58. *The Nature of Microseisms of Local Type.*

By Katsutada SEZAWA and Kiyoshi KANAI,

Earthquake Research Institute.

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1. It is well known that such forms of microseismic disturbance on a ground surface as are probably caused by local impulses in the atmosphere, generally point to undamped coupled free oscillations, their periods and amplitudes differing with localities as well as with meteorological conditions. Disturbances of this type that have been observed in the Kwanto region had very often periods ranging from 10 to 30 sec, with amplitudes of the order of from a few microns to a fraction of a milimeter. The question arises why these vibrations should be undamped in spite of their small amplitudes. For undamped free vibrations to be maintained, it is obviously necessary to postulate the existence of some surface strata, the whole thickness of which is a few kilometers, with the lowest surface bounded by another stratum whose elastic constant differs greatly from those of the upper strata,[1] although a part of the vibrational energy is dissipated into air. But, if the surface crust were really in such a condition, the amplitudes of forced vibrations under resonance would be abnormally large, as will be discussed presently. If, on the other hand, the lower boundary of the upper strata were not too rigid compared with these strata, the amplitudes of vibrations, even under resonance, would be within a certain range, but the free vibrations then would assume a quickly damping type. It is therefore of pressing importance to determine whether the microseisms under consideration are free vibrations or forced vibrations as well as to determine the state of stratification near the earth's surface.

2. We shall first consider the case in which a uniform layer rests on an extremely rigid substratum. Although the pulsatory disturbances on the earth's surface are as a matter of fact variations in dynamic pressure due to irregular flow of the atmosphere, for simplicity we take the case in which periodic pressure waves, whose approximate intensity is of the order of the variation in dynamic pressure, impinge

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 251~264, 484~495.

on the ground surface. The differential equation determining the pressure change $p_0 \partial\xi/\partial x$ in air may then be expressed by

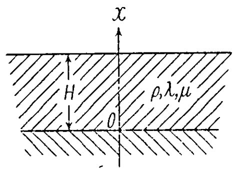

Fig. 1.

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}, \tag{1}$$

the axis of x being taken vertically upwards from the lower boundary of the layer. Since in this equation it is assumed that the pressure change takes place isothermally, owing to the relatively slow variations in the state of the air, we may put $c^2 = p_0/\rho_0$, so that the air pressure varies as $p_0 \partial\xi/\partial x$, where p_0, ρ_0, ξ are mean pressures as well as density in atmosphere and displacement of any point in the air in oscillation considered statically. The solution of (1), which is the result of vibration in air with amplitude a, is expressed by

$$\xi = 2a \sin(fx + \gamma) \cos pt, \tag{2}$$

where $f^2 = p^2/c^2 = \rho_0 p^2/p_0$. The vertical vibration of the surface layer of the earth is obtained from the equation

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2}, \tag{3}$$

ρ, λ, μ, u being density, elastic constants, and the displacement of the layer respectively. The solution of this equation satisfying the condition at the lower boundary of the layer, $x=0$, is expressed by

$$u = A \sin f'x \cos pt, \tag{4}$$

in which $f'^2 = \rho p^2/(\lambda+2\mu)$. The conditions at the upper surface of the layer are

$$u = \xi, \qquad (\lambda+2\mu)\partial u/\partial x = p_0 \partial\xi/\partial x. \tag{5), (6}$$

We thus finally get

$$\xi = 2a \sin\left[f(x-H) + \tan^{-1}\{\alpha \tan f'H\}\right] \cos pt, \tag{7}$$

$$u = \frac{2a\alpha \sin f'x \cos pt}{\sqrt{\alpha^2 \sin^2 f'H + \cos^2 f'H}}, \tag{8}$$

where $\alpha = \sqrt{\rho_0 p_0/\rho(\lambda+2\mu)}$. If we put $\rho_0 = 0{\cdot}00129$, $p_0 = 76 \,.\, 13{\cdot}6 \,.\, 980$, $\rho = 2$, $\lambda + 2\mu = 10^{11}$, we obtain $\alpha = 0{\cdot}809 \,.\, 10^{-4}$. It should be borne in mind however that, under resonance, the common boundary of the two media becomes loops of vibrations for the layer as well as for the air. We have calculated the values of u at the free surface for different values of $f'H$, that is, for different frequencies of the pulsations in air, the result being plotted in Fig. 2.

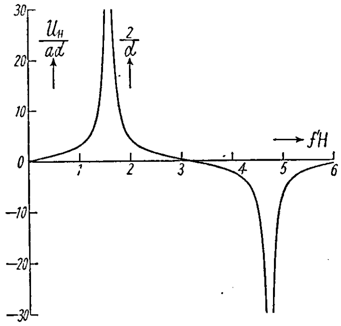

Fig. 2.

Let us take as an example the case, in which the maximum in pressure variation in air is 1/10,000 of the barometric pressure, and which roughly corresponds to a variation in wind velocity of 4·5m/sec, while the period of the same variation $2\pi/p$ is 10 sec. Then, if V be the velocity of sound waves, we have $f=p/V=(2\pi/10)/280.10^2=2{\cdot}24.10^{-5}\,\text{cm}^{-1}$, so that under resonance conditions, $f'H=\pi/2$ (roughly corresponding to $H=1{\cdot}5$ km$\sim$3 km in rocks of superficial nature), we have $2a=(\partial\xi/\partial x)/f=(1/10{,}000)/2{\cdot}24.10^{-5}=4{\cdot}5$ cm, whence the maximum value of $2u$ $(=4a)$ becomes 9 cm. This is obviously much too large to expect in microseisms under such small pulsations in the air pressure, thus rendering improbable the existence of an extremely rigid substratum.

3. In order to ascertain the nature of the free vibrations of the layer, generalization of a solution of types (7) and (8) by means of Fourier's integral was resorted to. The form of the free vibrations of the layer is such that

$$u=2a\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\left\{F\left[\beta\left\{Vt+x+H\left(\frac{1}{\beta}-\overline{2m+1}\right)\right\}\right]+F\left[\beta\left\{Vt-x+H\left(\frac{1}{\beta}-\overline{2m+1}\right)\right\}\right]\right\}, \tag{9}$$

where $\beta=f'/f$, corresponds to the initial disturbance

$$\xi_0=F(ct+x)\,, \tag{10}$$

which is supposed to act on the free surface. Thus, the damping factor is given by

$$(1-\alpha)^2/(1+\alpha)^2\,. \tag{11}$$

Since α is very small compared with unity, it is possible to expect undamped free vibrations in this case. But since for the reason that, in almost resonance conditions, the amplitudes of forced vibrations become abnormally large, the presence of an extremely rigid substratum under the surface layer is hardly possible.

4. We shall next take the case in which the ratio of the elasticity of the surface layer to that of the subjacent medium is finite, so that it is possible for the vibrational energy in the layer to dissipate not only in air but also in the subjacent medium under consideration. Let u, u', ρ, ρ', λ, μ, λ', μ' be the displacements, densities, and elastic constants of the surface layer and the subjacent medium respectively, H being the thickness of the layer, while ξ is the displacement of air particle such that the change in dynamic pressure in air may be expressed by $p_0 \partial\xi/\partial x$. The solutions for the vibrations of the respective media

$$\xi = ae^{i(pt+fx)} + Ae^{i(pt-fx)}, \tag{12}$$

$$u = Be^{i(pt+f'x)} + Ce^{i(pt-f'x)}, \tag{13}$$

$$u' = De^{i(pt+f''x)}, \tag{14}$$

where $f^2 = \rho p^2/p_0$, $f'^2 = \rho p^2/(\lambda+2\mu)$, $f''^2 = \rho' p^2/(\lambda'+2\mu')$, are to be substituted in the conditions

$$x=0; \qquad u=u', \qquad (\lambda+2\mu)\frac{\partial u}{\partial x} = (\lambda'+2\mu')\frac{\partial u'}{\partial x}, \qquad (15), (16)$$

$$x=H; \qquad u=\xi, \qquad (\lambda+2\mu)\frac{\partial u}{\partial x} = p_0\frac{\partial \xi}{\partial x}. \qquad (17), (18)$$

The final solutions for ξ, u, u' take the forms

$$\xi = a\cos(pt+fx)$$
$$+\frac{a\sqrt{\{(\alpha^2\alpha'^2-1)\cos^2 f'H + (\alpha^2-\alpha'^2)\sin^2 f'H\} + \{2\alpha(1-\alpha'^2)\cos f'H \sin f'H\}^2}}{\{(\alpha\alpha'+1)\cos f'H\}^2 + \{(\alpha+\alpha')\sin f'H\}^2}$$
$$\cdot\cos\left[pt - fx + 2fH + \tan^{-1}\left\{\frac{2\alpha(1-\alpha'^2)\cos f'H \sin f'H}{(\alpha^2\alpha'^2-1)\cos^2 f'H + (\alpha^2-\alpha'^2)\sin^2 f'H}\right\}\right], \tag{19}$$

$$u = \frac{2a\alpha\sqrt{\alpha'^2\cos^2 f'x + \sin^2 f'x}}{\sqrt{\{(\alpha\alpha'+1)\cos f'H\}^2 + \{(\alpha+\alpha')\sin f'H\}^2}}$$
$$\cdot\left\{pt + fH - \tan^{-1}\left(\frac{\alpha+\alpha'}{\alpha\alpha'+1}\tan f'H\right) + \tan^{-1}\left(\frac{1}{\alpha'}\tan f'x\right)\right\}, \tag{20}$$

$$u' = \frac{2a\alpha\alpha'}{\sqrt{\{(\alpha\alpha'+1)\cos f'H\}^2 + \{(\alpha+\alpha')\sin f'H\}^2}}$$
$$\cdot\left\{pt + f''x + fH - \tan^{-1}\left(\frac{\alpha+\alpha'}{\alpha\alpha'+1}\tan f'H\right)\right\}, \tag{21}$$

where $\alpha = \sqrt{\rho_0 p_0/\rho(\lambda+2\mu)}$, $\alpha' = \sqrt{(\lambda+2\mu)\rho/(\lambda'+2\mu')\rho'}$. We have calculated the surface displacements for two cases, namely (i) $\mu/\mu' = 1/4$, (ii) $\mu/\mu' = 1/16$, under the conditions that $\rho=\rho'$, $\lambda=\mu$, $\lambda'=\mu'$, the result being plotted in Figs. 3, 4. It will be seen that, owing to the great dissipation of vibrational energy into the subjacent medium, the amplitudes of vibrations at the surface are fairly small, even under resonance conditions.[2] Taking an example similar to that in the preceding section, we put $a=(\partial\xi/\partial x)/f=4\cdot5$ cm, so that the maximum value of $u(=2a\alpha C)$, C being the ordinates in Figs. 3, 4, become 14·6 microns and 29 microns for the respective cases of $\mu/\mu'=1/4$ and $\mu/\mu'=1/16$. These are more like the values probably to be expected in an actual microseism. If the period of pulsation in air be 30 sec, we have 44 microns and 60 microns for the amplitudes on the ground surface for the respective cases. The wind speed of 4·5 m/sec and its fluctuation period of 10 to 30 sec are rather underestimates, and we may expect larger amplitudes of microseisms in the case of strong wind, say 20 or 30 m/sec.

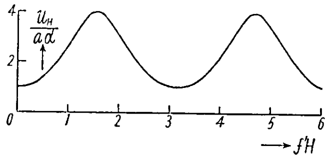

Fig. 3. $\mu/\mu'=1/4$.

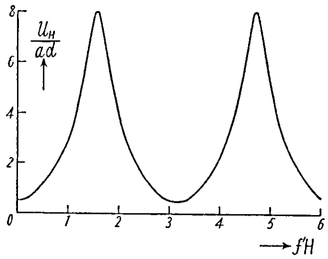

Fig. 4. $\mu/\mu'=1/16$.

5. The forms of damped free vibrations corresponding the above case are obtained as in the preceding problem by using elementary solutions and generalizing such solutions by means of Fourier's integrals, the result being

$$u=2\alpha(1+\alpha')\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m(1-\alpha')^m}{(1+\alpha)^{m+1}(1+\alpha')^{m+1}}F\left\{\beta\left(Vt+x+\frac{H}{\beta}-H\overline{2m+1}\right)\right\}$$

$$-2\alpha(1-\alpha')\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m(1-\alpha')^m}{(1+\alpha)^{m+1}(1+\alpha')^{m+1}}F\left\{\beta\left(Vt-x+\frac{H}{\beta}-H\overline{2m+1}\right)\right\}, \qquad (22)$$

$$u'=4\alpha\alpha'\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m(1-\alpha')^m}{(1+\alpha)^{m+1}(1+\alpha')^{m+1}}F\left\{\beta\beta'\left(V't+x+\frac{H}{\beta\beta'}-\frac{H}{\beta'}\overline{2m+1}\right)\right\}, \qquad (23)$$

where $\beta=f'/f$, $\beta'=f''/f'$, corresponding to the disturbance

$$\xi_0=F(ct+x)\,. \qquad (24)$$

2) The calculated microseismic amplitudes of the ground surface may be made small by introducing viscous resistance in the surface layer when the substratum is exceedingly rigid. However, the value of coefficient of resistance is then too large to be expected from the data of earthquake waves. Furthermore, it is impossible for microseismic amplitudes for any wave length to be equally small under such resistance.

The decay factor

$$\frac{(1-\alpha)^2(1-\alpha')^2}{(1+\alpha)^2(1+\alpha')^2} \tag{25}$$

in this case is much less than unity. This tells us that microseisms cannot be regarded as free vibrations of the surface layer even under the condition that the subjacent medium is not infinitely rigid.

6. The case of two surface layers resting on an exceedingly rigid medium could be solved in a similar way, although the nature of the answer thus found will not differ greatly from that of the case of one layer. The equations of displacements in air and in the first as well as the second layers are expressed by

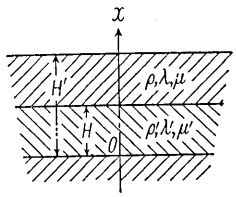

Fig. 5.

$$\xi = ae^{i(pt+fx)} + Ae^{i(pt-fx)}, \tag{26}$$

$$u = Be^{i(pt+f'x)} + Ce^{i(pt-f'x)}, \tag{27}$$

$$u' = De^{i(pt+f''x)} + Ee^{i(pt-f''x)}, \tag{28}$$

in which $f^2=\rho_0 p^2/p_0$, $f'^2=\rho p^2/(\lambda+2\mu)$, $f''^2=\rho' p^2/(\lambda'+2\mu')$, the conditions at the boundaries being

$$x=0; \quad u'=0, \tag{29}$$

$$x=H; \quad u'=u, \quad (\lambda+2\mu)\frac{\partial u}{\partial x}=(\lambda'+2\mu')\frac{\partial u'}{\partial x}, \tag{30), (31}$$

$$x=H'; \quad u=\xi, \quad (\lambda+2\mu)\frac{\partial u}{\partial x}=p_0\frac{\partial \xi}{\partial x}. \tag{32), (33}$$

The final solutions become

$$\xi=\frac{2a}{\sqrt{P^2+Q^2}}\cos\left\{f(x-H')-\tan^{-1}\frac{P}{Q}\right\}\cos\left\{pt+fH'+\tan^{-1}\frac{P}{Q}\right\}, \tag{34}$$

$$u=\frac{2a\alpha\sqrt{\alpha'^2\sin^2 f''H+\cos^2 f''H}}{\sqrt{P^2+Q^2}}$$
$$\cdot\cos\left\{f'(x-H)-\tan^{-1}\frac{\cot f''H}{\alpha}\right\}\cos\left\{pt+fH'+\tan^{-1}\frac{P}{Q}\right\}, \tag{35}$$

$$u'=\frac{2a\alpha\alpha'}{\sqrt{P^2+Q^2}}\sin f''x\cos\left\{pt+fH'+\tan^{-1}\frac{P}{Q}\right\}, \tag{36}$$

where

$$\left.\begin{aligned} P&=\cos f'(H'-H)\cos f''H-\alpha'\sin f'(H'-H)\sin f''H, \\ Q&=\alpha\{\sin f'(H'-H)\cos f''H+\alpha'\cos f'(H'-H)\sin f''H. \end{aligned}\right\} \tag{37}$$

Two cases, namely (i) $\mu/\mu'=1/4$, (ii) $\mu/\mu'=1/16$ $(H'=2H)$, other conditions being the same as in the preceding one, were calculated, the results being shown diagrammatically in Figs. 6, 7. In this case

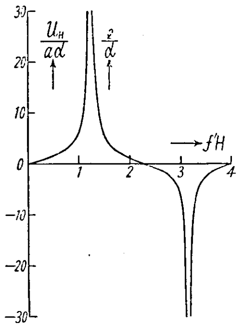

Fig. 6. $\mu/\mu'=1/4$.

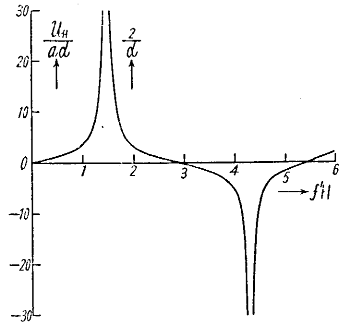

Fig. 7. $\mu/\mu'=1/16$.

too, owing to the fact that the energy is capable of flowing only into air but not into the deep stratum of the earth, the maximum values of the surface displacements become very large. It is nevertheless certain that for some ratios of μ/μ' as well as of H/H', the resonance periods for different types of vibrations approach each other, so that the selective forced vibrations would resemble the coupled free ones, at any rate, in the type of vibrations.

7. An approximate method of solving the problem of damped vibrations, due to dissipation of their energy into the air, is by integral equations. The method is similar to that employed in the previous paper.[3] Although the case in which the subjacent medium is infinitely rigid is of no practical importance, as mentioned in Section 2, we shall nevertheless show how integral equations may be applied to such dissipative system.

Take the axis of x directed upwards with its origin at the lower boundary of the surface layer, and let H, $\rho(x)$, $L(x)$ be the thickness, density, and elastic constant of the stratum respectively. Taking into consideration the special nature of the integral equation, we shall discuss the problem as being a static one. Let the reaction of the air at the free surface of the layer due to a unit load within that layer be A, its value being different according as the frequency of vibrations are different. The vertical displacement at x due to a unit load applied at $x=a$ in the layer is then expressed by Green's function such that

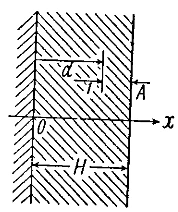

Fig. 8.

3) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 490.

$$\left.\begin{aligned} k(x, \alpha) &= (1-A)\int_0^x \frac{dz}{L(z)}, \qquad (x<\alpha) \\ k(x, \alpha) &= \int_0^\alpha \frac{dz}{L(z)} - A\int_0^x \frac{dz}{L(z)}. \qquad (x>\alpha) \end{aligned}\right\} \tag{38}$$

If in the vibration problem, the vertical displacement at $x=\alpha$ be $u(\alpha)\cos pt$ and the frequency of vibrations be p, then the load due to the inertia force acting on element $d\alpha$ at $x=\alpha$ will be

$$p^2\rho(\alpha)u(\alpha)\cos pt\, d\alpha. \tag{39}$$

The displacement at $x=x$ due to this load is therefore expressed by

$$p^2\rho(\alpha)u(\alpha)\cos pt\, d\alpha\, k(x, \alpha), \tag{40}$$

so that displacement $u(x)\cos pt$ at $x=x$ due to the total loads in the layer assumes the form

$$u(x) = p^2\int_0^H \rho(\alpha)k(x, \alpha)u(\alpha)d\alpha, \tag{41}$$

the time factor being omitted. The movement in air is taken only for boundary conditions at $x=H$. We now write

$$\rho(\alpha)k(x, \alpha) = K(x, \alpha), \tag{42}$$

whence (41) reduces to the homogeneous integral equation

$$u(x) = p^2\int_0^H K(x, \alpha)\,u(\alpha)d\alpha, \tag{43}$$

$K(x, \alpha)$ being its kernel. The boundary conditions at $x=H$ are such that

$$u(H) = 2\sin(k'H+\gamma), \tag{44}$$

$$p^2\int_0^H \frac{\partial K(H, \alpha)}{\partial H}u(\alpha)d\alpha = \frac{2p_0k'}{L_{x=H}}\cos(k'H+\gamma), \tag{45}$$

where p_0, $2\pi/k'$ are the barometric pressure and wave length of the disturbance in air respectively. Since

$$\left[\frac{\partial K(x, \alpha)}{\partial x}\right]_{x=H} = -\rho(H)\frac{A}{L(H)},$$

(45) reduece to

$$-p^2\int_0^H A\rho(\alpha)u(\alpha) = 2p_0k'\cos(k'H+\gamma). \tag{45'}$$

From (44), (45') we get

$$\left[u(H)\right]^2 + \left[\frac{p^2A}{p_0k'}\int_0^H \rho(\alpha)u(\alpha)d\alpha\right]^2 = 4. \tag{46}$$

By the trapezoidal rule for evaluating the integral, (43) and (46) are equivalent to

$$\frac{p^2H}{\nu}\sum_{q=0}^{\nu}\beta_q K\left(\frac{sH}{\nu}, \frac{qH}{\nu}\right)u\left(\frac{qH}{\nu}\right)=u\left(\frac{sH}{\nu}\right), \tag{43'}$$

$$\left[u(H)\right]^2+\frac{p^2A^2H^2}{p_0^2k'^2\nu^2}\left[\sum_{q=0}^{\nu}\beta_q\rho\left(\frac{qH}{\nu}\right)u\left(\frac{qH}{\nu}\right)\right]^2=4\,, \tag{46'}$$

where $\beta_0=1/2$, $\beta_1=\cdots\cdots=\beta_q=\cdots\cdots=\beta_{\nu-1}=1$, $\beta_\nu=1/2$. Eliminating $\nu+1$ values of u in (43′), it is possible to obtain the value of A. Substituting the value of A thus obtained in $\nu+1$ equations in (46′), we get the ratios

$$u(0):\, u\left(\frac{H}{\nu}\right):\,\cdots\cdots:\, u\left(\frac{sH}{\nu}\right):\,\cdots\cdots:\, u(H)\,. \tag{47}$$

Substituting these ratios in (46), we obtain the absolute values of

$$u(0)\,,\ u\left(\frac{H}{\nu}\right),\ \cdots\cdots,\ u\left(\frac{sH}{\nu}\right),\ \cdots\cdots,\ u(H)\,.$$

With a view to comparing the result of the present approximate method with that of an accurate one in Section 2, we selected the case

$$\rho(x)=\rho_0\,,\qquad \mu(x)=\mu_0\,, \tag{48}$$

when the Green's function becomes

$$\left.\begin{aligned} k(x,\,a)&=(1-A)\frac{x}{\mu_0}\,, &&(x<a)\\ k(x,\,a)&=\frac{a-Ax}{\mu_0}\,. &&(x>a)\end{aligned}\right\} \tag{49}$$

After substituting these values in (43′), (46′) for the two cases (i) $f'H=\pi/2$ (resonance condition), (ii) $f'H=1$, we calculated the distribution of displacements in the layer. If we were to take five ordinates, namely, $\nu=4$, A would take the form

$$A=\frac{1-32\phi+320\phi^2-1024\phi^3+512\phi^4}{1-32\phi+320\phi^2-1024\phi^3}\,, \tag{50}$$

where $\phi=4/(f'H)^2$. The result of the calculation is shown diagrammatically by the full lines in Figs. 9, 10; in which the accurate values determined by (8) are shown by broken lines.

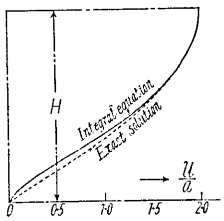

Fig. 9. $f'H=\pi/2$.

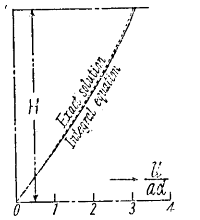

Fig. 10. $f'H=1$.

8. *Concluding Remarks.*

From a comparison of the results obtained in Section 2, 3, 4, 5 it seems now established that microseismic vibrations of local type are mainly formed by selective forced vibrations of the surface layers due to the pulsation in air pressure on the surface of the earth, and also that the rigidities of the underlying strata are not very large compared with those of the upper layers, the ratio of the rigidities for the two being probably in the range 3∼20. Under this condition the amplitudes of microseisms range from a few microns to a hundred microns when the variation of 4·5 m/sec in wind velocity takes place with period of the order of 10 to 30 sec. Even in such a state, no free oscillations can be maintained because of the fact that the energy is dissipated relatively rapidly into the subjacent medium. The flow of energy into air, when the underlying media are assumed to be extremely rigid, is too small to participate in the dissipation, in which case the vibratory motion of the surface layers under resonance becomes exceedingly large, say 9 cm in amplitude, even in a very slightly disturbed condition of the atmosphere, say a variation of wind velocity 4·5 m/sec with period 10 sec. It is also impossible to expect large damping due to solid viscosity in meterials. It follows therefore that microseims of local type cannot be free vibrations of the earth's upper layers unless we assume that no resonance conditions would ever arise. It would seem that the regular periodic motion in the microseisms under consideration has its origin only in atmospheric disturbances of such a character as would give rise to easy selective resonance of the surface layers. This implies that the variation of atmospheric condition is due to an aggregate of fairly regular periodic pulsations. Records of air conditions obtained by means of meteorological instruments, and data of vibrations of tall buildings in strong wind seem to confirm this conclusion.

58. 局所的脈動の性質

地震研究所 妹澤克惟
金井清

局所的脈動（和達博士の命名と思ふ）といはれてゐる種類の脈動はその原因が大體に於てその土地の地表上の大氣の狀態にあるとすることは大して異存がないと思はれる．しかし，その振動の形が殆ど減衰しない聯成振動の形をなす爲に，それが一つか數個の地表層の自由振動であるかのやうに思はれ易いものである．これを明瞭にするために地表層の臺をなしてゐる下層の彈性を地表層のそれに比して適當に大きくした場合と無限に大きな場合の夫々の自由振動と大氣の擾亂のための强制振動とを計算してみたのである．

假りに下層が極端に剛い場合を取つて見ると自由振動としてその振動性を永續させることは可能である．これは空中へ勢力逸散があつても大して影響をもたない．しかし强制振動で選擇共振をやると風速變化 4·5 m 位が 10 秒毎に起つても 9 cm 位の地表振幅が容易に出る． これを少くする爲に粘性抵抗を入れても普通の地震波位で豫期できる位の抵抗値では到底追付かぬし，のみならず，種々の週期の振動の振幅を比較して見るとその根本性が違つてゐることがわかる，それでどうしても下層を非常に剛くして置くことは許されぬことが知られる．

次に下層を地表層よりも適當に剛く，卽ち下層の剛度を地表層のそれの 3 倍乃至 20 倍位にして選擇共振の振幅を見ると，風速變化 4·5 m が 10 秒から 20 秒位の週期で變化しても數microns から數十 microns 位の地表振幅となり，風速變化が 20 m から 30 m にもなるともつと大きな地表振幅となる．之は實際の觀測とよく一致してゐる．しかし同樣な場合の自由振動を見ると，下層への勢力逸散の爲に振動が餘りにもよく減衰するから自由振動でないことは明かである．從て問題はこの場合の共振强制振動に落着くより外仕方がないと思ふ．

このやうにして見ると脈動は大體に於て强制振動であつて，その振幅の大なる規則正しい部分は選擇共振であり，その原因が大氣狀態の擾亂に存在することが知られるであらう．而して脈動の振幅が長く續くのは地表層が長く自由振動をなす爲でなく大氣の擾亂が比較的に規則正しい週期的振動の集りであつてそれが長く續く結果に過ぎないのである．氣象器械による大氣狀態の觀測や，强風中に於ける高層建物の振動結果等からもこの傾向を確め得ると思ふ．