56. *Vibrational Causes of the Overturning of Railway Carriages on the Setagawa Bridge in the Typhoon of Sept. 21, 1934.*

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(Read May 21, 1935.——Received June 20, 1935.)

1. The typhoon which struck the mid-western part of Japan on Sept. 21, 1934, caused considerable damage, among which the overturning of a railroad train on the Setagawa Bridge, crossing River Seta near Lake Biwa, is an example. Although information is scanty with respect to the details of that accident, it seems established that the overturning was due mainly to the strong winds that struck the train as it was crossing the bridge. Even were the origin of the accident of an aerodynamic nature, it is still possible to postulate one aerodynamical static cause and two aerodynamical vibrational causes. One of the vibrational causes is the turbulent wind in more or less periodic motion and the other the stationary strong wind that gives rise to unstable oscillation of the railway carriages. My intention here is to examine the two vibrational causes rather than the dynamical static cause.

2. The periodic turbulent wind will first be discussed. It is well known that when wind currents flow through a rigid obstacle, periodic Kármán vortices[1] or more general periodic turbulences[2] are formed behind or around that obstacle. According to Nagaoka's conclusion,[3] the overturning of the carriages was caused by just such wind action. It seems that the forced oscillation of the carriages was in resonance condition under the periodic wind force. Not many reliable experiments have been made concerning the periodicity of the growth of Kármán vortices behind an obstacle, but the experiment of Crausse and Baubiac[4] seems worth mentioning at this juncture. They found the relation between the velocity of the fluid and the frequency of

1) TH. v. KÁRMÁN, *Gött. Nachr.* 1911, 509; 1912, 547; *Phys. Z.*, **13** (1912).

2) H. NAGAOKA, *Proc. Math.-Phys. Soc.*, [iii], **1** (1919).

3) H. NAGAOKA's paper read at the monthly meeting of the Earthq. Res. Inst., Nov. 20, 1934; unpublished.

4) E. CRAUSSE et BAUBIAC, *C. R.*, **192** (1931), 1355~57, 1529~31.

vortex generation behind a cylindrical obstacle of circular section, using liquid jets of different kinds and measuring it with a hot wire anemometer. In order to make their result applicable to the present wind problem with sufficient similarity of conditions, I transformed the parameters into VT/D aud Reynolds's number VD/ν, where D is the diameter of the cylinder, V the velocity of the fluid, ν the kinematic coefficient of viscosity, and T the period of formation of vortices (or fluctuations in wind force). It is possible to prove that, if the Reynolds's number were the same for different fluid motions, the configuration of the vortices, including general turbulent flow, relative to the obstacle should be arranged always in similar types. The small dots in Fig. 1 were marked from data due to Crausse and Baubiac, using Reynolds's number of logarithmic scale. Data in Reynolds's number greater than 10,000 were not obtained, and I extrapolated the curve passing through these dots to as much of the same number as 10^7, and found that, for $V=20$ m/s, $D=2$ m, $\nu=0{\cdot}130$, T becomes 0·95 s. We shall find later that the period of free oscillation of the rolling type as that of a carriage of wall height 2 m (which is slightly less than actual) is 0·95 s. Even should Reynolds's numbers differ more or less for differently shaped bodies, it is still probable that the overturning under consideration took place under resonance conditions in the periodic vortical or turbulent motion of strong wind. Again, even were the extrapolated curve in Fig. 1 drawn in every other possible way, the resultant deviated period of vortices at the given wind speed would be of the same order as the original one. It is therefore obvious that the cause as explained by Nagaoka's conclusion is the principal one, at any rate for the accident on Setagawa Bridge.

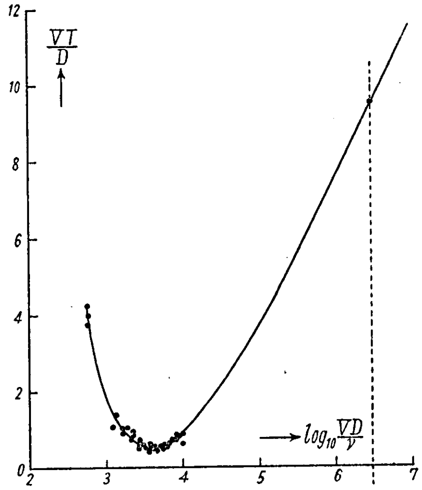

Fig. 1.

3. The unstable oscillations of a carriage due to strong wind will next be dealt with, although it may be rather a secondary cause. It is known that there are frequently downward (or upward) wind currents along a river bank. The vertical component of such a current imparts, particularly, such vibrational energy to a train of cars as to cause horizontal oscillation as well as angular rolling, both of which are generally coupled. The coupled oscillation becomes unstable (oscillatory unstable, but not aperiodic unstable) under certain conditions of the problem. Taking the case of a bogie and referring to Fig. 2, the notations used are as follows:

z, x, θ = vertical, horizontal, and angular displacements of the carriage with respect to O,

G = centre of gravity,

k = mass radius of gyration about G,

h = GO,

j = OE,

$2a$ = distance between helical springs on both sides of the bogie,

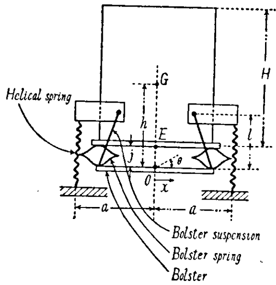

Fig. 2.

M = mass of bogie including bogie truck,

s = elastic resistance of all helical springs, including the effect of all bolster springs, on one side of bogie per unit vertical displacement,

$c_1 s\,\partial/\partial t$ = damping and dissipation resistance of . . . . ,

T/l = elastic resistance of all bolster suspensions on one side of bogie per unit angular displacement of suspension,

$(c_2 T/l)\partial/\partial t$ = damping and dissipation resistance of . . . . ,

l = effective length of bolster suspension,

L = length of bogie car,

H = wall height of bogie car,

c_m = moment coefficient of wind force about O,

c_f = sidewise dragging coefficient of wind force,

ρ = density of air,

V = speed of air current always flowing vertically.

The equations of motion of the carriage with respect to O which is fixed in the car are:

$$M\ddot{z}+2sz+2sc_1\dot{z}=0, \tag{1}$$

$$M\ddot{x}-Mh\ddot{\theta}+\frac{2T}{l}x+\frac{2T}{l}c_2\dot{x}\pm\frac{\partial c_f}{\partial\theta_0}\frac{\rho}{2}V^2LH\theta'=0, \tag{2}$$

$$M(k^2+h^2)\ddot{\theta}-Mh\ddot{x}+2sa^2\theta+2a^2c_1\dot{\theta}\pm\frac{\partial c_m}{\partial\theta_0}\frac{\rho}{2}V^2LH^2\theta'=0, \tag{3}$$

where the upper and lower ones of double signs belong to the respective cases of downward and upward wind currents, θ_0 specifies the rolling angle in the vicinity of $\theta=0$, while θ' is written in place of

$$\theta' = \theta + \frac{H/2+j}{V}\dot{\theta} \pm \frac{1}{V}\dot{x}, \tag{4}$$

the second and third terms in (4) denoting angular displacements of the carriage relative to the wind direction due to the respective angular and linear velocities of the same carriage.

Since the vertical oscillation given by equation (1) is not coupled with other oscillations, equation (1) is not necessary in the present problem. Now, by means of (4), equations (2) and (3) reduce to

$$a_{11}\ddot{\theta} + b_{11}\dot{\theta} + c_{11}\theta + a_{12}\ddot{x} + b_{12}\dot{x} = 0, \tag{3'}$$

$$a_{21}\ddot{\theta} + b_{21}\dot{\theta} + c_{21}\theta + a_{22}\ddot{x} + b_{22}\dot{x} + c_{22}x = 0, \tag{2'}$$

where

$$a_{11} = M(k^2+h^2), \qquad b_{11} = 2sa^2c_1 \pm \frac{\partial c_m}{\partial \theta_0}\frac{\rho}{2}LH^2\left(\frac{H}{2}+j\right)V,$$

$$c_{11} = 2sa^2 \pm \frac{\partial c_m}{\partial \theta_0}\frac{\rho}{2}LH^2V^2, \qquad a_{12} = -Mh,$$

$$b_{12} = \frac{\partial c_m}{\partial \theta_0}\frac{\rho}{2}LH^2V, \qquad a_{22} = M, \qquad b_{22} = \frac{2T}{l}c_2 + \frac{\partial c_f}{\partial \theta_0}\frac{\rho}{2}LHV,$$

$$c_{22} = \frac{2T}{l}, \qquad a_{21} = -Mh, \qquad b_{21} = \pm\frac{\partial c_f}{\partial \theta_0}\frac{\rho}{2}LH\left(\frac{H}{2}+j\right)V,$$

$$c_{21} = \pm\frac{\partial c_f}{\partial \theta_0}\frac{\rho}{2}LHV^2.$$

Putting

$$\left.\begin{aligned} \theta &= c_{11}e^{\lambda_1 t} + c_{12}e^{\lambda_2 t} + c_{13}e^{\lambda_3 t} + c_{14}e^{\lambda_4 t}, \\ x &= c_{21}e^{\lambda_1 t} + c_{22}e^{\lambda_2 t} + c_{23}e^{\lambda_3 t} + c_{24}e^{\lambda_4 t}, \end{aligned}\right\} \tag{5}$$

and substituting these in (2′), (3′), and eliminating the coefficients of (5), we get the following frequency equation for determining λ_1, λ_2, λ_3, λ_4

$$A_0\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0, \tag{6}$$

in which

$$\left.\begin{aligned} A_0 = |aa|, \quad A_1 &= |ab| + |ba|, \quad A_2 = |ac| + |ca| + |bb|, \\ A_3 &= |bc| + |cb|, \quad A_4 = |cc|, \end{aligned}\right\} \tag{7}$$

$|mn|$ signifying that $|mn| = \begin{vmatrix} m_{ii} & n_{ik} \\ m_{ki} & n_{kk} \end{vmatrix}$.

It will be seen from (5) that, if the real parts of all the λ_s are negative, the vibratory motion is stable, while, if the real part of any of the λ_s is positive, the motion is unstable. In the latter case, the amplitudes of vibrations become larger and larger with lapse of time t, regardless of whether the type of vibrations be periodic or aperiodic. This fact is strictly conditioned by Routh's criterion,[5] namely, if

$$A_0>0, \quad A_1>0, \quad A_2>0, \quad A_3>0, \quad A_4>0, \tag{8}$$

$$\Delta = A_1A_2A_3 - A_0A_3^2 - A_4A_1^2 > 0, \tag{9}$$

the vibratory motion is stable, while, if

$$\Delta < 0 \tag{10}$$

in place of (9), the motion is unstable. At the critical state of stability, $\Delta=0$, the frequency of vibrations (the numbers in radians) is expressed by

$$\sqrt{A_3/A_1}\,. \tag{11}$$

In a special case of coupled oscillations of two degrees of freedom, the type of motion becomes ternary[6] and there is a similar discriminant Δ for stability.

In order to get the critical wind speed to cause unstable oscillation of the railway carriage, some likely numerical values were assumed as follows:

$M=10^4$ kg mass, $k=1$ m, $h=1$ m, $a=1$ m $H=2$ m, $j=0{\cdot}5$ m,
$L=15$ m, $\rho=1{\cdot}2$ kg mass/m^3, $s=5.10^4$ kg$=5.10^4.9{\cdot}8$ kg m/s^2,
$T=5.10^3$ kg$=5.10^3.9{\cdot}8$ kg mass/s^2, $l=0{\cdot}05$ m,

whence we get

$$a_{11}=2.10^4 \text{ kg mass m}^2, \qquad b_{11}=\left\{10^6 c_1 \pm 54\frac{\partial c_m}{\partial \theta_0}V\right\} \text{ kg mass m}^2/\text{s},$$

$$c_{11}=\left\{10^6 \pm 36\frac{\partial c_m}{\partial \theta_0}V^2\right\} \text{ kg mass m}^2/\text{s}^2, \qquad a_{12}=-10^4 \text{ kg mass m},$$

$$b_{12}=36\frac{\partial c_m}{\partial \theta_0}V \text{ kg mass m/s}, \qquad a_{22}=10^4 \text{ kg mass},$$

$$b_{22}=2.10^6 c_2 + 18\frac{\partial c_f}{\partial \theta_0}V \text{ kg mass/s}, \qquad c_{22}=2.10^6 \text{ kg mass/s}^2,$$

5) E. J. ROUTH, *Advanced Rigid Dynamics* (London, 1905), 221.

6) In the special case that $A_0=0$ or $A_4=0$ in (9), the frequency equation becomes of third degree and the motion is ternary; if $A_4=0$, the discriminant for stability becomes

$$\Delta = A_1A_2 - A_0A_3 > 0,$$

the frequency of vibrations being then $\sqrt{A_2/A_0}$. I applied such a criterion to certain problems a few years ago: K. SEZAWA, *Journ. Aeron. Res. Inst.*, No. 74 (1930), 404~408; No. 87 (1931), 622~626.

$$a_{21} = \pm 21\frac{\partial c_f}{\partial \theta_0} V \text{ kg mass m/s}, \qquad c_{21} = \pm 18 \frac{\partial c_f}{\partial \theta_0} V^2 \text{ kg mass m/s}^2,$$

from which we obtain (omitting the names of the scale),

$$A_0 = 10^8, \quad A_1 = \left[4.10^{10}c_2 + 10c_1\right] + \left[\left.\begin{matrix}63\\9\end{matrix}\right\}.10^4\frac{\partial c_f}{\partial \theta_0}\left.\begin{matrix}+90\\-18\end{matrix}\right\}.10^4\frac{\partial c_m}{\partial \theta_0}\right]V,$$

$$A_2 = \left[5.10^{10} + 2.10^{12}c_1c_2\right] + \left[\pm 108.10^6 c_2 \frac{\partial c_m}{\partial \theta_0} + 18.10^6 c_1 \frac{\partial c_f}{\partial \theta_0}\right]V$$
$$+ \left[\pm 18.10^4 \frac{\partial c_f}{\partial \theta_0} \pm 36.10^4 \frac{\partial c_m}{\partial \theta_0}\right]V^2,$$

$$A_3 = \left[2.10^{12}c_1 + 2.10^{12}c_2\right] + \left[\pm 108.10^6 \frac{\partial c_m}{\partial \theta_0} + 18.10^6 \frac{\partial c_f}{\partial \theta_0}\right]V \pm 72.10^6 c_2 \frac{\partial c_m}{\partial \theta_0}V^2,$$

$$A_4 = 2.10^{12} \pm 72.10^6 \frac{\partial c_m}{\partial \theta_0}V^2,$$

the upper or lower one of each double-valued term corresponding to downward or upward current as the case may be. I have calculated from the above quantities the values of discriminants, $\Delta = A_1A_2A_3 - A_0A_3^2 - A_4A_1^2$, in five cases, namely,

(a) Downward wind,
$\frac{\partial c_m}{\partial \theta_0} = -0{\cdot}2, \quad \frac{\partial c_f}{\partial \theta_0} = 1, \quad c_1 = c_2 = 10^{-4},$

(b) Downward wind,
$\frac{\partial c_m}{\partial \theta_0} = -0{\cdot}2, \quad \frac{\partial c_f}{\partial \theta_0} = 1, \quad c_1 = c_2 = 10^{-5},$

(c) Downward wind,
$\frac{\partial c_m}{\partial \theta_0} = -0{\cdot}2, \quad \frac{\partial c_f}{\partial \theta_0} = 1, \quad c_1 = c_2 = 0,$

(d) Upward wind,
$\frac{\partial c_m}{\partial \theta_0} = -0{\cdot}1, \quad \frac{\partial c_f}{\partial \theta_0} = 1, \quad c_1 = c_2 = 10^{-4},$

(e) Upward wind,
$\frac{\partial c_m}{\partial \theta_0} = -0{\cdot}1, \quad \frac{\partial c_f}{\partial \theta_0} = 1, \quad c_1 = c_2 = 0,$

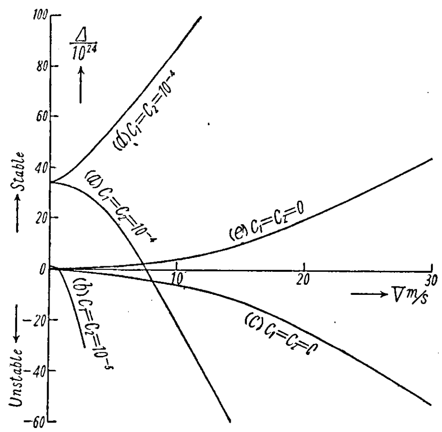

Fig. 3.

the result being plotted in Fig. 3. This figure shows that for downward current, instabilities in cases (a), (b), (c) take place respectively at wind speed $V = 7{\cdot}6$ m/s, $0{\cdot}76$ m/s, 0 m/s, while for upward current the vibrations are stable at any wind speed. It should be remarked that, unless damping or dissipation exists in the carriage, its free vibrations are unstable for any small downward wind current.

It is possible to determine accurately the equations of vibratory motion for the case without wind current[7] as well as for the one at critical wind speed, and fairly accurately for those at other wind speeds. In all cases we write

$$\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 = p \pm iq,\ p' \pm iq',$$

and put them in (6). The solutions of the free vibrations corresponding to the three kinds of downward current and under conditions $\partial c_m/\partial\theta_0 = -0{\cdot}2$, $\partial c_f/\partial\theta_0 = 1$, $c_1 = c_2 = 10^{-4}$, are

(i) $V=0$ (stable),

$$x = \alpha e^{-\cdot 0228\,t}\cos(21{\cdot}4\,t+\beta) + \gamma e^{-\cdot 00216\,t}\cos(6{\cdot}65\,t+\delta),$$

$$\theta = \alpha' e^{-\cdot 0228\,t}\cos(21{\cdot}4\,t+\beta') + \gamma' e^{-\cdot 00216\,t}\cos(6{\cdot}65\,t+\delta'),$$

(ii) $V=7{\cdot}6$ m/s (critical),

$$x = \alpha e^{-\cdot 0421\,t}\cos(21{\cdot}2\,t+\beta) + \gamma\cos(6{\cdot}65\,t+\delta),$$

$$\theta = \alpha' e^{-\cdot 0421\,t}\cos(21{\cdot}2\,t+\beta') + \gamma'\cos(6{\cdot}65\,t+\delta'),$$

(iii) $V=10$ m/s (unstable),

$$x = \alpha e^{-\cdot 0466\,t}\cos(21{\cdot}2\,t+\beta) + \gamma e^{\cdot 000686\,t}\cos(6{\cdot}65\,t+\delta),$$

$$\theta = \alpha' e^{-\cdot 0466\,t}\cos(21{\cdot}2\,t+\beta') + \gamma' e^{\cdot 000686\,t}\cos(6{\cdot}65\,t+\delta'),$$

α, β, α', β' being contants to be determined from initial conditions.

The first term of each solution belongs to principally horizontal oscillation and the second to mainly rolling osillation. The approximate periods of the two osillations are 0·29 s and 0·95 s respectively. The equations also show that, although the period of the carriage is not practically changed, the rolling oscillation will have negative damping beyond the critical speed. It is therefore now obvious that, under unstable conditions, the amplitudes of successive oscillations become larger and larger, the energy of large oscillations being continually supplied from air current.

In conclusion I wish to express my sincere thanks to Professor Nagaoka for his kind advices during the course of this investigation and also to Mr. Kanai for his valuable assistance in preparing this paper.

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7) K. SEZAWA, *Rep. Japanese Association for Applied Mechanics*, Tokyo, 1931; *Sindôgaku* (Tokyo, 1932), 357.

56. 瀬田川鐵橋に於ける車輛顚覆の振動的原因

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昭和9年9月21日の颱風による被害は數々あるが，瀬田川鐵橋に於て鐵道車輛が顚覆したのもその一例である．鐵橋上で而も烈風中での事故であるから，原因を風力に持つて行くべきは當然であるけれども，尙それを空氣力學上の靜力學的原因と空氣力學上の振動的原因とに分けることができる．靜力學的の原因卽ち飛行機の翼を持上るやうな力や壁に風があたるやうな作用も見逃し難いけれども，玆には特に振動的原因のみを考へることにした．

振動的原因には二つのものが考へられ，一つは列車を直角に通過する空氣の流れの中に生ずる週期的擾流である．これは颱風後間もなく長岡博士が其說明に用ひられた原因であつて，筆者はそれを確める爲に，既にある實驗から Reynolds 數を用ひて比較して見た所が，或風速に於けるその擾流變化の週期が車體の動搖の週期と大體一致することがわかつた．それで車體は共振的の振動によつて顚覆したとして先づ差支ないと思はれる．

次に振動力學上の興味から，空氣の流れの中に週期的の變化のない場合でも高い風速で車體が不安定な動搖をなして顚覆することがないかといふことをしらべて見た．車體に數種のばねがあり，そのばねの位置，車體の重心の位置，風壓のかゝり具合によつて車體の自由振動が種々の型式を取り，或風速に於てはその自由振動が不安定になることがあるのである．而してその不安定は普通の單一振子の不安定などと異なり，もつと高次の方程式から決定できるものであるし，且つそれが不安定となつてからでも週期的振動をなすのであるから，原位相にあることが不安定といふことなどとは大いに異なり，顚覆に最も適當した不安定となるものである．但しこのやうに自由振動をなすものが大振幅になる爲には必ず勢力の供給がなくてはならぬのであるが，現在の場合には風がその勢力を供給するわけである．果してこのやうな原因で車輛が顚覆したとすると擾流中の週期的力が殆どなくても必ずある風速で危險となるから，車體形の少し位の變化ではこの作用を免れることができぬ譯であつて，それよりも寧ろばねや重心の方を適當に變へた方が効果が多いのである．