34. The M_2 Seismic Waves.[1]

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The equation of dispersion curves of Rayleigh waves transmitted through a stratified body,[2] namely

$$\frac{4r's'}{f^2}\left(2-\frac{k'^2}{f^2}\right)\eta-\frac{r's'}{f^2}\left\{4\vartheta+\left(2-\frac{k'^2}{f^2}\right)^2\zeta\right\}\cosh r'H\cos s'H$$

$$+\frac{r'}{f}\varphi\left\{\frac{4rs'^2}{f^3}+\frac{s}{f}\left(2-\frac{k'^2}{f^2}\right)^2\right\}\cosh r'H\sin s'H$$

$$+\frac{s'}{f}\varphi\left\{-\frac{4sr'^2}{f^3}+\frac{r}{f}\left(2-\frac{k'^2}{f^2}\right)^2\right\}\sinh r'H\cos s'H$$

$$+\left\{-\frac{4r'^2s'^2}{f^4}\zeta+\left(2-\frac{k'^2}{f^2}\right)^2\vartheta\right\}\sinh r'H\sin s'H=0, \qquad (1)$$

where

$$\left.\begin{aligned}&\varphi=\frac{\mu'k^2k'^2}{\mu f^4},\quad \zeta=\frac{4rs}{f^2}\left(\frac{\mu'}{\mu}-1\right)^2-\alpha^2,\quad \eta=\frac{2rs}{f^2}\left(\frac{\mu'}{\mu}-1\right)\beta-\alpha\gamma,\\ &\vartheta=\frac{rs}{f^2}\beta^2-\gamma^2,\ \alpha=\frac{2\mu'}{\mu}-\left(2-\frac{k^2}{f^2}\right),\quad \beta=\frac{\mu'}{\mu}\left(2-\frac{k'^2}{f^2}\right)-2,\\ &\gamma=\frac{\mu'}{\mu}\left(2-\frac{k'^2}{f^2}\right)-\left(2-\frac{k^2}{f^2}\right),\\ &r^2=f^2-h^2,\quad s^2=f^2-k^2,\quad r'^2=f^2-h'^2,\quad s'^2=k'^2-f^2,\\ &h^2=\rho p^2/(\lambda+2\mu),\quad h'^2=\rho'p^2/(\lambda'+2\mu'),\quad k^2=\rho p^2/\mu,\quad k'^2=\rho'p^2/\mu',\end{aligned}\right\} \qquad (2)$$

in which H is the thickness of the stratum, $2\pi/f$ the wave length, ρ, λ, μ, ρ', λ', μ' the densities and elastic constants of the stratum and the subjacent medium respectively, throws considerable light on the nature of seismic waves as well as on the structure of the earth crust. The M_2 seismic waves that we have newly found from this equation and the application of these waves to the determination of the super-

1) Preliminary report published in the *Proc. Imp. Acad.*, **11** (1935), 96.
2) K. SEZAWA and K. KANAI, "Discontinuity in the Dispersion Curves of Rayleigh Waves," *Bull. Earthq. Res. Inst.*, **13** (1935), 245~250.

ficial structure of the earth may be of some interest.

After a prolonged study of papers by Matuzawa[3] and a number of other writers we came to the conclusion[4] that it is urgently necessary to obtain, mathematically, the dispersion curves of Rayleigh waves that are transmitted through stratified bodies in such cases as (i) $\rho'/\rho=1{\cdot}14$, $\mu'/\mu=1{\cdot}65$, $\lambda=\mu$, $\lambda'=\mu'$, (ii) $\rho'/\rho=1$, $\mu'/\mu=2$, $\lambda=\mu$, $\lambda'=\mu'$, the former case and the latter approximately corresponding to the stratifications of the Eurasian Continent and the Pacific Ocean respectively. Thus, by means of equation (1) we determined the dispersion curves of Rayleigh waves in bodies whose densities and elastic constants are specified by (i) and (ii), the result being shown in Table I and plotted in Figs. 1, 2.

Table I. ($\lambda=\mu$, $\lambda'=\mu'$)

$\rho'/\rho=1{\cdot}14$, $\mu'/\mu=1{\cdot}65$				$\rho'/\rho=1$, $\mu'/\mu=2$			
1st phase		2nd phase		1st phase		2nd phase	
$\frac{2\pi}{fH}$	$\frac{p}{f}\sqrt{\frac{\rho}{\mu}}$	$\frac{2\pi}{fH}$	$\frac{p}{f}\sqrt{\frac{\rho}{\mu}}$	$\frac{2\pi}{fH}$	$\frac{p}{f}\sqrt{\frac{\rho}{\mu}}$	$\frac{2\pi}{fH}$	$\frac{p}{f}\sqrt{\frac{\rho}{\mu}}$
1·285	0·9325	0·5075	1·049	2·082	1·000	1·030	1·225
1·620	0·949	0·951	1·149	8·910	1·225	1·319	1·296
2·460	1·000	1·162	1·174	225·0	1·296	1·460	1·319
3·920	1·049	1·513	1·196			1·892	1·364
64·7	1·082			Other values from prev. paper[5]		2·400	1·400
						2·780	1·414

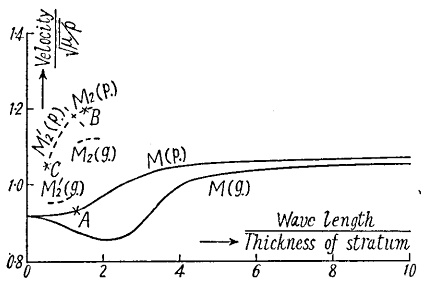

Fig. 1. $\frac{\rho'}{\rho}=1{\cdot}14$, $\frac{\mu'}{\mu}=1{\cdot}65$.

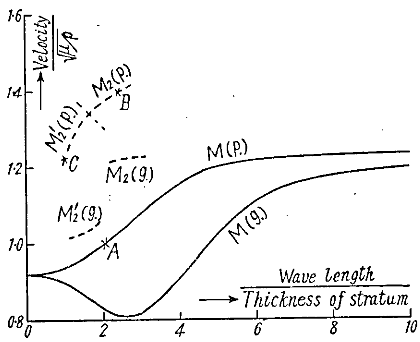

Fig. 2. $\frac{\rho'}{\rho}=1$, $\frac{\mu'}{\mu}=2$.

3) T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **6** (1929), 213~229.
4) K. SEZAWA, "Rayleigh- and Love-waves transmitted through the Pacific Ocean and the Continents," *Bull. Earthq. Res. Inst.*, **13** (1935), 245~250.
5) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **3** (1927), 1.

It was found in the course of our previous study[6] that there are two dispersion curves of Rayleigh waves of different types for a stratified body, particularly when the elastic constants of the stratum differ greatly from those of the subjacent medium. Our present result shows that it is still possible for two dispersion curves to exist even in the case in which the difference in the elastic constants as well as that in the densities of the two media are not very marked. The curves in full lines in Figs. 1, 2 give the velocities of transmission for the usual type of Rayleigh waves, while the curves in broken lines give that for another type of the same waves. The two wave types may now be called M waves and M_2 waves respectively.

As will be seen from Figs. 3, 4, which show the distribution of displacements in the body, the nature of M_2 waves differs somewhat from that of waves corresponding to the second dispersion curves found and described in the previous study.[7] The vibrational energy of the present M_2 waves, instead of being accumulated on the free surface, is concentrated mainly in the vicinity of the boundary between the stratum and the subjacent medium, the horizontal component of the displacement at the surface being too small compared with the vertical component. Thus, it is probable that the M_2 waves, if they were to be observed, would not generally give rise to large displacements at the surface, while moreover their horizontal displacement would be exceedingly small.

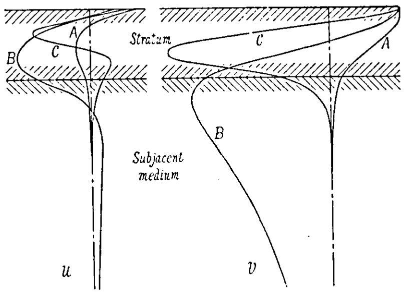

Fig. 3. $\frac{\rho'}{\rho}=1{\cdot}14$, $\frac{\mu'}{\mu}=1{\cdot}65$.

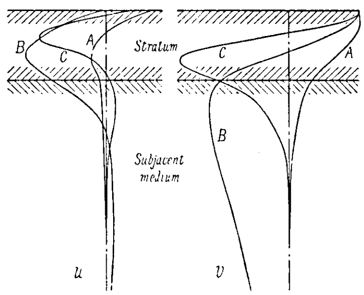

Fig. 4. $\frac{\rho'}{\rho}=1$, $\frac{\mu'}{\mu}=2$.

Careful examination of the dispersion curves for M_2 waves shows that the curve consists of two parts, namely, one for M_2 waves and the other for M_2' waves (if we may so call it), the group velocities of M_2 and M_2' are respectively greater and less than that of the maxim-

6) K. SEZAWA and K. KANAI, *loc. cit.* 2).

7) K. SEZAWA and K. KANAI, *ditto.*

um group velocity of M. The distributions of displacements of cases corresponding to points A, B, C (in Figs. 1, 2) which belong respectively to the M, M_2, M'_2 waves, were calculated in the same way as that described in the previous paper[8] and diagrammatically shown in Figs. 3, 4. Since the displacement of the waves corresponding to M_2 is distributed relatively uniformly within a certain range of depth in the subjacent layer, while that of M'_2 is accumulated rather in the very vicinity of the stratum, it would be a more easy matter to identify the M'_2 phase than the M_2 phase from a seismic record, which after all is the result of measurement of motion only on the surface of the earth. It is, however, certain that the periods of oscillation in the M_2 and M'_2 phases are relatively short and limited to a certain range, as will be seen from Figs. 1, 2.

In order to confirm the existence of these M_2 and M'_2 waves in actual seismic disturbances, we studied the seismic record[9] of the great India Earthquake of January 15, 1934, which was obtained at Tokyo, as shown in Fig. 5. The times of arrival of M_2, M, M'_2 are in fairly

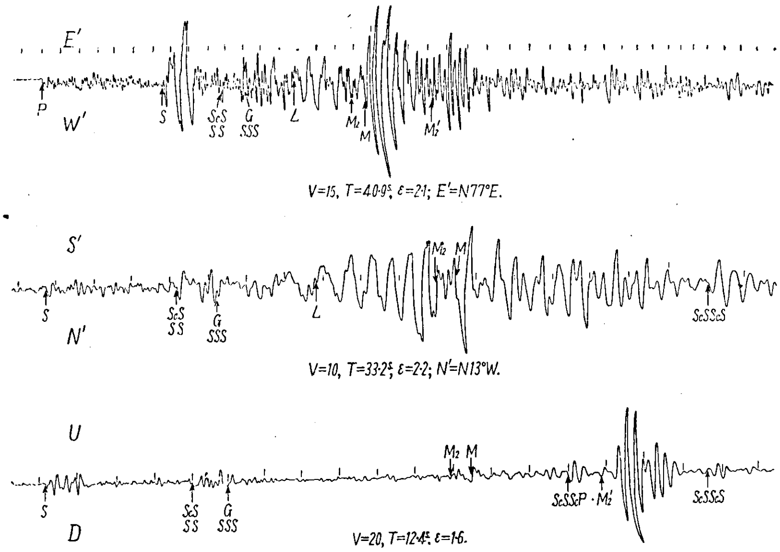

Fig. 5. $\Delta=48°$; India Earthquake, 1934, I 15. Original×1/5.

good accordance with those estimated from Fig. 1; the feature that the oscillations of waves in the M_2 and M'_2 phases are of quick type is also clearly recognized. M_2 and M'_2 may also be distinguished from each other by the relative amplitudes of their respective oscillations. The identification of M_2 and M'_2 was therefore not difficult; not at any rate in the present example.

8) K. SEZAWA and K. KANAI, *loc. cit.* 2).
9) *Seism. Rep. Earthq. Res. Inst.*, 1934, Pt. 1.

34. M_2 地震波

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彙報第 13 號第 2 册に於て，地表層がある場合の上下層の彈性が著しく異なる場合の Rayleigh 波に二種類あることを指摘しておいたが，上下層の彈性比がそれ程違はぬ場合にも二種類あるかどうかを吟味して見たのである．しかし，實際の場合に最も近いものを見る爲に，(i) $\rho'/\rho=1{\cdot}14$，$\mu'/\mu=1{\cdot}65$，$\lambda=\mu$，$\lambda'=\mu'$，(ii) $\rho'/\rho=1$，$\mu'/\mu=2$，$\lambda=\mu$，$\lambda'=\mu'$ といふ二つの場合を取つて見た．之等が非常に大切な事は旣に我々が研究しておいた所である．この研究の結果，普通の Rayleigh 波以外にも比較的短波長のものに別個の Rayleigh 波が存在し，且つ之は地表層の下面に振動勢力がよく集中してゐることがわかるのである．之を M_2 波と名づけ，地表では上下動の割合に水平動が非常に小さい．その中でも波長によつて群速度が普通の Rayleigh 波の最高群速度よりも速いものと，それよりも遲いものとの二種があり，前者を M_2 波，後者を M_2' 波として見た．而して M_2' 波の方が M_2 波よりも地表の振幅が大きくなる傾向がある．M_2 波は M_2' 波よりも地中深く勢力が分布されてゐるから，振幅も小さくなる譯である．

大陸を通過した遠地地震の例として，昭和 9 年 1 月 15 日の印度地震の東京に於ける觀測結果をしらべて見た所が，M_2，M，M_2' 波ともに存在し，それ等の波は上記の各性質を具備してゐることがわかつた．從つて M_2，M_2' 波は，場合によつて實在し得るものであると思はれるのである．たとひ，M_2，M_2' 波が存在しない場合があつても，それは地震の機構によつたものとしてもよいであらう．

彙報第 13 號第 2 册中の別の論文で述べたやうに，歐亞大陸の最上層の地表層の厚さは Rayleigh 波の速度から出すことが最もよいのは明かであるが，只今の M_2 波，M 波，M_2' 波のすべてを用ひて恰度 12km（次層は L 波によつて 28km と出る）となる譯であるから，遠地地震の研究は地震なき地方の學者の道樂などといつてゐる譯には行かない．我々の地表下 12km の狀態がこれでわかるとすれば，我々も大いに考へて然るべきであると思ふ．M_2，M_2' 波は勿論の事，この問題に應用できるやうな M 波の場合でさへも七八年前の我々の研究までは何もなかつたのである．尤もそれよりも前にあつた記號だけの方程式位では應用物理學上に殆ど役に立たぬことは勿論である．