5. *Approximate Determination of the Spheroid most nearly Representing the Surface of Japan.**

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I. Theory, observation and measurement agree in showing that the figure of the earth is very closely represented by an ellipsoid of revolution about the axis of the earth. Strictly speaking, however, that is not so; the visible irregularities of the physical surface, variations of density of the materials composing the crust, irregular perturbative forces arising from the motion of the planets, etc., all these superinduce on the ellipsoidal form undulations that cannot be expressed by any mathematical formulae. The result of both extensive geodetical operations and astronomical observations enable us however to determine a spheroid to which the mathematical surfaces of the earth may be very conveniently referred, the deviations from which are very small. We shall call this ellipsoid of reference R, and the actual mathematical surface of the earth M. Our problem is to determine the surfaces R and M which satisfy the condition that surface R shall most closely represent the surface M of Japan.

As will be seen from Table I, a number of astronomical observations have been made by the officials of the Japanese Geodetic Commission, the Land Survey Department, the Hydrographic Department, and others. The accuracy of these observations generally differs according to the purposes of their observations: the Geodetic Commission to find the deviation of the plumb line, the Land Survey Department to compute the new base line in the course of their geodetic work, the Hydrographic Department in their cartographic views.

The deviations of the plumb line are expressed by the well known formulae

$$\xi = B_g - B_a, \tag{1}$$

$$\eta = (L_g - L_a) \cos B_g, \tag{2}$$

$$\eta = (A_g - A_a) \cot B_g, \tag{3}$$

where B, L and A are latitude, longitude and azimuth, respectively, reckoned in the usual manner; a and g the respective astronomical and

* Communicated by T. TERADA.

geodetic indices. These quantities are computed in the last column of Table I.

Although the deviation of the plumb line at any given station is affected by local attractions of the surrounding mass, as is evident from conditions in the neighbourhood of Tanzawayama and Mt. Aso, they are still too large and too systematic to be interpreted as arising solely from these local topographic deflections. They occurred in all probability from errors with respect to the situation, shape, and dimension of the reference ellipsoid we have taken in the geodetic work in Japan.

A short time ago, Mr. K. Atumi,[1] after examining the deviations of plumb lines at 68 astronomical stations, determined what he considered the most suitable reference ellipsoid and geoid undulations in Japan. Although, to simplify the calculation of the coefficients in his equations of condition, he assumed the earth to be a sphere, those coefficients differ more and more from the values exactly based on the ellipsoid of revolution with increase of distance. In particular, it will be apparent from the later part of this paper that the coefficients of da and de^2, the corrections that must be applied to the semi-major axis and the eccentricity of the meridian ellipse, increase in proportion to the distance, with the result that these coefficients differ widely. For this reason his computations should be regarded as determinations from these erroneous values alone. Since, moreover, he has assigned an equal weight to all the observations, not taking into account their accuracy, the results of his computations do not appear to be very accurate.

In the following, in the first place I have computed the coefficients exactly on the ellipsoid of rotation, and in the second place I took into consideration the different weights in making out the conditional equations, although roughly, taking into account Laplace's condition which must exist in the components of the deviation of the plumb line as deduced from the longitudinal and azimuthal observations.

II. Geodetic Data.

Before entering into details it is necessary to explain how the geodetic data in this report were obtained.

Geodetic work in Japan, since 1870, has been done by the Land Survey Department. The instrument used in measuring the horizontal angles of the primary triangulation is the theodolite made by Carl Bamberg, the diameter of the horizontal circle of which is 27 cm, the circle being divided into five minutes and read by two microscopes. The instrument rests on a pedestal provided with three levelling screws. One

1) Captaine du génie K. Atumi, "La déviation de la varticale au Japon," *Jap. Jour. Astr. Geophys.*, **10** (1933), 305.

Table I

No.	Station	Height	B_a, L_a, A_a	B_g L_g A_g	Epoch	Observer	Direction of azimuth
1	Handa-sawa		49°59′60″·28±0″·17 142 46 — —	50″·89 16·18 —	1930	Author	—
2	Higasi-yama		49 59 46·90±0·18 143 7 57·30±0·68 269 11 4·29	44·61 78·79 13·96	1930	Author	Aimidake
3	Syana		45 13 46·1 147 52 30·0±0·33 —	28·88 55·84 —	1904	Hydro. Depart.	—
4	Abasiri		44 1 10·7 144 16 26·3 —	9·70 35·77 —	1905	”	—
5	Nemuro		43 20 23·41±0·038 145 34 52·0 ±0·225 —	28·22 54·15 —	1911	”	—
6	Sapporo	m 15·0	43 4 34·4 141 21 — 349 53 58·2±0·85	20·9 26·67 54·7	1900	Land survey	Northern point of Sapporo base
7	Urakawa		42 9 15·0 142 46 29·0 —	36·2 49·5 —	?	Hokkaido-tyo	—
8	Hakodate		41 45 67·0 140 43 16·6 —	57·21 35·15 —	1884	Hydro-Depart.	—
9	Aomori		40 49 63·86±0·059 140 46 17·52±0·135 —	43·58 40·61 —	1914	”	—
10	Turukotai	43·1	40 42 76·8 141 9 — 244 15 11·5	59·3 26·8 8·3	1898	M. Sugi-yama.	Western point of Turukotai base
11	Sionohara	221·7	38 49 61·1 140 21 — 266 18 16·2	50·8 47·7 28·7	1894	M. Yazi-ma	Western point of Sionohara base
12	Matuga-saki	26·4	37 56 70·57±0·249 139 8 19·89±0·75 —	54·17 45·45 —	1923	T. Naka-sima	—
13	Toriya-yama	580·6	37 34 66·09±0·127 139 33 — —	53·24 43· —	1923	”	—
14	Keizyô	—	37 33 43·48 126 58 29·94 —	32·7 43·2 —	1907	M. Yama-da	—
15	Wazima	—	37 23 67·12 136 53 25·46 —	51·09 49·80 —	1888	Hydro. Depart.	—
16	Sakato-yama	633·9	37 3 46·21±0·211 138 53 36·20±3·600 149 22 59·20±0·90	21·56 65·73 22·00	1923	T. Naka-sima	—

Instruments	Method of observation	$Ba-Bg$	$La-Lg$	$Aa-Ag$
Bamberg's transit	15 pairs in 3 nights by Talcott's method. Resistering micrometer in 2 nights.	+ 9″·39	+16″·37	—
Bamberg's transit 27 cm theodolite	21 pairs in 3 nights by Talcott's method. Resistering micrometer in 3 nights. 24 times by circumpolar stars.	+ 2·29	– 21·49	– 9″·67
	Zinger's method in 10 nights.	+ 17·22	– 25″84	—
		+ 1″00	– 9·47	—
	Talcott's method in 5 nights.	– 4″81	– 2·15	—
Bamberg's transit	54 pairs in 7 nights by Talcott's method (Time ; equal altitude of sun) 6 times in 3 nights by any hour angle of polaris.	+ 13·5	—	+ 3·5
		– 21·2	– 20·5	—
		+ 9·79	– 18·55	—
	Talcott's method	+ 20·28	—	– 23·09
Universal theodolite	Altitude of Polaris in 4 nights (Time: equal altitude of sun by sextant) Elongation of polaris or 51 Cephei in 4 nights.	+ 17·5	-	+ 3·2
21 cm. universal theodolite	Altitude of polaris in any time or meridian altitude of any stars (Time: equal altitude of sun) 6 times in elongation of polaris.	+ 10·3	—	– 12·5
Bamberg's transit	17 pairs in 4 nights by Talcott's method. Chronograph and Key in 4 nights.	+ 16·40	– 25·56	—
„	15 pairs in 3 nights by Talcott's method.	+ 12·75	—	—
13·5 cm theodolite	Talcott's and sing. altitude of polaris. Equal altitude	+ 10·78	– 13·26	—
—	—	+ 16·03	– 24·34	—
Bamberg's transit 13·5 cm theodolite	18 pairs in 5 nights by Talcott's method. Chronograph and key in 3 nights. Any hour angle of polaris.	+ 24·56	– 29·53	– 22·80

(to be continued.)

Table I.

No.	Station	Height	*Ba, La, Aa,*	*Bg* *Lg* *Ag*	Epoch	Observer	Direction of azimuth
		m					
17	Nanao	—	37°2′58″·0 — 136 57 31·6 —	39″·52 47·94 —	1887	Hydro. Depart.	—
18	Suzaka	353·3	36 40 37·0 138 18 — 102 44 15·3±0·70	17·8 29·2 29·5	1896	Land Survey	Eastern point of the Suzaka base
19	Takasuzu-yama	623·7	36 37 3·53±0·098 140 35 — —	4·22 28·2 —		N. Itinoe	—
20	Utuno-miya	58·7	36 34 9·0 139 53 24·0 —	0·7 25·9 —		Land Survey	—
21	Akagi	1673·9	36 32 17·57±0·054 139 10 —	16·37 50·0 —	1903	N. Itinoe K. Sôtome	—
22	Teruisi-yama	419·1	36 21 26·71±0·091 139 40 — —	25·63 43·0 —	1907	N. Itinoe	—
23	Kariyado	1096·3	36 21 21·04±0·072 138 33 44·1±0·260 —	13·76 55·3 —	1921	T. Matu kuma S. Kanda	—
24	Tukuba-san	709·0	36 13 22·0 140 5 55·0 —	22·3 67·2 —	1902	K. Sotome S. Tasiro R. Ootani	—
25	Kiriga-mine	1021·1	36 2 49·00±0·093 138 2 40·6 ±1·01 —	30·61 56·8 —	1921	T. Matu-kuma S. Kanda	—
26	Dôhira-yama	857·8	36 0 21·08±0·053 139 11 — —	9·26 35·73 —	1903	N. Itinoe K. Sotome	—
27	Takagami-mura	73·6	35 42 5·96±0·071 140 51 — —	11·03 24·4 —	1907	N. Itinoe	—
28	Ako	685·4	35 42 16·97±0·056 137 55 49· 5±0·38 —	5·38 60·4 —	1921	T. Matu-kuma S. Kanda	—
29	Siozaki-mura	349·6	35 40 25·55 138 30 22·5 —	21·54 26·3 —	1921	T. Matu-kuma S. Kanda	—
30	Mitaka	58·0	35 40 21·0 139 32 31·5 —	20·6 24·1 —		Astronomical Observatory	—
31	Turuga	—	35 39 14, 136 4 15, —	7· 28·8 —	1890	Hydro. Depart.	—
32	Sakae	—	35 32 49·70±0 037 133 14 31·77±0·330 —	39 77 41·96 —	1920	Hydro. Depart.	—

(conitnued.)

Instruments	Method of observation	$Ba-Bg$	$La-Lg$	$Aa-Ag$
		+18″·48	−16″·34	—
21 cm universal theodolite	86 pairs in 3 nights by Talcott's method (Time; meridian transit by theodolite or eqal altitude of sun by sextant). 17 times in 4 nights by elongation of circumpolar stars.	+ 19·2	—	− 14·2
—	29 pairs in 5 nights by Talcott's method.	− 0·69	—	—
—		+ 8·3	− 1·9	—
Bamberg's transit	60 pairs in 6 nights by Talcott's method.	+ 1·2	—	—
Bamberg's transit	40 pairs in 4 nights by Talcott's method.	+ 1·08	—	—
Bamberg's transit	34 pairs in 3 nights by Talcott's method. Chronograph and key in 3 nights.	+ 7·28	− 11·2	—
—		− 0·3	− 12·2	—
Bamberg's transit	31 pairs in 4 nights by Talcott's method. Chronograph and key in 2 nights.	+ 18·39	−16·20	—
Bamberg's transit	39 pairs in 6 nights by Talcott's method.	+ 11·82	—	—
Bamberg's transit	34 pairs in 5 nights by Talcott's method.	− 5·07	—	—
Bamberg's transit	39 pairs in 4 nights by Talcott's method. Chronograph and key in 4 nights.	+ 11·59	− 10·9	—
Bamberg's transit	26 pairs in 3 nights by Talcott's method.	+ 4·01	− 3·8	—
Bamberg's transit	Numerous pairs in 3 years. Resistering micrometer.	+ 0·4	+ 7·4	—
		+ 7·	− 13·8	—
	Talcott's method. Transit observation (Zinger ?)	+ 9·93	− 10·19	—

(to be continued.)

Table I.

No.	Station	Height	*Ba La Aa*	*Bg Lg Ag*	Epoch	Observer	Direction of azimuth
33	Tobio-yama	m 234·8	35°30′10″·44±0″·280 139 19 61·38±0·375 183 11 14·79±0·790	4″42 40·47 3·26	1927	T. Tiba N. Saito	Sengenyama
34	Maiduru	—	35 28 39·43±0·035 135 23 3·54±0·135 —	20·86 14·15 —	1918	Hydro. Depart.	—
35	Tanzawa-yama	1567·4	35 28 20·48±0·111 139 9 74·72±0·600 178 34 65·59±0·18	15·75 57·07 56·7	1926	M. Kaga-wa	Togadake
36	Rokudizô	173·2	35 26 47·19±0·035 140 12 — —	39·72 31·0 —	1903	N. Itinoe K. Soto-me	—
37	Tenzinno	52·2	35 24 73·1 133 47 — 214 32 35·1	51·5 48·8 37·1	1883	Land Survey	Yakebayasi-mura
38	Kenasi-yama	1954·4	35 24 41·45±0·087 138 32 17·04±0·252 17 1 2·15±0·57	38·59 28·99 7·76	1926	M. Kaga-wa	Kokusidake
39	Aibano	233·7	35 23 26·9 135 59 — 273 43 4,7	2·6 48·0 4·0	1888	M. Yazi-ma	Western point of Aibano base.
40	Kanda-mura	138·5	35 21 35·68 136 17 16·41 —	28·89 31·10 —	1922	T. Naka-sima M. Tyo	—
41	Sengen-yama	181·3	35 19 9·67±137 139 18 — —	7·93 55·9 —	1907	N. Itinoe	—
42	Kanôzan	353·3	35 15 25·09±0·038 139 57 — —	6·10 32·3 —	1903	K. Sotome	—
43	Sanomura	—	39 9 20·85 138 55 20·25 156 7 59·38	17·35 33·32 55·77	1931	D. Saida	Daibamura
44	Daiba-mura	—	35 6 27·79 138 56 57·45 336 8 55·98	21·45 68·04 50·28	1931	D. Saida	Sanomura
45	Kwasan	221·3	34 59 36· 135 47 33·4 —	25· 48· —	1930	Kwasan Obser-vatory	—
46	Bôdaisan	—	34 57 58·78±0·075 139 46 — —	39·14 51·4 —	1907	N. Itinoe	—
47	Mera	—	34 54 46·97±0·038 139 49 59·64 —	28·91 56·34 —	1916	Hydro. Depart.	—
48	Hamada	—	34 53 53·2±0·04 132 54 4·4±0·38 —	36·39 12·71 —	1920	Hydro. Depart.	—

(continued.)

Instruments	Method of observation	$Ba-Bg$	$La-Lg$	$Aa-Ag$
Bamberg's transit 21 cm theodolite	10 pairs in 3 nights by Talcott's method. Chronograph and key in 3 nights. Any hour angle of polaris in 2 nights.	+ 6″·02	+ 20·91	+ 11·54
	Talcott's method. Transit Observation.	+ 18·57	− 10·61	—
Bamberg's transit 27·5 cm theodolite	35 pairs in 5 nights by Talcott's method. Chronograph and key in 3 nights. Any hour angle of polaris in 2 nights.	+ 4·74	+ 17·65	+ 8·89
Bamberg's transit	53 pairs in 3 nights by Talcott's method.	+ 7·47	—	—
	17 pairs by Talcott's method. 14 times by any hour angle of circum-polar stars.	+ 21·6	—	− 2·0
Bamberg's transit 27·5 cm theodolite	38 pairs in 6 nights by Talcott's method. Chronograph and key in 3 nights. Any hour angle of polaris in 3 nights.	+ 2·86	− 11·95	− 5·61
	Altitude in meridian of any stars in 1 night. Greatest elongation of polaris in 1 night.	+ 24·3	—	+ 0·7
		+ 6·79	− 14·69	—
Bamberg's transit	30 pairs in 4 nights by Talcott's method.	+ 1·74	—	—
Bamberg's transit	67 pairs in 5 nights by Talcott's method.	+ 18·99	—	—
		+ 3·50	− 13·07	+ 3·61
		+ 6·34	− 10·59	+ 5·70
		+ 11·	− 14·6	—
Bamberg's transit	51 pairs in 5 nights by Talcott's method.	+ 19·64	—	—
	Talcott's method. Transit observatien.	+ 18·06	+ 3·3	—
	Talcott's method. Transit observation (Zinger ?).	+ 16·81	− 8·31	—

(to be continued.)

Table I.

No.	Station	Height	Ba La Aa	Bg Lg Ag	Epoch	Observer	Direction of azimuth
49	Rokkôzan	m 932·1	34°46′40″·19±0″·29 135 15 51·40±1·25 224 24 39· 3±0·53	28″·83 59·49 43·7	1928	M. Takasaki T. Matui	Kumogaiwa
50	Sakabe	151·1	34 45 62.52±0·121 138 12 15·49±0·292 318 51 19·53±0·494	57·86 22·52 21·74	1931	T. Urano K. Kirihara	Hakkozan
51	Oosaka-zyô	44·2	34 41 21·0 135 31 28·0 —	3·37 42·71 —		Land Survey	—
52	Mikomo-tozima	—	34 34 27·2 138 56 31·05 —	19·2 46·72 —	1928	Hydro. Depart.	—
53	Kasaoka	—	34 29 61·5 133 30 12·2 —	52·30 32·19 —	1893	Hydro. Depart.	—
54	Edazima	—	34 14 37·0 132 28 27·6 —	31·20 39·86 —	1888	Hydro. Depart.	—
55	Nisibayasi	37·0	34 4 31·3 134 11 — 97 33 33·0±0·77	9·3 41·0 31·7	1890	Land Survey	Eastern point of Nisibayasi base.
56	Simono-seki	—	33 57 22·6±0·04 130 56 16·2±0·08 —	8·89 26·72 —	1917	Hydro. Depart.	—
57	Saeki	—	32 54 10·11±0·048 131 57 51·42±0·320 —	3·10 46·51 —	1920	Hydro. Depart.	—
58	Nagasaki	—	32 43 58· 129 52 10·5 —	42·2 21·2 —	1917	Hydro. Depart.	—
59	Miyazaki	4·4	31 54 62·67±0·048 131 25 37·96±0·315 —	53·75 22·56 —	1920	Hydro. Depart.	—
60	Kagosima	—	31 35 41·9 130 33 33·0 —	41·7 29·1 —		Hydro. Depart.	—
61	Kasano-hara	106·3	31 23 15·6 130 51 — 38 29 26·1	3·9 54·4 30·2	1892	M. Yazima	Northern point of Kasanohara base

revolution of the micrometer, when in perfect adjustment, is equal to 2′. As the micrometer head has 60 divisions, it can be read to 2″, but by taking the mean of two micrometers A and B, it can be read down to 1″, and by mentally dividing the space down to one-tenth of a second.

One angle was measured twenty-four times by the angle method, not by that of direction. To eliminate collimation errors, one-half of

(continued.)

Instrument	Method of observation	$Ba-Bg$	$La-Lg$	$Aa-Ag$
Bamberg's transit Universal theodolite	18 pairs in 5 nights by Talcott's method. Chronograph and key in 3 nights. Any hour angle of polaris in 1 night.	+11″·36	− 8″·09	− 4″·4
Bamberg's transit 27·5 cm theodolite	52 pair in 8 nights by Talcott's method. Resistering micrometer in 7 nights. Any hour angle of polaris in 3 nights.	+ 4·66	− 7·03	− 2·21
		+ 17·63	− 14·71	—
		+ 8·0	− 15·67	—
		+ 9·20	− 19·99	—
		+ 5·8	− 12·26	—
12 cm universal theodolite	25 pairs in 2 nights by Talcott's method. 5 times in 2 nights by elongation of polaris.	+ 22·0	—	+ 1·3
		+ 13·71	− 10·52	—
	Zinger's method.	+ 7·01	+ 4·91	—
		+ 5·8	− 10·7	—
	Talcott's method. Transit observation.	+ 8·92	+ 15·40	—
		+ 0·2	+ 3·9	—
21 cm universal theodolite	4 times by altitude at any hour angle of polaris or meridian altitude of circumpolar stars (Time: equal altitude of sun by sextant). 24 times at greatest elongation of polaris.	+ 11·7	—	− 4·1

these twenty-four observations was made in one position of the telescope and the remaining half in the reversed position. At each of these twenty-four observations, the position of the circle was systematically altered according to the law of O. Schreiber, to eliminate errors arising from the eccentricity and graduation of the circle. Since the run of the microscope is apt to get out of order in transporting the instrument from mountain to mountain, it was always accurately adjusted upon arrival of the observer at any station, and rechecked after completion of the observation at that station.

As the distances separating the primary triangulation points in Japan generally exceed 30 kilometers, and as the zenith distance makes nearly a right angle, the error that arises from the axes is considered very small when the axes are almost fully adjusted. For this reason the error that arises from the axes, or the correction for errors that arise from the height of the observing stations owing the disagreement of the vertical sections of the rotation ellipsoid, may generally be neglected, so long as the instrument is adjusted frequently during observations. The value of one division of the striding level used in these adjustment is generally $3'' \sim 4''$.

The triangulation net of the whole country, except Taiwan and Tyôsen, therefore was divided into twenty-four individual nets as shown in the map appended. With a few exceptions, each of them has generally one base line.

The base line, the length of which is usually several kilometers, was measured with a 5-meter Guillaume apparatus, or a 4-meter Hilgard contact slide apparatus; instruments similar to those used in previous surveys. Recently, 25-meter Jäderin wires, the length of which was determined just before and after field survey work by comparing it with comparison base line at Mitaka Astronomical Observatory, was used. It was ascertained when re-measuring the Aibano and Tenzinno base lines, that previous measurements of them were almost as accurate as those made much later. These base lines were projected on the mean sea level of Tôkyô Bay by means of precise levellings.

Were the observed angles of a triangulation free from error, calculation of the distance between pairs of points would present no difficulty, but the errors with which every observed angle is vitiated lead to conflicting results. These contradictions in the measured angles however were removed and a consistent figure was obtained after adjusting the net so as to satisfy the following two conditions:—

(I) Station adjustment:—Those arising at a station from the relation of the angles to one another at that station.

(II) General adjustment:—Those due to the geometrical relations needed to form a closed figure.

The general adjustment is based on two conditions:—

(a) Angle equation:—That the sum of the angles of each triangle in the figure should be equal to 180° plus spherical excess of the triangle.

(b) Side equation:—Even if the angle equations be satisfied, the figure does not close, so that the length of any side as computed from the known base line or the side, should be the same whatever the route chosen.

The trigonometrical nets were adjusted step by step as geodetic work proceeded in accordance with the conditions laid down as just mentioned. It should be mentioned here that, in expanding the net, observations at the station on the boundary of two nets were made with utmost care in order to eliminate errors in connection with the orientation of the new net, a way of observing that differed somewhat from those at other stations. The above mentioned twenty-four observations were divided into two groups, one-half of them being measurements from one known point and the remaining half from the other known point. The result was that we obtained a consistent figure.

After Bessel's spheroid

$$a = 6377397 \cdot 15 \text{ meters},$$

$$b = 6356078 \cdot 96 \text{ meters},$$

$$\alpha = 1 : 299 \cdot 152813,$$

had been adopted as the reference ellipsoid and all the angles and lengths had been completely fixed, it was still necessary, before the calculation of latitudes, longitudes, and azimuths could be made, to adopt a standard latitude and longitude for a special station and a standard azimuth of a line from that station. For convenience of reference, the adopted standard position of a given station, together with the adopted standard azimuth of a line from that station, is called the geodetic datum.

One geodetic datum in our survey was the center of the meridian circle at Azabu Astronomical Observatory, Tôkyô, and the other the astronomical azimuth to the primary triangulation point, Kanôzan, from that station.

$$B = 35°39'17''\cdot 5148,$$

$$L = 139°44'40''\cdot 5020,$$

$$A = 156°25'30''\cdot 156.$$

The site, which is shown in Fig. 1, served as the basis for all the geodetic values in this report.

Using these values and results of the adjustment, we calculated the latitude, longitude, and azimuth of all the stations according to O. Schreiber's[2] formulae, which is exact up to $0''\cdot 0001$ for latitude and longitude and up to $0''\cdot 001$ for azimuth applied to a point, the distance

2) O. SCHREIBER, "Formeln und Tafeln zur Berechnung der geographischen Koordinaten aus den Richtungen und Längen der Dreiecks seiten," Berlin (1878).

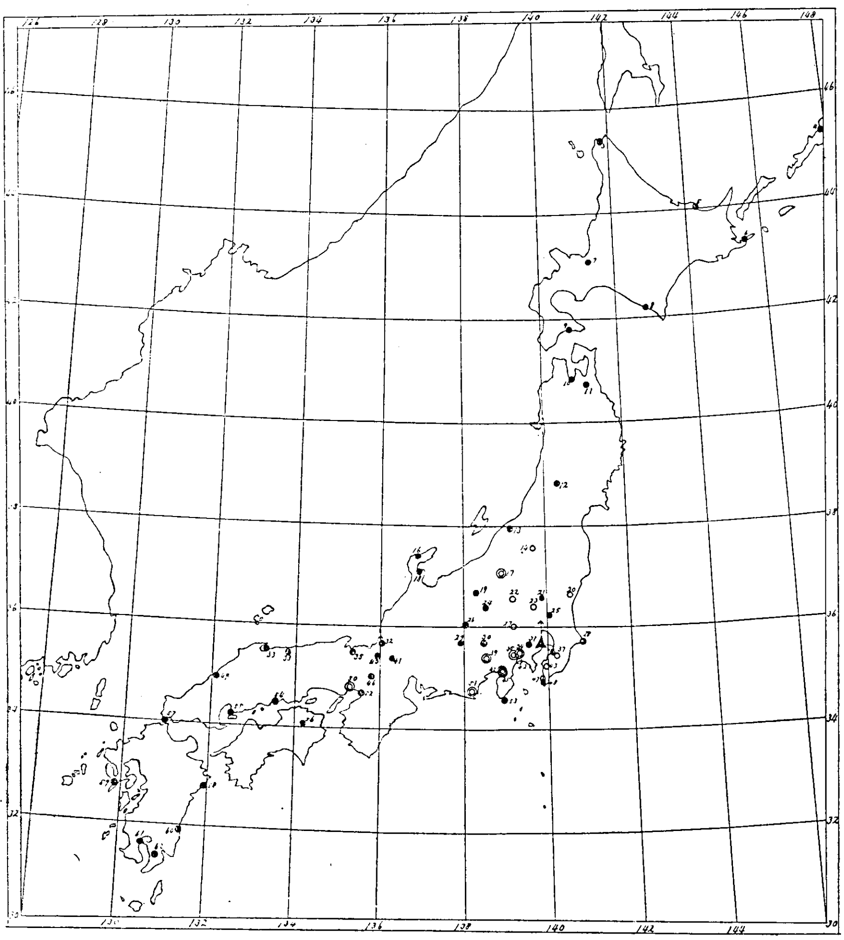

Fig. 1. Distribution of astronomical stations.

▲: Standard datum.
◎: Latitude, longitude and azimuth.
●: Latitude and one of longitude or azimuth.
○: Latitude only.

of which is 120 km. These quantities were always calculated from two directions and controlled.

III. Determination of the Surface *R*.

Since measured base lines are reduced to the surface of the mean sea level of Tôkyô Bay, and since the angles measured by the theodolite are the same as were measured on that surface, it follows that the trigonometrical operations may be considered as having been virtually conducted on the surface *M*. But, since the angles measured on *M* are not identical with the angles measured on *R*, the former must be reduced to the surface *R*. Meanwhile, for purposes of reduction, it is necessary that the form of the geoid shall be known before beginning the geodetic work, and even if that were known, it is almost impossible to calculate the triangulation on a surface so complexed. Fortunately, however, since in the case of a distant object observed in a trigonometrical survey, the angles of elevation of the signal are generally small, and since in ordinary cases, ξ's and η's are very small angles, we may infer that the angle observed on M is not sensibly different from the corresponding angle on R. It may therefore be concluded that the larger part of ξ's and η's arises from absolute deviations of the plumb line at the geodetic datum ξ_0 and η_0, and from the differences of magnitude and shape da and de^2 of the ellipsoid, adopted as reference and assumed to represent most closely the surface of M of Japan. In consideration of the foregoing we can now state that, at every point from which the astronomical observations were made, the ξ_i and η_i belonging to that point can be expressed in terms of $\xi_0\,\eta_0$, da and de^2.

The equation of condition thus obtained are

$$a_i\xi_0+b_i\eta_0+c_i\frac{da}{a}+d_i\frac{1}{2}\frac{de^2}{1-e^2}-\xi_i=0, \tag{4}$$

$$a_i'\xi_0+b_i'\eta_0+c_i'\frac{da}{a}+d_i'\frac{1}{2}\frac{de^2}{1-e^2}-\eta_i=0, \tag{5}$$

$$a_i''\xi_0+b_i''\eta_0+c_i''\frac{da}{a}+d_i''\frac{1}{2}\frac{de^2}{1-e^2}-\eta_i=0, \tag{6}$$

of which (4), (5), (6) were deduced from the observations of latitude, longitude, and azimuth respectively. When longitude is measured eastward and the azimuth reckoned from the south, the value of the coefficient[3] may be expressed as follows:

$$\left.\begin{aligned}
a_i&=\frac{W_i^3}{W_0^3}\left\{\cos\Delta\alpha+\left[1-\left(\frac{dm}{ds}\right)_{i0}\right]\sin\alpha_{0i}\sin\alpha_{i0}\right\},\\
b_i&=-\frac{m}{a}\frac{W_i^3}{1-e^2}\tan B_0\sin\alpha_{i0},\\
c_i&=-\rho''\frac{S_{i0}}{a}\frac{W_i^3}{1-e^2}\cos\alpha_{i0},\\
d_i&=\rho''\frac{W_i^{\,3}}{1-e^2}\left(\frac{D}{a}\cos\alpha_{i0}+\frac{E}{a}\sin\alpha_{i0}\right),\\
a_i'&=\frac{(1-e^2)W_i}{W_0^3}\left\{\sin\Delta\alpha-\left[1-\left(\frac{dm}{ds}\right)_{i0}\right]\sin\alpha_{0i}\cos\alpha_{i0}\right\},\\
b_i'&=\left\{1+\frac{m}{a}W_i\sin B_0\sec B_i\cos\alpha_{i0}\right\}\sec B_0\cos B_i,\\
c_i'&=-\rho''\frac{S_{i0}}{a}W_i\sin\alpha_{i0},
\end{aligned}\right\} \tag{7}$$

3) F. R. Helmert, "Theorien der Höheren Geodasie," I Teil.

$$\left.\begin{aligned}
d_i' &= \rho'' W_i\left(\frac{D}{a}\sin\alpha_{i\iota} - \frac{E}{a}\cos\alpha_{\iota 0}\right),\\
a_i'' &= -\frac{a}{m}\frac{1-e^2}{W_0^3}\left\{\left[1-\left(\frac{dm}{ds}\right)_{0\iota} + 1 - \left(\frac{dm}{ds}\right)_{\iota 0}\right]\cos\Delta\alpha\sin\alpha_{\iota 0} - \frac{\sin^2\Delta\alpha}{\sin\alpha_{0\iota}}\right.\\
&\quad \left. + \left[1-\left(\frac{dm}{ds}\right)_{0\iota}\right]\left[1-\left(\frac{dm}{ds}\right)_{\iota 0}\right]\sin^2\alpha_{\iota 0}\sin\alpha_{0\iota}\right\}\cot B_\iota,\\
b_i'' &= \left\{\left(\frac{dm}{ds}\right)_{0\iota} + \frac{m}{a}W_\iota\tan B_\iota\cos\alpha_{\iota 0}\right\}\tan B_0\cot B_\iota,\\
c_i'' &= -\rho''\frac{S_{\iota 0}}{a}W_\iota\sin\alpha_{\iota 0},\\
d_i'' &= \rho'' W_i\left(\frac{D}{a}\sin\alpha_{\iota 0} - \frac{E}{a}\cos\alpha_{\iota 0} + \frac{F}{W_\iota}\right),
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\text{where}\qquad W = \sqrt{1-e^2\sin^2 B},\quad B = \frac{1}{2}(B_0 + B_\iota),\\
&m:\ \text{reduced length of the geodesic line}\\
&\quad = s\left(1 - \frac{S^2}{6a^2}K + \ldots\right),\\
&K = \frac{a^2}{\rho_m\rho_n},\ \rho_m, \rho_n:\ \text{radii of curvature in meridian and parallel},\\
&D = \frac{S}{W^2}\left(2\cos^2 B\cos^2\frac{\alpha_{\iota 0}+\alpha_{0\iota}-180^\circ}{2} - [1-e^2]\sin^2 B + \ldots\right),\\
&E = S\left\{\frac{1}{W_0^2}\cos^2 B_0\cos\alpha_{0\iota}\sin\alpha_{0\iota}\left[1 - \frac{1}{6}\frac{S^2}{a^2}\right]\right.\\
&\qquad\qquad \left. + \frac{1}{W_\iota^2}\cos^2 B_\iota\cos\alpha_{\iota 0}\sin\alpha_{\iota 0} + \ldots\right\},\\
&F = \frac{S}{2a^2}B - \frac{S^3}{6a^3}\sin B_0\cos B_0\sin\alpha_{0\iota} + \ldots.
\end{aligned}\right\}\quad (8)$$

The first step in the work is the calculation of the azimuth, the backazimuth, and the distance to the astronomical stations. After slight modification of the spheroidal formulae given by Jordan,[4] we have

$$\left.\begin{aligned}
\log S\sin\alpha &= \log\frac{(L_i''-L_0'')\cos B}{[2]}\\
&\qquad - [3](L_i''-L_0'')^2\sin^2 B + [4]\quad (B_i''-B_0'')^2\\
\log S\cos\alpha &= \log\frac{B_i''-B_0''}{[1]} - [5](L_i''-L_0'')^2\cos^2 B - [6]\quad (B_i''-B_0'')^2,\\
\log\Delta\alpha &= \log(L_i''-L_0'')\sin B + [7](L_i''-L_0'')^2\cos^2 B + [8](B_i''-B_0'')^2,
\end{aligned}\right\}\quad (9)$$

From these we have

4) JORDAN-EGGERT, "Handbuch der Vermessungskunde," Band III.

$$\left.\begin{aligned} \alpha_{i0} &= \alpha + \frac{\Delta\alpha}{2}, \\ \alpha_{0i} &= \alpha - \frac{\Delta\alpha}{2} - 180°. \end{aligned}\right\} \qquad (10)$$

In the above

$$\left.\begin{aligned} &[1] = \frac{\rho}{N}V^2, \quad [2] = \frac{\rho}{N}, \quad [3] = \frac{\mu}{24\rho^2}, \quad [4] = \frac{\mu}{24\rho^2}\frac{1+\eta^2-9\eta^2t^2}{V^4}, \\ &[5] = \frac{\mu}{24\rho^2}(2+3t^2+2\eta^2), \quad [6] = \frac{\mu}{8\rho^2}\eta^2\frac{t^2-1-\eta^2-4\eta^2t^2}{V^4}, \quad [7] = \frac{\mu}{12\rho^2}V^2, \\ &[8] = \frac{\mu}{24\rho^2}\frac{3+8\eta^2+5\eta^4}{V^4}, \\ &V = \sqrt{1+\frac{e^2}{1-e^2}\cos^2 B}, \quad N = \frac{a}{W}, \quad \rho = \frac{1}{\sin 1''}, \\ &\qquad\qquad \mu = \text{modulus of natural logarithm}, \\ &\eta^2 = \frac{e^2}{1-e^2}\cos^2 B, \quad t = \tan B. \end{aligned}\right\} \qquad (11)$$

(Argument of all quantities are $B = \frac{1}{2}(B_0 + B_i)$)

The following Table II contains an abstract from the result of the computation, together with the reduced length of the geodetic lines.

There is no general principle by means of which we can assign to the equations derived from longitude and azimuth their values relative to the equations derived from the observed latitude. Since the probable errors in the astronomical observations of longitude or azimuth are considerably greater than in the case of latitude, in view of the nature of such observations, the longitudinal and azimuthal equations are entitled to much less weight than the latitude equations, although what the exact ratios are cannot generally be determined. In addition to these difficulties, the azimuth equations in particular are directly affected by cumulative errors in the observed angles of triangulation or by errors arising from observations of the geodetic datum. But we now lack the necessary means of examining the error of the azimuth in the geodetic datum owing to changes brought about as the result of the great Kwantô Earthquake of 1923, making it very difficult to estimate how much these errors accumulate at each station. But, with respect to the errors that arise in the observations of the intervening trigonometrical stations, they could be inferred to a certain extent from the mean value of the closing errors of the intervening triangles, or of the closing error of sides between the two base lines, although it would be a question to what extent the weights obtained in this manner can be trusted. On the other hand, η's that were deduced from the longitudinal and the azimuthal observations must, in the existence of the two observations, be equal. So that in making out the error equations, the mean of the two equations were taken and also given the same weight unity as that given to the latitude equations, and to the remaining equations of the half weight.

Table II.

No.	Station	log S in meters	log m in meters	α_{0i}			α_{i0}		
1	Handasawa	6·2073440	5·99772	187°	49′	32″	9°	54′	9″
2	Higasiyama	6·2085437	5·99742	188	43	43	11	3	1
3	Syana	6·1022009	5·98519	210	24	45	35	42	36
4	Abasiri	6·0025528	5·93232	201	13	52	24	8	29
5	Nemuro	5·9136407	5·86829	215	56	54	39	40	22
6	Sapporo	5·9222801	5·87498	189	31	2	10	32	32
7	Urakawa	5·6302909	5·61845	217	4	9	38	58	33
8	Hakodate	5·8347983	5 80373	187	50	13	7	26	56
9	Aomori	5·7643472	5·74212	188	35	2	9	13	14
10	Turukotai	5·7598381	5·73808	191	59	32	12	52	0
11	Sionohara	5·5522737	5·54403	188	39	54	9	2	23
12	Matugasaki	5·4152475	5·41088	168	11	36	347	49	50
13	Toriyayama	5.3314592	5·32850	175	17	44	355	10	37
14	Keizyô	6·0633378	5·96761	104	17	12	276	39	1
15	Wazima	5·5057784	5·49914	127	57	4	306	15	7
16	Sakatoyama	5·23859	5·23854	154	5	8	333	34	52
17	Nanao	5·4679065	5·46234	122	28	44	300	49	40
18	Suzaka	5·23424	5·23419	131	31	28	310	40	36
19	Takasuzuyama	5·11804	5·11801	215	14	49	35	44	45
20	Utunomiya	5·00873	5·00871	187	20	10	7	25	18
21	Akagi	5·04275	5·04273	152	46	10	332	26	14
22	Teruisiyama	4·89286	4·89285	175	39	49	355	37	29
23	Kariyado	5·11992	5·11989	126	23	13	305	41	31
24	Tukubasan	4·84907	4·84906	206	45	57	26	58	25
25	Kirigamine	5·20259	5·20255	106	7	2	285	7	18
26	Dôhirayama	4·79925	4·79924	127	55	40	307	36	20
27	Takagamimura	5·00354	5·00352	266	38	7	87	17	3
28	Ako	5·21562	5·21557	92	20	0	271	16	30
29	Siozakimura	5·04962	5·04960	90	38	51	270	38	51

(*to be continued.*)

Table II. (*continued.*)

No.	Station	log S in meters	log m in meters	α_{oi}			α_{io}		
30	Mitaka	4·26568	4·26568	96°	6′	42″	275°	59′	39″
31	Turuga	5·5219365	5·51478	91	7	36	268	59	6
32	Sakae	5·7702356	5·74734	90	42	11	266	54	55
33	Tobioyama	4·61225	4·61225	65	31	23	245	17	3
34	Maiduru	55974241	5·58725	88	20	21	265	48	8
35	Tanzawayama	4·74722	4·74722	68	45	40	248	25	38
36	Rokudizô	4·68240	4·68240	298	53	40	119	9	52
37	Tenzinno	5·7323292	5·71320	88	53	56	265	26	24
38	Kenasiyama	5·05192	5·05192	76	26	42	255	44	38
39	Aibano	5·5330280	5·52549	86	2	25	263	51	41
40	Kandamura	5·4986779	5·49225	85	0	38	263	0	9
41	Sengenyama	4·73309	4·73309	46	33	42	226	18	38
42	Kanôzan	4·68824	4·68824	336	25	9	156	32	37
43	Sanomura	4·96861	4·96859	53	37	58	233	9	22
44	Daibamura	4·97539	4·97537	50	6	28	229	38	50
45	Kwasan	5·5642584	5·55554	79	32	54	257	15	46
46	Bôdaisan	4·88683	4·88682	357	30	42	177	31	58
47	Mera	4·92034	4·92033	354	25	3	174	28	7
48	Hamada	5·8471207	5·81412	85	19	49	260	53	33
49	Rokkôzan	4·68240	4·68240	298	53	40	119	9	52
50	Sakabe	5·23414	5·23413	55	20	29	234	27	13
51	Oosakazyô	5·6011358	5·59079	75	34	30	253	8	36
52	Mikomotozima	5·14811	5·14808	31	34	15	211	6	33
53	Kasaoka	5·7659060	5·74349	79	5	56	255	30	35
54	Edazima	5·8338318	5·80287	78	49	40	254	39	28
55	Nisibayasi	5·7299492	5·71103	72	28	54	249	18	26
56	Simonoseki	5·9341555	5·88394	72	8	47	249	6	42
57	Saeki	5·8912259	5·85044	69	8	0	244	44	44
58	Nagasaki	5·9848552	5·92041	73	11	57	247	38	19
59	Miyazaki	5·9417458	5·88960	64	2	38	237	24	39
60	Kagosima	5·9837233	5·91958	64	45	15	239	39	25
61	Kasanohara	5·9779584	5·91563	62	38	53	237	44	4

Table III. Equation of condition derived from observed latitudes (Square root of the remarked weight were already multiplied)

No.	Station			[10000 ×]	[10000 ×]	$-\xi_i$	Weight
1	Handasawa	+0·996 ξ_0	−0·019 η_0	−5·130 $\frac{da}{a}$	+3·199 $\frac{de^2}{1-e^2}$	+ 9·39 = ξ_1	1
2	Higasiyama	+0·987 〃	−0·021 〃	−5·129 〃	+3·192 〃	+ 2·29 = ξ_2	1
3	Syana	+0·988 〃	−0·064 〃	−3·329 〃	+2·583 〃	+17·22 = ξ_3	1
4	Abasiri	+0·996 〃	−0·039 〃	−2·974 〃	+2·128 〃	+ 1·00 = ξ_4	1
5	Nemuro	+0·994 〃	−0·053 〃	−2·045 〃	+1·674 〃	− 4·81 = ξ_5	1
6	Sapporo	+0·998 〃	−0·015 〃	−2·665 〃	+2·129 〃	+13·50 = ξ_6	1
7	Urakawa	+0·997 〃	−0·029 〃	−1·076 〃	+0·909 〃	−21·20 = ξ_7	1
8	Hakodate	+0·999 〃	−0·009 〃	−2·197 〃	+1·821 〃	+ 9·79 = ξ_8	1
9	Aomori	+0·999 〃	−0·010 〃	−1·860 〃	+1·530 〃	+20·28 = ξ_9	1
10	Turukotai	+0·999 〃	−0·014 〃	−1·818 〃	+1·551 〃	+17·50 = ξ_{10}	1
11	Sionohara	+1·000 〃	−0·006 〃	−1·142 〃	+1·055 〃	+10·30 = ξ_{11}	1
12	Matugasaki	+1·000 〃	+0·006 〃	−0·825 〃	+0·393 〃	+16·40 = ξ_{12}	1
13	Toriyayama	+1·000 〃	+0·002 〃	−0·693 〃	+0·641 〃	+12·75 = ξ_{13}	1
14	Keizyô	+0·975 〃	+0·104 〃	−0·435 〃	−0·290 〃	+10·78 = ξ_{14}	1
15	Wazima	+0·999 〃	+0·029 〃	−0·615 〃	+0·600 〃	+16·03 = ξ_{15}	1
16	Sakatoyama	+1·000 〃	+0·009 〃	−0·503 〃	+0·480 〃	+24·56 = ξ_{16}	1
17	Nanao	+0·999 〃	+0·028 〃	+1·588 〃	+0·496 〃	+18·48 = ξ_{17}	1
18	Suzaka	+1·000 〃	+0·015 〃	−0·363 〃	+0·352 〃	+19·20 = ξ_{18}	1
19	Takasuzuyama	+1·000 〃	−0·009 〃	−0·346 〃	+0·333 〃	− 0·69 = ξ_{19}	1
20	Utunomiya	+1·000 〃	−0·001 〃	−0·328 〃	+0·314 〃	+ 8·30 = ξ_{20}	1
21	Akagi	+1·000 〃	+0·006 〃	−0·317 〃	+0·306 〃	+ 1·20 = ξ_{21}	1
22	Teruisiyama	+1·000 〃	+0·001 〃	−0·253 〃	+0·252 〃	+ 1·08 = ξ_{22}	1
23	Kariyado	+0·999 〃	+0·012 〃	−0·249 〃	+0·244 〃	+ 7·28 = ξ_{23}	1
24	Tukuba	+1·000 〃	−0·004 〃	−0·204 〃	+0·199 〃	− 0·30 = ξ_{24}	1
25	Kirigamine	+0·999 〃	+0·017 〃	−0·136 〃	+0·137 〃	+18·39 = ξ_{25}	1
26	Dôhirayama	+1·000 〃	+0·006 〃	−0·125 〃	+0·122 〃	+11·82 = ξ_{26}	1
27	Takagamimura	+1·000 〃	−0·011 〃	−0·016 〃	+0·019 〃	− 5·07 = ξ_{27}	1
28	Ako	+0·999 〃	+0·019 〃	−0·012 〃	+0·019 〃	+11·59 = ξ_{28}	1
29	Siozakimura	+1·000 〃	+0·013 〃	+0·004 〃	+0·000 〃	+ 4·01 = ξ_{29}	1
30	Mitaka	+1·000 〃	+0·002 〃	−0·006 〃	+0·006 〃	+ 0·40 = ξ_{30}	1
31	Turuga	+0·998 〃	+0·037 〃	+2·094 〃	+0·062 〃	+ 7·00 = ξ_{31}	1
32	Sakae	+0·994 〃	+0·063 〃	+3·704 〃	−0·021 〃	+ 9·93 = ξ_{32}	1
33	Tobioyama	+1·000 〃	+0·004 〃	+0·056 〃	−0·005 〃	+ 6·02 = ξ_{33}	1
34	Maiduru	+0·997 〃	+0·044 〃	+2·485 〃	−0·041 〃	+18·57 = ξ_{34}	1
35	Tanzawayama	+1·000 〃	+0·006 〃	+0·067 〃	−0·066 〃	+ 4·73 = ξ_{35}	1
36	Rokudizô	+1·000 〃	−0·005 〃	+0·076 〃	−0·074 〃	+ 7·47 = ξ_{36}	1

(to be continued.)

Table III. (*continued.*)

No.	Station			[10000 ×]	[10000 ×]	$-\xi_i$	Weight
37	Tenzinno	$+0{\cdot}994\,\xi_0$	$+0{\cdot}058\,\eta_0$	$+3{\cdot}388\,\frac{da}{a}$	$-0{\cdot}083\tfrac{1}{2}\,\frac{de^2}{1-e^2}$	$+21{\cdot}60''=\xi_{37}$	1
38	Kenasiyama	+1·000 〃	+0·012 〃	+0·090 〃	−0·087 〃	$+\ 2{\cdot}86=\xi_{38}$	1
39	Aibano	+0·998 〃	+0·038 〃	+2·136 〃	−0·083 〃	$+24{\cdot}30=\xi_{39}$	1
40	Kandamura	+0·991 〃	+0·035 〃	+1·971 〃	−0·103 〃	$+\ 6{\cdot}79=\xi_{40}$	1
41	Sengenyama	+1·000 〃	+0·004 〃	+0·121 〃	−0·163 〃	$+\ 1{\cdot}74=\xi_{41}$	1
42	Kanôzan	+1·000 〃	−0·002 〃	+0·145 〃	−0·145 〃	$+18{\cdot}99=\xi_{42}$	1
43	Sanomura	+1·000 〃	+0·008 〃	+0·181 〃	−0·178 〃	$+\ 3{\cdot}50=\xi_{43}$	1
44	Daibamura	+1·000 〃	+0·008 〃	+0·199 〃	−0·168 〃	$+\ 6{\cdot}34=\xi_{44}$	1
45	Kwasan	+0·998 〃	+0·040 〃	+0·262 〃	+0·393 〃	$+11{\cdot}00=\xi_{45}$	1
46	Bôdaizan	+0·999 〃	−0·000 〃	+0·250 〃	−0·250 〃	$+19{\cdot}64=\xi_{46}$	1
47	Mera	+1·000 〃	+0·001 〃	+0·269 〃	−0·267 〃	$+18{\cdot}06=\xi_{47}$	1
48	Hamada	+0·991 〃	+0·073 〃	+4·372 〃	−0·248 〃	$+16{\cdot}81=\xi_{48}$	1
49	Rokkôzan	+0·999 〃	+0·045 〃	+0·346 〃	−0·331 〃	$+11{\cdot}36=\xi_{49}$	1
50	Sakabe	+1·000 〃	+0·016 〃	+0·321 〃	−0·480 〃	$+\ 4{\cdot}66=\xi_{50}$	1
51	Oosakazyô	+0·997 〃	+0·042 〃	+2·405 〃	−0·352 〃	$+17{\cdot}63=\xi_{51}$	1
52	Mikomotozima	+1·000 〃	+0·008 〃	+0·391 〃	−0·391 〃	$+\ 8{\cdot}00=\xi_{52}$	1
53	Kasaoka	+0·994 〃	+0·061 〃	+3·556 〃	+0·704 〃	$+\ 9{\cdot}20=\xi_{53}$	1
54	Edazima	+0·992 〃	+0·069 〃	+4·141 〃	−0·497 〃	$+\ 5{\cdot}80=\xi_{54}$	1
55	Nisibayasi	+0·996 〃	+0·054 〃	+3·162 〃	−0·580 〃	+22·[illegible]0 $=\xi_{55}$	1
56	Simonoseki	+0·998 〃	+0·080 〃	+4·987 〃	−0·952 〃	$+13{\cdot}71=\xi_{56}$	1
57	Saeki	+0·991 〃	+0·072 〃	+4·434 〃	−0·994 〃	$+\ 7{\cdot}01=\xi_{57}$	1
58	Nagasaki	+0·986 〃	+0·087 〃	+5·626 〃	−1·506 〃	$+\ 5{\cdot}80=\xi_{58}$	1
59	Miyazaki	+0·990 〃	+0·075 〃	+4·740 〃	−0·828 〃	$+\ 8{\cdot}92=\xi_{59}$	1
60	Kagosima	+0·988 〃	+0·081 〃	+5·235 〃	−0·467 〃	$+\ 0{\cdot}20=\eta_{60}$	1
61	Kasanohara	+0·989 〃	+0·079 〃	+5·063 〃	−0·932 〃	$+11{\cdot}70=\xi_{61}$	1

Equation of condition derived from observed longitudes and azimuths
Table IV. (Square root of the remarked weight were already multiplied.)

No.	Station			[10000 ×]	[10000 ×]	$-\eta_i$	Weight
62	Handazawa	—	—	—	—	—''	—
63	Higasiyama	$-0{\cdot}284\,\xi_0$	$+0{\cdot}710\,\eta_0$	$-1{\cdot}950\,\frac{da}{a}$	$+1{\cdot}503\tfrac{1}{2}\,\frac{de^2}{1-e^2}$	$-10{\cdot}96=\eta_{63}$	1
64	Syana	+0·071 〃	+0·675 〃	−3·287 〃	+1·893 〃	$-12{\cdot}88=\eta_{64}$	½
65	Abasiri	+0·038 〃	+0·688 〃	−1·831 〃	+1·421 〃	$-\ 4{\cdot}95=\eta_{65}$	½
66	Nemuro	+0·048 〃	+0·677 〃	−2·328 〃	+1·237 〃	$-\ 1{\cdot}10=\eta_{66}$	½
67	Sapporo	−0·022 〃	+0·648 〃	−0·142 〃	+0·697 〃	$+\ 2{\cdot}52=\eta_{67}$	½
68	Urakawa	+0·024 〃	+0·671 〃	−1·195 〃	+0·670 〃	$-10{\cdot}74=\eta_{68}$	½

(*to be continued.*)

Table IV. (*continued.*)

No.	Station			[10000 ×]	[10000 ×]	$-\eta_l$	Weight
69	Hakodate	+0·008 ξ_0	+0·699 η_0	−0·395 $\frac{da}{a}$	+0.407½ $\frac{de^2}{1-e^2}$	$-\ 9''\!\cdot\!79 = \eta_{69}$	½
70	Aomori	+0·008 ″	+0·702 ″	−0·414 ″	+0·161 ″	$-12\cdot35 = \eta_{70}$	½
71	Turukotai	−0·019 ″	+0·686 ″	−0·620 ″	+0·618 ″	$+\ 2\cdot12 = \eta_{71}$	½
72	Sionohara	−0·009 ″	+0·715 ″	−0·272 ″	+0·301 ″	$-\ 7\cdot75 = \eta_{72}$	½
73	Matugasaki	+0·004 ″	+0·706 ″	+0·244 ″	−0·204 ″	$-14\cdot37 = \eta_{73}$	½
74	Toriyayama	—	—	—	—	—	—
75	Keizyô	+0·092 ″	+0·698 ″	+5·117 ″	−0·889 ″	$-\ 7\cdot43 = \eta_{75}$	½
76	Wazima	+0·021 ″	+0·706 ″	+1·150 ″	−0·336 ″	$-13\cdot69 = \eta_{76}$	½
77	Sakatoyama	+0·008 ″	+1·162 ″	+0·789 ″	+0·084 ″	$-26\cdot88 = \eta_{77}$	1
78	Nanao	−0·021 ″	+0·706 ″	+1·123 ″	−0·233 ″	$-\ 9\cdot23 = \eta_{78}$	½
79	Suzaka	+0·030 ″	+0·960 ″	+0·579 ″	+0·032 ″	$-13\cdot48 = \eta_{79}$	½
80	Takasuzuyama	—	—	—	—	—	—
81	Utunomiya	+0·001 ″	+0·707 ″	−0·059 ″	−0·010 ″	$-\ 1\cdot08 = \eta_{81}$	½
82	Akagi	—	—	—	—	—	—
83	Teruisiyama	—	—	—	—	—	—
84	Kariyado	−0·008 ″	+0·707 ″	+0·477 ″	+0·083 ″	$-\ 6\cdot38 = \eta_{84}$	½
85	Tukuba	+0·003 ″	+0·707 ″	−0·143 ″	−0·025 ″	$-\ 6\cdot95 = \eta_{85}$	½
86	Kirigamine	−0·012 ″	+0·707 ″	+0·684 ″	+0·119 ″	$-\ 9\cdot35 = \eta_{86}$	½
87	Dôhirayama	—	—	—	—	—	—
88	Takagamimura	—	—	—	—	—	—
89	Ako	−0·013 ″	+0·707 ″	+0·731 ″	+0·160 ″	$-\ 6\cdot25 = \eta_{89}$	½
90	Siozakimura	−0·009 ″	+0·707 ″	+0·499 ″	+0·169 ″	$-\ 2\cdot19 = \eta_{90}$	½
91	Mitaka	−0·001 ″	+0·707 ″	+0·081 ″	+0·014 ″	$+\ 4\cdot25 = \eta_{91}$	½
92	Turuga	−0·028 ″	+0·769 ″	+1·613 ″	+2·809 ″	$-\ 8\cdot63 = \eta_{92}$	½
93	Sakae	−00 50 ″	+0·768 ″	+2·852 ″	+0·492 ″	$-\ 5\cdot86 = \eta_{93}$	½
94	Tobioyama	+0·004 ″	+1·002 ″	+0·234 ″	+0·040 ″	$+16\cdot59 = \eta_{94}$	1
95	Maiduru	−0·034 ″	+0·769 ″	+1·914 ″	+0·333 ″	$-\ 6\cdot67 = \eta_{95}$	½
96	Tanzawayama	+0·006 ″	+1·002 ″	+0·327 ″	+0·057 ″	$+13\cdot42 = \eta_{96}$	1
97	Rokudizô	—	—	—	—	—	—
98	Tenzinno	+0·143 ″	+0·770 ″	+2·606 ″	+0·444 ″	$-\ 1\cdot09 = \eta_{98}$	½
99	Kenasiyama	+0·012 ″	+1·003 ″	+0·688 ″	+0·120 ″	$-\ 8\cdot82 = \eta_{99}$	1
100	Aibano	+0·088 ″	+0·773 ″	+1·642 ″	+0·254 ″	$+\ 0\cdot38 = \eta_{100}$	½
101	Kandamura	−0·027 ″	+0·770 ″	+1·518 ″	+0·238 ″	$-\ 9\cdot22 = \eta_{101}$	½
102	Sengenyama	—	—	—	—	—	—
103	Kanôzan	—	—	—	—	—	—
104	Sanomura	+0·008 ″	+1·210 ″	+0·469 ″	+0·080 ″	$-\ 2\cdot78 = \eta_{104}$	1
105	Daibamura	+0·008 ″	+1·212 ″	+0·453 ″	−0·381 ″	$-\ 0\cdot28 = \eta_{105}$	1

(*to be continued.*)

Table IV. (*continued.*)

No.	Station			[10000 ×]	[10000 ×]	$-\eta_i$	Weight
106	Kwasan	$+0·028\,\xi_0$	$+0·707\,\eta_0$	$+1592\,\dfrac{da}{a}$	$+0·190\frac{1}{2}\,\dfrac{de^2}{1-e^2}$	$-\ 8''·45=\eta_{106}$	½
107	Bôdaisan	—	—	—	—	—	—
108	Mera	+0·001 ″	+0·707 ″	−0·036 ″	−0·006 ″	$+\ 1·92=\eta_{108}$	½
109	Hamada	−0·059 ″	+0·769 ″	+3·367 ″	+0·492 ″	$-\ 5·25=\eta_{109}$	½
110	Rokkôzan	+0·075 ″	+1·010 ″	+2·552 ″	+0·247 ″	$-\ 6·49=\eta_{110}$	1
111	Sakabe	+0·016 ″	+1·220 ″	+0·878 ″	−0·065 ″	$-\ 4·48=\eta_{111}$	1
112	Oosakazyô	−0·032 ″	+0·769 ″	+1·852 ″	+0·111 ″	$-\ 9·32=\eta_{112}$	½
113	Mikomotozima	−0·006 ″	+0·707 ″	+0·324 ″	+0·054 ″	$-\ 9·12=\eta_{113}$	½
114	Kasaoka	−0·047 ″	+0·769 ″	+2·738 ″	+3·601 ″	$-12·68-\eta_{114}$	½
115	Edazima	−0·055 ″	+0·769 ″	+3·189 ″	+0·270 ″	$-\ 7·80=\eta_{115}$	½
116	Nisibayasi	+0·133 ″	+0·797 ″	+2·440 ″	+0·032 ″	$+\ 0·52=\eta_{116}$	½
117	Simonoseki	−0·065 ″	+0·760 ″	+3·840 ″	−0·048 ″	$-\ 6·72=\eta_{117}$	½
118	Saeki	−0·056 ″	+0·769 ″	+3·415 ″	−0·222 ″	$-\ 3·17=\eta_{118}$	½
119	Nagasaki	−0·056 ″	+0·770 ″	+4·333 ″	−0·048 ″	$-\ 6·95=\eta_{119}$	½
120	Miyazaki	−0·059 ″	+0·770 ″	+3·650 ″	−0·635 ″	$+10·05=\eta_{120}$	½
121	Kagosima	−0·059 ″	+0·771 ″	+4·031 ″	−0·127 ″	$+\ 2·57=\eta_{121}$	½
122	Kasaoka	+0·300 ″	+0·859 ″	+3·900 ″	−0·842 ″	$-\ 2·00=\eta_{122}$	½

The solution of the normal equation:—

[10000 ×] [10000 ×]

$$60·8651\,\xi_0+1·2973\,\eta_0+34·7293\,\frac{da}{a}+16·9588\,\frac{1}{2}\,\frac{de^2}{1-e^2}=-582·5337,$$

$$1·2973\,\xi_0+3·27452\,\eta_0+49·9064\,\frac{da}{a}+9·9784\,\frac{1}{2}\,\frac{de^2}{1-e^2}=+181·6181,$$

$$34·7293\,\xi_0+4·99064\,\eta_0+587·1374\,\frac{da}{a}-107·2847\,\frac{1}{2}\,\frac{de^2}{1-e^2}=-359·4006,$$

$$16·9588\,\xi_0+9·9784\,\eta_0-107·2847\,\frac{da}{a}+93·9366\,\frac{1}{2}\,\frac{de^2}{1-e^2}=-2·9009$$

gives

$$\xi_0=-9''363,$$

$$\eta_0=+6''665,$$

$$10000\,\frac{da}{a}=-0·5552,$$

$$10000\,\frac{1}{2}\,\frac{de^2}{1-e^2}=+0·3664.$$

From these we have

$$da=-354 \text{ meter},$$

$$\frac{a-b}{a}=\frac{1}{296}.$$

IV. Determination of Geoid.

By substituting these in the equations of condition we get the following residuals:—

Table V.

No.	Station	ξ	η	No.	Station	ξ	η
1	Handasawa	+ 3″·8	—	32	Sakae	− 1″·2	− 1″·7
2	Higasiyama	− 3·2	− 1″·8	33	Tobioyama	− 3·5	+23·1
3	Syana	+10·2	− 6·5	34	Maiduru	− 7·9	− 2·2
4	Abasiri	− 6·3	+ 0·8	35	Tanzawayama	− 4·8	+19·9
5	Nemuro	−12·9	+ 4·7	36	Rokudizô	− 2·2	—
6	Sapporo	+ 6·2	+ 7·7	37	Tenzinno	+10.6	+ 1·4
7	Urakawa	−30·0	− 5·6	38	Kenasi	− 6·6	− 4·6
8	Hakodate	+ 2·1	− 4·8	39	Aibano	+13·6	+ 3·9
9	Aomori	+12·3	− 7·4	40	Kandamura	− 3·6	− 4·6
10	Turukotai	+ 9·5	+ 7·5	41	Sengenyama	− 7·9	—
11	Sionohara	+ 1·8	− 2·6	42	Kanôzan	+ 9·3	—
12	Matugasaki	+ 7·5	− 9·9	43	Sanomura	− 6·1	+ 5·0
13	Toriyayama	+ 3·8	—	44	Daibamura	− 3·3	+ 7·3
14	Keizyô	+ 3·3	− 6·9	45	Kwasan	+ 1·7	− 4·8
15	Wazima	+ 7·3	− 9·9	46	Bôdaizan	+ 9·9	—
16	Sakatoyama	+15·6	−19·6	47	Mera	+ 8·3	+ 6·6
17	Nanao	+ 8·4	− 5·0	48	Hamada	+ 5·3	− 1·3
18	Suzaka	+10·1	− 7·7	49	Rokkôzan	+ 1·7	− 1·8
19	Takasuzuyama	−10·0	—	50	Sakabe	− 5·1	+ 3·0
20	Utunomiya	− 0·9	+ 3·7	51	Oosakazyô	+ 6·9	− 4·9
21	Akagi	− 8·0	—	52	Mikomotozima	− 1·8	− 4·5
22	Teruisiyama	− 8·2	—	53	Kasaoka	− 1·6	− 7·3
23	Kariyado	− 1·9	− 1·8	54	Edazima	− 3·6	− 3·9
24	Tukubasan	− 9·7	− 2·2	55	Nisibayasi	+10·6	+ 3·2
25	Kirigamine	+ 9·1	− 4·9	56	Simonoseki	+ 2·0	− 3·2
26	Dôhirayama	+ 2·4	—	57	Saeki	− 4·1	+ 6·8
27	Takagami	−14·6	—	58	Nagasaki	− 6·8	− 3·8
28	Ako	+ 2·2	− 1·8	59	Miyazaki	− 3·0	+13·4
29	Siozakimura	− 5·4	+ 2·4	60	Kagosima	−11·8	+ 5·9
30	Mitaka	− 9·1	+ 8·9	61	Kasanohara	− 0·4	− 1·6
31	Turuga	− 3·4	− 3·3				

The quantities in this table have the property that the sum of their squares is less than that of any other system that can be obtained in connection with any spheroidal surface whatsoever. These are therefore to be understood as the actual inclinations of the surface to that of the spheroid, we have above determined. Assuming then that the distance between geoid and reference ellipsoid is uniformly increasing or decreasing between two successive stations, it is possible to trace the profile in the meridian from ξ_s and the profile in the parallel from η_s, as shown in Fig. 3.

Fig. 2. Triangulation in Japan.

As shown in Fig. 1, the small number of astronomical stations in the intervening regions between the Kwantô and the Kwansai renders it doubtful if the linearity above assumed does actually hold throughout such a long distance. The errors of observation are particularly large in η, which was extensively used in determining the relative height of the geoid in Kwantô and Kwansai, and this makes the obtained height of Kwansai somewhat questionable. As however there are a number of astronomical stations in the vicinity of both Kwantô and Kwansai, the contours drawn for the vicinities of these two regions should be fairly correct.

It is interesting to note that the vertical displacements of bench marks that were detected after the great Kwantô earthquake in 1923, seem to be somewhat correlated with the geoid undulations, ie., the regions where the geoid is higher than the reference ellipsoid seem to have been depressed, while the regions where the geoid is lower than the reference ellipsoid seem to have upheaved.

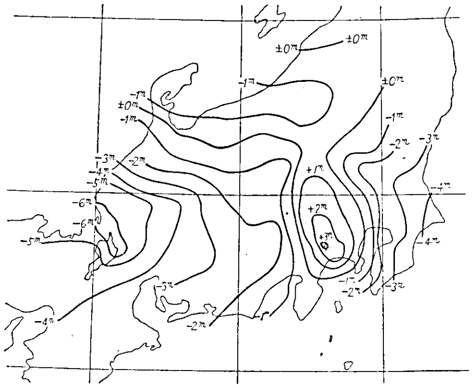

Fig. 3. Geoid in Central Japan.

V. Conclusion.

The conclusion arrived at is as follows :—

(a) The elements of the spheroid that most closely represent the geoid in Japan are

$$da = -354 \text{ meter},$$

$$\alpha = 1 : 296.$$

In other words, the equatorial length is shorter about 354 meter and the ellipticity is somewhat larger compared to the Bessel's ellipsoid.

The absolute deviation of the plumb line at the standard datum is

$$\xi_0 = -9''363,$$

$$\eta_0 = +6''665,$$

where the sign of these quantities is defined by the equation (1), (2), (3).

(b) The geoid in Central Japan, referred to the ellipsoid just determined, is highest at Tanzawayama and lowest at Lake Biwa.

The writer here expresses his best thanks to Professor T. Terada for interest shown in this study.

5. 日本に適合する回轉楕圓體の近似的計算

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日本に於ける垂直線偏倚の量は可成り大きい.

これを測地的計算に採用した準據楕圓體が不適當であつたために起つたものと考へて問題を解いて見る. 卽ち原點に於ける垂直線偏倚の成分 ξ_0, η_0 及び準據楕圓體の赤道半經及び偏率の修正量 da 及び $d\alpha$ を未知數として最小二乘法にかける. その際これらの未知數にかゝる係數は地球を回轉楕圓體と見做した場合と簡單に球と見做した場合とではかなり相違しその相違は原點を遠ざかれば遠ざかる程著しくなるから係數を出來る丈け嚴密に算定する必要がある. 又天體觀測による經度及び方位角の觀測は緯度の觀測に比べて極めて困難を伴ふからこれらを同一重量と考へないで緯度を一樣に 1 とし他を $\frac{1}{2}$ とする. 但し經度及方位角の觀測が同時に存在する樣な點では兩方から求めた η が同じでなければならないから之を平均して緯度と同一重量と見做す. 一方測地的座標は原點に於ける觀測誤差途中に蓄積された觀測の誤差或は geoid が回轉楕圓體でないために起る水平角觀測の誤差など色々な量を含むから大體原點からの距離に逆比例する樣な重量を想定する必要があるがそれらは個人的の研究としては困難である. 斯くして方程式を解いて見ると原點に於ける垂直線偏倚は $\xi_0=-9''36$, $\eta_0=+6''67$ (但し符號は本文方程式 (1) (2), (3) を參照せられたい) となり赤道半經は Bessel の値より 354 m. 短くなり偏率は 1/296 となる.

これらの値を各方程式に入れてその剩餘を求めて見ると, それは今算定した準據楕圓體に對する垂直線偏倚をあらはすから之から geoid を概略畫いて見ると第 3 圖の樣になつて丹澤山で一番高くビワ湖附近で一番低いと言ふ結果が得られる.