.) and abstain from the detailed discussion. We shall only give here the analysis of the P-curve expressed as cubic equation of Δ, to see the accuracy of the present determination of velocity and path of seismic waves.

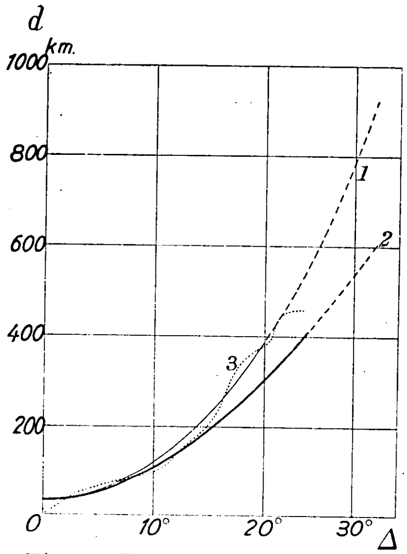

Fig. 3. The depth of vertex of sciomic ray.

For this purpose we must know the structure of the surface layers. But having no data to determine it from the observations, we have, for the present, assumed the two surface layers with the velocities of P-waves 5·0 and 6·1 km/sec respectively as determined by Matuzawa,[27] and determined the thickness of these layers so as the delay in starting becomes 7·9 sec under the assumption that the second layer is twice as thick as the first layer. Thus we adopted 11·3 km as the thickness of the first layer and 33·9 km as the total thickness of the surface crust.

If we substitute by calculating time curves of $\overline{P}$, P*, etc. for the lacking observations, we can determine the velocity within the earth by means of the Herglotz-Wiechert's method integrating along the path found by Slichter[28] in the plane problem and proved by the present writers[29] for the spherical problem. But we shall here adopt the usual procedure of reducing the time curve on the surface of subcrust.

$$T = t - \delta t\,, \qquad \theta = \Delta - \delta\Delta\,, \tag{4}$$

where δt and $\delta\Delta$ are the time and distance corresponding to the parts of seismic ray within the surface layers. The corrections δt and $\delta\Delta$ vary from 18 sec to 15 sec and from 0·7° to 0·5° in the range up to $\Delta=20°$, but the variation is so small that we may consider them to be constant. Because if we take the variation of δt and $\delta\Delta$ into consideration and expressing, for simplicities sake, in

27) T. MATUZAWA, *Bull. Earthq. Res. Inst.*

28) L. B. SLICHTER, *Physics.*, **3** (1932), 273.

29) H. KAWASUMI and S. HONMA, Read at the meeting of the Earthquake Research Institute on Sept. 17, 1935, and will be published shortly.

$$\delta t = \left(18 - \frac{3}{20}\Delta\right)\sec, \qquad \delta\Delta = \left(0{\cdot}7 - \frac{0{\cdot}2}{20}\Delta\right), \tag{5}$$

and carry out the transformation into

$$t - 18 = d(\Delta - 0{\cdot}7) - \beta(\Delta - 0{\cdot}7)^3 - \left(\frac{3}{20} - \frac{0{\cdot}2}{20}\alpha\right)\Delta - 3 \times \frac{0{\cdot}2}{20}\beta\Delta(\Delta - 0{\cdot}7)^2, \tag{6}$$

the last two terms are almost negligible from the accuracy of the present determination of the coefficient of time-distance curve. We can therefore use, for the P-curve on the level of subcrust

$$T = \alpha\theta - \beta\theta^3, \tag{7}$$

where α and β are the same as those of P-curve on the earth's surface.

Then the Herglotz-Wiechert's formula

$$\log\frac{R}{r} = \frac{1}{180}\int_0^{\theta_1}\cosh^{-1}u\,d\theta, \qquad \text{where } u = \frac{dt}{d\theta}\Big/\frac{dT}{d\theta_1} \tag{8}$$

reduces to

$$\log\frac{R}{r} = \frac{1}{90}\sqrt{\frac{2}{\gamma}}\frac{K - E}{\sqrt{1-k^2}}, \tag{9}$$

[30]

where $\gamma = \frac{3\beta}{\alpha}$, and $K = \int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{1 - k^2\sin^2\varphi}}$ and $E = \int_0^{\frac{\pi}{2}}\sqrt{1 - k^2\sin^2\varphi}\,d\varphi$ are complete elliptic integrals with modulus $k^2 = \frac{\gamma\theta_1^2}{2 - \gamma\theta_1^2}$.

The velocity at the deepest point of the ray is then given by

$$v = \frac{\pi}{180}\frac{r}{(\alpha - 3\beta\theta_1^2)} = \frac{\pi}{180\alpha}\frac{1 + k^2}{1 \quad k^2}r. \tag{10}$$

Calculations were made by taking $R = (6371 - 34)$ km. The depth of vertex ($d = 6371 - r$) of the ray emerging at θ is tabulated in Table

---

29) Examination problem set by Prof. Matuzawa in 1934, and solved by one of the writers. The brief proof is as follows. Since $\frac{dT}{d\theta} = \alpha - 3\beta\theta^2$, $u = \frac{1 - \gamma\theta^2}{1 - \gamma\theta_1^2}$. Integrating (8) by parts, we have

$$\log\frac{R}{r} = \frac{1}{180}\Big[\theta\cosh^{-1}u\Big]_0^{\theta_1} - \frac{1}{180}\int_0^{\theta_1}\frac{\theta}{\sqrt{u^2-1}}\frac{du}{d\theta}d\theta = \frac{2\gamma}{180(1-\gamma\theta_1^2)}\int_0^{\theta_1}\frac{\theta}{\sqrt{u^2-1}}d\theta,$$

and changing variables into $\theta = \theta_1\sin\varphi$ and putting k^2 as above we have

$$\log\frac{R}{r} = \frac{1}{90}\sqrt{\frac{2 - \gamma\theta_1^2}{\gamma}}\int_0^{\frac{\pi}{2}}\frac{k^2\sin^2\varphi}{\sqrt{1 - k^2\sin^2\varphi}}d\varphi,$$

which reduces to (9).

II. The values corresponding to θ beyond 20° might be meaningless, but we have calculated for the sake of comparison with the following cases discussed in the next chapter.

Table IV. The depth (d) and velocity (v) at the vertex of a seismic ray emerging at epicentral distance θ.

θ	0°	4°	8°	12°	16°	20°	24°	28°
d km	33·9	46·9	88·3	157·8	257·9	392·0	568·6	787·4
v km/sec	7·56	7·69	7·82	8·04	8·38	8·87	9·57	10·61

Next we have to examine the accuracy of the values here determined. From the value of the coefficient of Δ in the P-curve we have

$$v_0 = 7{\cdot}65 \pm 0{\cdot}07 \text{ km/sec},$$

where v_0 is the velocity at 34 km level. The probable error of v will increase with the increasing depth of vertex of seismic ray. So we will examine it with the ray emerging at $\theta = 20°$. For this purpose we have calculated four cases answering to the four combinations of signs of errors of α and β. The errors of α and β obtained above from this earthquake are smaller than those obtained by Matuzawa, and nearly equal to those of Jeffreys and Bullen. So we have also calculated the cases corresponding to Matuzawa's values.

Table V. Examination of accuracies of determination of v and d corresponding to the errors of determination of time-distance curve, in cases of I, the Formosa earthquake ($\delta\alpha = \pm 0{\cdot}13$, $\delta\beta = \pm 0{\cdot}00033$) and II, the Sanriku earthquake ($\delta\alpha = \pm 0{\cdot}35$, $\delta b = \pm 0{\cdot}0015$) for the ray emerging at $\theta = 20°$.

	I				II			
	1 (++)	2 (+−)	3 (−+)	4 (−−)	1 (++)	2 (+−)	3 (−+)	4 (−−)
v km/sec	9·03	8·54	9·24	8·71	9·86	7·70	10·59	10·46
d km	420·2	359·2	424·3	362·5	513·9	227·6	534·6	630·1

Comparing these values with those in Table II the accuracies of the present determination are $v = (8{\cdot}87 \pm 0{\cdot}33)$ km/sec, $d = (392 \pm 33)$ km in case I, while the ranges of v and d in case II are $(-1{\cdot}17 \sim +1{\cdot}59)$ km/sec and $(-154 \sim +238)$ km respectively. Thus the determination of the discontinuity of velocity, if any, cannot attain higher accuracies.

The values obtained are shown in Figs. 2 and 3 (curve 1) in which the results of Sagisaka and Miura are also indicated (curve 3) for comparison.

3. Examination of the Assumption of the Discontinuity of First Order.

As we have already stated the observations beyond $\Delta=20^\circ$ seem to be subjected to large errors to draw definite conclusion, but the writers were struck by the fact that when they first draw the time-distance curve using the primary epicentral distance, smooth curve tentatively drawn was nearly coincident to the usual P-curves up to $\Delta=20^\circ$ca. which suddenly deviate beyond that distance. This is easily explained by considering the branch as the late P-phase found by Lehmann, the first phase being missed owing to the smallness of amplitude.

We shall therefore proceed upon this assumption to test the existence of first order discontinuity. Then we shall be able to examine the depth and amount of discontinuity as well as the form of the time distance curve.

As the assumption itself is in question and the observational accuracy is not so good as we have seen in the preceding chapter, we shall be contented with the simple procedure on the assumption that the seismic ray is circular, and consequently the velocity distribution in the subcrust and time distance curve are given by

$$v=a-br^2, \tag{11}$$

and

$$T=\frac{1}{\sqrt{ab}}\sinh^{-1}\frac{\sqrt{ab}\,2\sin\frac{1}{2}\theta}{a-b} \quad ^{31)} \tag{12}$$

respectively.

Selecting 42 stations including Kôhu, Oiwake, Toyama, Yokahama, Tokyo, Kumagaya, Utunomiya and Kakioka on the branch beyond $\Delta=18^\circ$, we have determined T_0, a and b by the method of least squares. The formula used was

$$T_{ob}-T_{cal}-T_0=\delta T_0+\frac{\partial T}{\partial(a-b)}\partial(a-b)+\frac{\partial T}{\partial\sqrt{ab}}\delta\sqrt{ab}, \tag{13}$$

in which T and θ is reduced value on the 34 km level. And primary values adopted were $a-b=0{\cdot}001207$ and $\sqrt{ab}=0{\cdot}002338$ which was

31) H. KAWASUMI, *Bull. Earthq. Res. Inst.*, **10** (1032), 94~129; *Jap. Journ. Astro. Geophys.*, **9** (1931), 15~22.

determined by using the cotangent constant $\frac{a+b}{a-b}=\cot i \cot\frac{1}{2}\theta=4$.
The results were

$$\left.\begin{aligned} a-b&=0{\cdot}001227\pm0{\cdot}000008 \text{ (P. E.)},\\ \sqrt{ab}&=0{\cdot}001750\pm0{\cdot}000103,\\ T_0&=22\text{ h } 1\text{ m } 53{\cdot}7\pm0{\cdot}6\text{ s}, \end{aligned}\right\} \tag{14}$$

the probable errors of individual observation being $\rho=1{\cdot}3$ sec and consequently

$$\left.\begin{aligned} a&=0{\cdot}0024678\pm0{\cdot}000018,\\ b&=0{\cdot}0012409\pm0{\cdot}000018. \end{aligned}\right\} \tag{15}$$

The velocity distribution $v=a-br^2$ and the depth of the vertex of the seismic ray emerging at θ or $\Delta=\theta+\delta\Delta$ are shown in Table VI, Fig. 2 and Fig. 3.

Table VI. v and d.

θ	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°
d km.	45	62	80	106	137	175	212	257	306	357	412	473	540
v km/sec	7·81	7·85	7·89	7·95	8·03	8·12	8·21	8·32	8·43	8·56	8·69	8·83	8·98

The residues ε_2 of all the observations calculated from the above values of T_0, a and b are tabulated in the last column of Table I. The residues beyond 18° are of course very large, and some of these may be explained by the hypothetical three branches (Fig. 4) of time-distance curve, the individual, reflected waves and the transmitted wave partly through the lower layer. We shall now proceed to determine these three branches of time-distance curve in order to have some clue on the discontinuity of velocity. Since we know the velocity distribution outside the hypothetical discontinuity surface, we can calculate two branches, i. e. the individual and reflected waves, provided we know the depth of discontinuity surface, by the formula (11) and

$$T=\frac{1}{\sqrt{ab}}\sinh^{-1}\frac{\sqrt{ab}\,d}{V}, \tag{16}$$ [32]

where $V=\sqrt{(a-b)\{a-b(1-h)^2\}}$ and $d=\sqrt{h^2+4h\sin^2\frac{1}{2}\theta}$,

32) H. KAWASUMI, *loc. cit.*, 30).

in which h denotes the depth of the discontinuity surface measured in fraction of the earth's radius.

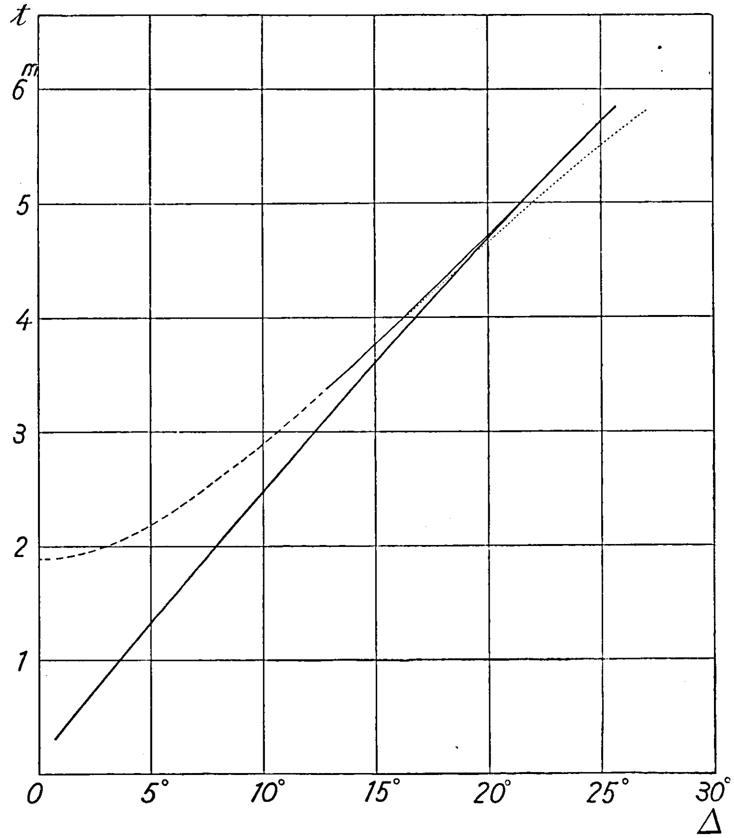

Fig. 4. Calculated three branches of P-waves using the velocity determined upon the assumption (II) that the first order discontinuity is at the depth of 450 km.

Assuming three values 400, 450, and 500 km as the thickness of the discontinuity surface we have first calculated the constant K of the seismic ray defined by

$$K = \frac{r}{v} \sin i = \frac{dT}{d\theta} \qquad (\theta \text{ in radian}) \qquad (17)$$

for each of these branches. (Fig. 5.) As we were not able to determine the branch of the transmitted wave from the observations of the present earthquake, the mean P-curve deduced from the Sanriku earthquake and the deep-focus earthquake of Feb. 20, 1931 was substituted in this place. (Table II.) The branch in the lower part and beyond

$\theta=20°$ denotes the value obtained from the mean P-curve.

We can now take off the effect due to the upper part of the discontinuity surface by simply subtracting the distance θ_r of the reflected wave from θ_t of the tansmitted wave corresponding to the same value of K. The results are shown in the lower left of Fig. 5.

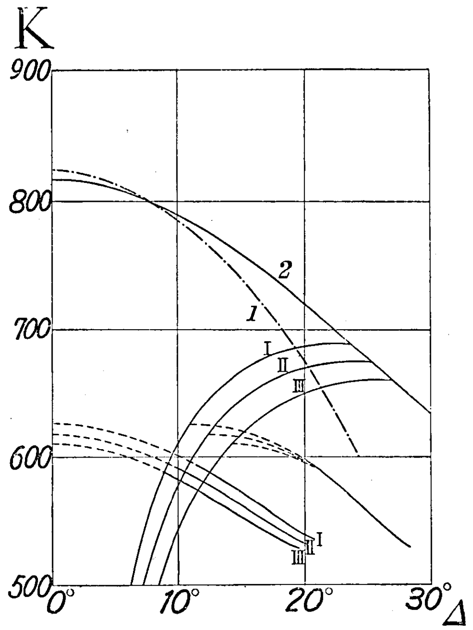

Fig. 5.

As we have no means at present to determine the lacking part of this reduced $K(\theta)$ curve, we are compelled to extrapolate the $K(\theta)$ curve to $\theta=0°$. For this purpose we again assumed the velocity distribution of the form $v_1=a_1-b_1r_1^2$ where r_1 is the distance from the centre of the earth measured in fraction of the radius of the discontinuity surface. Then

$$K=\frac{dT}{d\theta}=\frac{\cos\frac{1}{2}\theta}{\sqrt{(a_1-b_1)^2+4a_1b_1\sin^2\frac{1}{2}\theta}} \tag{18}$$

and we have from this equation

$$(a_1-b_1)^2+4a_1b_1\sin^2\tfrac{1}{2}\theta=\frac{\cos^2\frac{1}{2}\theta}{K^2}. \tag{19}$$

Since we know K as a function of θ we can determine $(a_1-b_1)^2$ and $4a_1b_1$ by means of this formula. Least square solutions using 9 values for the above three cases were obtained. The results are tabulated in Table VII.

Table VII. Constants of velocity formula for the inside of discontinuity surface.

d (km)	I 400	II 450	III 500
$(a_1-b_1)^2\times10^6$	26·162	27·912	30·814
$4\,ab\times10^6$	25·49	26·18	26·83
$a\times10^3$	3·477	3·572	3·713
$b\times10^3$	1·881	1·954	2·075

Table VIII. Velocity distribution within the discontinuity surface. (km/sec)

d	400	450	500	550	600	650	700	750	800	850
I	9.53	9·72	9·91	10·09	10·27	10·45	10·63	10·81	10·99	11·16
II		9·58	9·78	9·97	10·16	10·35	10·54	10·72	10·91	11·09
III			9·62	9·82	10·03	10·23	10·43	10·63	10·83	11·03

The values thus extrapolated are given in Table IX, and indicated by broken lines in Fig. 5.

Table IX. Extrapolated *K*.

θ	0	2	4	6	8	10
I	626·4	625·3	622·1	616·9	609·8	601·0
II	618·0	616·9	613·6	608·3	601·1	592·1
III	610·5	609·3	605·9	600·3	592·6	583·2

Now that we have complete $K(\theta)$ curve on the level surface of assumed discontinuity, we can deduce them to the values on the earth's surface or any other level surface by reversing the above process. Thus we could have all three branches of $K(\theta)$ curve on the 34 km level as shown in Fig. 5. The corresponding P-curves can also be calculated from the values we have obtained. But the $K(\theta)$ curve offers in itself a conspicuous means for the selection of the most suitable case among the assumed cases. The epicentral distances of the three joints of the three branches, if we have observation, are very convenient. We can also obtain the P-curves by integrating this $K(\theta)$ curve along the three branches.

Table X. Calculated 3 branches of P-curve for case II, $d=450$ km.

Individual wave	θ	0°43′	2°42′	4°41′	6°40′	8°39′	10°38′	12°37′	14°36′	16°34′	18°33′	20°32′	22°31′	24°29′	26°28′
	t	18·2s	46·2	74·5	102·6	130·3	157·8	184·9	211·5	237·6	263·2	287·3	312·9	336·9	360·2
Reflected wave	θ	0°	2°42′	4°41′	6°40′	8°39′	10°38′	12°37′	14°36′	16°34′	18°33′	20°32′	22°31′	24°29′	26°28′
	t	112·5s	46·2	74·6	102·8	124·6	160·0	186·6	214·6	242·2	268·6	297·0	324·2	351·1	377·9

Transmited wave	θ	12°46′	14°41′	16°25′	18°04′	19°34′	21°03′	22°31′	23°56′	25°23′	26°52′
	t	202·8s	223·5	242·3	259·4	275·7	291·0	305·8	320·0	334·5	348·5

The three branches of P-curve for the case II ($d=450$ km) indicated in Table X and Fig. 5 are calculated rigorously from the values of velocity, while the curves for cases I and III were deduced by the integration of $K(\theta)$ curves. The branch of the transmitted wave thus obtained is nearly conformal with the observed mean curve above used. The point of intersection of this branch with the curve of the individual wave and the constant difference between the observed and calculated curve for each case are as follows:

d (km)	I 400	II 450	III 500
Δ	17°·5	19°·3	20°·4
t_0-t_c	2ˢ·2	−1ˢ·4	−3ˢ·6

It seems from the value of t_0-t_c that the case II is nearest to the observed value. Moreover the epicentral distance of the point of intersection for this case is nearly coincident with the value 19° which was found by Neumann, Lehmann, Jeffreys and Bullen. If interpolation is permissible we have from the value of t_0-t_c the depth of the discontinuity surface of about 430 km from the earth's surface.

It is also to be noted that a number of points excluded in the preceeding analysis can be explained by the three branches calculated (Fig. 5), and especially the correspondence of the late observations from 13° to 18° with the late P-phase (probably the reflected wave) is worthy of some further examination even if it were mere coincidence.

The amounts of discontinuity of velocity thus worked out are

	I	II	III
v_0 (km/sec)	8·66	8·78	8·89
v_i (km/sec)	9·53	9·58	9·62
v_0-v_i (km/sec)	0·87	0·80	0·73

all within 1 km/sec, while the ranges of triplication of P- curve are

I. $\Delta=11°·3-23°·9$, II. $\Delta=12°·6-25°·6$ and III. $\Delta=14°·6-27°·3$

respectively.

These values are the results of our assumption that the six stations beyond 18° observed late P-phase. It also depends on the adopted travel time curve beyond 20°. But if we were to take the velocity determined in chapter II, the amount of velocity discontinuity and the range of triplication of P-curve would be smaller, while the depth of discontinuity would be still larger.

The amount of discontinuity and range of triplication are thus unexpectedly smaller than the values anticipated by various authorities.

Another word to be added is on the amplitude variation of early P-phases near the joint at about 19°. Since the transmitted wave passes twice the discontinuity surface, the amplitude must diminish as the effect of refraction. Now the angles of emergence at the discontinuity surface of the waves emerging at $\Delta=20°$, 22°, and 24° are 13°, 19° and 23° respectively (for the case II, $d=450$ km), considerable decrease of amplitude of initial motion is to be expected in passing the joint at abouts=19°, but the increase of amplitude with increase of the angle of emergence is so large in the range concerned, that the amplitude will increase rapidly with increasing distance.

The variation of amplitude due to the variation of $\frac{dt}{d\theta}$ is also to be considered[33]. But we see from Fig. 4 that the $K(\theta)$ curves of the individual and transmitted waves are nearly parallel, the effect will be almost negligible. And if we were to use the P-curve used in Chapter II, the decrease of amplitude from this effect is only about $1/\sqrt{1{\cdot}5}$ at about 20°. The larger the variation of $\frac{dt}{d\theta}$ of the individual wave, or the smaller the variation of $\frac{dt}{d\theta}$ of the transmitted wave, the more decrease of amplitude is to be expected.

Thus we see, as already discussed by Lehmann and Gutenberg, essential difference of the consequences of the two assumptions of the discontinuity of the first and second order. But we must take care, in the examination of initial motion, of the fact that the appearance of smaller transmitted wave is restricted to very small range, which may be subjected to more complication by the superposition of late P-phases.

Concluding Remark.

We have discussed the problem on the existence of discontinuity surface in the mantle of the earth from the examination of P-curve due to the Formosa earthquake of April 20, 1935.

33) K. ZÖPPRITZ, *Nachr. d. Kgl. Geo. d. Wiss. zu Göttingen*, Math.-phys. Kl. (1912), 123~143.
E. WIECHERT, *ibid.* foot note on page 127.
L. GEIGER and B. GUTENBERG, *do*, 144~206.
H. JEFFREYS, *M. N. R. A. S. Geophys. Suppl.*, **1** (1926), 334~348.
H. KAWASUMI, *Bull. Earthq. Res. Inst.*, **10** (1933), 403~453.

The epicentre determined by the method of least squares came out at

$$\varphi_0 = 24^\circ\ 19'.6 \pm 2'.5\ \mathrm{N},$$

$$\lambda_0 = 120^\circ\ 37'.6 \pm 2'.6\ \mathrm{E}.$$

And velocity distribution within the earth as well as the depth of vertex of the seismic ray emerging at any epicentral distance up to about 20 or so are determined, upon the two assumptions of the existence of the velocity discontinuity of first and second order.

The quantitative basis of these results are examined specially, and could see that discontinuity of second order may be adopted if correction amounting only to two tenths of a second is applied to the time distance curves used. While the assumption of the first order discontinuity seems in some respects more favourable, the amount of discontinuity may be smaller than 1 km/sec, which is much smaller than expected. The depth of discontinuity surface, if any, is about 430 km from the earth's surface. There may exist some correlation with the fact that the deep-focus earthquake occurs outside this surface.

The range of triplication of P-curve was discussed, but other possibility of this cause of triplications than the first order discontinuity is not considered in this paper. The difference of triplication due to the existence or nonexistence of the first order discontinuity is to be examined by the change of sense of motion in the reflexion in the former case. The examination of the reflected wave at shorter epicentral distance may also be used for this purpose, though it will be difficult owing to the smallness of amplitude.

In conclusion the writers wish to acknowledge their cordial thanks to Prof. T. Matuzawa for the generosity in placing the data of the Sanriku Earthquake at our disposal.

18. 昭和10年4月21日臺灣地震の走時曲線と地殼構造上の一問題

地震研究所 河 角 廣
本 間 正 作

上記の地震 P-波走時曲線を吟味して地下約 400 粁にありと云はれて居る不連續面を吟味した. 此の爲に最少二乘法により發震時を用ひて震央及び走時曲線を定めた.

$$\text{卽} \quad \varphi_0 = 24^\circ\ 19'\!\cdot\!6 \pm 2'\!\cdot\!5 \ \ \mathrm{N}$$

$$\lambda_0 = 120\ 37\!\cdot\!6 \pm 2\!\cdot\!6 \ \ \mathrm{E}$$

$$t = 20^{\mathrm{d}}\ 22^{\mathrm{h}}\ 2^{\mathrm{m}}\ 2^{\mathrm{s}}\!\cdot\!5 + 14^{\mathrm{s}}\!\cdot\!337\Delta - 0^{\mathrm{s}}\!\cdot\!00217\Delta^3$$

$$\pm 0\!\cdot\!9 \quad \pm 0\!\cdot\!13 \quad \pm 0\!\cdot\!00033.$$

此の震央は大甲街の南大甲溪岸に當る.

地球內部に於ける第2種及び第1種の不連續面の存在を假定し，各の場合に於ける速度分布及び震波線最深點の深さ等を計算した. 特に其等數値の誤差の程度に注意し. 各假定の適否は現在尙走時のみよりは決定不可能なりしも，其等の數量的根據を明にした.

卽速度分布に有限な不連續のないと云ふ假定に合はせるには走時曲線に僅か 2/10 秒の補正を加へれば十分であるが，若し有限の不連續がありと假定しても其の不連續の分量は 0·8 粁の程度に過ぎず. 從來考へられたものより著しく小いものであり，其の不連續面の深さは地下約 430 粁の程度である.

尙第1種の不連續面でなくても速度分布の狀態によつては走時曲線が3重になる事があり得るが此の點は其の觀測がないので吟味出來なかつた. 此れは初動の方向を吟味すれば直ちに解る問題である. 卽第1種の不連續面がある場合には反射波は 180° の位相の變化を受けるからである.

第1種の不連續面か，第2種の不連續面かの區別は走時曲線の形だけでなく初動の大きさの吟味によつて完全に決定される筈であるが，其の際極めて細心の注意を要する事を知つた.