64. *The Air Damper.*

By Takahiro HAGIWARA,

Earthquake Research Institute.

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1. The air damper, which consists of a movable piston enclosed in a fixed cylinder, has been long in use for damping the pendulums of seismographs while recently it has been put into use for damping pendulums of short period, such as that of an acceleration seismograph. Hitherto the proportion between the resistance and the velocity of the piston was theoretically deduced under the assumption that the air in the cylinder is incompressible. Actually however the air must be more or less compressed and expanded as the pressure in the cylinder is increased and decreased by the motion of the piston. This paper discusses the effect of the compressibility of the air on the resisting force of the damper.

2. The air damper of a seismograph generally has the form of Fig. 1, a and b, the section of the piston being either a circle or a rectangle. Since type b is equal to two of type a joined together, we need consider only type a. Generally when an incompressible viscous fluid flows steadily through a tube with different pressures at the two ends of the tube, the outflow of the fluid from one of the ends per unit time is proportional to the pressure difference. This relation also holds when the flow of the fluid is not steady, provided the effect of the inertia of the fluid can be neglected, that is,

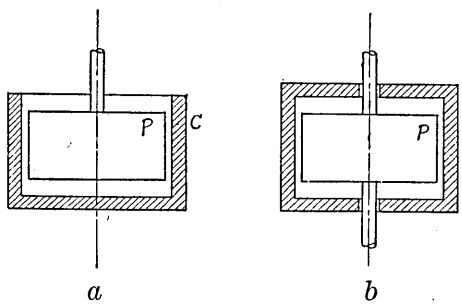

Fig. 1. Two types of the air damper. P piston, C cylinder.

$p=\alpha\dfrac{dv}{dt}$, where p is the pressure difference, v the volume of outflow, t the time, and α the constant determined by the viscosity of the fluid and the geometrical size of the tube. When the tube is of simple form the value of α can be calculated, some examples of which follow.

(a) Straight pipe of uniform rectangular section, of which one side a is very small compared with the other b. $\alpha=\frac{12\eta l}{a^3 b}$ (where η is the viscosity coefficient of the fluid, l the length of the pipe.

(b) Straight pipe of uniform circular section. $\alpha=\frac{8\eta l}{\pi r^4}$ (where r is the radius).

(c) Straight pipe formed of two co-axial circular cylinders. $\alpha=\frac{6\eta l}{\pi e^3 r}$ (where r is the radius of one cylinder, e the difference in radius of the two cylinders and $r \gg e$).

As regards the air damper $\frac{dv}{dt}=S\frac{d\xi}{dt}$, where S is the sectional area of the piston and ξ the displacement of the piston from the neutral position. The pressure increase of the air in the damper is therefore given by $p=\alpha S\frac{d\xi}{dt}$, and the resisting force exerted on the piston

$$f=pS=\alpha S^2\frac{d\xi}{dt}\quad, \tag{1}$$

that is, the resisting force is proportional to the velocity of the piston. Since, in this case, the air flows through the clearance surrounding the piston, the tangential viscous force acts on the side of the piston, although it is negligible compared with the normal force that is exerted on the base[1].

Next the compressibility of the air will be considered. We denote the pressure and the volume of air contained in the damper at the outset by P_0 and V_0 respectively, P_0 being equal to the pressure of

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1) If, for example, the section of the damper is circular $\alpha=\frac{6\eta l}{\pi c^3 r}$ and $S=\pi r^2$, where r is the radius of the piston, c the clearance, l the length of the piston along the axis, and η the viscosity coefficient of air. Hence $f=\frac{6\pi\eta l r^3}{c^3}\times\frac{d\xi}{dt}$.

The tangential force is given by $f'=\frac{6\pi\eta l r^2}{c^2}\times\frac{d\xi}{dt}=\frac{c}{r}\times f$.

If $r=4$ cm and $c=0{\cdot}1$ mm, $f'\doteqdot 0{\cdot}0025\times f$.

free air. After time t, P_0 and V_0 become P_0+p and V respectively, and the total outflow of air through the clearance amounts to volume V'. If we assume for simplicity that the phenomenon is isothermal, then

$$\left.\begin{aligned} &P_0(V_0-V')=(P_0+p)V, \\ &V_0=Sh, \\ &V\;=S(h-\xi), \\ &V'=\int_0^t \frac{p}{\alpha}dt, \end{aligned}\right\} \qquad (2)$$

where h is the distance between theb ase of the piston and the cylinder, α the constant determined by the viscosity and the geometrical size of the clearance. Hence

$$P_0\left(\xi S-\int_0^t \frac{p}{\alpha}dt\right)=Sph-Sp\xi. \qquad (3)$$

Generally p and ξ are very small compared with P and h respectively, so that in the foregoing equation we ignore the product $p\xi$. Rewriting the equation then in the differential form,

$$\frac{dp}{dt}+\frac{P_0}{\alpha Sh}p=\frac{P_0}{h}\frac{d\xi}{dt}. \qquad (4)$$

This equation shows the so-called visco-elastic property. The pressure increase in the damper (consequently the resisting force of the piston) is proportional to the velocity of the piston if the motion is comparatively slow, whereas it becomes proportional to the displacement if the motion rapid.

We shall next see when the motion of the piston assumes the sine form. Putting $\frac{d\xi}{dt}=\xi_0\omega\sin\omega t$ in equation (4), we get

$$\frac{dp}{dt}+\frac{P_0}{\alpha Sh}p=\frac{P_0\xi_0\omega}{h}\sin\omega t. \qquad (5)$$

The solution of the equation for the stationary state is

$$p=\frac{\xi_0\omega\alpha S\sin\omega t}{1+\left(\frac{\omega\alpha Sh}{P_0}\right)^2}-\frac{\frac{\xi_0 P_0}{h}\cos\omega t}{1+\left(\frac{P_0}{\omega\alpha Sh}\right)^2} \qquad (6)$$

$$\left.\begin{aligned} \text{or} \qquad p &= \frac{\xi_0 \omega \alpha S P_0 \sin(\omega t - \delta)}{\sqrt{P_0^2 + (\omega \alpha S h)^2}}, \\ \tan\delta &= \frac{\omega \alpha S h}{P_0}. \end{aligned}\right\} \qquad (7)$$

As will be seen from equation (6), pressure increase p has one term in phase with the velocity of the piston and another out of phase. In equation (7), δ is the phase difference between the resultant force and the velocity of the piston. If ω is very small (that is to say, the period of motion of the piston is large), the second term on the righthand side of equation (6) vanishes and p is exactly in phase with the velocity of the piston. If ω is very large the first term vanishes and p is proportional to the displacement of the piston, in which case the restitutive force of the pendulum increases by the retarding force of the damper and the proper period of the pendulum apparently shortens. We notice, however, that such disturbances can be avoided to a certain extent by making h (the distance between the base of the piston and the cylinder) sufficiently small.

3. We investigated the foregoing theory by an experiment. A sketch of the experimental arrangement is given in Fig. 2. The damper used here is the same as that used in the Ishimoto acceleration seismograph. The section of the piston perpendicular to its axis is circular with a diameter of 8 cm and a length along the axis of 4 cm; the clearance being 0·1 mm. The piston was set in simple harmonic motion by means of a D. C. motor. A manometer (M) was attached to the damper. The tube connecting the cylinder and the manometer is about 5 cm long, with an inside diameter of 0·5mm. The volume of the air contained in the manometer and the tube is negligible compared with that contained in the damper. The manometer consists of a stretched circular rubber membrane with a diameter of 1 cm and a thickness of 0·2 mm. A fine phosphorbronze wire is attached to the centre of the membrane, which is stretched

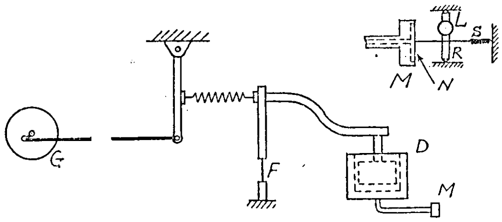

Fig. 2. Sketch of the experimental arrangement.
D damper, F flat spring, G D. C. motor, M manometer, R roller, L lens mirror, S weak herical spring, N rubber membrane.

by a weak spring. The wire, contacting with the roller (R), translates the motion of the membrane into a rotation of the mirror about its vertical axis. The displacement of the piston is also translated into a rotation of the plane mirror about its horizontal axis. The ray of light from a point source, which at first falls on the lens mirror, is reflected to the plane mirror and then reflected back again to the photographic plate, on which an image of the point source is formed. In this way we obtained Lissajous' figures on the plate, in which the pressure is taken as abscissa and the displacement of the piston as ordinate. The sensibility of the manometer was calibrated statically with a known pressure. Since the free vibration of the membrane is sufficiently rapid, the sensibility is not affected by the vibration period of the change of pressure as far as this experiment went.

The experiments covered the case where h is 0·55 and 0·20 cm. The vibration period of the piston ranged from 0·1 sec to 1 sec. One of the records obtained are shown in

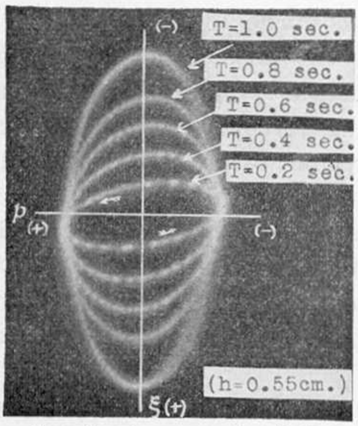

Fig. 3. An example of the records (Lissajous' figures).

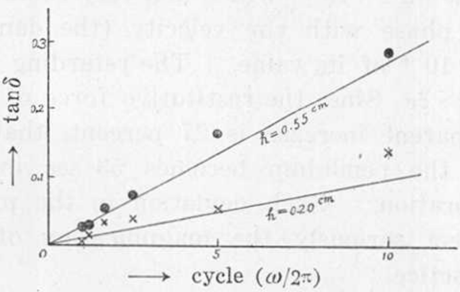

Fig. 4. The full lines are theoretical values, while the plottings are experimental.

Fig. 3. We calculated the phase differences from the records with respect to various periods and compared them with the values that were calculated by means of equation (7), the results of which are shown in Fig. 4, in which the full lines are the theoretical values and the plottings those obtained by experiment. It will be seen that the two values, theoretical and experimental, are in fair accord.

4. We shall see how the compressibility of the air in the damper affects the constants of the seismograph in the following two cases.

a) An acceleration seismograph (short period pendulum). An acceleration seismograph with the following conditions is assumed. Mass of pendulum 15 kg; proper period 0·15 sec. Damper of circular section and area 50 cm² (diameter about 8 cm). In this case, the value of α will be required to be 500 to damp the pendulum in the critical state. It is further assumed that the distance between the base of the piston and the cylinder (h) is 0·2 cm and the period of earthquake motion is 0·1 sec. The effects are then calculated by equation (6) and (7). The phase difference between the retarding force of the damper and the velocity of the piston (δ) is 17°30′. The retarding force in phase with the velocity decreases about 1 percent of its value as compared with the case of a very slow motion of the piston. The retarding force out of phase becomes $2{\cdot}3\times10^6\times\xi$, but as the restitutive force of the pendulum itself is $2{\cdot}6\times10^7\times\xi$, there is an apparent increase in this force of about 10 percent.

b) A seismograph with long period. A seismograph with the following conditions is assumed. Mass of pendulum 40 kg; proper period of pendulum 60 sec . As to the damper, $S=50\,\text{cm}^2$ and $h=2\,\text{cm}$. The value of α for the critical damping state is then 3·36. It is further assumed that the pendulum is made to vibrate owing to earthquake motion with a period of 1 sec . By calculation we get $\tan\delta=2{\cdot}1\times10^{-9}$, which is a very small quantity. The retarding force in phase with the velocity (the damping force) decreases by only 4×10^{-6} of its value. The retarding force out of phase becomes $1{\cdot}1\times10^2\times\xi$. Since the restitutive force of the pendulum is $4{\cdot}0\times10^2\times\xi$, its apparent increase is 27 percent, that is to say, the apparent period of the pendulum becomes 53 sec in stead of 60 sec, for the given vibration. Such deviation in the proper period, however, does not affect seriously the magnification of the seismograph in ordinary practice.

The foregoing two examples show that the effects of compressibility of the air can be avoided to a certain extent by a proper design of the damper.

In conclusion, the present writer wishes to express his thanks to Professor M. Ishimoto for his kind advices and criticisms in the course of these studies.

64. 空氣制振器に就て

地震研究所 萩原尊禮

1. 空氣制振器は從來比較的長週期の地震計の制振器として使用せられて來たが，最近に於ては加速度地震計の如き短週期の地震計の制振にも利用せられる樣になつた．空氣制振器は一般にピストン型を爲したものであるが，ピストンの速度と之に働く反抗力（制振力）との比例的關係は理論的には空氣の不可壓縮性を假定することに依つて求められる．然し實際にはピストンの運動に從ひ內部の空氣の壓力は増大又は減少せしめられるのであるから，空氣の膨脹壓縮の生ずることは當然である．本文は此の空氣の膨脹壓縮の現象が制振力に對し如何なる影響を持つかを理論的に吟味し，此の結果を實驗に依つて確めたものである．

2. 結果の概略を次に列擧する．

ピストンに働く反抗力は比較的緩慢な運動に對してはピストンの速度に比例するが，極めて急激な運動に對してはピストンの變位に比例する．

今ピストンが正弦運動を行ふ場合を考へると，反抗力はピストンの速度と同位相のものと，之と 90° だけ相違した位相（卽ちピストンの變位と同位相）のものとに分解出來る．振動週期の大なる場合は異位相の力は同位相の力に比較して小となり，極端に大なる週期に對しては異位相の力は消失し速度と同位相の反抗力のみとなる．此の場合はピストンの反抗力は完全に其の速度に比例するのであつて空氣の不可壓縮性を假定した場合と同じである．然し振動週期が小となるに從つてピストンの速度と同位相の反抗力は次第に減少し，之に反し異位相の反抗力は増大する．極端に小なる振動週期に對しては同位相の力は消失し異位相の力のみとなる．卽ち反抗力はピストンの變位に全く比例し，見かけ上振子の復元力を増大する．之はピストンの間隔を通して空氣の出入することなく空氣の彈性に依り反抗力を生ずる場合に相當する．

空氣の可壓縮性は制振器の反抗力に如上の影響を齎すが，之はピストンの底と壔との距離を適當に小とすることに依り或る程度迄免れることが可能である．

3. 以上の結果は實驗的にも確めることが出來た．

4. 實際問題としては，加速度地震計にあつては現在の如く 0·1 秒程度より大なる週期の地動を目標としてゐる限り上に述べた影響は餘り問題とならないが，之より小なる週期の地動の記錄を必要とする場合は此の影響から免れる樣に制振器を設計することは相當困難であらう．

又長週期地震計にあつては比較的少なる週期の地震に對しては制振器の反抗力に依り見かけ上振子の週期は減少するが，餘りに少なる週期の地震の記錄は長週期地震計の目的とする所では無く，又多少振子週期が見かけ上減少しても地動週期が振子週期に比較して相當小である以上地震計倍率に左程の影響はない．從つて現在より遙かに精密を要する樣な測定でない限り問題にならない．