12. *Energy Dissipation in Seismic Vibrations of a Six-storied Structure. Coincidence of Resonance and Corresonance.*

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1. Introduction.

After a protracted investigation we have at last succeeded in ascertaining the nature of energy dissipation, etc., in the seismic vibrations of a framed multistoried structure, but not exceeding 5-stories.[1)~6)] With this reservation in the number of stories, various problems in seismic vibrations, namely, the periods of free vibrations, the nature of forced vibrations both with and without energy dissipation, the same problems as applied to actual buildings, and the effect of structural conditions on the energy dissipation, etc., have been solved. For actual examples, the following buildings, owned by the Mitsubishi Co. are here discussed, namely, the annexe of the Middle 7th House, Annexe of the Middle 8th House, and the Middle 13th House for the case of a 3-storied structure; No. 6 of the Middle 12th Lot of Houses for that of a 4-storied structure, and the Annexe of the East 7th House for that of a 5-storied one. Mathematical formulae for treatment of the problem in the case of buildings of less than five stories have also been worked out to our satisfaction.

But we had not yet formulated a theory of dissipation for the

1) K. SEZAWA and K. KANAI, "Some New Problems of Free Vibrations of a Structure", *Bull. Earthq. Res. Inst.*, **12** (1934), 804~822.

2) Ditto, "Some New Problems of Forced Vibrations of a Structure", *ibid.*, **12** (1934), 823~853.

3) Ditto, "Decay in Seismic Vibrations of a Simple or Tall Structure by Dissipation of their Energy into the Ground", *ibid.*, **13** (1935), 681~697.

4) Ditto, "Energy Dissipation in Seismic Vibrations of a Framed Structure", *ibid.*, **13** (1935), 698~714.

5) Ditto, "Energy Dissipation in Seismic Vibrations of Actual Buildings.," *ibid.*, **13** (1935), 921~941.

6) Ditto, "Energy Dissipation in Seismic Vibrations of Actual Buildings in Different Structural Conditions," *ibid.*, **14** (1936), 110–120.

case of actual buildings higher than six stories. Through the courtesy of the Mitsubishi co., we had access to data regarding their Middle 28th House, and these enabled us to proceed with the problem for a 6-storied structure, the important mathematical formulae having been worked out specially for the purpose.

2. Mathematical Formulae for a 6-storied Structure with Rigid Floors and Clamped Base.

We have already given in a previous paper the general formula for an n-storied structure with rigid floors and clamped base[7]. Using equations (13)$\sim$(20) from that paper, it is possible to obtain A_1, B_1, C_1, D_1, ..., A_6, B_6, C_6, D_6 as follows. In these equations, $\gamma_S = m_S p^2 l_S^3 / E_S I_S$, and m_S is the mass concentrated on the s th floor.

$$
\left.
\begin{aligned}
A_1\Phi = {} & \eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3(\gamma_4-12)\Big[\eta_2(\gamma_3 \\
& -12)\{\eta_1(\gamma_1-12)(\gamma_2-12)-12\gamma_2\}-12\gamma_3\{\eta_1(\gamma_1-12)-12\}\Big] \\
& -12\gamma_4\Big[\eta_2\{\eta_1(\gamma_1-12)(\gamma_2-12)-12\gamma_2\}-12\{\eta_1(\gamma_1-12)-12\}\Big]\Big\} \\
& -12\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3\Big[\eta_2(\gamma_3-12)\{\eta_1(\gamma_1-12)(\gamma_2-12) \\
& -12\gamma_2\}-12\gamma_3\{\eta_1(\gamma_1-12)-12\}\Big]-12\Big[\eta_2\{\eta_1(\gamma_1-12)(\gamma_2-12) \\
& -12\gamma_2\}-12\{\eta_1(\gamma_1-12)-12\}\Big]\Big\}, \\
D_1 l_1^3\Phi = {} & -2\Big[\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3(\gamma_4 \\
& -12)\Big[\eta_2(\gamma_3-12)\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12\gamma_3(\eta_1\gamma_1-12)\Big] \\
& -12\gamma_4\Big[\eta_2\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12(\eta_1\gamma_1-12)\Big]\Big\}-12\{\eta_5\gamma_5(\gamma_6 \\
& -12)-12\gamma_6\}\Big\{\eta_3\Big[\eta_2(\gamma_3-12)\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12\gamma_3(\eta_1\gamma_1 \\
& -12)\Big]-12\Big[\eta_2\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12(\eta_1\gamma_1-12)\Big]\Big\}\Big],
\end{aligned}
\right\} \quad (1)
$$

7) K. SEZAWA and K. KANAI, *loc. cit.* 5).

$$\left.\begin{aligned}
A_2\Phi = &-12\eta_1\Big\{\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)\\
&\qquad -12\gamma_6\}\Big[\eta_3(\gamma_4-12)\{\eta_2(\gamma_2-12)(\gamma_3-12)-12\gamma_3\}\\
&\qquad -12\gamma_4\{\eta_2(\gamma_2-12)-12\}\Big]-12\{\eta_5\gamma_5(\gamma_6-12)\\
&\qquad -12\gamma_6\}\Big[\eta_3\{\eta_2(\gamma_2-12)(\gamma_3-12)-12\gamma_3\}\\
&\qquad\qquad -12\{\eta_2(\gamma_2-12)-12\}\Big]\Big\},\\
D_2l_2^3\Phi = &\,2.12\eta_1\Big\{\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\Big[\eta_3(\gamma_4\\
&\qquad -12)\{\eta_2\gamma_2(\gamma_3-12)-12\gamma_3\}-12\gamma_4(\eta_2\gamma_2-12)\Big]\\
&\qquad -12\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big[\eta_3\{\eta_2\gamma_2(\gamma_3-12)\\
&\qquad\qquad -12\gamma_3\}-12(\eta_2\gamma_2-12)\Big]\Big\},
\end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned}
A_3\Phi = &\,12^2\eta_1\eta_2\Big[\eta_4\{\eta_3(\gamma_3-12)(\gamma_4-12)\\
&\qquad -12\gamma_4\}\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\\
&\qquad -12\{\eta_3(\gamma_3-12)-12\}\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big],\\
D_3l_3^3\Phi = &-2.12^2\eta_1\eta_2\Big[\eta_4\{\eta_3\gamma_3(\gamma_4-12)\\
&\qquad -12\gamma_4\}\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\\
&\qquad -12(\eta_3\gamma_3-12)\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big],
\end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned}
A_4\Phi = &-12^3\eta_1\eta_2\eta_3\Big[\eta_4(\gamma_4-12)\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\\
&\qquad -12\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big],\\
D_4l_4^3\Phi = &\,2.12^3\eta_1\eta_2\eta_3\Big[\eta_4\gamma_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\\
&\qquad -12\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big],
\end{aligned}\right\} \quad (4)$$

$$\left.\begin{aligned} &A_5\Phi=12^4\eta_1\eta_2\eta_3\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\,,\\ &D_5l_5^3\Phi=-2.12^4\eta_1\eta_2\eta_3\eta_4\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\,,\end{aligned}\right\}(5)$$

$$\left.\begin{aligned} &A_6\Phi=-12^5\eta_1\eta_2\eta_3\eta_4\eta_5(\gamma_6-12)\,,\\ &D_6l_6^3\Phi=2.12^5\eta_1\eta_2\eta_3\eta_4\eta_5\gamma_6\,,\end{aligned}\right\}(6)$$

$$B_S=0\,,\qquad C_S=\frac{3}{2}\,l_SD_S\,,\qquad (S=1\,,2\,,\ldots,6) \qquad (7)$$

$$\begin{aligned}\Phi(Q+3ik\varepsilon P)=Q\Bigg[&\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3(\gamma_4\\ &-12)\Big[\eta_2(\gamma_3-12)\{\eta_1(\gamma_1-12)(\gamma_2-12)-12\gamma_2\}\\ &-12\gamma_3\{\eta_1(\gamma_1-12)-12\}\Big]-12\gamma_4\Big[\eta_2\{\eta_1(\gamma_1-12)(\gamma_2-12)\\ &-12\gamma_2\}-12\{\eta_1(\gamma_1-12)-12\}\Big]\Big\}-12\{\eta_5\gamma_5(\gamma_6-12)\\ &-12\gamma_6\}\Big\{\eta_3\Big[\eta_2(\gamma_3-12)\{\eta_1(\gamma_1-12)(\gamma_2-12)-12\gamma_2\}\\ &-12\gamma_3\{\eta_1(\gamma_1-12)-12\}\Big]-12\Big[\eta_2\{\eta_1(\gamma_1-12)(\gamma_2-12)\\ &-12\gamma_2\}-12\{\eta_1(\gamma_1-12)-12\}\Big]\Big\}\Bigg]\\ +\frac{36E_1\varepsilon j_1^2}{\rho l_1^3}P\Bigg[&\eta_4\{\eta_5(\gamma_5-12)(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3(\gamma_4\\ &-12)\Big[\eta_2(\gamma_3-12)\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12\gamma_3(\eta_1\gamma_1-12)\Big]\\ &-12\gamma_4\Big[\eta_2\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12(\eta_1\gamma_1-12)\Big]\Big\}\\ &-12\{\eta_5\gamma_5(\gamma_6-12)-12\gamma_6\}\Big\{\eta_3\Big[\eta_2(\gamma_3-12)\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}\\ &-12\gamma_3(\eta_1\gamma_1-12)\Big]-12\Big[\eta_2\{\eta_1\gamma_1(\gamma_2-12)-12\gamma_2\}-12(\eta_1\gamma_1-12)\Big]\Big\}\Bigg],\end{aligned} \qquad (8)$$

$$\left.\begin{aligned}P&=(h\varepsilon k\varepsilon+4)-i(h^2\varepsilon^2k\varepsilon+2h\varepsilon+2k\varepsilon),\\ Q&=2\Big[\Big\{\frac{\lambda}{\mu}h^2\varepsilon^2(h\varepsilon k\varepsilon+4)-2(k\varepsilon-h\varepsilon)(h^2\varepsilon^2k\varepsilon+2h\varepsilon+4k\varepsilon)\Big\}\\ &\qquad+2i(k\varepsilon-h\varepsilon)\Big(\frac{\lambda}{\mu}h^2\varepsilon^2+2h^2\varepsilon^2+4\Big)\Big].\end{aligned}\right\}\quad(9)$$

By putting $E_1=\ldots=E_6=E,\quad I_1=\ldots=I_6=I,\quad l_1=\ldots=l_6=l,\quad m_1=\ldots=m_6=m,\quad h\varepsilon\to 0,\quad k\varepsilon\to 0,$ we obtain

$$\left.\begin{aligned}A_1\Phi&=\{\gamma(\gamma-24)^2-12(\gamma-12)^2\}^2-12\gamma(\gamma-12)^2(\gamma-36)^2,\\ D_1l^3\Phi&=-2\gamma(\gamma-12)(\gamma-24)(\gamma-36)\{(\gamma-12)^2-24\gamma\},\end{aligned}\right\}\quad(10)$$

$$\left.\begin{aligned}A_2\Phi&=-12\Big\{(\gamma-12)\{(\gamma-12)^2-12\gamma\}^2\\ &\qquad-12\gamma(\gamma-24)\Big[\{(\gamma-12)^2-12\gamma\}+(\gamma-12)(\gamma-36)\Big]\Big\},\\ D_2l^3\Phi&=2.12\gamma\Big[\{(\gamma-12)^2-12\gamma\}^2-24(\gamma-24)\{(\gamma-12)^2-12\gamma\}\Big],\end{aligned}\right\}\quad(11)$$

$$\left.\begin{aligned}A_3\Phi&=12^2\Big[\{(\gamma-12)^2-12\gamma\}^2-12\gamma(\gamma-24)^2\Big],\\ D_3l^3\Phi&=-2.12^2\gamma(\gamma-24)\Big[\{(\gamma-12)^2-12\gamma\}-12(\gamma-12)\Big],\end{aligned}\right\}\quad(12)$$

$$\left.\begin{aligned}A_4\Phi&=-12^3\Big[(\gamma-12)\{(\gamma-12)^2-12\gamma\}-12\gamma(\gamma-24)\Big],\\ D_4l^3\Phi&=2.12^3\gamma(\gamma-12)(\gamma-36),\end{aligned}\right\}\quad(13)$$

$$A_5\Phi=12^4\{(\gamma-12)^2-12\gamma\},\qquad D_5l^3\Phi=-2.12^4\gamma(\gamma-24),\quad(14)$$

$$A_6\Phi=-12^5(\gamma-12),\qquad D_6l^3\Phi=2.12^5\gamma,\quad(15)$$

$$B_S=0,\qquad C_S=\frac{3}{2}l_SD_S,\qquad (S=1,\,2,\ldots\ldots,\,6)\quad(16)$$

$$\left.\begin{aligned}\Phi&=4\Big\{\Big(\sqrt{\frac{\lambda+2\mu}{\mu}}-1\Big)M-9i\sqrt{\frac{\lambda+2\mu}{\mu}}\frac{Ej^2}{\mu kl^3}N\Big\}\Big(7\sqrt{\frac{\lambda+2\mu}{\mu}}-4\Big)^{-1},\\ M&=\{\gamma(\gamma-24)^2-12(\gamma-12)^2\}^2-12\gamma(\gamma-12)^2(\gamma-36)^2,\\ N&=\gamma(\gamma-12)(\gamma-24)(\gamma-36)\{(\gamma-12)^2-24\gamma\},\end{aligned}\right\}\quad(17)$$

In the same special case of a 6-storied structure, the ratio of the bending moment in the columns on each floor to the product of ml by the acceleration of the ground (on which no structure stands) is expressed by equations of the forms

$$\left.\begin{aligned}
&EI\frac{\partial^2 y_1}{\partial x_1^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{-6}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}}\\
&\qquad\qquad .(\gamma-12)(\gamma-24)(\gamma-36)\{(\gamma-24)^2-432\},\\
&EI\frac{\partial^2 y_2}{\partial x_2^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{6.12}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}}\\
&\qquad\qquad .\{(\gamma-18)^2-180\}\{(\gamma-30)^2-180\},\\
&EI\frac{\partial^2 y_3}{\partial x_3^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{-6.12^2}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}}\\
&\qquad\qquad .(\gamma-24)\{(\gamma-24)^2-288\},\\
&EI\frac{\partial^2 y_4}{\partial x_4^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{6.12^3}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}}(\gamma-12)(\gamma-36)\\
&EI\frac{\partial^2 y_5}{\partial x_5^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{-6.12^4}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}}(\gamma-24),\\
&EI\frac{\partial^2 y_6}{\partial x_6^2}\bigg/ ml\left(2\frac{\partial^2 u_0}{\partial t^2}\right)=\frac{6.12^5}{\sqrt{\mathfrak{A}^2+\mathfrak{B}^2}},
\end{aligned}\right\}\quad(18)$$

where

$$\left.\begin{aligned}
&\mathfrak{A}=\{\gamma(\gamma-24)^2-12(\gamma-12)^2\}^2-12\gamma(\gamma-12)^2(\gamma-36)^2,\\
&\mathfrak{B}=12\left(\frac{1}{a}\sqrt{\frac{mEI}{l^3\rho\mu}}\right)\sqrt{\gamma}(\gamma-12)(\gamma-24)(\gamma-36)\{(\gamma-24)^2-432\}.
\end{aligned}\right\}\quad(19)$$

The form of displacement of the ground surface, on which no structure is standing, is

$$u_0=2\cos pt.$$

Since $\mathfrak{A}=0$ gives the frequency equation of the free vibrations of the structure, it follows that, when $\gamma=mp^2l^3/EI$ satisfies this equation, $\mathfrak{A}=0$, namely, $\gamma=0{\cdot}7005$, $6{\cdot}038$, $15{\cdot}49$, $26{\cdot}9$, $37{\cdot}63$, $45{\cdot}25$, we have the resonance condition of the vibrations in the usual sense. It is possible to consider the condition, $\mathfrak{B}=0$. We shall call the case in which $\gamma=mp^2l^3/EI$ satisfies the equation, $\mathfrak{B}=0$, as *corresonance*.

If γ were to satisfy the conditions, $\mathfrak{A}=0, \mathfrak{B}=0$, then the amplitudes of vibrations or bending moments in the columns would assume infinitely large values. At certain vibration frequencies in the present problem, such conditions are almost, but not entirely, satisfied.

3. Middle 28 th House (*Naka* 28 *gô-kan*).

This is a six-storied reinforced concrete structure that was built in January 1926, a photographic view of the profile and the general plan of the first floor being shown in Figs. 1, 2. The basement being

Fig. 1. Naka 28 gô-kan.

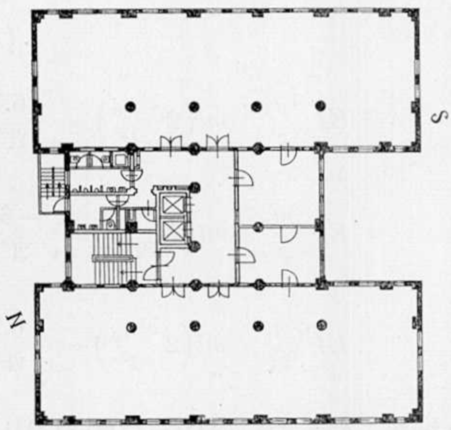

Fig. 2. Naka 28 gô-kan. Scale 1/500.

of very strong construction and its floor height much less than those of the other floors, we applied a correction by adding 1/6 of the basement height to the actual mean between floor height from the ground floor to the fifth floor. It was also assumed that all floors, including the roof, have live loads of intensity 370 kg/m^2 of the floor area, or a live load of $2{\cdot}442.10^5$ kg on every floor. The calculated results of the important elements are given in Table I.

Table I. Middle 28 th House (Naka–28 gô–kan).

Floor		1 st	2 nd	3 rd	4 th	5 th	Roof	Mean
Height of floor below l (m)		4·015	3·483	3·483	3·483	3·483	3·787	3·622 +0·518
Area of vertical members a (m²)		38·461	38·461	31·702	27·201	24·873	24·004	30.784
Mt. inertia of vertical members I (m⁴)		2·641	2·641	1·934	1·562	1·450	1·429	1·943
Beams	volume (m³)	75·40	60·45	60.45	60·45	60·45	60·35	62·93
	weight (kg)	$1·81.10^5$	$1·45.10^5$	$1·45.10^5$	$1·45.10^5$	$1·45.10^5$	$1·449.10^5$	$1·510.10^5$
Floors	volume (m³)	50	50	50	50	50	50	50
	weight (kg)	$1·2.10^5$	$1·2.10^5$	$1·2.10^5$	$1·2.10^5$	$1·2.10^5$	$1·2.10^5$	$1·2.10^5$
Columns	volume (m³)	74·32	46·00	39·55	32·22	26·23	23·37	40·28
	weight (kg)	$1·783.10^5$	$1·104.10^5$	$0·950.10^5$	$0·774.10^5$	$0·630.10^5$	$0·561.10^5$	$0·967.10^5$
Walls	volume (m³)	68·40	59·32	49·45	49·45	49·45	53·70	54·96
	weight (kg)	$1·641.10^5$	$1·423.10^5$	$1·187.10^5$	$1·187.10^5$	$1·187.10^5$	$1·289.10^5$	$1·319.10^5$
Total mass (kg)		$6·434.10^5$	$5·177.10^5$	$4·787.10^5$	$4·611.10^5$	$4·467.10^5$	$4·499.10^5$	$4·996.10^5$
Area of floor (m²)		660·7	660·7	660·7	660·7	660·7	660·7	660·7
$\frac{1}{a}\sqrt{\frac{mEI}{l^3\rho\mu}}$		1·436	1·436	1·492	1·563	1·648	1·693	1·540

The periods of free vibrations are determined by the equation, $\mathfrak{A}=0$, namely

$$\{\gamma(\gamma-24)^2-12(\gamma-12)^2\}^2-12\gamma(\gamma-12)^2(\gamma-36)^2=0, \qquad (20)$$

where $\gamma=mp^2l^3/EI$. This equation gives us

$$\gamma=0·7005,\quad 6·038,\quad 15·49,\quad 26·90,\quad 37·63,\quad 45·25,$$

from which we obtain the periods

$$T(\text{sec})=0·295,\quad 0·1004,\quad 0·0627,\quad 0·0475,\quad 0·0402,\quad 0·0367,$$

$E=2·1.10^9$ kg/m² $=2·1.10^9.9·8$ kg mass m/s²/m² being used in Table I as well as in the determination of T. These periods are somewhat longer than what they actually would be. The values of the present case come from the assumption that the floors are extremely rigid.

Although such smallness of the periods may be inconvenient for investigators aiming only at finding the periods of natural vibrations, which however are rather meaningless in our present investigation, the nature of the energy dissipation in seismic vibrations is not greatly affected by such changes in periods.

The maximum values of the bending moments $M=E_S I_S(d^2y_S/dx_S^2)$ at each end of the columns corresponding to the maximum values of the acceleration of the ground (on which no structure is standing and in which $\lambda=14\mu$) due to the movement in NS-direction, were determined from equation (18), the results being shown in Figs. 3 a, 3 b. Since resonance conditions exist at the periods indicated by the vertical strips, we may conclude that the bending moments

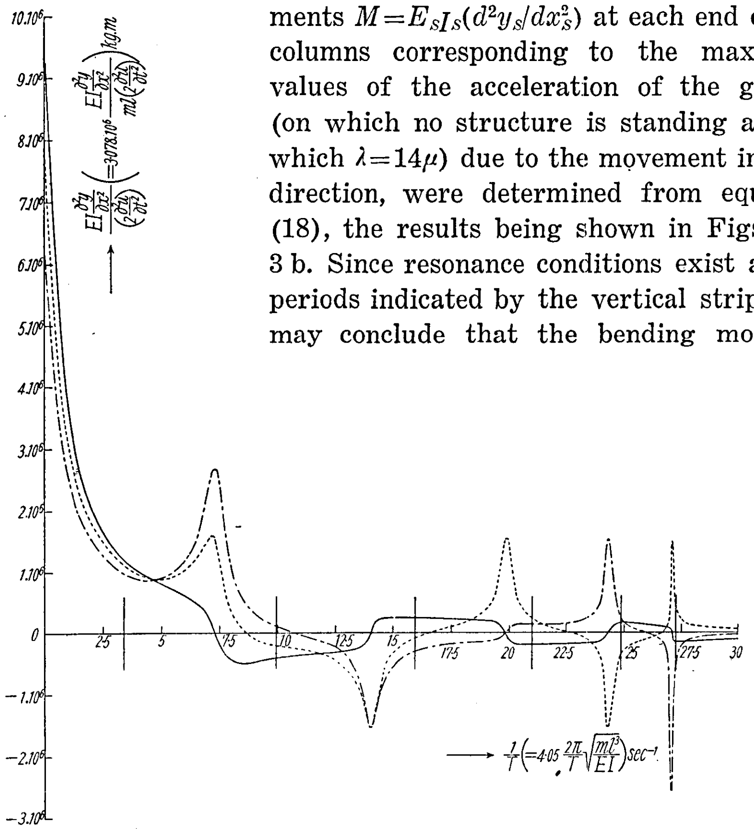

Fig. 3 a. Full, broken, and chain lines represent moments in columns of ground, first, and second floors respectively.

under resonance conditions are rather less than those at periods out of resonance. It is also an important fact that the bending moments in the columns below the first floor are generally very small for any vibration period of the disturbance, excepting those near where the vibration period is very long, namely, near zero frequency of vibrations. Another remarkable fact is that, at the highest natural frequency, resonance and corresonance virtually coincide, tending to cause each of the denominators in (17) to vanish. This will be referred to again in the next section.

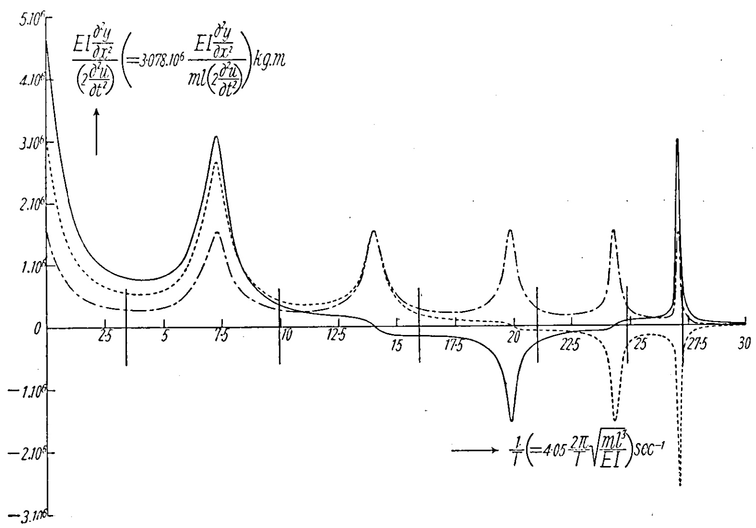

Fig. 3 b. Full, broken, and chain lines represent moments in columns of third, fourth, and fifth floors respectively.

Finally, the induced stresses in the columns and walls will be calculated. Assuming that the acceleration of the ground, (on which no structure is standing), is $g/10$, the stresses in the lowest columns at zero frequency become

maximum stress in walls running in NS-direction $=36.0$ kg/cm^2,
maximum stress in columns attached the same walls $=27{\cdot}4$ kg/cm^2.

4. Coincidence of Resonance and Corresonance.

It was said in the last section that, since at period, 0·0367 sec, resonance and corresonance approximately coincide, the bending moments in the columns below the 3 rd, 4 th, and 5 th floors (namely those between 3 gai and 5 kai in the plan), assume fairly large values. If resonance and corresonance were to coincide exactly, which, in the present problem, is mathematically impossible, infinitely large vibrations would result. Nevertheless, at such a short vibration period as 0·0367 sec, the material itself damping as in solid viscosity,[8] etc., plays an important part in energy dissipation, as the term is generally understood, actual large vibrations being thus improbable from the physical point of view.

The manner in which the period for resonance approaches that for corresonance at the period corresponding to the sixth natural frequency never changes provided the number of stories, in the present case six, is stipulated. Thus, even if resonance and corresonance were to coincide, the condition of the coincidence cannot be modified, no matter what the changes may be in the geometrical and physical constants, such as l, m, E, I, μ, ρ. This condition however arises from the fact that we have always concentrated mass on every floor, whereas if the masses were distributed in any other way, the relation between resonance period and corresonance period would differ.

5. Bending Moments in the Columns below the lst Floor.

we have seen in Section 3 that, notwithstanding their abnormal largeness in zero frequency, the bending moments in columns below the lst floor (1–kai in the plan), become smaller with increase in frequency of the ground vibrations beyond the frequency above the first (namely, principal) resonance. As a matter of fact, this contingency exists even in 3–, 4–, 5–storied structures, as will be seen from Figs. 2, 3, 5, 6, 7, 8 in our last paper.[9] This seems to agree with the fact as observed that damage to a building in a great earthquake occurs in the vertical members on the lst or 2 nd floor rather than in those on the ground floor. Without invoking the theory of dissipation (for instance, considering the structure with hinged base), it would be very difficult[10] to explain this apparent anomaly.

In conclusion we wish to express our sincere thanks to the members of the staff of the Mitsubishi Co., for having supplied us with valuable data for the present investigation, and also to the Council of the Foundation for the Promotion of Scientific and Industrial Research of Japan, with whose aid the work was begun.

Added Note *to Vol. 13, Paper 54, (pp. 681~697) Equation(2′).*

The form of Δ was approximate from the reason of its mathematical simplicity and also of its practical unimportance in such engineering problem. We forgot to remark this fact in that paper. The accurate formulae will be seen in foot note (4) in the same paper.

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8) K. SEZAWA, The part, "Waves propagated in Beam...", which was appended at the end of the paper, "On the Decay of Waves in Visco-elastic Solid Bodies", *Bull. Earthq. Res. Inst.*, **3** (1927), 50, etc.

9) K. SEZAWA and K. KANAI, *loc. cit.* 5).

10) Ditto, *loc. cit.* 2).

12. 6階建築に於ける震動勢力の逸散性・共振と餘共振とが一致する問題

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これまでの研究によつて5階までの高さの建物の地震による振動勢力の逸散性その他の性質がわかつた. 何れの場合にも三菱地所課の御好意によつて實際の建物にあてはめて見ることができたのである. しかし6階以上の場合までやつてなかつたので, ここにあらためてその計算を試み, 更に三菱仲28號館の場合を借用してそれと比較研究をやつてみた.

大體の性質はこれまでの場合やつた研究と似たものであるから, この抄錄に於て委しく述べるのを省くけれども, 特に注意したいことは1階の柱や壁に於ける屈曲モーメントが zero frequency では非常に大きいのに拘らず, 第1次共振以上では frequency とともに非常に小さくなることである. このことは建物の大地震に於ける被害が1階よりも2階3階に多いといふことと一致してゐる. このやうな建物も風壓等に對してはやはり1階の方が被害の多いことが豫期されるから, 震動逸散性を用ひることなく, 單に床が剛く建物脚部が鉸脚になつてゐるやうな場合を考へても, この問題の說明がむづかしいやうに思はれる. 末廣博士が考へられたやうに建物の rocking motion のやうなことを導入してこの事實を說明しやうとする行方もあるけれども, (rocking motion の面白さは別として) 地震の上下動の成分までも適當に入れなければならぬ上に, 問題それ自體に種々の難點 (殊に高層建築では) がある. しかし, rocking motion を入れたからといつて, この問題の性質には殆ど變化がなく, それに相當して再び別個の震動逸散性が增加するのみである.

6階建築の問題に於て特に著しい現象は第6次の共振が餘共振 (corresonance) と略一致することである. 若し完全に一致するものであるとするとその場合に無限大の屈曲モーメントが得られる譯であるけれども, 理論的にそのやうにはならぬ. 共振と餘共振とが略一致する事柄は, 質量を各床に集め且つ6階といふことから生ずるのであつて, 質量を別の方法で分布すると違つた結果になる. 又, この一致することとは建物の彈性, 質量, 階高, 土地の性質などに何等關係がないものである. 但し注意すべきことは, たとひ, 共振と餘共振とが一致するにしても, frequency の非常に高い所であるから, 物質に關係する粘性抵抗などがよく働き, 實際問題としては恐れるに足らぬといふことである.