## 31. Experiments on the Visco-elastic Properties of Pitch-like Materials. (II).

By Kumizi Iida.

### Introduction.

In the preceding paper,[1] we described and discussed the results of experiments on the visco-elastic properties of substances, such as pitch, paraffin, sandstone, and water-glass by statical methods of experiments. In discussing the results of these experiments with reference to the geophysical phenomena, namely, the folding of mountains, the chronic deformations of the earth's crust, and the propagation of seismic waves, it was considered desirable to carry out further experiments by dynamical methods to be described here.

When these substances were subjected to forces for a long period of time, though from appearances they seem to be elastic solids, they behaved mostly like a liquid of high viscosity. However we do not know as yet how these substances will behave when subjected to sudden forces for only a short period of time. In the present experiments, therefore, we imparted sudden forces for only a short period of time to these substances in order to ascertain whether or not they behave like an elastic solid. By means of the same method we examined the properties of an artificial visco-elastic substance, as will be shown later. A comparison of the properties of substances such as pitch, paraffin, and sandstone, with those of an artificial visco-elastic substance, is given later in connection with the structure of these substances.

S. Kusakabe[2] obtained his moduli of the elasticity of rocks by kinetic measurements as well as by statical measurements as also the relation between the kinetic and the static moduli. The elastic properties of rocks therefore are now understood, but their viscous properties in their solid states seem to be unknown as yet. The author intends to study the visco-elastic properties of rocks, the present experiment being the first step in that direction.

1) K. Iida, Bull. Earthq. Res. Inst., 13 (1935), 198~212.
2) S. Kusakabe, Publ. Imp. Earthq. Inv. Comm., 22 (1906~1908), 27~49.
   Proc. Phys.-Math. Soc., Japan, 2 (1903~1905), 197~206, 341~352.

## Part II.  The Dynamical Methods of Experiment.

## Experiment I.  The Relation between the Moduli of Rigidity and the Magnitude of the Torsional Couple.

**1.  Methods of Experiment.**  The apparatus constructed is as show in Fig. 1, its principal part being almost the same as that shown in Fig. 1, in the preceding paper,[3] the main difference being the contrivance for the twisting motion.  In order to impart torque to the specimen to be tested, the following device was adopted.  The moving coil C, to which the clamp $A_2$ is connected for clamping the specimen T, rotates freely between the poles P of an electromagnet that is magnetized by the coil K.  By means of the key connected to the circuit of the moving coil C, an electric current is made to flow through it for a certain length of time.  The intensity of the current is read off from the ammeter A.  The motion of the image produced by deflection of the lens mirror M is photographically recorded on a rapidly rotating drum on which a bromide paper is wound, and which is placed about one meter distant from the lens mirror M.  The elastic property of the specimen is ascertained from this record.  Another lens mirror $M_A$ is attached to a small piece of soft iron that rotates through a small angle around one of its ends. When a current flows through the coil of the small magnet, the coil of which is connected to the circuit of the moving coil C, the small piece of iron is pulled to another small magnet. The deflection of the mirror $M_A$ is photographically recordedwith that of mirror M, the length of time during which the current flowed being known.

When a current is passed through the moving coil C in the magnetic field, a torsional couple is imparted to the lower end of the specimen
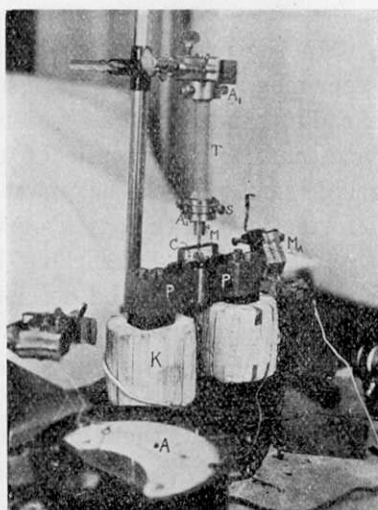


Fig. 1.  Twisting apparatus.

T, specimen;  $A_1$, $A_2$, clamps;  S, screw;  M, lens mirror;  C, moving coil;  $M_A$, lens mirror;  P, pole of magnet;  K, solenoid coil;  A, ammeter.

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3) K. IIDA, *loc. cit.* It is quoted in proper places.

T, and this twists the specimen.  In order to find the magnitude of this couple, instead of the specimen, a steel wire of known rigidity is clamped, and its twisted angle observed.  The torsional couples were calculated from this angle by using the formula $C = \dfrac{\pi r^4 n \varphi}{2l}$, where $C$ is the torsional couple, $n$ the modulus of rigidity, $l$ the length, $r$ the radius, $\varphi$ the angle of the twisted wire; the last-named angle, which is caused by the electric current, being observed by means of a telescope and scale.

The calibration of the torsional couple is carried out for the following two cases:

1).  When the current passing through the moving coil is kept constant, the intensity of the magnetic field is changed.

2).  When the intensity of the magnetic field is kept constant, the current passing through the moving coil is changed.

Next, instead of the steel wire, the specimen is clamped.  The specimen, which at first is motionless, is twisted as soon as the current is passed through the moving coil C in the magnetic field.  If the specimen possesses any elastic property at all, it will return to its initial position by virtue of its elasticity as soon as the circuit connecting to the moving coil C or the solenoid K is opened, so that by observing the elastic recovery of the specimen, we know whether or not it behaves like an elastic solid when subjected to sudden forces for only a short period of time.  The temperature is kept constant during the experiment.

**2.  Results of the Experiments.**  The actual records obtained by the apparatus are shown in Figs. 2~4.  From the phenomenon of elastic recovery of the specimens shown by these records, we know that these substances possess the property of elastic solids.  The same experiment was repeated with water-glass, the coefficient of its viscosity $\eta$, which is of the order of $10 \sim 10^3$ poises at $15° \sim 50°$C, with the lower end of the moving coil C connected to the inner cylinder of the concentric cylinder apparatus, but no phenomenon of elastic recovery could be seen (Fig. 6), whereas upon adding to it very small pieces of black rubber the phenomenon of elastic recovery became plainly evident (Fig. 5).  From this fact we may conclude that even the suspension of an elastic substance in the viscous liquid will impart elasticity to a visco-elastic substance when sudden forces are applied for a short period of time.  The interest of this investigation lies in the fact that a large number of colloidal solutions have been proved to behave in a manner closely resembling that just described.

The moduli of rigidity $n$ of pitch, paraffin, and sandstone can be calculated from the well-known formula $n = \dfrac{2lC}{\pi r^4 \varphi}$ by measuring the
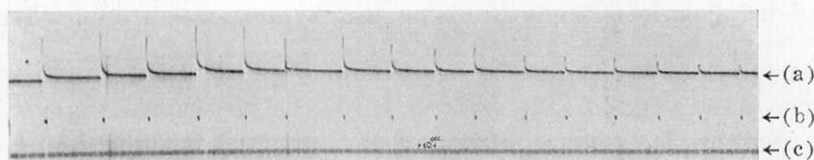


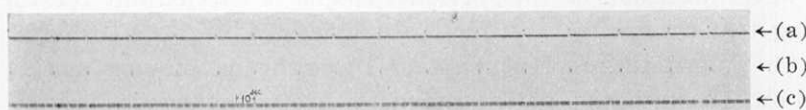Fig. 2.  Record of twisted paraffin cylinder.
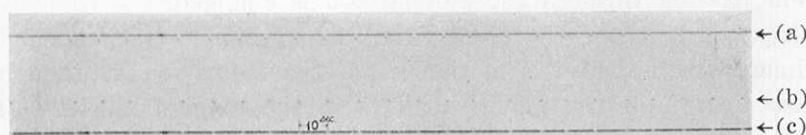


Fig. 3.  Record of twisted pitch cylinder.
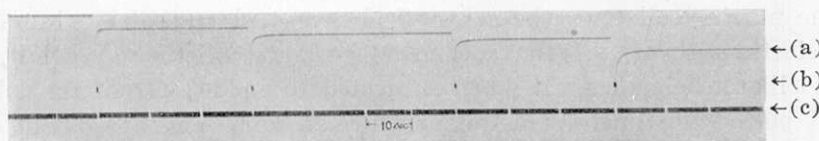


Fig. 4.  Record of twisted sandstone cylinder.



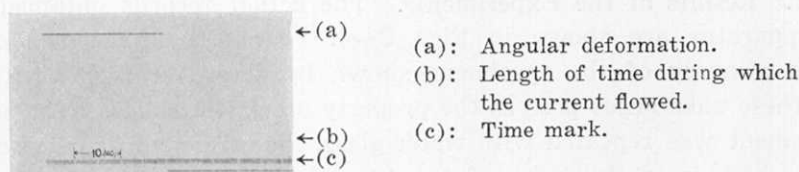Fig. 5.  Record of twisted mixture of rubber and water-glass.



(a):  Angular deformation.
(b):  Length of time during which the current flowed.
(c):  Time mark.

Fig. 6.  Record of twisted water-glass.

amount of torsion given the specimen by applying a torsional couple, the symbols having the same meaning as before.

The curves showing the relation between the moduli of rigidity of the substances thus obtained and the magnitude of the couple are shown in Figs. 7~9 and Tables I~III. In these figures the modulus of rigidity of the substances is taken as ordinate and the magnitude of the couple as abscissa. In this case the length of time for which the couple was applied was one second. From these figures it will be
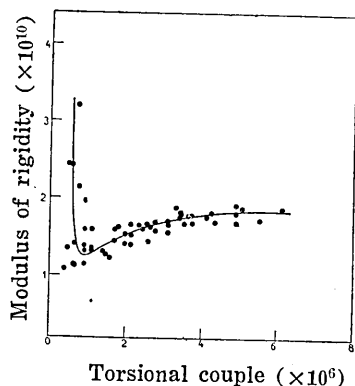
Fig. 7.   Relation between the modulus of rigidity of pitch and the torsional couple (temp. 24·8°C).
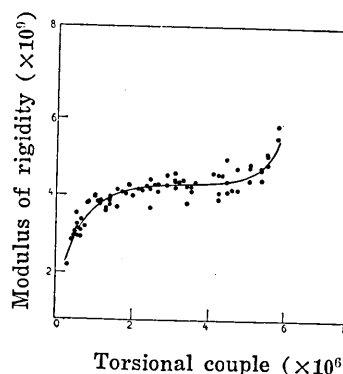


Fig. 8.   Relation between the modulus of rigidity of paraffin and the torsional couple (temp. 24·5°C).

seen that at first the moduli of rigedity $n$ of pitch and sandstone, diminishes slightly with decrease in the torsional couple $C$, the minimum being at a certain value of the couple $C$.   When the torsional couple decreases beyond this certain value of $C$, $1·0 \times 10^6$ in c. g. s. unit, the moduli of their rigidity increase rapidly.   The modulus of rigidity of paraffin, however, does not increase within this small range of the couple.
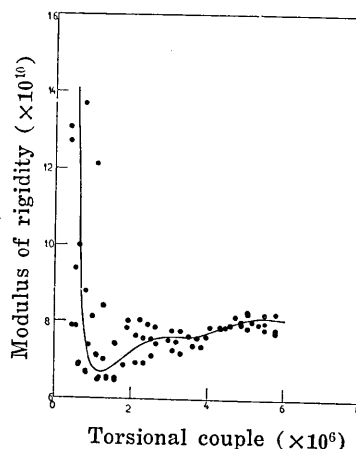


Fig. 9.   Relation between the modulus of rigidity of sandstone and the torsional couple (temp. 24·5°C).

Table I.   Modulus of Rigidity of Pitch and Magnitude of Torsional Couple.

(Diameter of pitch cylinder, $2r=2·13$ cm.   Length of pitch cylinder, $l=10·5$ cm.)

| $\varphi$ $\times 10^{-3}$(radian) | $C$ $\times 10^6$(c. g. s.) | $n$ $\times 10^{10}$(c. g. s.) | $\varphi$ $\times 10^{-3}$(radian) | $C$ $\times 10^6$(c. g. s.) | $n$ $\times 10^{10}$(c. g. s.) |
|---|---|---|---|---|---|
| 1·70 | 6·13 | 1·88 | 0·73 | 2·13 | 1·52 |
| 1·40 | 5·06 | 1·88 | 0·67 | 2·13 | 1·66 |
| 1·21 | 4·10 | 1·77 | 0·61 | 2·13 | 1·82 |
| 1·03 | 3·63 | 1·78 | 0·55 | 1·91 | 1·81 |
| 0·97 | 3·42 | 1·83 | 0·36 | 1·49 | 2·15 |
| 0·91 | 3·31 | 1·89 | 0·24 | 0·92 | 1·97 |
| 0·85 | 2·77 | 1·69 | 0·12 | 0·74 | 3·21 |
| 0·79 | 2·55 | 1·68 | | | |

Table II. Modulus of Rigidity of Paraffin and Magnitude of Torsional Couple.

$(2r=1{\cdot}83\ \text{cm}, \quad l=10{\cdot}5\ \text{cm})$

| $\varphi$ $\times 10^{-3}$(radian) | $C$ $\times 10^{6}$(c.g.s.) | $n$ $\times 10^{9}$(c.g.s.) |
|---|---|---|
| 10·02 | 5·80 | 5·52 |
| 10·80 | 5·36 | 4·74 |
| 8·56 | 4·47 | 4·98 |
| 10·32 | 4·24 | 3·93 |
| 8·19 | 3·10 | 3·61 |
| 7·89 | 2·89 | 3·49 |
| 6·31 | 2·45 | 3·70 |
| 5·65 | 2·55 | 4·30 |
| 5·22 | 1·99 | 3·64 |
| 4·25 | 1·91 | 4·28 |
| 3·95 | 1·70 | 4·10 |
| 3·64 | 1·59 | 4·17 |
| 3·52 | 1·39 | 3·78 |
| 3·34 | 1·39 | 3·97 |
| 2·73 | 1·10 | 3·84 |
| 2·43 | 1·01 | 3·97 |
| 2·06 | 0·83 | 3·83 |
| 2·00 | 0·74 | 3·53 |
| 1·82 | 0·65 | 3·40 |
| 1·52 | 0·47 | 2·95 |

Table III. Modulus of Rigidity of Sandstone and Magnitude of Torsional Couple.

$(r=0{\cdot}75\ \text{cm}, \quad l=10\ \text{cm})$

| $\varphi$ $\times 10^{-3}$(radian) | $C$ $\times 10^{6}$(c.g.s.) | $n$ $\times 10^{10}$(c.g.s.) |
|---|---|---|
| 1·23 | 5·06 | 8·26 |
| 1·17 | 4·74 | 8·15 |
| 1·17 | 4·60 | 7·92 |
| 1·11 | 4·34 | 7·86 |
| 1·05 | 4·10 | 7·86 |
| 1·05 | 3·98 | 7·62 |
| 0·99 | 3·74 | 7·57 |
| 0·93 | 3·52 | 7·62 |
| 0·86 | 3·31 | 7·76 |
| 0·80 | 3·10 | 7·78 |
| 0·72 | 2·89 | 8·05 |
| 0·68 | 2·66 | 7·88 |
| 0·62 | 2·45 | 7·94 |
| 0·56 | 2·24 | 8·04 |
| 0·56 | 2·13 | 7·64 |
| 0·49 | 1·91 | 7·84 |
| 0·43 | 1·59 | 7·44 |
| 0·37 | 1·29 | 7·02 |
| 0·31 | 1·10 | 7·14 |
| 0·25 | 1·01 | 8·15 |
| 0·25 | 0·83 | 6·67 |
| 0·19 | 0·65 | 6·87 |
| 0·12 | 0·56 | 9·39 |
| 0·06 | 0·38 | 12·73 |

The same phenomenon, however, may be seen within a smaller couple range than this. According to professor T. Terada, this phenomenon is similar to that of "Thixotropie"[4] ("Die isotherme umkerbare Sol-Gel-Umwandlung") in colloids.

4) FREUNDLICH, „Kolloid-Chemie" Bd. II, 615.
This work explains "Thixotropie" as follows; „Diese bei konstanter Temperatur verlaufende Sol-Gel-Umwandlung wird als Thixotropie bezeichnet; Sol und Gel, die ein solches Verhalten zeigen, nennt man thixotrop. Der durch Schutteln bewirkte krasse Ubergang von Gel zu Sol ist nur ein gut gekennzeichneter Grenzfall der Thixotropie."

## Experiment II.   The Relation between Rigidity and Viscosity and the Period of the Repeated Twists.

**1.   The Hysteresis Curve.**   With most substances Hooke's law does not hold even for very small stresses; for upon force being applied, whether in the form of a load or as a twist, and then removed, the after-effect (fatigue) are considerable.   Since however this is chiefly due to the inferior limits of both elasticity and visco-elasticity,[5] we need to experiment within only narrow limits of loading or twisting in order to find the stress-strain relation.   From the stress and strain produced, stress-strain diagrams could be drawn by taking the strain developed as ordinate and the stress as abscissa.

We shall now consider the mechanism involved, a subject that has occupied the attention of many investigators in the past.   The length of time that a substance has been subjected to considerable load generally affects the strain caused, and the length of time that a strained substance has been freed from load also generally affects the extent of the elastic recovery.   The latter effect, discovered by W. Weber[6] in 1835, is called "Elastische Nachwirkung" or "elastic after-working". The former effect appears to have been noticed by Vicat,[7] in 1834. After a substance has been strained by a load exceeding the limit of perfect elasticity and is then set free, the set gradually diminishes. Since the substance never returns to its original condition, the ultimate deformation is the "permanent set", while part of the strain that gradually disappears is called the "elastic after-strain." Experiments on the gradual flow of solids under great stress have been made by H. Tresca.[8]   According to these investigators we can understand that the hysteresis curve is the result of the elastic after-working.

All hysteresis phenomena show that the condition of the substance at any instant depends on its previous states as well as on the external conditions (forces, temperature, etc.) that obtain at that instant.   Hysteresis, which always implies irreversibility in the sequence of the states through which a substance passes, is generally traceable to the molecular structure of matter, with the result that theories of molecular action have been invoked to account for the viscosity and elastic

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5)   T. H. C. Thompson, "On the Theory of Visco-elasticity". *Phil. Trans. Roy. Soc., London.* [A], 231 (1933), 360.

6)   W. Weber, *Ann. Phys. Chem.*, (1835) and (1841).
A. E. H. Love, "*A Treatise on the Mathematical Theory of Elasticity*", 4th ed., 116.

7)   Vicat, *ibid,*

8)   H. Tresca, *ibid.*

after-working by various investigators, for example, J. C. Maxwell,[9] 1890; J. G. Butcher,[10] 1877; O. E. Meyer,[11] 1874. L. Boltzman,[12] 1878, showed that the ultimate condition to which a substance, subjected to constant stress for a very long period of time, must be reduced is plastic flow, which phenomenon, he discussed mathematically with Maxwell. He traced this phenomenon to plasticity and called it "hereditary hysteresis".

Hysteresis is a phenomenon that operates even within the limit of perfect elasticity. The earliest account of this phenomenon is in a paper by Ewing,[13] describing experiments with a steel wire. He showed that the effect in question, which he called "hysteresis", is more marked when the loading and unloading are rapid than when they are slow, but that it is also sensible when they are performed very slowly, so that there appears to be such a thing as "statical hysteresis". According to the studies of B. Hopkinson and G. T. Williams,[14] it appears that with very rapid alternations of stress the observable amount of hysteresis is less than in the case of statical hysteresis. A certain amount of energy seems to be dessipated in putting a specimen through a cycle of stress changes. Ewing poined out the bearing this conclusion has on Wöhler's experiments[15] relating to repeated loading and alternating stress. The subject, however, is still obscure.

Statical hysteresis was the special phenomenon noticed by Ewing, because, as has been already observed, elastic after-working, plasticity, and viscosity are also properties indicating hysteresis. The same may also be said of the property Lord Kelvin[16] calls *fatigue*.

The results of Adams and Coker,[17] and Zisman[18] show clearly that there is an increase in resistance of rock to deformation from repeated loading and unloading of forces. P. W. Bridgman[19] observed a linear hysteresis, the stress-strain diagram of which is a closed reproducible curve.

S. Kusakabe,[20] 1903, studied the torsional hysteresis of rocks and

9)  J. C. MAXWELL, *Scientific Papers*, **2** (1890).
10)  J. G. BUTCHER, *Proc. Math. Soc., London*, **8** (1877).
11)  O. E. MEYER, *J. F. Math.*, **78** (1874).
12)  L. BOLTZMAN, *Ann. Phys. Chem.*, **7** (1878).
13)  J. A. EWING, *Brit. Assoc. Rep.*, (1859), 502.
14)  B. HOPKINSON and G. T. WILLIAMS, *Proc. Roy. Soc., London*, **87** (1912), 502.
15)  A. WÖHLER, ,,*Ueber Festigkeits versuche mit Eisen und Stahl,*" Berlin, (1870).
16)  LORD KELVIN, *Math. and Phys. Papers*, **3** (1890), 22.
17)  F. D. ADAMS and E. G. COKER, *Carnegie Inst., Washington Publ.*, **46** (1906).
18)  W. A. ZISMAN, *Proc. Nat. Acad. Sci.*, **7** (1933), 653.
19)  P. W. BRIDGMAN, *Am. J. Sci.*, **7** (1924), 81 (rocks), *ibid.*, **10** (1925), 359 (tachylite).
20)  S. KUSAKABE, *Publ. Imp. Earthq. Inv. Comm.*, **14** (1903), 1~73.

discussed mathematically the hysteresis curves. The hysteresis curves obtained by his experiments were generally complicated shapes, but when the specimens were twisted about cyclically within definite limits of couple, the twist curves were nearly closed ones of simple and regular forms. T. Taniguti and M. Imai,[21] 1934, subjected concrete blocks to repeated compression and obtained hysteresis curves. Their experimental method is similar to Kusakabe's.

Upon going through the two last-named studies, it will be seen that these investigators experimented with the idea that the substances tested possess elasticity only, with the result that the length of time the substances were subjected to load seems to have been left out of consideration. Since these substances possess viscosity also, experiments with then should have been made under continuous stress with relation to time, i. e., change of stress should be shown in the form of simple harmonic motion. And since experiments with these substances differ from those relating to magnetic induction, and as their deformation under stress depends greatly on the length of time involved, experiments should take into consideration the change in stress with regard to time.

Judging from the numerous studies that have so far been made on the subject of hysteresis, there are at least two causes that result in these curves, the one being the elastic after-effect, i. e., the plastic flow of the materials, and the other the elastic hysteresis or statical hysteresis, the phenomenon of which last is exhibited by elastic substances.

In the present experiments, the author studied the phenomenon of hysteresis due to elastic after-effect, carefully noting the length of time that the substance was subjected to stress.

**2. Experiments in Hysteresis.** *The Method of Experiment.* When we repeatedly twist the substance to be tested, the strain produced will differ according to the length of time during which the twisting has been repeated, although the torsional couple is constant. We therefore twist the specimens repeatedly by applying torsional couples of various periods, and thus obtain a relation between the stress and the strain of such substances as pitch, paraffin, and sandstone. The curve showing the relation between the stress and the strain is the hysteresis curve.

The apparatus used was the same as that shown in Fig. 1, although we used, besides, a sensible galvanometer. The change in the

21) T. TANIGUTI and M. IMAI, "Hysteresis of Concrete and Reinforced Concrete (Part I)", *Bull. Tokyo Univ. Engin.*, **3** (1934), 587.

electric current — the change in stress — is known from the deflection
of the mirror attached to the galvanometer.

Since a relation between stress and
strain is required in order to obtain the
hysteresis curve, we arranged that the
axis of rotation of the moving coil C,
which is exactly the same as that des-
cribed in Experiment I, section 1, shall be
perpendicular to that of the galvanometer
G. This arrangement is shown in Fig. 10.
A lens mirror is attached to the lower end
of the specimen that is to be tested by
being twisted by the rotation of the mov-
ing coil C.   A plane mirror is attached
to the galvanometer G. The ray of light
from a slit, which at first falls on the
lens mirror, is reflected to a plane mirror,
from which it is re-reflected, the image
of the slit being formed about one meter
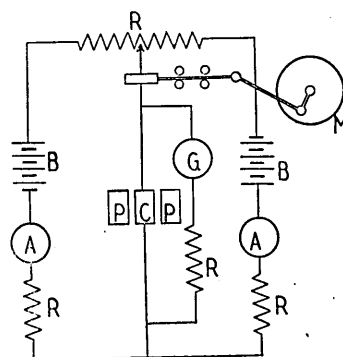distant from the lens mirror. The image



Fig. 10.  Arrangement of appara-
tus for obtaining the hysteresis
curve.

A, ammeter;  B, battery;  G,
galvanometer;  C, moving coil;
M, brass plate rotated by mo-
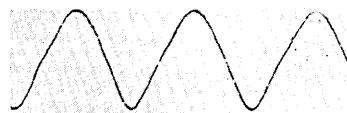tor;  P, pole of magnet;  R, re-
sistance.

of the slit thus formed is photographically recorded on bromide paper.
This record is the hysteresis curve desired. In the record, the stress
(torsional couple) is taken as ordinate and the strain (angular defor-
mation caused) as abscissa.

The galvanometer circuit being connected to that of the moving
coil C, any change in stress is given by the deflection of the plane
mirror that is attached to the galvanometer.

By the movement of the piston attached to the plate M that moves
with the motor, the electric resistance R is enabled to change into one
of sine-form, with the result that the curve of the torsional couple
imparted to the lower end of the specimen alters to one of sine-form
since the change in the torsional couple is proportional to that of the
current passing through the moving coil
C.  For this reason the specimen is twist-
ed repeatedly.  The time variation in the
electric current is shown in Fig. 11.  The
resistence R connected to the ammeter A
is of 200 ohms.  The electromotive force
of the two B batteries are each 100 volts.



Fig. 11.  Variation in magnitude
of torsional couple of sine-form.

The resistance R, which varies as a simple harmonic as the result of
the piston attached to the plate M, is 200 ohms.

**3. Results of the Experiment.** Examples of successive forms of hysteresis curves obtained by repeated twists are shown in Figs. 12~ 14. Some actual photographs are shown in Figs. 15~17. In the photographs, the stress (torsional couple) is taken as ordinate and the strain (angular deformation of the specimen) produced as abscissa. As shown in Table IV, the length of time for which the twistings were repeated ranges from a few seconds to about 12 minutes.

It will be seen that although such substances as pitch, paraffin, and sandstone, were twisted two or three times, all their hysteresis curves, which are ellipses, close regularly. These curves are almost alike, but with increase in the period of the repeated twists, the major
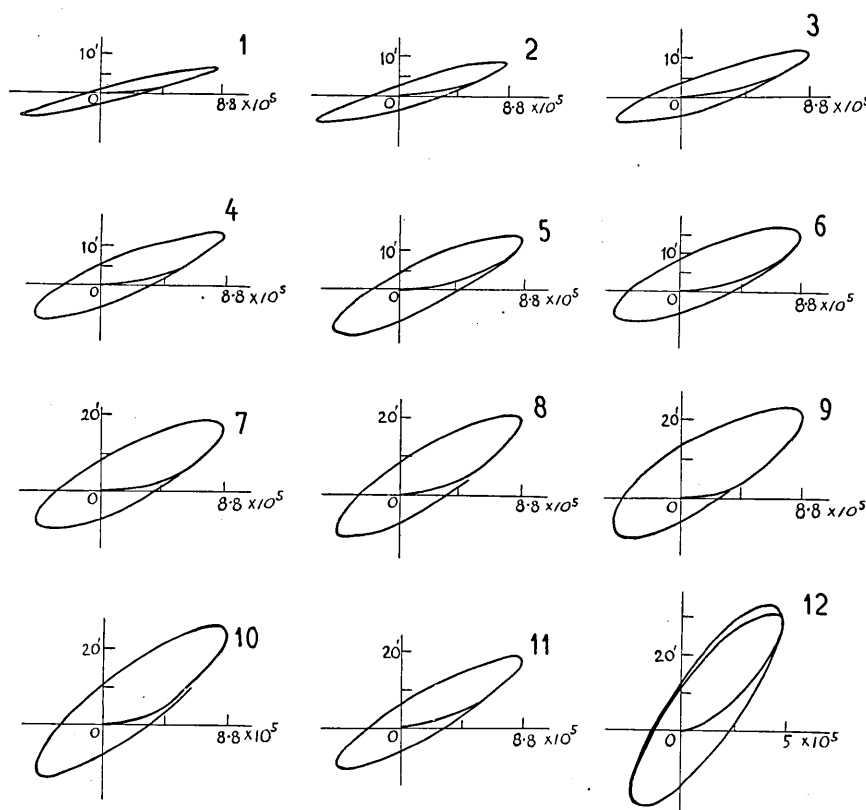


Fig. 12. Hysteresis curves of paraffin (24·8°C).
Ordinate, angular deformation; abscissa, torsional couple.
Diameter of paraffin cylinder=2·13 cm, length=10·5 cm.

(1) Period  T=9 sec.  (2) T=44 sec.  (3) T=1 m. 23 sec.  (4) T=3 m.
(5) T=4 m.  (6) T=5 m. 35 sec.  (7) T=6 m. 50 sec.  (8) T=8 m. 5 sec.
(8) T=9 m. 20 sec.  (10) T=11 m. 25 sec.  (11) T=2 m. 55 sec.
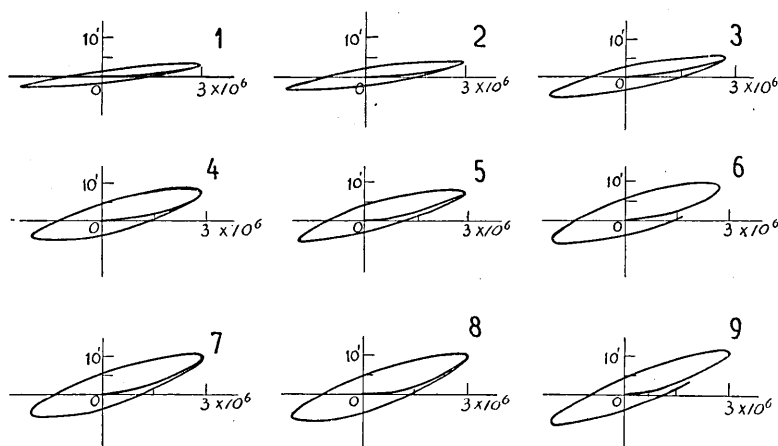(12) T=1 m. 23 sec. (Diameter of cylinder=1·83 cm).

Fig. 13.  Hysteresis curves of pitch (24·8°C).
Ordinate, angular deformation; abscissa, torsional couple.
Diameter of pitch cylinder=2·13 cm, length=10·5 cm.

(1)  T=30 sec.  (2)  T=50 sec.  (3)  T=1 m. 23 sec.  (4)  T=4 m.  (5)  T=5 m. 45 sec.
(6)  T=6 m. 50 sec.  (7)  T=8 m.  (8)  T=9 m. 20 sec.  (9)  T=12 m. 30 sec.
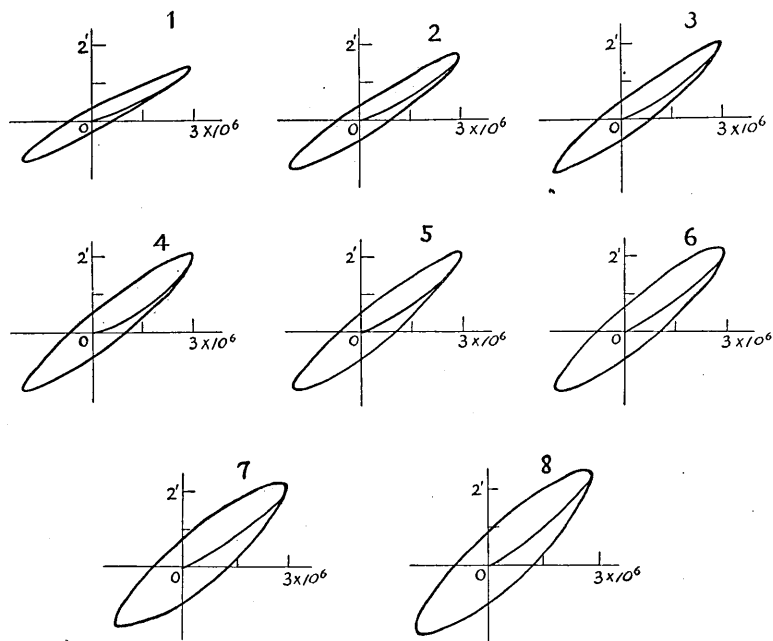


Fig. 14.  Hysteresis curves of sandstone (25°C).
Ordinate, angular deformation; abscissa, torsional couple.
Diameter of sandstone cylinder=1·5 cm, length=10·0 cm.

(1)  T=1 m. 23 sec.  (2)  T=2 m. 10 sec.  (3)  T=2 m. 48 sec.  (4)  T=4 m. 10 sec.
(5)  T=5 m. 20 sec.  (6)  T=6 m. 50 sec.  (7)  T=8 m. 35 sec.  (8)  T=11 m. 45 sec.

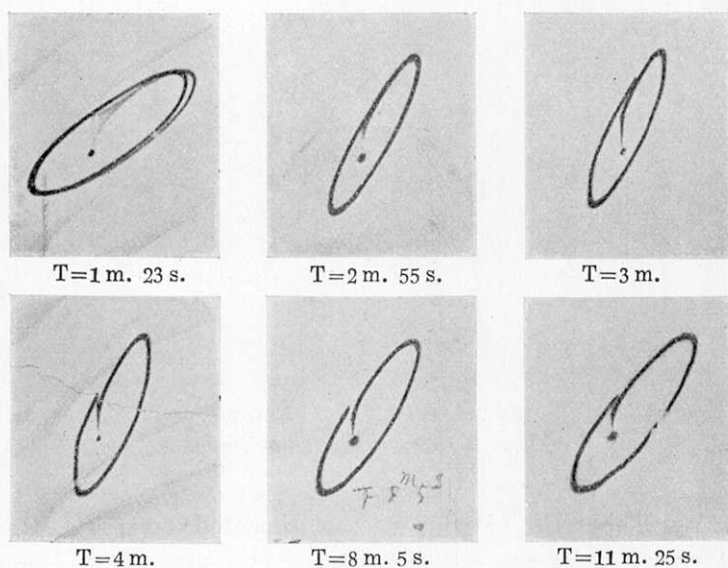T=1 m. 23 s.　　　　T=2 m. 55 s.　　　　T=3 m.

T=4 m.　　　　T=8 m. 5 s.　　　　T=11 m. 25 s.

Fig. 15.　Hysteresis curves of paraffin.



T=1 m. 23 s.　　　　T=3 m.　　　　T=4 m.

T=6 m. 50 s.　　　　T=8 m.　　　　T=9 m. 20 s.

Fig. 16.　Hysteresis curves of pitch.

T=2 m. 10 s.　　　T=2 m. 48 s.　　　T=6 m. 50 s.　　　T=8 m. 35 s.
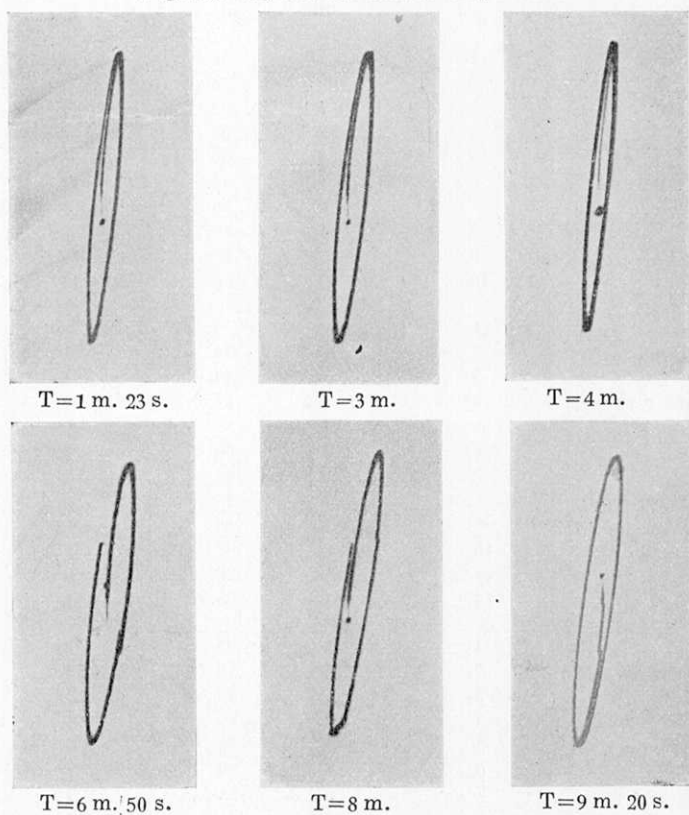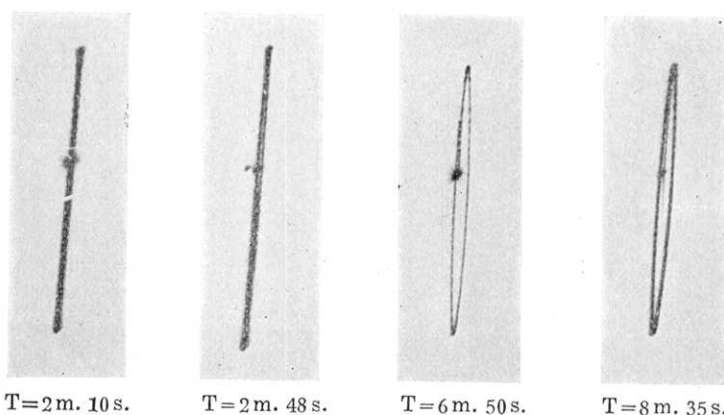
Fig. 17.　Hysteresis curves of sandstone.

## Table IV.　Period of the Repeated Twists.

| Substance (dimension) | No. | Period m. | Period s. | Total time m. | Total time s. |
|---|---|---|---|---|---|
| Pitch  Diameter=2·13 cm.  Length=10·5 cm. | 1 |  | 30 | 1 | 20 |
|  | 2 |  | 50 | 1 | 30 |
|  | 3 | 1 | 23 | 7 | 05 |
|  | 4 | 3 | 00 | 6 | 11 |
|  | 5 | 4 | 00 | 8 | 34 |
|  | 6 | 5 | 45 | 11 | 38 |
|  | 7 | 6 | 50 | 7 | 20 |
|  | 8 | 8 | 00 | 8 | 40 |
|  | 9 | 9 | 20 | 12 | 02 |
|  | 10 | 12 | 30 | 14 | 00 |
| Paraffin  Diameter=2·13 cm.  Length=10·5 cm.  *Diameter=1·83 cm.  Length=10·5 cm. | 1 |  | 9 |  | 20 |
|  | 2 |  | 35 | 1 | 29 |
|  | 3 |  | 44 | 1 | 30 |
|  | 4* | 1 | 23 | 4 | 25 |
|  | 5 | 2 | 55 | 9 | 06 |
|  | 6 | 3 | 00 | 3 | 30 |
|  | 7 | 4 | 00 | 4 | 30 |
|  | 8 | 5 | 35 | 5 | 56 |
|  | 9 | 6 | 50 | 7 | 15 |
|  | 10 | 8 | 05 | 8 | 19 |
|  | 11 | 9 | 20 | 9 | 54 |
|  | 12 | 11 | 25 | 11 | 55 |
| Sandstone  Diameter=1·50 cm.  Length=10·0 cm. | 1 | 1 | 23 | 2 | 00 |
|  | 2 | 2 | 10 | 4 | 24 |
|  | 3 | 2 | 48 | 3 | 17 |
|  | 4 | 4 | 10 | 4 | 10 |
|  | 5 | 5 | 20 | 6 | 12 |
|  | 6 | 6 | 50 | 7 | 33 |
|  | 7 | 8 | 35 | 9 | 10 |
|  | 8 | 11 | 45 | 12 | 26 |

axes of the ellipses tend to become vertical (Figs. 12, 13, 14.). This has a direct relation to the fact that the moduli of rigidity of the substances diminish as the period lengthens. The relation between their moduli of rigidity or viscosity and the periods of the repeated twists may be found by analysing these curves. These analytical results will be dealt with in a later section.

**4. Mathematical Treatment of the Hysteresis Curve.** We shall now take up the mathematical theory in connection with the hysteresis curve due to repeated twisting. In this case, variation in the magnitude of the torsional couple is of the simple-harmonic type, so that external force applied to the specimen is expressed by the form of $C \sin pt$, in which $C$ and $2\pi/p$ is respectively the maximum value of the torsional couple and the period of the repeated twists. The equation of motion of this torsional system at any time $t$, therefore, may be written

$$\varepsilon \frac{d\theta}{dt} + k\theta = C \sin pt, \tag{1}$$

or

$$\varepsilon \frac{d\theta}{dt} + \theta\left(k_1 + k_2 \frac{d\theta}{dt}\right) = C \sin pt, \tag{2}$$

where $\theta$ is the angular deformation of the substance, $\varepsilon$ a constant that is proportional to the viscosity of the substance, $k$ an elastic constant that is the torsional moment of the substance per unit angle twisted. $k_1$ and $k_2$ have the same meaning as $k$. We shall now solve these equations. If the value of $k_2$ is negligible compared with that of $k_1$, equation (2) is equivalent to (1). To simplify matters we shall therefore take equation (1).

The solution of equation (1) under the condition $\theta = 0$ at $t = 0$ is given by

$$\theta = \frac{C}{k^2 + \varepsilon^2 p^2}\left\{\varepsilon p e^{-\frac{k}{\varepsilon}t} + k \sin pt - \varepsilon p \cos pt\right\}. \tag{3}$$

Let us consider the following three cases which can arise from this equation. Using rectangular coordinates, we take the torsional couple as abscissa and the angular deformation of the substance caused as ordinate, i. e.,

$$y = \theta, \qquad x = C \sin pt. \tag{4}$$

(1) When $k \gtrless 0$, $\varepsilon = 0$. The substance possesses no viscosity; only elasticity. Substituting (4) in (3), we get

$$y = \frac{1}{k}x. \tag{5}$$

This equation represents *a straight line through the origin.*

(2) When $k=0$, $\epsilon \gtrless 0$. The substance possesses no elasticity; only viscosity. Substituting (4) in (3), we get the equation

$$y = \frac{1}{\epsilon p}\left\{ C - \sqrt{C^2 - x^2} \right\},$$

or

$$\frac{x^2}{C^2} + \frac{\left(y - \dfrac{C}{\epsilon p}\right)^2}{\left(\dfrac{C}{\epsilon p}\right)^2} = 1. \tag{6}$$

This equation represents *an ellipse whose center is at the point* $\left(0, \dfrac{C}{\epsilon p}\right)$.

(3) When $k \gtrless 0$, $\epsilon \gtrless 0$. The substance possesses both elasticity and viscosity. Substituting (4) in (3) as before, we get

$$y = \frac{1}{k^2 + \epsilon^2 p^2}\left\{ C\epsilon p e^{-\frac{k}{\epsilon}t} + kx - \epsilon p\sqrt{C^2 - x^2} \right\}, \tag{7}$$

in which $e^{-\frac{k}{\epsilon}t}$ tends to become zero for large values of $t$.

Hence, after a long period of time, equation (7) is reduced to the general form

$$(k^2 + \epsilon^2 p^2)x^2 - 2k(k^2 + \epsilon^2 p^2)xy + (k^2 + \epsilon^2 p^2)^2 y^2 - \epsilon^2 p^2 C^2 = 0. \tag{8}$$

If we put

$$\left.\begin{array}{ll} k^2 + \epsilon^2 p^2 = a, & -k(k^2 + \epsilon^2 p^2) = h \\ (k^2 + \epsilon^2 p^2)^2 = b, & -\epsilon^2 p^2 C^2 = C' \end{array}\right\}, \tag{9}$$

equation (7) becomes

$$ax^2 + 2hxy + by^2 + C' = 0. \tag{10}$$

We shall now determine the curve represented by equation (10), for which purpose the following calculation is necessary:

$$\Delta \equiv \begin{vmatrix} a & h & 0 \\ h & b & 0 \\ 0 & 0 & C' \end{vmatrix} = C'(ab - h^2) = C'a^2 p^2 \epsilon^2. \tag{11}$$

Since $C'$ is negative, $a\Delta$ is negative, and $ab - h^2$ is positive. Equation (10) represents, therefore, *an ellipse whose center is at the coordinate origin.*

In order to obtain a simple form of equation (10), we transfer the rectangular axes $(x, y)$ to $(X, Y)$ making an angle $\theta'$ with the old. The relations between the old and the new axes are given by

$$x = X\cos\theta' - Y\sin\theta' \atop y = X\sin\theta' + Y\cos\theta' \Big\} . \tag{12}$$

Substituting (12) in (10) as well as the relation

$$\tan 2\theta' = \frac{2h}{a-b} , \tag{13}$$

equation (10) is reduced to the simple form

$$a'X^2 + b'Y^2 + C' = 0,$$

or

$$\frac{X^2}{-\dfrac{C'}{a'}} + \frac{Y^2}{-\dfrac{C'}{b'}} = 1, \tag{14}$$

where $a'$ and $b'$ are the roots of the equation

$$z^2 - (a+b)z + (ab - h^2) = 0, \tag{15}$$

in which we have to take $h$, $a'$, and $b'$ such that $a' - b'$ is positive or negative according as $h$ is positive or negative. Hence, by determining the values of $a'$, $b'$, and $C'$, the values of the two constants ($k$ and $\varepsilon$) can be obtained.

Again, since equation (13) is given by

$$\tan 2\theta' = \frac{2h}{a-b}$$

$$= \frac{2k}{k^2 + \varepsilon^2 p^2 - 1} , \tag{16}$$

we can see that $\theta'$ becomes large when the values of $\varepsilon p$ and $k$ are small, and also that the major axis of the ellipse tends to become vertical as the period of the twists increases. Consequently the rigidity of the substance diminishes as the period of the twists is increased when $\theta'$ is large.

As a matter of fact $k$ depends also on the velocity of the angular deformation of the substance, so that the term $k_2$ in equation (2) cannot be neglected. The following two cases refered to in the preceding section may, however, be explained without difficulty by the mathematical treatment just given.

(1) The hysteresis curve is an ellipse.

(2) The major axis of the ellipse tends to become vertical as the period of the twist is increased.

The fact that both equation (1) and (2) clearly show how these hysteresis curves are formed will serve as a clue in problems connected with the structure of matter.

**5. Analytical Results of the Experiment.** Since equation (10) represents an ellipse, as just stated, we use this equation for determining

the properties of the substances from their hysteresis curves given in section 3.

We measure the length of the major and minor axis of each curve and the angle $\theta'$ showing the inclination of the major axis of the ellipse, and insert these values in equations (9), (14), (15) and (16). We can thus find the values of the two constants, $k$ and $\epsilon$. Using the well-known relation, $n = \dfrac{2lk}{\pi r^4}$, we determine the moduli of rigidity $n$

of the substances. As the area A of the hysteresis curve represents the loss of internal energy of the substance, we measure it with a planimeter. Area A being a measure of the viscosity of the substance, it is possible to determine the relation the change of viscosity has to the period T of the repeated twists. We obtain then the following three relations:

(a). *Relation Between n and T.* From the data just obtained, a diagram is constructed on which the values of $n$ are taken as ordinates and the corresponding values of T as abscissae. Figs. 18~20 are diagrams
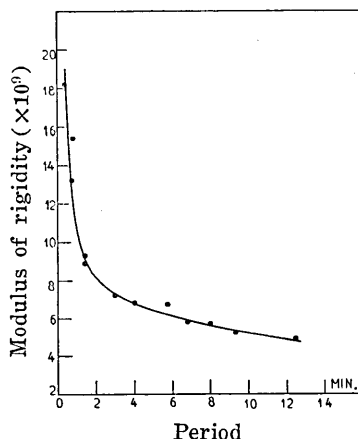


Fig. 18.  Relation between the modulus of rigidity of pitch and the period of the repeated twists.
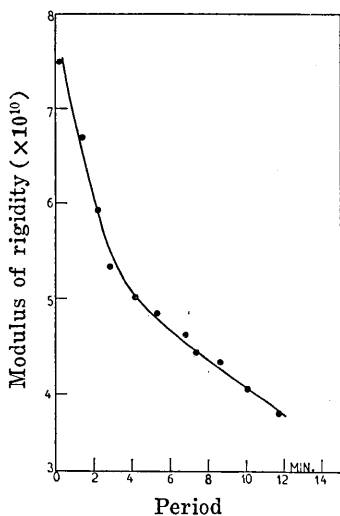


Fig. 19.  Relation between the modulus of rigidity of sandstone and the period of the repeated twists.
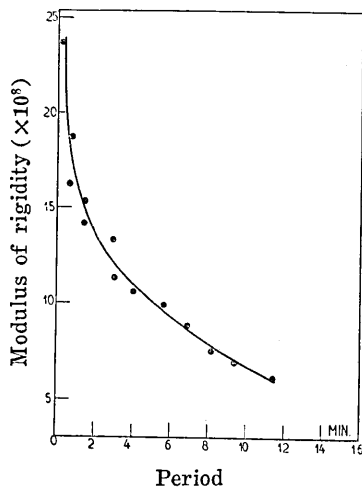


Fig. 20.  Relation between the modulus of rigidity of paraffin and the period of the repeated twists.

showing these relations. The values of $n$ are shown in Table V. From these figures we can see that the moduli of rigidity of all these substances decrease with increase in the period of the repeated twists, though the manner in which they decrease differs according to the particular substance.

(b). *Relation Between the Apparent Viscosity and the Period T.*

We take the period T of the repeated twists as abscissa and the apparent viscosity as ordinate. The relations between the two thus obtained are shown in Figs. 21~23 and Table V. It will be seen that the apparent viscosity of all these substances increases somewhat rapidly with the increasing period T.

(c). *Relation Between Loss of Internal Energy and Angular Deformation 0 of the Substance.* We take the loss of internal energy (the area of the hysteresis curve) of the sub-
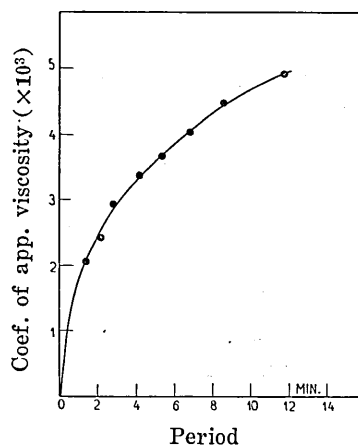


Fig. 21. Relation between the coefficient of apparent viscosity of sandstone and the period of the repeated twists.
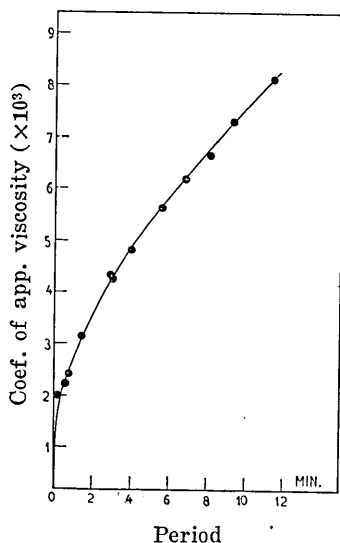


Fig. 22. Relation between the coefficient of apparent viscosity of paraffin and the period of the repeated twists.
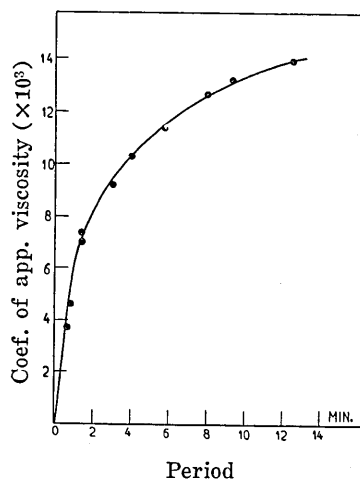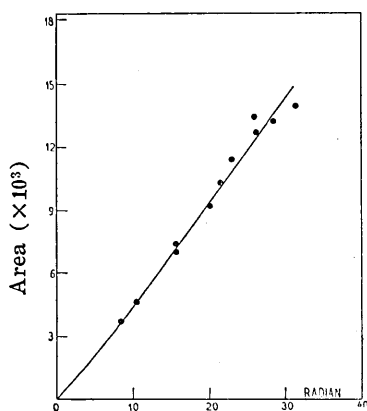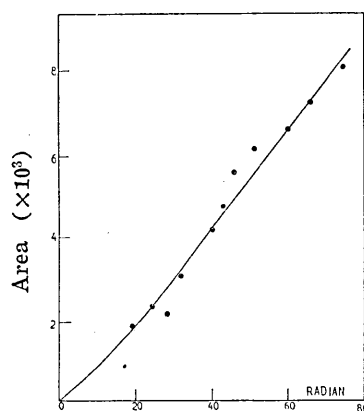


Fig. 23. Relation between the coefficient of apparent viscosity of pitch and the period of the repeated twists.

stance as ordinate and the angular deformation as abscissa, as shown
in Figs. 24～26.    From these figures it will be seen that in the case



Angular deformation $\theta$ ($\times 10^{-4}$)

Fig. 24.  Relation  between  the
  strain and the internal energy
  loss of pitch.



Angular deformation $\theta$ ($\times 10^{-4}$)

Fig. 25.  Relation  between  the
  strain and the internal energy
  loss of paraffin.
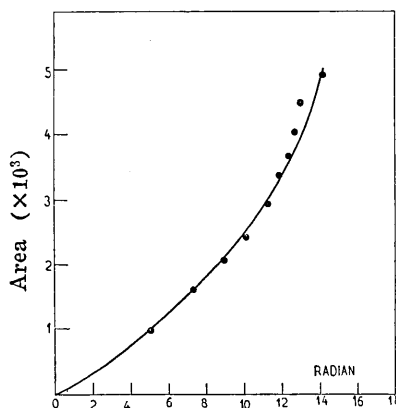


Angular deformation $\theta$ ($\times 10^{-4}$)

Fig. 26.   Relation  between  the
  strain and the internal energy
  loss of sandstone.

of pitch and paraffin, loss of their internal energies increases linearly
with their increasing angular deformations (strain), while in the case
of sandstone these changes seem to take place somewhat rapidly.

Table V.  Modulus of Rigidity and Apparent Viscosity
of Pitch, Paraffin, and Sandstone.

| Substance | Period m.  s. | | $n$ (c. g. s.) | $\varepsilon$ (c. g. s.) $\times 10^3$ | Couple(max.) (c. g. s.) | $\theta$ (max.) radian $\times 10^{-3}$ |
|---|---|---|---|---|---|---|
| Pitch | | 40 | $18\cdot24 \times 10^9$ | 3·68 | $2\cdot98 \times 10^6$ | 0·85 |
| | | 50 | 15·43 $''$ | 4·60 | $''$ | 1·05 |
| | 1 | 23 | 8·88 $''$ | 7·37 | 2·68 $''$ | 1·57 |
| | 3 | 00 | 7·22 $''$ | 9·21 | 2·80 $''$ | 2·01 |
| | 4 | 00 | 6·81 $''$ | 10·32 | $''$ | 2·15 |
| | 5 | 45 | 6·76 $''$ | 11·42 | $''$ | 2·30 |
| | 6 | 50 | 5·82 $''$ | 13·45 | $''$ | 2·59 |
| | 8 | 00 | 5·72 $''$ | 12·71 | 2·89 $''$ | 2·62 |
| | 9 | 20 | 5·24 $''$ | 13·26 | $''$ | 2·85 |
| | 12 | 30 | 4·94 $''$ | 14·00 | 2·98 $''$ | 3·14 |
| Paraffin | | 9 | $2\cdot37 \times 10^9$ | 1·82 | $4\cdot87 \times 10^5$ | 1·92 |
| | | 35 | 1·87 $''$ | 2·11 | $''$ | 2·82 |
| | | 44 | 1·62 $''$ | 2·30 | $''$ | 2·44 |
| | 1 | 23 | 1·42 $''$ | 3·07 | $''$ | 3·20 |
| | 2 | 55 | 1·31 $''$ | 4·32 | $''$ | 5·47 |
| | 3 | 00 | 1·13 $''$ | 4·22 | $''$ | 4·01 |
| | 4 | 00 | 1·06 $''$ | 4·80 | $''$ | 4·30 |
| | 5 | 35 | $9\cdot92 \times 10^8$ | 5·66 | $''$ | 4·59 |
| | 6 | 50 | 8·83 $''$ | 6·23 | $''$ | 5·15 |
| | 8 | 05 | 7·53 $''$ | 6·71 | $''$ | 6·02 |
| | 9 | 20 | 6·91 $''$ | 7·39 | $''$ | 6·60 |
| | 11 | 25 | 6·13 $''$ | 8·25 | $''$ | 7·45 |
| Sandstone | 1 | 23 | $6\cdot70 \times 10^{10}$ | 2·06 | $2\cdot98 \times 10^6$ | 0·89 |
| | 2 | 10 | 5·92 $''$ | 2·43 | $''$ | 1·01 |
| | 2 | 48 | 5·33 $''$ | 2·94 | $''$ | 1·12 |
| | 4 | 10 | 5·07 $''$ | 3·38 | $''$ | 1·18 |
| | 5 | 20 | 4·85 $''$ | 3·68 | $''$ | 1·23 |
| | 6 | 50 | 4·73 $''$ | 4·05 | $''$ | 1·27 |
| | 8 | 35 | 4·34 $''$ | 4·49 | 2·80 $''$ | 1·30 |
| | 11 | 45 | 3·81 $''$ | 4·93 | 2·68 $''$ | 1·41 |

## The Structure of the Substance.

As already described in the preceding paper and in this one, substances, such as pitch, paraffin, and sandstone, behave as if they were a liquid of high viscosity or as an elastic solid according to the length of time that they have been subjected to forces.  There seems to be

little doubt that these substances are formed of matter possessing elasticity and another of viscosity. They may be regarded as mixtures of elastic and viscous matter. Our experiments show that when these substances are subjected to forces long applied, the viscous constituent alone is affected and exhibits the property of viscosity, whereas when they are subjected to forces of sudden short duration, the elastic component alone reacts to the force applied and exhibits elastic properties. In the former case, the substance behaves as if it had no elastic component and in the latter as if it had no viscous constituent. For example, according to the experiment with the mixture described in section 2, Experiment I, a mixture of elastic and viscous substances, such as small pieces of rubber and water-glass, behaves as if it were an elastic substance (rubber) when it is subjected to sudden forces for a short period of time. To forces long applied, the mixture behaves of course as if it were a viscous liquid (water-glass).

That these substances show dual properties of viscosity and elasticity may well be explained by imagining the following two alternative models of their structure. The one assumes that the structure of these substances is somewhat similar to that of a sponge, the cavities of the elastic skeleton of which are filled with viscous liquid, as already stated in the preceding paper, while the other is that the structure of these substances is similar to that in which an elastic substance is suspended in a viscous liquid.

The former conception is useful in explaining the properties of pitch, paraffin, and sandstone; for these substances, when subjected to forces for a long period of time, exhibit the properties of an elastic solid as well as those of a viscous liquid, while the latter is useful in explaining the properties of certain kinds of pitch, since they exhibit merely the properties of a viscous liquid when subjected to forces for a long period of time. Substances having both structures exhibit of course the properties of an elastic solid when subjected to sudden forces for only a short period of time.

If the material composing the earth's crust were assumed to have mechanical properties similar to substances having the structures just described, the earth's crust would behave like a liquid of high viscosity when subjected to forces for a long period of time and like an elastic solid when subjected to sudden forces for only a short period of time, so that mountain folding, chronic deformations of the earth's crust, and propagation of seismic waves could be explained without difficulty.

## Summary and Conclusion to Part II.

(1). In the foregoing experiments, the elastic behaviour of substances, such as pitch, paraffin, and sandstone were investigated.

(2). The moduli of rigidity of substances ascertained by the dynamical method are larger than those obtained by the statical method. The moduli of rigidity of substances obtained by the dynamical method alter with variation in the magnitude of the torsional couple.

(3). The stress-strain curve of these substances for repeated twists is an ellipse, the shape of which however varies according to the period of the repeated twists; the longer the period of the repeated twists, the more the major axis of the ellipse tends to become vertical.

(4). The stress-strain curve may be expressed by solving the equation

$$\epsilon \frac{d\theta}{dt} + k\theta = C \sin pt,$$

where the symbols in the equation have the same meaning as before. By means of this equation, the viscous and the elastic characteristics of substances can be known separately.

(5). The moduli of rigidity of these substances decrease with increase in the period of the repeated twists, while their viscosity increases with increase in the period of the repeated twists. The elastic after-effect and the hysteresis are very marked.

(6). That these substances show dual properties of viscosity and elasticity may well be explained by imagining the following two alternative models of their structure; the one being that the structure of these substances is somewhat similar to that of a sponge, the cavities of the elastic skeleton of which are filled with a viscous liquid, as already described in the preceding paper and the other being that the structure of these substances is similar to that in which an elastic substance is suspended in a viscous liquid.

(7). If the material composing the earth's crust were assumed to have properties similar to the substances investigated in this paper, the folding of mountains, the chronic deformations of the earth's crust, and the propagation of seismic waves could be easily explained.

## 31. 物質の粘彈性性質に關する實驗(II)

飯　田　汲　事

　靜力學的方法の實驗によつて得られた，ピッチ，パラフイン，砂岩等の物質の粘彈性性質に關しては，既に第1報さして此の彙報の前號に報告してある．その實驗の結果を地球物理學的現象に應用する場合，山脈の褶曲及びこれに關聯した地殼の漫性變動等は容易に説明し得られるが，これらの物質が短時間の間比較的急激に作用する力に對して如何なる性質を示すかに關する實驗的調査の結果でなければ，地震波傳播等の急激な力の作用によつて起る現象の説明には不十分である．こゝに述べた實驗はこの方面の自然現象を對照さして行はれたものであるさ共に，物質の粘性及び彈性の二性質を考慮に入れて物質の履歴現象を研究したのである．

　物質の履歴現象に關する研究の中最も重要さ考へられる事は，凡そ物質は粘性及び彈性の二性質を有する故物體に歪力が加へられた場合には，歪力の時間的變化に左右される事が顯著である．從つてこの歪力の時間に對する變化を定義して研究さるべき事である．

　實驗に關して得られた主な結果を擧げるさ次の如くである．

　(1) ピッチ，パラフイン，砂岩等の物質は短時間急激に作用する力に對しては，剛性ある固體の性質のみが有力に認められる．

　これらの物質の剛性率は振力の函數さして變化し，ある振力の範圍內ではその剛性率には左程の變化がないが，振力がある値より小さくなるさ急激に增加する．これらの物質の剛性率は靜力學的に得られたものに比してその値が比較的大きく求められる．又これらの物質はコロイドの性質に似た所が多い．

　(2) これらの物質の履歴曲線は殆んざ橢圓形をなしてゐて，加へられた正弦的に變化する振力の周期が比較的小であるさ橢圓の長軸は小なる傾斜をなし，周期が長くなるさ橢圓の長軸が直立してくる．この事は物質の彈性がその加へられる振力の周期の長短によつて著しく變化し，周期の短い場合には比較的大きな彈性を有するものである．又粘性は之に反し周期が長くなるにつれて增加する傾向がある．

　これらの物質の履歴曲線は次の方程式をさく事によつて容易に得られる．

$$\varepsilon \frac{d\theta}{dt} + k\theta = C \sin pt .$$

　この式から粘性及び彈性の二性質が別々に求められる．

　(3) 物質の構造は，前號に述べた如く海綿構造の外に，粘性物質の中に彈性體が浮游してゐるやうな "suspension" の構造も考へられる．地殼がこれらの物質さ同じ性質を有する物質から成立してゐるならば，緩漫に變化する力に對しては全く粘性液體の如く流動して褶曲現象を示すに反し，地震波の如き急激な力の作用の下に生じた波動に對しては全く彈性體さして地震波をも傳播せしめ得らるのである．

　岩石の性質に關しては次の機會に述べる考へである．