38. Pulsatory Oscillations of the Earth's Crust due to Surface[1] *Force.**

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(Read March 19, 1935.—Received June 20, 1935.)

1. Oscillations of the earth's crust, exceedingly minute, but having no connection with ordinary earthquakes, are matters of constant observation. This subject was first discussed by Milne[2], and later by Omori[3], who classified the pulsations as observed in Tokyo into three types, q, Q_1, and Q_2, according to their periods of oscillations. Since then Prof. T. Matuzawa[4] and Dr. K. Wadati[5] have made detailed studies of these types of pulsations. The Q_1-type has recently been studied by Dr. F. Kishinouye[6].

Although, in the hope of learning the causes of these pulsations, a number of investigators[7] have attempted to trace the connections, if any, between them and meteorological conditions and other agencies that might be at work, the whole subject is still wrapped in mystery. But whatever the primary causes may be, there is no doubt that they are greatly influenced by the geologic and topographic conditions of the regions in which they occur. They seem to be proper oscillations peculiar to the particular locality, combined with forced oscillations as the result of disturbances propagated through the earth's crust. Since this idea was first mooted by F. Omori, problems relating to the proper period of the earth's crust have come to play a very important rôle in seismology, with the result that problems of this kind are being

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* Communicated by M. Ishimoto.

1) For lack of a more suitable word, the writer uses the words *surface force* here to mean forces that operate on the earth's surface, such as winds, etc.,

2) J. MILNE, *Trans. Seism. Soc., Japan,* **6** (1883); **7** (7884); **11** (1887); **13** (1889); **17** (1893); **18** (1893); **10** (1894).

3) F. OMORI, *Bull. Earthq. Inv. Comm.,* **1** (1907); **2** (1908); **3** (1909); **5** (1913); **94** (1921), (in Japanese).

4) T. MATUZAWA, *Journ. Fac. Sci., Tokyo,* [ii], **2** (1927), 205~263.

5) K. WADATI, *Geophys. Mag.,* **1** (1926~1928), 35.

6) F. KISHINOUYE, *Bull. Earthq. Res. Inst.,* **13** (1935), 146.

7) For example, H. SCHÜNEMANN, *Z. f. Geophys.,* **8** (1932), 216.
B. GUTENBERG, „Die Seismishe Bodenunruhe", (1924), 43.
S. K. BANERJI, *Phil. Trans. Roy. Soc., London,* **229** (1930), 287.

attacked by many investigators[8] from the standpoint of both theory and observation.

To explain the period of pulsations, E. Wiechert[9] considers the pulsations as stationary waves in the upper layer of the earth's crust, assuming a surface of discontinuity for its bottom. The upper layer of the earth's crust oscillates in such a way that the bottom of that layer forms the nodal plane of the oscillation, the upper surface of the layer being the loop-plane of the said oscillation. He calculated the depth of the layer by means of the formula $D=\frac{1}{4}TV$, where D is the depth of the layer, V the velocity of the transversal wave in the layer, T the period of the pulsation. By a similar treatment, K. Wadati[10] estimated the thickness of the surface layer in Tokyo to be about $1\sim 2$ km.

Prof. H. Nagaoka[11] regards the pulsations as stationary surface tremors of Rayleigh type, assuming the existence of appropriate boundaries. T. Matuzawa[12] thinks these pulsations are coupled oscillations of two oscillating systems. H. Honda[13], who believes these pulsations to be stationary surface tremors of Love type, obtained the mode of oscillation for the Q_2-type, assuming a suitable boundary covering the hypothetical case that the surface layers in Tokyo may be only some ten metres thick. The Q_1-type pulsation seems to be an overtone of that of the Q_2-type.

All these studies just mentioned concern problems of free oscillations of the earth's crust. Recently G. NISHIMURA[14], in discussing the problem of vibrations of a heterogeneous elastic solid due to *surface force*, applied the problem to pulsation. By similar treatment the writer calculated the pulsatory oscillations of the earth's crust due to *surface*

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8) K. SUYEHIRO, *Publ. Imp. Acad.*, **4** (1929), 2; *Bull. Earthq. Res. Inst.*, **7** (1930), 467.
M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **9** (1931), 316; **10** (1932), 171; **17** (1934), 234.
T. SAITA and S. SUZUKI, *Bull. Earthq. Res. Inst.*, **12** (1934), 517.
W. INOUYE, *ibid.*, **12** (1934), 712.
K. SEZAWA, *ibid.*, **8** (1930), 1.
K. SEZAWA and G. NISHIMURA, *ibid.*, **8** (1930), 321.
K. SEZAWA and K. KANAI, *ibid.*, **10** (1932), 1.

9) E. WIECHERT, *Beiträge zur Geophysik, Ergänzungs Band.*, **2** (1904).

10) K. WADATI, *loc. cit.*

11) H. NAGAOKA, *Publ. Earthq. Inv. Comm.*, **22** (1906).

12) T. MATUZAWA, *loc. cit.*

13) H. HONDA, *Geophys. Mag.*, **3** (1930), 177.

14) G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **12** (1934), 403. *Disin*, **5** (1933), 677; **7** (1935), 78, (in Japanese).

force. In the present paper the writer gives a method for calculating the pulsatory oscillations with the aid of integral equations. In discussing the growth of these pulsations and the structure of the earth's crust, there are admittedly many unknown facts, but the pulsations are here treated as shearing vibrations of the upper layer of the earth's crust, it being assumed for simplicity that the medium that lies next below the upper layer is infinitely rigid. The periods of both the free and forced oscillations of the layer (in some simple cases) are calculated and compared with those of pulsations actually observed.

2. Take the x- and y-axis horizontally and the z-axis vertically downwards, the origin being on the surface of the layer (Fig. 1). The equations of motion in the layer are expressed by

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2}+2\epsilon\frac{\partial u}{\partial t}&=\mu\frac{\partial^2 u}{\partial z^2}+\frac{d\mu}{dz}\frac{\partial u}{\partial z}\,, \\ \rho\frac{\partial^2 v}{\partial t^2}+2\epsilon\frac{\partial v}{\partial t}&=\mu\frac{\partial^2 v}{\partial z^2}+\frac{d\mu}{dz}\frac{\partial u}{\partial z}\,, \end{aligned}\right\} \qquad (1)$$

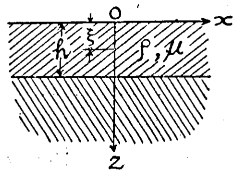

Fig. 1.

where u, v denote displacement components in the x- and y-axis, ϵ the coefficient of damping, μ the rigidity, ρ the density of the layer. Since u is independent of v, the two equations in (1) with respect to u or v can be solved in the same way. We therefore treat only the equation that involves u. In order to obtain the free oscillation of the layer, we must solve the equation

$$\mu\frac{\partial^2 u}{\partial z^2}+\frac{d\mu}{dz}\frac{\partial u}{\partial z}=0\,. \qquad (2)$$

By integrating equation (2) with respect to z, we have

$$\mu\frac{du}{dz}=C\,. \qquad (3)$$

Integrating further, we get

$$u(z)=\int_{(z)}\frac{C\,d\xi}{\mu(\xi)}+D\,, \qquad (4)$$

where C and D are the integration constants.

In the present case, we consider the simplest problem in which the medium next below the surface layer is infinitely rigid, so that the boundary conditions are merely denoted by

$$\left.\begin{aligned} &\text{(i)} \qquad z=0\,, \quad \frac{du}{dz}=0\,, \\ &\text{(ii)} \qquad z=h \quad u=0\quad , \end{aligned}\right\} \qquad (5)$$

where h is the thickness of the layer.

If we assume that unit force per unit mass at point $z=\xi$, we get

$$\left|\mu\frac{du}{dz}\right|_{\xi+0}^{\xi-0}=1,\ \text{or}\ \frac{du}{dz}=-\frac{1}{\mu(\xi)}\ . \tag{6}$$

By means of condition (i) in (5), equation (3) is reduced to $u=$const., since $C=0$. By means of (ii) in (5), equation (4) becomes $u=\int_z^h\frac{Cd\xi}{\mu(\xi)}$, whence

$$\left|\frac{du}{dz}\right|_{z=\xi}=C\int_\xi^h\frac{d}{dz}\left(\frac{d\eta}{\mu(\eta)}\right)dz-\frac{1}{\mu(\xi)}=-\frac{C}{\mu(\xi)}. \tag{7}$$

Since equation (6) is equivalent to (7), $C=1$.

Thus

$$u=\int_z^h\frac{d\xi}{\mu(\xi)}. \tag{8}$$

We assume $G(z,\xi)$ equal $u(z)$ and take it Green's function, which function, of course, satisfies conditions (5). The product of $G(z,\xi)$ and the unit force that is applied at the point $z=\xi$, represents the displacement at that point.

We now introduce a symmetric kernel (Kern) $K(z,s)$

$$\left.\begin{aligned}K(z,s)&=\int_z^h\frac{\rho(\xi)}{\mu(\xi)}d\xi,\quad z>s,\\&=\int_s^h\frac{\rho(\xi)}{\mu(\xi)}d\xi,\quad z<s.\end{aligned}\right\} \tag{9}$$

When the displacement of the layer is given by $\Phi(z,t)=\phi(z)e^{i\lambda t}$, the inertial force produced at point $z=s$ is given by $\rho\lambda^2\phi(s)e^{i\lambda t}ds$. The product of the inertial force and $G(z,\xi)$ equals the displacement produced at point z due to the concentrated mass at $z=\xi$. As there are many concentrated masses at various points between $z=0$ and $z=h$, integration must be made from $z=0$ to $z=h$. We have therefore the displacement $\phi(z)$ given by the homogeneous integral equation

$$\phi(z)=\lambda^2\int_0^h K(z,s)\phi(s)ds. \tag{10}$$

Assuming next that force is concentrated at point $z=\xi$ and that the force per unit mass is $p(\xi)$, the force at the surface of the layer is given by

$$\lim_{\xi\to0}\int_{\xi-\varepsilon_1}^{\xi+\varepsilon_1}p(\xi)d\xi=T, \tag{11}$$

where T is the tangential stress of the wind or other agencies, ε_1 being a very small quantity.

The force at $z=s$ is given by

$$\rho\lambda^2\phi(s)e^{i\lambda t}ds+\rho f(s)e^{i\lambda t}ds+2i\lambda\varepsilon'\rho\phi(s)e^{i\lambda t}ds,$$

assuming that

$$p(\xi)=f(\xi)e^{i\lambda t}, \quad \varepsilon'\rho=\varepsilon. \tag{12}$$

Then the integral equation governing the forced oscillation of the layer is given by

$$\phi(z)=(\lambda^2+2i\lambda\varepsilon')\int_0^h K(z,s)\phi(s)ds+F(z), \tag{13}$$

where

$$F(z)=\lim_{\xi\to 0}\int_{\xi-\varepsilon_1}^{\xi+\varepsilon_1} p(\xi)K(z,\xi)d\xi=K(z,0)\int_0^{z+0} p(\xi)d\xi \tag{14}$$

$$=K(z,0)T.$$

The above equation is a Fredholm's integral equation of the second kind.

Should condition $\int_0^h\int_0^h[K^2(s,z)]^2dzds<1$ be not satisfied, as is generally the case, we must then resort to Schmidt's solution. If we define $\phi_n(z)$ as the normalised principal solutions (normal functions; Eigenfunktionen) of the homogeneous integral equation

$$\phi(z)=\lambda^2\int_0^h K(z,s)\phi(s)ds.$$

It is easily proved that

$$\left.\begin{aligned}\int_0^h[\phi_n(z)]^2dz=1, \quad \int_0^h\phi_n(z)\phi_m(z)dz=0,\\ K(z,s)=\sum_{n=1}^{\infty}\frac{\phi_n(z)\phi_n(s)}{\lambda_n},\end{aligned}\right\} \tag{15}$$

$$(n=1,\ 2,\cdots\cdots,\quad m=1,\ 2,\cdots\cdots,\ n\neq m.)$$

where $\lambda_n's$ are the characteristic numbers (Eigenwerte). We have moreover

$$K^2(z,s)=\int_0^h K(z,\xi)\sum_{n=1}^{\infty}\frac{\phi_n(\xi)\phi_n(s)}{\lambda_n}d\xi$$

$$=\sum_{n=1}^{\infty}\frac{\phi_n(s)}{\lambda_n}\int_0^h K(z,\xi)\phi_n(\xi)d\xi$$

$$=\sum_{n=1}^{\infty}\frac{\phi_n(s)\phi_n(z)}{\lambda_n^2}. \tag{16}$$

Now that $K(z,s)$ and $F(z)$ are all continuous, (13) is easily solved. By means of Schmidt's solution, equation (13) can be solved as follows:

$$\phi(z)=F(z)+\sum_{n=1}^{\infty}\frac{\lambda^2+2i\lambda\varepsilon'}{\lambda_n^2-(\lambda^2+2i\lambda\varepsilon')}\,\phi_n(z)\int_0^h F(z)\phi_n(z)dz. \tag{17}$$

While

$$\frac{\lambda^2+2i\lambda\varepsilon'}{\lambda_n^2-(\lambda^2+2i\varepsilon'\lambda)}=\frac{(\lambda_n^2-\lambda^2)\lambda^2-4\lambda^2\varepsilon'^2+2i\lambda\varepsilon'\lambda_n^2}{(\lambda_n^2-\lambda^2)^2+4\lambda^2\varepsilon'^2},$$

whence equation (17) becomes

$$\phi(z)=F(z)+\sum_{n=1}^{\infty}\frac{(\lambda_n^2-\lambda^2)\lambda^2-4\lambda^2\varepsilon'^2}{(\lambda_n^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\,\phi_n(z)\int_0^h F(z)\phi_n(z)dz$$
$$+i\sum_{n=1}^{\infty}\frac{2\varepsilon'\lambda\lambda_n^2\phi_n(z)}{(\lambda_n^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\int_0^h F(z)\phi_n(z)dz. \tag{18}$$

3. We shall now give some examples in the following cases:

(*1*) *The case in which there is only one surface layer.*

If ρ and μ are all constant, the kernel $K(z,s)$ becomes

$$\left.\begin{aligned}K(z,s)&=\frac{\rho}{\mu}(h-z),\ [z>s],\\ &=\frac{\rho}{\mu}(h-s),\ [z<s].\end{aligned}\right\} \tag{19}$$

In this case the "Eigenwerte" and "Eigenfunktionen" of the homogeneous integral equation (10) are given by

$$\left.\begin{aligned}\phi_n(z)&=\sqrt{\frac{2}{h}}\cos\frac{2n-1}{2h}\pi z,\\ \lambda_{2n-1}&=\frac{(2n-1)\pi}{2h}\sqrt{\frac{\mu}{\rho}}.\quad n=1,2,\cdots\end{aligned}\right\} \tag{20}$$

From these we can calculate the periods and amplitudes of oscillations of the layer. The manner of free oscillation of the layer is shown in Fig. 2.

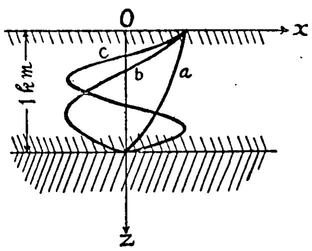

Fig. 2. a, $n=1$; b, $n=2$; c, $n=3$.

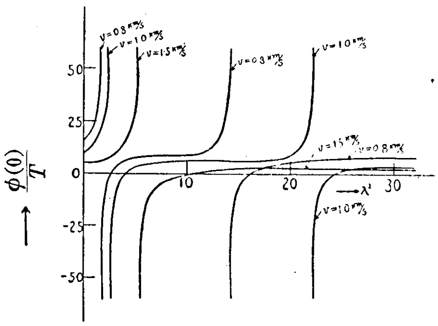

Fig. 3. Relation between $\left[\frac{\phi(z)}{T}\right]_{z=0}$ and λ^2 ($h=1$ km).

Since we obtain in this case

$$\int_0^h F(z)\phi_n(z)dz = T\sqrt{\frac{2}{h}}\frac{\rho}{\mu}\int_0^h (h-z)\cos\frac{(2n-1)\pi}{2h}z dz,$$

equation (18) for determining the forced oscillation of the layer becomes

$$\frac{\phi(z)}{T} = \frac{\rho}{\mu}(h-z) + \frac{2\rho}{h\mu}\sum_{n=1}^{\infty}\frac{(\lambda_{2n-1}^2-\lambda^2)\lambda^2-4\lambda^2\varepsilon'^2}{(\lambda_{2n-1}^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\frac{\cos\frac{2n-1}{2h}\pi z}{\frac{(2n-1)^2\pi^2}{4h^2}}$$

$$+i\frac{2\rho}{h\mu}\sum_{n=1}^{\infty}\frac{2\varepsilon'\lambda\lambda_{2n-1}^2}{(\lambda_{2n-1}^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\frac{\cos\frac{2n-1}{2h}\pi z}{\frac{(2n-1)^2\pi^2}{4h^2}}. \tag{21}$$

The curves of the forced oscillations showing the displacement at $z=0$ for $h=1$ km when $v\left(\sqrt{\frac{\mu}{\rho}}\right)=0{\cdot}8$ km/sec, 1 km/sec, 1·5 km/sec, $\varepsilon'=0$ are plotted in Fig. 3. When the period of the forced oscillation coincides with that of the free oscillation of the layer, the amplitude of the oscillation at the surface becomes infinitely great (Fig. 3).

(2) *The case of two superficial layers.*

As to the symmetric kernel, we have two cases: (i) $z>s$, (ii) $z<s$. Furter, each case involves the two cases (a) and (b) (Figs. 4, 5).

(i) When $z>s$.

(a) When $h_1\geqq s\geqq 0$, the "Kern" $K(z,s)$ is given by

$$K(z,s)=\int_z^{h_2}\frac{\rho(\xi)}{\mu(\xi)}d\xi=\int_{h_1}^{h_2}\frac{\rho(\xi)}{\mu(\xi)}d\xi+\int_z^{h_1}\frac{\rho(\xi)}{\mu(\xi)}d\xi$$

$$=\frac{\rho'}{\mu'}(h_2-h_1)+\frac{\rho}{\mu}(h_1-z),\ [z<h_1] \tag{22}$$

$$K(z,s)=\frac{\rho'}{\mu'}(h_2-z).\ [z>h_1] \tag{23}$$

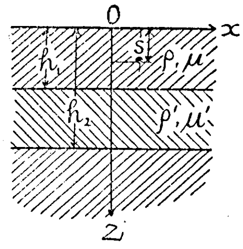

Fig. 4.

(b) When $h_2\geqq s\geqq h_1$, the "Kern" is given by

$$K(z,s)=\int_z^{h_2}\frac{\rho(\xi)}{\mu(\xi)}d\xi=\frac{\rho'}{\mu'}(h_2-z). \tag{24}$$

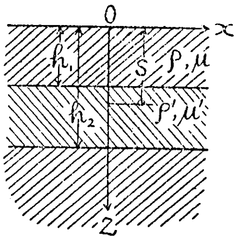

Fig. 5.

(ii) When $z<s$, there are two cases as follows:

(a) When $h_1\geqq s\geqq 0$, the "Kern" is given by

$$K(z,s)=\int_s^{h_2}\frac{\rho(\xi)}{\mu(\xi)}d\xi=\frac{\rho}{\mu}(h_1-s)+\frac{\rho'}{\mu'}(h_2-h_1)\,. \tag{25}$$

(b) When $h_2 \geqq s \geqq h_1$, the "Kern" is given by

$$K(z,s)=\int_s^{h_2}\frac{\rho(\xi)}{\mu(\xi)}d\xi=\frac{\rho'}{\mu'}(h_2-s). \tag{26}$$

Since the "Kern" in all cases just mentioned are now obtained, we can find the "Eigenwerte" and "Eigenfunktionen" of the integral equation

$$\phi(z)=\lambda^2\int_0^{h_2}K(z,s)\phi(s)ds, \tag{27}$$

governing the free oscillation of the layer in the two cases, such as $h_2 \geqq z \geqq h_1$ and $h_1 \geqq z \geqq 0$. $\phi_{2,n}(z)$ and $\phi_{1,n}(z)$ stand for the "Eigenfunktionen" of the former and the latter of the above two cases respectively.

When $h_2 \geqq z \geqq h_1$, the homogeneous integral equation

$$\phi(z)=\lambda^2\int_0^{h_2}K(z,s)\phi(s)ds\,,$$

is written

$$\begin{aligned}\phi_2(z)&=\lambda^2\left\{\int_0^{h_1}K(z,s)\,\phi_2(s)ds+\int_{h_1}^{z}K(z,s)\,\phi_2(s)ds+\int_z^{h_2}K(z,s)\phi_2(s)ds\right\}\\&=\lambda^2\left\{\int_0^{h_1}\frac{\rho'}{\mu'}(h_2-z)\phi_2(s)ds+\int_{h_1}^{z}\frac{\rho'}{\mu'}(h_2-z)\phi_2(s)ds+\int_z^{h_2}\frac{\rho'}{\mu'}(h_2-s)\phi_2(s)ds\right\}\\&=\lambda^2\frac{\rho'}{\mu'}\left[\int_0^{z}(s-z)\phi_2(s)ds+\int_0^{h_2}(h_2-s)\phi_2(s)ds\right]\\&=\frac{\rho'}{\mu'}\lambda^2\left[\int_0^{z}(s-z)\phi_2(s)ds+\text{const.}\right],\end{aligned} \tag{28}$$

where, $\text{constant}=\int_0^{h_2}(h_2-s)\phi(s)ds$.

The solution of (28) with the boundary condition $\phi_2(z)=0$ at $z=h_2$ is given by

$$\phi_{2,n}(z)=B\sin\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-z), \tag{29}$$

where B is a constant.

The values of $\phi_{2,n}(z)$ that corresponds to that of λ_n can be determined.

When $h_1 \geqq z \geqq 0$, we have similarly

$$\phi_1(z)=\lambda^2\int_0^{h_2}K(z,s)\phi_1(s)ds$$

$$=\lambda^2\left[\int_0^z\left\{\frac{\rho'}{\mu'}(h_2-h_1)+\frac{\rho}{\mu}(h_1-z)\right\}\phi_1(s)\,ds+\int_z^{h_1}\left\{\frac{\rho}{\mu}(h_1-s)+\frac{\rho'}{\mu'}(h_2-h_1)\right\}\cdot\right.$$

$$\left.\cdot\,\phi_1(s)\,ds+\int_{h_1}^{h_2}\frac{\rho'}{\mu'}(h_2-s)\phi_1(s)\,ds\right]$$

$$=\lambda^2\left[\frac{\rho}{\mu}\int_0^z(s-z)\phi_1(s)\,ds+c_1+c_2\right], \tag{30}$$

where

$$c_1=\int_0^{h_1}\left\{\frac{\rho}{\mu}(h_1-s)+\frac{\rho'}{\mu'}(s-h_1)\right\}\phi_1(s)\,ds,$$

$$c_2=\int_0^{h_2}\frac{\rho'}{\mu'}(h_2-s)\phi_1(s)\,ds.$$

The solution of (30) is given by

$$\phi_{1,n}(z)=A\cos\lambda_n\sqrt{\frac{\rho}{\mu}}\,z, \tag{31}$$

where A is a constant: $A=\lambda^2(c_1+c_2)$.

Next, the boundary conditions are expressed by

$$\left.\begin{aligned}&\phi_{1,n}(z)=\phi_{2,n}(z),\\&\mu'\frac{\partial\phi_{2,n}(z)}{\partial z}=\mu\frac{\partial\phi_{1,n}(z)}{\partial z},\end{aligned}\right\} \tag{32}$$

at $z=h_1$, by $\phi_{2,n}(z)=0$ at $z=h_2$, and by $\frac{\partial\phi_{1,n}(z)}{\partial z}=0$ at $z=0$.

Substituting (29) and (31) in (32), we have

$$A\cos\lambda_n\sqrt{\frac{\rho}{\mu}}\,h_1=B\sin\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1), \tag{33}$$

$$\mu\lambda_n\sqrt{\frac{\rho}{\mu}}\,A\sin\lambda_n\sqrt{\frac{\rho}{\mu}}\,h_1=\mu'\lambda_n\sqrt{\frac{\rho'}{\mu'}}\,B\cos\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1), \tag{34}$$

whence the ratio A/B is expressed by

$$\frac{A}{B}=\frac{\sin\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1)}{\cos\lambda_n\sqrt{\frac{\rho}{\mu}}\,h_1},\ \text{or}\ \frac{\sqrt{\rho'\mu'}\cos\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1)}{\sqrt{\rho\mu}\sin\lambda_n\sqrt{\frac{\rho}{\mu}}\,h_1}, \tag{35}$$

and

$$\sqrt{\frac{\rho\mu}{\rho'\mu'}}\tan\lambda_n\sqrt{\frac{\rho}{\mu}}\,h_1\tan\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1)=1. \tag{36}$$

By (36) the "Eigenwerte" λ_n for determining the periods of the free oscillation of the layer can be obtained.

Thus, (29) and (31) are readily written

$$\phi_{1,\,n}(z)=B\frac{\sin\lambda_n\sqrt{\dfrac{\rho'}{\mu'}}(h_2-h_1)}{\cos\lambda_n\sqrt{\dfrac{\rho}{\mu}}h_1}\cos\lambda_n\sqrt{\frac{\rho}{\mu}}z\,, \tag{31'}$$

$$\phi_{2,\,n}(z)=B\sin\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-z). \tag{29'}$$

Since $\phi_{1,\,n}(z)$ and $\phi_{2,\,n}(z)$ are the normalized functions, constant B can be determined by the condition

$$\int_0^{h_1}[\phi_{1,\,n}(z)]^2dz+\int_{h_1}^{h_2}[\phi_{2,\,n}(z)]^2dz=1\,. \tag{37}$$

From this condition, we find that

$$\frac{1}{B^2}=\frac{\sin^2\lambda_n\sqrt{\dfrac{\rho'}{\mu'}}(h_2-h_1)}{\cos^2\lambda_n\sqrt{\dfrac{\rho}{\mu}}h_1}\frac{1}{2\lambda_n}\sqrt{\frac{\mu}{\rho}}\left(\lambda_n\sqrt{\frac{\rho}{\mu}}h_1+\frac{1}{2}\sin 2\lambda_n\sqrt{\frac{\rho}{\mu}}h_1\right)$$
$$+\frac{1}{2\lambda_n}\sqrt{\frac{\mu'}{\rho'}}\left(\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1)-\frac{1}{2}\sin 2\lambda_n\sqrt{\frac{\rho'}{\mu'}}(h_2-h_1)\right). \tag{38}$$

Thus, the "Eigenwerte" and "Eigenfunktionen" of the integral equation in the case of two stratified layers can be obtained. An integral equation to determine the forced oscillation of the two stratified layers can be obtained by substituting λ_n, $\phi_{1,\,n}(z)$, and $\phi_{2,\,n}(z)$ in that of (18). Further, in this case, $F(z)$ in (18) is expressed by

$$F(z)=K(z,0)\,T=\left\{\frac{\rho'}{\mu'}(h_2-h_1)+\frac{\rho}{\mu}(h_1-z)\right\}T. \tag{39}$$

Thus, we finally get

$$\frac{\phi(z)}{\dfrac{T}{v^2}}=\left\{\frac{m}{m'}(h_2-h_1)+(h_1-z)\right\}+\sum_{n=1}^{\infty}\frac{(\lambda_n^2-\lambda^2)\lambda^2-4\lambda^2\varepsilon'^2}{(\lambda_n^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\phi_{n'}(z)\,\cdot$$
$$\cdot\int_0^{h_2}\left\{\frac{m}{m'}(h_2-h_1)+(h_1-z)\right\}\phi_{n'}(z)\,dz$$
$$+\,i\sum_{n=1}^{\infty}\frac{2\varepsilon'\lambda\lambda_n^2\phi_{n'}(z)}{(\lambda_n^2-\lambda^2)^2+4\lambda^2\varepsilon'^2}\int_0^{h_2}\left\{\frac{m}{m'}(h_2-h_1)+(h_1-z)\right\}\phi_{n'}(z)\,dz\,, \tag{40}$$

where

$$\left.\begin{aligned}&v=\sqrt{\frac{\mu}{\rho}}\,,\quad m=\frac{\rho'}{\rho}\,,\quad m'=\frac{\mu'}{\mu}\,,\\&\phi_{n'}(z)=\phi_{1,\,n}(z)+\phi_{2,\,n}(z)\,.\end{aligned}\right\} \tag{41}$$

Let us now compute the following simple cases and ascertain the manner of the oscillations.

Example 1. Let the thickness of the first and the second layer be each 1 km and $\rho'=1{\cdot}5\rho$, $\mu'=3\mu$.

From (29′), (31′), (36), (38), we get the values of λ_n, $\phi_{1,n}(z)$ and $\phi_{2,n}(z)$ such that

$$\lambda_1=\frac{64{;}3}{180}v\pi,\quad \phi_{1,1}(z)=1{\cdot}24\cos 64{\cdot}3\,z,\qquad \phi_{2,1}(z)=0{\cdot}75\sin 45{\cdot}65(2-z),$$

$$\lambda_2=\frac{149{\cdot}1}{180}v\pi,\quad \phi_{1,2}(z)=-1{\cdot}08\cos 149{\cdot}3\,z,\quad \phi_{2,2}(z)=0{\cdot}96\sin 105{\cdot}86(2-z),$$

$$\lambda_3=\frac{265{\cdot}9}{180}v\pi,\quad \phi_{1,3}(z)=1{\cdot}26\cos 265{\cdot}9\,z,\qquad \phi_{2,3}(z)=0{\cdot}58\sin 188{\cdot}79(2-z),$$

$$\lambda_4=\frac{465}{180}v\pi,\quad \phi_{1,4}(z)=1{\cdot}26\cos 465\,z,\qquad \phi_{2,4}(z)\doteq 0{\cdot}65\sin 330{\cdot}15(2-z),$$

$$\lambda_5=\frac{591{\cdot}1}{180}v\pi,\quad \phi_{1,5}(z)=-1{\cdot}16\cos 591{\cdot}1\,z,\quad \phi_{2,5}(z)=0{\cdot}85\sin 419{\cdot}68(2-z).$$

Thus we get the curves of the free oscillations of the layer in this case for $n=1, 2, 3$, and 4 as shown in Fig. 6. Since the periods of the free oscillations are the function of v, we can determine the values of λ_n, that is, T_n for various values of v. The values of T_n are as follows:

v (km/s)	λ_1	λ_2	λ_3	λ_4	T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)
0·8	0·89	2·08	3·72	6·49	7·04	3·01	1·69	0·67
1·0	1·12	2·60	4·65	8·12	5·62	2·42	1·35	0·78
1·5	1·68	3·90	6·95	12·15	3·74	1·61	0·91	0·52
2·0	2·24	5·20	9·30	16·24	2·80	1·21	0·68	0·39

The manner of free oscillation of the layer in the case of two stratified layers is more complicated than that in the case of one surface layer, especially as the forced oscillations at $z=0$ differ entirely from one another as shown in Fig. 7. As will be seen from Figs. 3

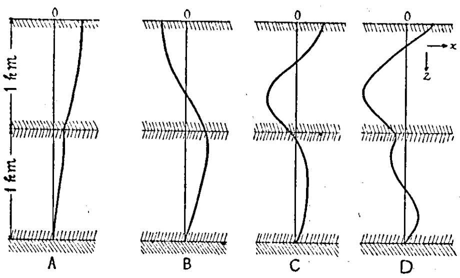

Fig. 6. $h_1=1$km, $h_2=2$km, $\dfrac{\rho'}{\rho}=1{\cdot}5$, $\dfrac{\mu'}{\mu}=3$. A, $n=1$; B, $n=2$; C, $n=3$; D, $n=4$.

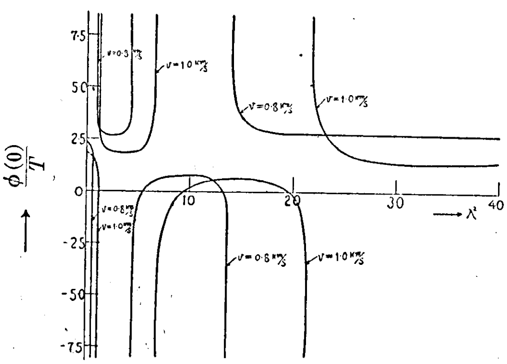

Fig. 7. Relation between $\left[\dfrac{\phi(z)}{T}\right]_{z=0}$ and λ^2. $\left(h_1=1\text{km},\ h_2=2\text{km},\ \dfrac{\rho'}{\rho}=1{\cdot}5,\ \dfrac{\mu'}{\mu}=3.\right)$

and 7, in the case of one surface layer the amplitudes of oscillations change from $\phi(z)\to-\infty$ to $\phi(z)\to+\infty$, while in the case of two stratified layers, its amplitudes change from $\phi(z)\to+\infty$ to $\phi(z)\to+\infty$, or from $\phi(z)\to-\infty$ to $\phi(z)\to-\infty$.

Example 2. Let $h_1=0{\cdot}5\text{km}$, $h_2=1\text{km}$, $\rho=\rho'$, $\mu'=2\mu$. By similar treatment as Example 1, we have

$$
\begin{aligned}
&\lambda_1=2{\cdot}02v, && \phi_{1,1}(z)=1{\cdot}56\cos 116z, && \phi_{2,1}(z)=1{\cdot}25\sin 81{\cdot}9(1-z),\\
&\lambda_2=5{\cdot}38v, && \phi_{1,2}(z)=-1{\cdot}44\cos 308z, && \phi_{2,2}(z)=1{\cdot}37\sin 217{\cdot}5(1-z),\\
&\lambda_3=9{\cdot}22v, && \phi_{1,3}(z)=1{\cdot}25\cos 529z, && \phi_{2,3}(z)=1{\cdot}17\sin 372{\cdot}9(1-z),\\
&\lambda_4=12{\cdot}90v, && \phi_{1,4}(z)=-1{\cdot}16\cos 740z, && \phi_{2,4}(z)=1{\cdot}16\sin 521{\cdot}8(1-z),\\
&\lambda_5=16{\cdot}38v, && \phi_{1,5}(z)=1{\cdot}44\cos 939z, && \phi_{2,5}(z)=1{\cdot}07\sin 662{\cdot}0(1-z).
\end{aligned}
$$

The curves showing the free oscillations of the layer for $n=1, 2, 3$, and 4 are shown in Fig. 8, and those of the forced oscillations in Fig. 9. In this case, the periods of the free oscillations are as follows:

v (km/s)	λ_1	λ_2	λ_3	λ_4	T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)
0·8	1·62	4·30	7·38	10·33	3·87	1·46	0·85	0·61
1·8	2·02	5·38	9·22	12·90	3·10	1·17	0·68	0·49
1·5	3·02	8·06	13·84	19·34	2·08	0·78	0·46	0·33
2·0	4·04	10·76	18·44	25·80	1·56	0·59	0·34	0·24

In this case the values of T_n are smaller than that of the Example 1.

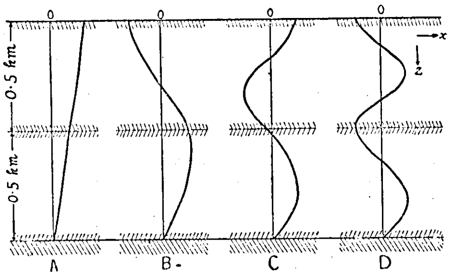

Fig. 8. $h_1=0{\cdot}5\text{km}$, $h_2=1{\cdot}0\text{km}$, $\dfrac{\rho'}{\rho}=1$, $\dfrac{\mu'}{\mu}=2$.

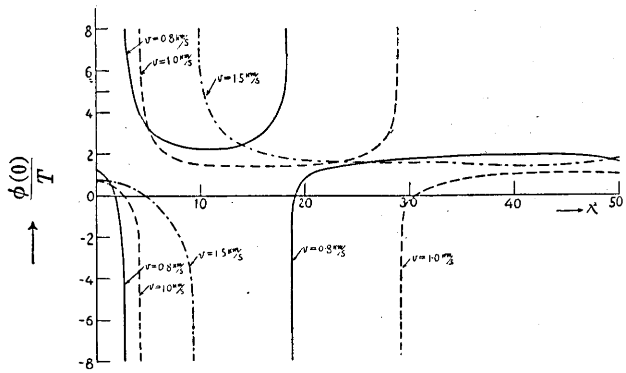

Fig. 9. Relation between $\left[\dfrac{\phi(z)}{T}\right]_{z=0}$ and λ^2.

$\left(h_1=0{\cdot}5\text{km},\ h_2=1\text{km},\ \dfrac{\rho'}{\rho}=1,\ \dfrac{\mu'}{\mu}=2.\right)$

Example 3. Let $h_1=0{\cdot}2$ km, $h_2=1{\cdot}2$ km, $\rho'=1{\cdot}1\rho$, $\mu'=2{\cdot}2\mu$. As before, we have

$$
\begin{array}{lll}
\lambda_1=1{\cdot}65v, & \phi_{1,1}(z)=1{\cdot}25\cos 112z, & \phi_{2,1}(z)=1{\cdot}18\sin 79{\cdot}5(1{\cdot}2-z),\\
\lambda_2=5{\cdot}45v, & \phi_{1,2}(z)=-1{\cdot}70\cos 312{\cdot}5z, & \phi_{2,2}(z)=1{\cdot}18\sin 221{\cdot}8(1{\cdot}2-z),\\
\lambda_3=8{\cdot}59v, & \phi_{1,3}(z)=1{\cdot}50\cos 492{\cdot}5z, & \phi_{2,3}(z)=1{\cdot}22\sin 349{\cdot}6(1{\cdot}2-z),\\
\lambda_4=12{\cdot}04v, & \phi_{1,4}(z)=-1{\cdot}32\cos 690z, & \phi_{2,4}(z)=1{\cdot}28\sin 633{\cdot}6(1{\cdot}2-z),\\
\lambda_5=15{\cdot}57v, & \phi_{1,5}(z)=1{\cdot}30\cos 892{\cdot}5z, & \phi_{2,5}(z)=1{\cdot}30\sin 784{\cdot}2(1{\cdot}2-z).
\end{array}
$$

The curves showing the free oscillations of the layer for $n=1$, 2, 3, and 4 are plotted in Fig. 10 and those of forced oscillations in Fig. 11. In this case the periods of the free oscillations are as follows:

v (km/s)	λ_1	λ_2	λ_3	λ_4	T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)
0·8	1·32	4·36	6·87	9·62	4·75	1·44	0·92	0·65
1·0	1·65	5·45	8·59	12·04	3·80	1·15	0·73	0·52
1·5	2·47	8·17	12·88	18·06	2·55	0·77	0·49	0·35
2·0	3·30	10·90	17·15	24·08	1·90	0·62	0·37	0·25

4. In our present study, we regard the plusations as stationary waves caused by the force due to wind. In order to find the relation between the periods of the pulsation and $\frac{\phi(0)}{T}$, we analysed the records of pulsations observed at the Earthquake Research Institute, Tokyo Imperial University, and determined the periods of the pulsations and the ratio $\frac{\phi(0)}{T}$, $\phi(0)$ being the amplitudes of the pulsations, and T the square of the wind velocity, the values of which were taken from the weather-charts of the Central Meteorological Observatory. The instrument used was designed and constructed by Prof. M. Ishimoto, the magnification and the proper

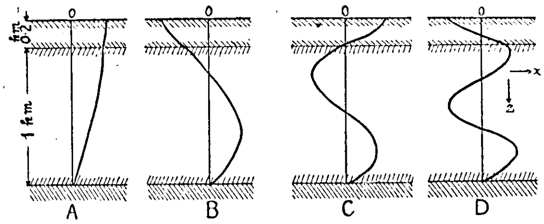

Fig. 10. $h_1=1\,\text{km}$, $h_2=1{\cdot}2\,\text{km}$, $\frac{\rho'}{\rho}=1{\cdot}1$, $\frac{\mu'}{\mu}=2{\cdot}2$. A, $n=1$; B, $n=2$; C, $n=3$; D, $n=4$.

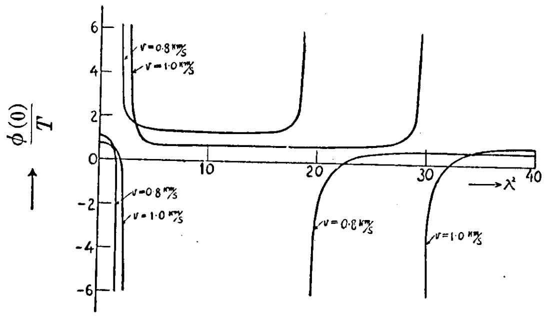

Fig. 11. Relation between $\left[\frac{\phi(z)}{T}\right]_{z=0}$ and λ^2. $\left(h_1=0{\cdot}2\,\text{km},\ h_2=1{\cdot}2\,\text{km},\ \frac{\rho'}{\rho}=1{\cdot}1,\ \frac{\mu'}{\mu}=2{\cdot}2.\right)$

period of which are 400 and 1·0 sec respectively.

We used the most pronounced records obtained the instrument during the eight months, from August, 1934 to March, 1935. An example is shown in Fig. 12. Since the records represent the acceleration of the pulsatory oscillations of the earth's crust, we compute the displacements from the records by taking the constants of the instrument into consideration and assuming the actual displacement to be of the form of a simple harmonic motion. We take the periods of the pulsations as abscissa and the ratio $\frac{\phi(0)}{T}$ as ordinate, an example of which is shown in Fig. 13. We drew this curve by tracing the maximum values of $\frac{\phi(0)}{T}$ that correspond to a certain value of period by means of such consideration that all the points were included in the space between the curve and the abscissa. From this it will be seen that the maximum values of $\frac{\phi(0)}{T}$ correspond to the periods 0·3, 0·8, about 2·5 and 5 sec (or about 4·28 sec and about 5·20 sec). Of these periods, the value of $\frac{\phi(0)}{T}$ for about 5 sec period is the greatest. It is believed that the state that the value of $\frac{\phi(0)}{T}$ is the greatest is in that of resonance, where resonance means that the amplitude of pulsation becomes very great when the proper period of the layer agrees with the period of the applied forces. The pulsations of about 2·5 and 5 sec here seem to correspond to Omori's q- and Q-type.

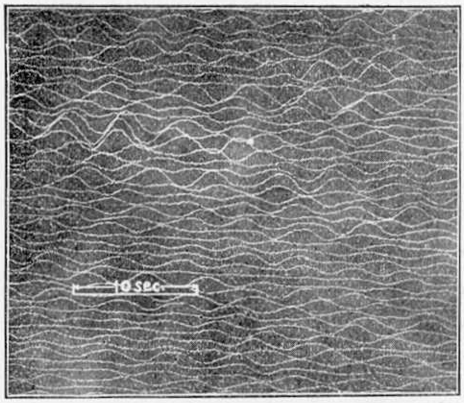

Fig. 12. Portion of actual record obtained by the Ishimoto seismograph. Jan. 1, 1935.

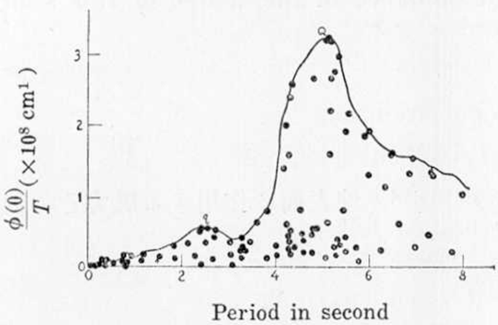

Fig. 13. Relation between $\frac{\phi(0)}{T}$ and period of the pulsation.

When we compare the relation between the periods of the pulsations and the corresponding value $\frac{\phi(0)}{T}$ with that in the case of the examples just mentioned, we shall be able to calculate the constant that is proportional to the square of the wind velocity, and the thickness of the layers besides. As will be seen from the foregoing examples, the larger the thickness of the layers, or the smaller the transverse velocity $\sqrt{\frac{\mu}{\rho}}$ in the layer, the longer is the proper period of their oscillations. Therefore, if the surface layers of the earth's crust were to consist of two layers, the pulsatory oscillations of the earth's crust may be explained by assuming that the thickness of the upper and lower layer are some hundreds of meters and about 1 km respectively, but if the values of $\sqrt{\frac{\mu}{\rho}}$ and $\sqrt{\frac{\mu'}{\rho'}}$ in the layers were smaller than those of the examples, the thickness of these two layers may become smaller than the said values.

In conclusion, the writer desires to express his cordial thanks to Professor Mishio Ishimoto and Dr. R. Takahasi for their kind advices and guidance in the course of this study.

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38. 地表面に作用する風力その他によつて起される脈動

飯　田　汲　事

この論文は脈動を地表層の歪振動に關係づけて，理論的に地表面の振動問題を取扱つたのである．表面に作用する力によつて起される不均質彈性體の振動問題については，既に西村學士の詳しい計算がある．筆者はこれと同じ問題を積分方程式を用ひて解き，表面層が二つある場合をも求め，二三の簡單な例についてその解法を示した．最後に實際觀測された脈動の周期，振幅，風速との關係を圖示し，これらの圖と計算から得られた値とを比較する事によつて，地表面に働く摩擦力，及び表面層の厚さが夫々求め得られる事を附加しておいた．