*15. Experiments on the Visco-elastic Properties of Pitch-like Materials. (I)**

By Kumizi IIDA.

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Introduction.

As is generally noticed in natural phenomena, to forces long applied, the materials composing the earth's crust exhibit the properties of viscous liquids, and behaving as elastic solids when they are subjected to sudden forces for a short period of time. The former property will be noticed in sedimentary rocks once deposited at the bottom of the sea, but which have since been subjected to folding and in the chronic deformations of the earth's crust as revealed by levellings, and the latter is seen in the propagation of seismic waves.

It will thus be seen that the materials forming the earth's crust possess two different properties, according to the length of time that they are subjected to forces; differences in pressure and temperature being, of course, powerful factors in producing these changes. The author undertook the experiments here described with a view to looking at the matter that is connected with the above described two properties. As a matter of expediency, the experiments were made with such readily accessible substances as commercial black pitch, paraffin, water-glass, and sandstone. The experiments on these substances have in the past been made in order to obtain one of their elastic and viscous properties. The present experiments are different from the past on the point that the author distinguishes clearly the elastic from the viscous properties of a substance.

The physical properties of rocks have been studied by H. Nagaoka[1] (1900), S. Kusakabe[2] (1903), and F. D. Adams and E. G. Coker[3] (1906), so that their elastic properties are now understood. The nature of rocks being so different to that of common metals, no perfectly elastic rock has yet been found, so that Hooke's law does not seem to hold even for

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* Communicated by M. Ishimoto.

1) H. NAGAOKA, *Pub. Imp. Earthq. Inv. Comm.*, 4 (1900), 47~67; *Phil. Mag.*, 50 (1900), 53.

2) S. KUSAKABE, *Publ. Imp. Earthq. Inv. Comm.*, 14 (1903) 1~73; 17 (1904), 1~48, etc.

3) F. D. ADAMS and E. G. COKER., *Carnegie Inst., Washington Publ.*, 46 (1906).

very small forces. Water-glass[4] and "Miduame",[5] the Japanese rice jelly, are substances typical of a viscous liquid, the coefficient of viscosity of which was measured by Stokes's method at various temperatures.

Of pitch-like substances, such as pitch, paraffin, etc., there have been many investigations. All these studies concern their viscosity and seldom their elasticity. The rigidity as well as the viscosity of colloidal solutions have, however, been determined by some investigators. The first investigator to determine it was Th. Schwedoff,[6] who in 1889, examined a single gelatin sol, the method used being that with the concentric cylinder apparatus, as will be described later. He found that the modulus of rigidity in a sol of 0·5 per cent gelatin was 0·535 c. g. s. Later, E. Hatschek and R. S. Jane[7], in 1926, also made detailed investigations by the method just mentioned and found the modulus of rigidity for a sol of 0·3 per cent benzopurpurin at a temperature of 14° C to be 0·75 c. g. s. They later determined the coefficient of viscosity using the Maxwell relation.[8] The apparent viscosity of many colloidal solutions exhibited anomalies in the order of the centipoise when determined by ordinary methods.

The various methods which have been proposed for measuring the internal friction of solids, such as pitch are the following:

B. Weinberg[9] used the method of spiral form torsion or flow in an inclined channel under the action of gravity and obtained $2{\cdot}0\times10^8$ at 15°C or $1{\cdot}9\sim2{\cdot}0\times10^7$ for the internal friction. V. Obermayer,[10] by the shearing of a paralleropiped and compression of a cylindrical disc, obtained $1{\cdot}0\times10^8$ at 15°C and $2{\cdot}7\times10^8$ at 12°C respectively. Trouton[11] obtained $1{\cdot}1\times10^{10}$ by the method of longitudinal extension of a cylinder. The shearing of two adjacent parallelopipeds and tearing of asphalt gave Dudeckij[12] $8{\cdot}1\sim8{\cdot}8\times10^{10}$ at 15°C, while Segel,[13] by the axial shearing of a cylindrical layer of sealing wax, obtained $1{\cdot}1\times10^{10}$ at 19°C.

According to the experimental results of F. T. Trouton and E. S. Andrews,[14] pitch behaves differently from that of an ideally or purely

4) Prof. M. ISHIMOTO has measured the coefficient of viscosity of water-glass at various temperatures and its values are almost the same of the author's results.

5) N. MIYABE, *Bull. Earthq. Res. Inst.*, 12 (1934), 199.

6) Th. SCHWEDOFF, *J. de Physique*, 8 (1889), 341.

7) E. HATSCHEK and R. S. JANE, *Koll. Zeitsch.*, 39 (1926), 300.

8) J. C. MAXWELL, *Phil. Mag.*, 35 (1868), 133.

9) B. WEINBERG, *Proc. Ind. Assoc. Cult. Sci.*, 9 (1926), 215; *Ind. J. Phys.*, 1 (1926~1927), 279.

10) V. OBERMAYER, *Sitzber. Wien. Akad.*, 75 (1877), 665.

11) TROUTON, *Proc. Roy. Soc. Lond.*, 77 (1900), 426.

12) V. D. DUDECKIJ, *J. R. Ph. S.*, 45 (1913), 499.

13) SEGEL, *Phys. Zts.*, 4 (1903), 493.

14) F. T. TROUTON and E. S. ANDREWS, *Proc. Phys. Soc. Lond.*, 19 (1909), 47.

viscous body. They measured the viscosity of pitch at constant temperature by clamping cylinders of it at both ends, one of which was kept fixed while a constant torque was applied to the other. They found also the elastic viscous recovery, but did not give the value of the elastic moduli. They obtained $1{\cdot}3\times10^{10}$ at 15°C for the coefficient of viscosity of pitch. The most complete investigation, as far as temperature range is concerned, was that by Pockettino[15] in 1914 on ordinary black pitch. He determined not only the viscosity coefficient, but also the Young's modulus and the Thomson and Tait coefficient of elastic restitution. Three different methods such as the concentric cylinder method, Stokes', and the capillary tube method, were used in three ranges of temperature, from 9°C to 50°C, 34°$\sim$80°C, and above 80°C, respectively, the results of which fall accurately on a smooth curve, the value of the viscosity coefficient being from $2{\cdot}35\times10^{10}$ at 9°C to $1{\cdot}19\times10^{2}$ at 99·9°C. In elasticity, by comparing the vibrations of tubes filled with pitch at different temperatures, he found the modulus falls from 720 at 9·8°C, to 330 at 32·3°C, to 64 at 44·3°C, and to 0 at 51·8°C. The coefficient of elastic restitution falls continuously, at first slowly, then more and more rapidly, from 0·92 at 10°C to 0·44 at 28·1°C, to 0·12 at 31°C and to 0·0 at 32°C. Between 32°C and 75°C, pitch is plastic, neither a proper solid nor a liquid.

The viscosity of typical pitches has been recently measured by A. B. Manning[16] (1933), using three different methods to cover the range 30°$\sim$110°C. All the coal-tar pitches tested behaved as a truly viscous liquid according to his results. Experiments made with bitumen, that is, petroleum pitch, showed properties differing from those of colloids, such as gelatine, rubber, etc. Bitumen therefore appears to have the structure of a gel just described.

Some investigators[17] studied the effect of pressure on the viscosity of pitch-like substances and found that it increased with pressure.

Since, as shown by the studies of certain investigators, the substances just mentioned possess two different properties, elasticity and viscosity, the author desired to know their physical properties, and to distinguish clearly the elastic from the viscous properties of a substance the present experiments therefore were carried out at certain constant temperature and its effect on their properties was taken into consideration, but not the pressure effect.

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15) A. POCKETTINO, *Nuovo Cimento*, 8 (1914), 77.

16) *Nature Dec.*, 30 (1933), 1009.

17) For example, W. C. RÖNTGEN, *Wied. Ann.*, 45 (1892), 98; C. BARUS, *Phil. Mag.*, 29 (1890), 337

Part I. The Statical Method of Experiments.

1. Theory. We shall first examine the substance by means of the most readily available method—that with the concentric cylinder apparatus.

The upper end of the wire from which an inner cylinder is suspended, is twisted through a constant angle φ; if the substance is merely a viscous liquid, the cylinder rotates with the wire with decreasing velocity until it has moved through the angle φ, i. e. until no torsion is left in the wire. If, however, the substance possesses rigidity, the cylinder does not do so, but only rotates through an angle $\omega(<\varphi)$, and maintains this position for a short time, the elastic deformation of the substance balancing the torque of the wire, which is $k(\varphi-\omega)=k\delta\theta$, k, the torsional moment of the wire per unit angle. It will be noticed that the substance is subjected to a force that is proportional to the rotating angle of the wire.

If the substance is perfectly elastic, the cylinder does not follow the wire, but rotates through an angle θ at the same time that the wire is twisted, and maintains this position for a long time, i. e. θ remains constant.

When the force therefore is proportional to the rotating angle of the wire imparted to the substance, the angular deformation that results must correspond with the property of the substance. This may be held to apply also to the torsional experiment of the cylindrical bar, which, will be dealt with in the next section. It therefore follows that it will be possible to know the property that a substance possesses if we know its angular deformation.

2. The First Experiment. In view of the simple theory mentioned in the preceding section, experiments were carried out statically in order to see whether or not such substances as pitch, paraffin, and a sandstone from Yamaguti, a village in Nagasaki prefecture show viscous property when they are subjected to forces that are proportional to the rotating angle of the wire, for a prolonged period.

(*a*) *The Apparatus.* The apparatus is shown in Fig. 1. The specimen to be tested was prepared in the form of a cylindrical bar by casting the substance in a mould except the sandstone which was cut. The dimensions of the specimens are shown in Table I. The squared end of the specimen (T), was firmly clamped in a vertical position at its two ends to two similar brass parts (A_1), (A_2). A_2 is fixed to the solid frame, A_1 is connected to the steel wire (W), the lower end of which is twisted by the twister lever (R) attached to the wire. The amount of torsion caused in the specimen due to the twisted wire was measured by optically recording the deflection of two lens mirrors (M_1) (M_2), one attached to the rod which moves with the twister lever (R) which is clamped to the lowor end of the wire to enable observation of the fixed image, and the other attached to the pivot to which a magnet (C) is connected in order to magnify the angular deformation of the specimen, assuming that it rotates uniformly. The angular deformation of the specimen was photographically recorded on the drum, 1 meter away from the mirrors.

(*b*) *Observation.* In order to see the angular deformation of the specimen owing to forces that are proportional to the rotating angle of the wire for a prolonged period, the lower end of the wire was twisted at first through a constant angle φ by the twister lever, and

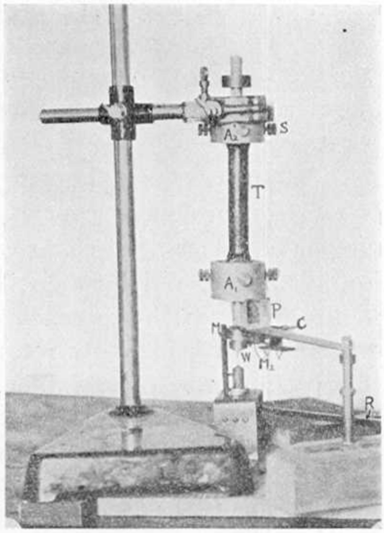


Fig. 1. Twisting apparatus.

T specimen, S screw, A_1, A_2 clamps, W steel wire, M_1, M_2 lens mirrors, P rectangular prism, C magnet, R twister lever.

Table I. Modulus of Rigidity and Apparent Viscosity of Pitch, Paraffin, and Sandstone.

Substance (dimension)	No.	Temp. (°C)	n (c. g. s.)	ε (c. g. s.)
Pitch	1	19·0	$7{\cdot}69\times10^{6}$	$1{\cdot}02\times10^{12}$
(Diameter = $3{\cdot}66^{cm}$	2	19·7	3·71 〃	1·12 〃
Length = $10{\cdot}5^{cm}$)	3	20·5	1·07 〃	$7{\cdot}70\times10^{11}$
	4	20·9	0·90 〃	7·50 〃
Paraffin	1	18·8	$7{\cdot}58\times10^{7}$	$9{\cdot}70\times10^{12}$
	2	20·3	4·37 〃	8·40 〃
(Diameter = $3{\cdot}66^{cm}$	3	20·8	3·26 〃	3·85 〃
Length = $10{\cdot}5^{cm}$)	4	22·6	$7{\cdot}34\times10^{6}$	2·88 〃
	5	22·8	5·61 〃	2·21 〃
Sandstone	1	21·0	$2{\cdot}46\times10^{9}$	$1{\cdot}85\times10^{14}$
	2	21·9	2·38 〃	1·57 〃
(Diameter = $1{\cdot}5^{cm}$	3	22·1	2·34 〃	1·56 〃
Length = $10{\cdot}0^{cm}$)	4	22·5	2·31 〃	1·55 〃

Substance	Water-glass	Pitch	Paraffin	Sandstone
Density	1·69	1·16	0·87	2·51

then allowed to remain in that state for a week. A week's record was thus obtained. The experiment was made in the basement of the Earthquake Research Institute in order to eliminate temperature variation as far as possible. Though it was difficult to keep the temperature constant for a long time, its variation during one week, the duration of the experiment, did not exceed 0·5°C. Since the rigidity of the wire was determined by observing the period of a torsional pendulum, the torsional couple c could be calculated by the formula $c=\dfrac{\pi r^4 n}{2l}\varphi$, where n is the modulus of rigidity of the wire, and l and r the length and radius of the wire respectively.

3. Results. Some examples of relaxation curves (the curves of the angular deformation) of twisted cylinders are shown in Fig. 6, and some

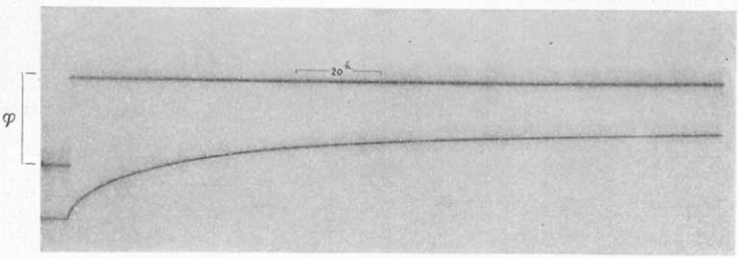


Fig. 2. Record of twisted pitch cylinder (No. 3).

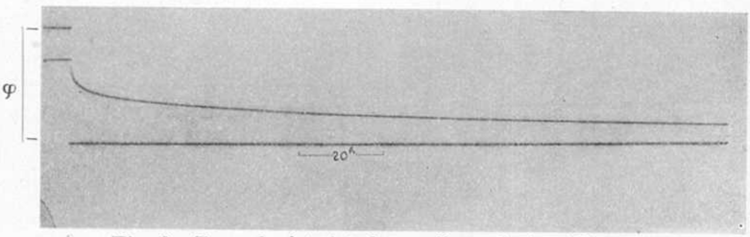


Fig. 3. Record of twisted paraffin cylinder (No. 2).

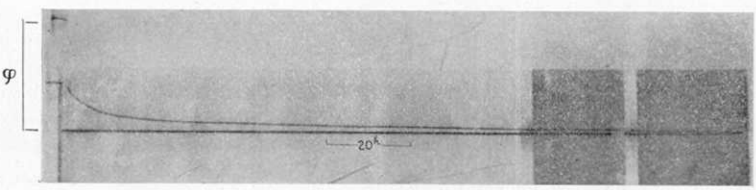


Fig. 4. Record of twisted sandstone cylinder (No. 1).

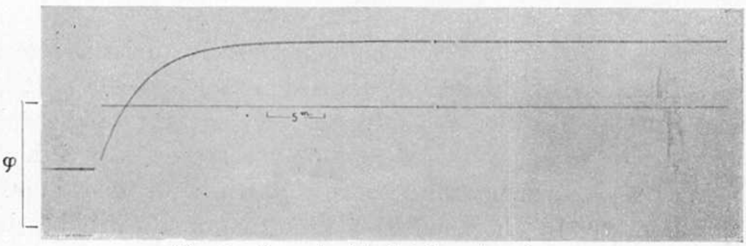
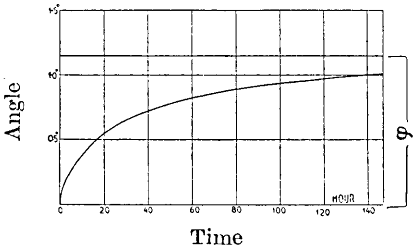


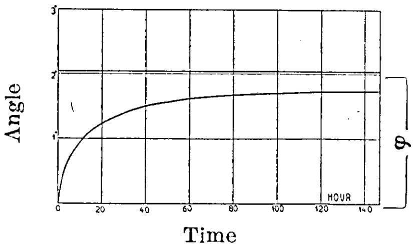
Fig. 5. Record of twisted water-glass.

of their actual records obtained by the apparatus are shown in Figs. 2~5.

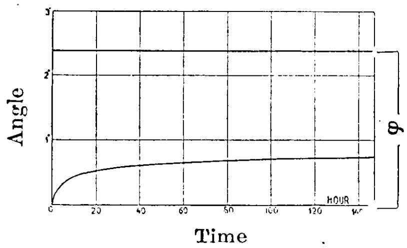
When the torque is first applied there is a rapid flow or movement that gradually subsides and eventually reaches a steady state.



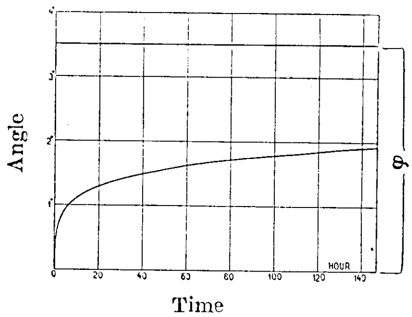
a. Pitch cylinder (No. 1) at 19·0°C. The twisted constant angle $\varphi=1.15°$.



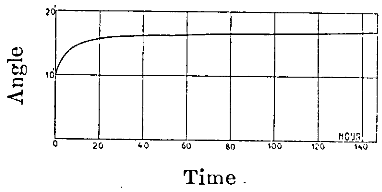
b. Pitch cylinder (No. 4) at 20·9°C. The twisted constant angle $\varphi=2·04°$.



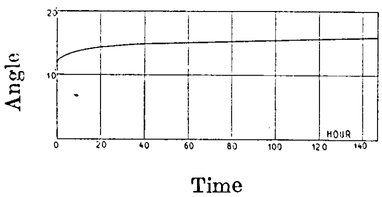
c. Paraffin cylinder (No. 1) at 18.8°C. The twisted constant angle $\varphi=2·39°$



d. Paraffin cylinder (No. 2) at 20·3°C. The twisted constant angle $\varphi=3·51°$.



e. Sandstone cylinder (No. 1) at 21·0°C. The twisted constant angle $\varphi=1·44°$



f. Sandstone cylinder (No. 4) at 22·5°C. The twisted constant angle $\varphi=1·95°$

Fig. 6. Relaxation curves of twisted cylinders. The straight line shows the twisted constant angle φ. In Figs. e and f, the straight line is omitted owing to that the value of φ is very large.

This steady state of deformation would apparently continue indefinitely, but the final state of it approaches the asymptotic definite value, balancing the torque of the wire according to the property of the substance. The mode of deformation therefore differs more or less with different substances. On comparing the above figures with one another, we notice a difference in the modes of deformation as will be described later in detail.

In order to enable comparison with these figures, the deformation of water-glass is shown in Fig. 7. This curve was obtained by means of the concentric cylinder apparatus. Water-glass is so viscous that, in the final state, the inner cylinder follows the quartz fibre, that is, no torsion is left in the quartz fibre. In this, it is like pitch, but not paraffin and sandstone, so that pitch is more viscous than paraffin, while sandstone is the most elastic. When the torque is applied to sandstone the initial stage of angular deformation is so rapid that it will rotate through a certain angle at the same time that the wire is twisted, after which gradual deformation occurs.

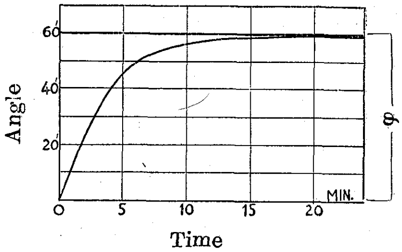
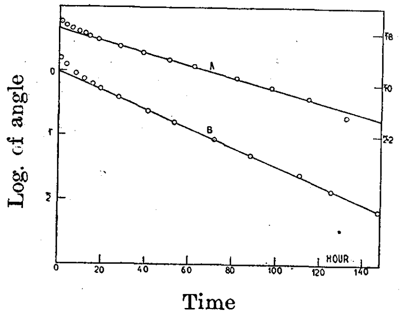


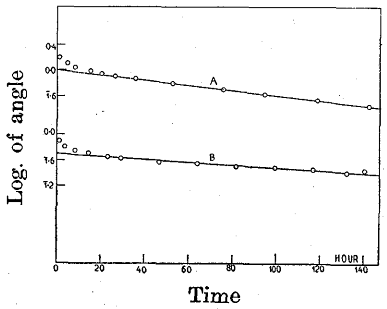
Fig. 7. Relaxation curve of twisted water-glass. The straight line shows the twisted constant angle $\varphi=59'$

It will be seen that all the specimens that were tested possessed more or less the property of a viscous liquid to forces long applied.

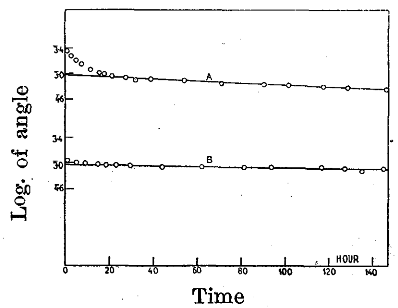
4. Analysis of the Curves. We shall denote the angular deformation of the specimen by θ, and the final value by $\theta_{t=\infty}$. We first tried to see whether or not the angular deformation decreases exponentially



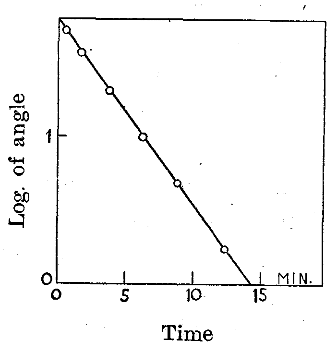
a. Pitch cylinder. A No. 2, B No. 4.



b. Paraffin cylinder. A No. 2, B No. 1.



c. Sandstone cylinder.
A No. 1, B No. 4.



d. Water-glass

Fig. 8. Curves of time and the log. of angle are almost a straight line.

with time. For this purpose the logarithm of θ was plotted as ordinate and the time as abscissa, an example of which is shown in Fig. 8. It will be seen that, except in the initial stage, all the points lie on a straight line, as will be understood, it seems, by considering that part of k that is proportional to the speed of deformation. We may conclude from this the initial curve may be expressed by

$$\theta=\theta_{t=\infty}(1-e^{-at}), \tag{1}$$

where a is a constant and t the time.

In the case of water-glass, $\theta_{t=\infty}$ equals φ, because no torsion is left in the quartz fibre, but $\theta_{t=\infty}$ does not equal the φ of parrafin, sandstone, and pitch, whose difference between $\theta_{t=\infty}$ and φ is very small. We therefore see that these substances possess rigidity as well as viscosity.

5. Mathematics. Assuming that the resistance of the viscosity of the substance is proportional to the velocity of the deformation, we have in this torsional system to deal with the equation of motion. Its elastic force is proportional to the angular deformation, while the external force is proportional to the rotating angle of the wire. We therefore have the equation

$$\varepsilon\frac{d\theta}{dt}+k\theta=C(\varphi-\theta), \tag{2}$$

where ε is the constant that is proportional to the viscosity of the substance, k the elastic constant which is the torsional moment of the substance per unit angle twisted, and C the torsional couple of the wire per unit angle.

The solution of (2) is given by

$$\theta=\frac{C}{C+k}\varphi(1-e^{-\frac{k+C}{\varepsilon}t}) \tag{3}$$

under the initial condition $\theta=0$ at $t=0$.

If now we put

$$\frac{C}{k+C}\varphi=\theta_{t=\infty},\quad \frac{k+C}{\varepsilon}=a \tag{4}$$

equation (3), then changes to

$$\theta=\theta_{t=\infty}(1-e^{-at}).$$

This equation is exactly the same as that of (1) in the preceding section. From (4)

$$k=\frac{C(\varphi-\theta_{t=\infty})}{\theta_{t=\infty}},\qquad \varepsilon=\frac{k+C}{a}. \tag{5}$$

If therefore the values of a, $\theta_{t=\infty}$ and C are known, the apparent rigidity and viscosity will easily be found. Further the modulus of rigidity of a cylindrical bar, n, is calculated by the formula

$$n=\frac{2lk}{\pi r^4}, \tag{6}$$

where l and r are the length and radius of the cylindrical bar, respectively.

6. Calculation of the Coefficients. The modulus of rigidity and the apparent viscosity coefficient can be calculated from equations (5) and (6). For example the calculation of ε and n due to paraffin shown in Fig. 6. are as follows.

On paraffin (No. 1) at 18·8°C

$\varphi=1{\cdot}39°$, $\theta_{t=\infty}=1{\cdot}000°$, $C=5{\cdot}72\times10^6$, $l=10{\cdot}5^{\text{cm}}$, $r=1{\cdot}83^{\text{cm}}$,

$$k=5{\cdot}72\times10^6\times\frac{1{\cdot}39}{1{\cdot}00}=7{\cdot}95\times10^6,\ n=\frac{2l}{\pi r^4}k=9{\cdot}54\times7{\cdot}95\times10^6$$

$$=7{\cdot}58\times10^7\quad a=0{\cdot}00499,\ \varepsilon=\frac{k+C}{a}=\frac{13{\cdot}67\times10^6}{0{\cdot}00499}=9{\cdot}70\times10^{12}.$$

The values obtained by these calculations are shown in Table 1.

7. The Second Experiment. As is generally known, since temperature greatly affects viscosity and rigidity, it is important to see what these effects are. If the temperature rises the cylinder cannot be used, since the cylinder will melt at the high temperature, in which case the concentric cylinder apparatus is used.

(*a*) *Apparatus for the Second Experiment to show Effect of Temperature on Viscosity and Rigidity.* As shown in Fig. 9, the brass cylinder is suspended from a steel wire or quartz fibre (W) coaxially in a cylindrical brass vessel (B), containing the substance to be tested. With rise in temperature, the wire is substituted by one of small diameter or a quartz fibre of diameter 0·41 mm. The upper end of the wire is clamped by the buckle (C) and is twisted through an angle φ by the screw (D), or tangent screw (T). In order to see both the twisted angle of the wire and the rotating angle of the inner cylinder, the lens mirrors (M_1), (M_2) are used. The deflection angles of the mirrors are photographically recorded on the drum, 1 metre away from the mirrors.

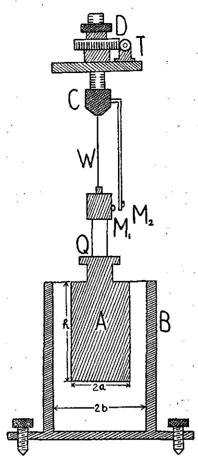


Fig. 9. Concentric cylinder apparatus.

A inner cylinder, B outer cylinder, C buckle, D screw, T tangent screw, M_1, M_2 lens mirrors, W steel wire, Q quartz cylinder, a=1·2 cm., b=1·9cm. h=6 cm.

As the rigidity of the wire or quartz fibre depends on temperature the cylindrical quartz bar (Q) is used to eliminate such effects. In order to regulate the temperature so that it shall remain constant during an experiment, the regulator is connected to the electric utensils, by which the water is heated. Into the water the outer cylinder is submerged, the water being stirred by the stirrer connected to the motor. The temperature was adjusted to be constant within the limits of 0·2°∼0·5°C. The torsional couple of the wire and quartz fibre were determined by the vibrational method.

(b) *Results.* We have treated this experimental results in the same way as shown in section 3. Since in the case of the concentric cylinder apparatus, the moment due to viscous resistance, M, equals $\frac{4\pi h a^2 b^2}{b^2-a^2}\eta\dot{\theta}$, the constant that is proportional to the viscosity of the substance, ε, equals $\frac{4\pi\eta h a^2 b^2}{b^2-a^2}$, where a and b are respectively the radii of the inner and outer clylinder, h the effective height of the inner cylinder, η the coefficient of viscosity, whence

$$\eta=\frac{1}{4\pi h}\frac{b^2-a^2}{a^2b^2}\varepsilon.$$

By means of this equation the viscosity coefficient η can be cal-

Table II. Viscosity Coefficient and Apparent Rigidity of Paraffin.

Temp. (°C)	η (c. g. s.) (Poises)	Log η	k (c. g. s.)	Log k
18·8	14·86×10¹¹	12·17	7·95×10⁶	6·90
20·3	8·00 〃	11·90	4·58 〃	6.66
20·5	5·35 〃	11·73	3·44 〃	6.54
20·8	4·99 〃	11·70	3·42 〃	6·53
21·6	2·43 〃	11·54	1·76 〃	6·25
22·6	3·84×10¹⁰	10·58	7·69×10⁵	5·89
22·8	2·93 〃	10·47	5·88 〃	5·77
29·0	1·22×10⁸	8·09	2·81×10⁴	4·52
32·0	1·81×10⁷	7·26	1·52 〃	4·18
34·0	1·10 〃	7·04	3·88×10³	3·59
37·1	2·18×10⁶	6·34	2·98 〃	3·47
40·3	9·89×10⁵	6·00	3·72×10²	2·57
42·0	6·29 〃	5·80	2·51 〃	2·40
44·5	5·32 〃	5·73	1·59 〃	2·20
46·0	1·66 〃	5·22	1·26 〃	2·10
48·0	1·47 〃	5·17	—	—
50·0	0·42	$\bar{1}$·62	—	—
51·8	0·30	$\bar{1}$·48	—	—
52·5	0·24	$\bar{1}$·38	—	—

Table III. Viscosity Coffiecient and Apparent Rigidity of Pitch.

Temp. (°C)	η (c. g. s) (Poises)	Log η	k (c. g. s)	Log k
19·0	$7{\cdot}57\times10^{11}$	11·88	$1{\cdot}48\times10^{6}$	6·17
19·7	3·92 〃	11·59	1·14 〃	6·06
20·5	3·14 〃	11·50	$9{\cdot}87\times10^{5}$	5·99
20·9	3·10 〃	11·49	9·42 〃	5·97
22·0	2·04 〃	11·31	4·13 〃	5·62
22·4	1·37 〃	11·14	4·12 〃	5·61
30·0	$2{\cdot}08\times10^{10}$	10·32	$5{\cdot}13\times10^{4}$	4·71
34·0	1·29 〃	10·11	2·56 〃	4·41
37·1	$1{\cdot}52\times10^{9}$	9·18	1·59 〃	4·20
44·0	$1{\cdot}69\times10^{8}$	8·23	$6{\cdot}37\times10^{3}$	3·80
48·0	$5{\cdot}31\times10^{7}$	7·73	5·05 〃	3·70
55·1	$8{\cdot}55\times10^{6}$	6·93	3·01 〃	3·48
60·2	2·62 〃	6·42	2·02 〃	3·31
65·0	1·78 〃	6·25	1·63 〃	3.21
70·0	$5{\cdot}72\times10^{5}$	5·76	1·13 〃	3·05
76·5	1·52 〃	5·18	$7{\cdot}73\times10^{2}$	2·89
83·0	$4{\cdot}08\times10^{4}$	4·61	—	—
87·0	2·49 〃	4·40	—	—
90·5	1·66 〃	4·22	—	—
95·1	1·04 〃	4·02	—	—

Table IV. Viscosity Coefficient of Water-glass.

Temp. (°C)	η (c. g. s) (Poises)	Log η	Temp. (°C)	η (c. g. s) (Poises)	Log η
14·3	$6{\cdot}05\times10^{3}$	3·78	32·6	$0{\cdot}23\times10^{2}$	2·37
17·0	3·48 〃	3·54	33·3	0·20 〃	2·31
19·0	2·47 〃	3·39	36·3	0·14 〃	2·16
19·7	2·21 〃	3·34	38·3	0·12 〃	2·07
23·4	1·19 〃	3·08	40·2	$0{\cdot}94\times10^{1}$	1·97
27·3	$0{\cdot}66\times10^{2}$	2·82	40·8	0·87 〃	1·94
27·5	0·56 〃	2·75	42·5	0·75 〃	1·87
28·1	0·50 〃	2·70	44·0	0·61 〃	1·78
31·2	0·34 〃	2·53	47·3	0·44 〃	1·64
31·8	0·30 〃	2·47	48·9	0·37 〃	1·57
32·3	0·25 〃	2·39	50·4	0·33 〃	1·52

culated. The values of the viscosity coefficient and the apparent modulus of rigidity for various temperatures are shown in Tables II–IV

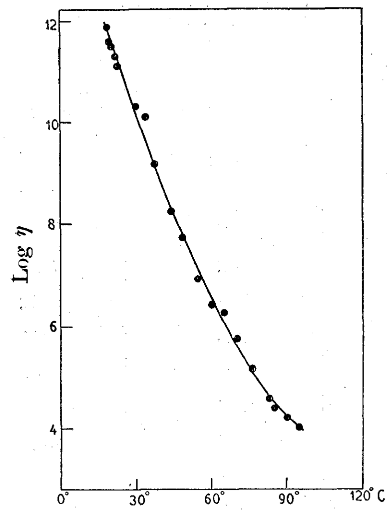


Fig. 10. Curve of temperature and the log. of viscosity coefficient of pitch.

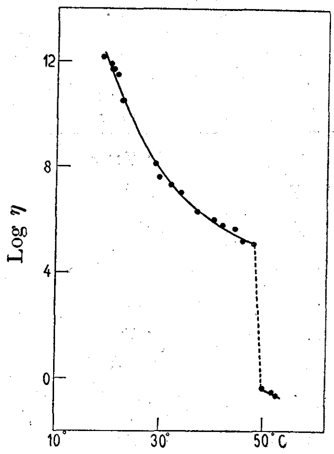


Fig 11. Curve of temperature and the log. of viscosity coefficient of Paraffin.

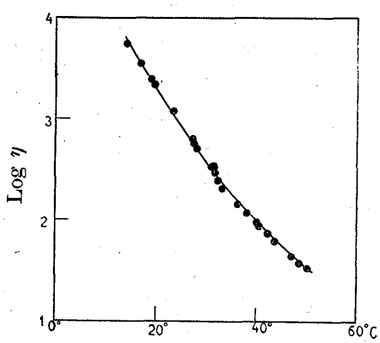


Fig. 12. Curve of temperature and the log. of viscosity coefficient of water-glass.

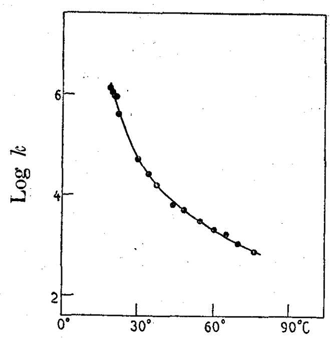


Fig. 13. Curve of temperature and the log. of apparent rigidity of pitch.

and Figs. 10~14. In the figures, the common logarithms of the viscosity are taken as ordinates and the temperature as abscissa. The study with rocks will be described at a future opportunity. As will be seen from these figures, the curve of the temperature and that of the log. of the viscosity of water-glass is almost straight line, whereas those of paraffin and pitch are not so. The rigidities of paraffin and pitch disappear when the temperature reaches values of about 49°C for paraffin and about 80°C for pitch, at which temperatures they change from the solid to the liquid state. It can be noticed that the viscosity of paraffin discontinuously decreases when its melting point is arrived at, whereas no such phenomenon is observed for pitch. When in the course of cooling, the temperature reaches 50·5°C, the solid part of the paraffin that resembles cotton is suspended in the liquid part, but which as soon as the temperature descends below 50°C, solidifies throughout and becomes hard. From this fact, it is ascertained that paraffin is a mixture of substances having various melting points, and so is pitch, although it differs from paraffin in certain respects.

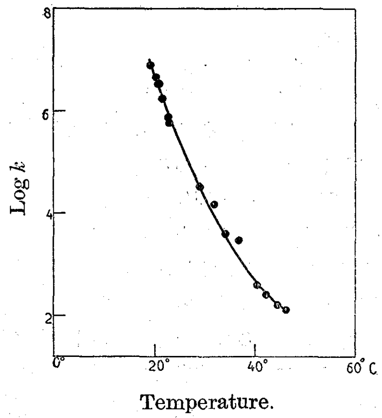


Fig. 14. Curve of temperature and the log. of apparent rigidity of paraffin.

8. Summary and Conclusion to Part I.

1) In the present experiments, the elastic and plastic behaviors of substances such as water-glass, pitch, paraffin, and sandstone were investigated.

2) The angular deformation of these substances caused by a torque that is proportional to the rotating angle of the wire can be expressed by the equation

$$\theta = \frac{c}{c+k}\varphi\left(1 - e^{-\frac{k+c}{\varepsilon}t}\right),$$

where the symbols in the equation have the same meaning as before. By means of this equation, the viscous and elastic characteristics of the substances can separately be known.

3) Both the modulus of rigidity and the viscosity coefficient of the substances decrease with the increasing temperature, the former vanishing at the melting points. The viscosity of paraffin discontinuously decreases when its melting point is arrived at, whereas no such phenomenon is observed for pitch.

4) That these substances show dual properties of viscosity and elasticity may well be explained by assuming that the structure of these substances is somewhat similar to that of sponge, the cavity of elastic skeleton of which is filled with viscous liquid.

5) If the material of the earth's crust is assumed to have similar mechanical properties as the materials investigated in this paper, no sensible stress accumulation will take place in it and the folding of mountains and other kindred phenomena may be explained without difficulty.

In conclusion, the present writer wishes to express his best thanks to Professor Mishio Ishimoto under whose kind guidance and encouragement the present work has been carried out and to Professor Torahiko Terada, Drs. Chûji Tsuboi and Ryûtarô Takahasi for their kind advices.

15. 物質の粘彈性性質に關する實驗 (I)

飯 田 汲 事

地球上に於て觀察する地球物理學關係の現象の中，地殼變動に關する問題及び地震波傳播に關する問題の解決は地殼物質の性質の論議が根本問題となる．從つてこの地殼物質の性質に關する研究が古くから行はれてゐる．

地殼物質は長い間作用する力に對しては粘性ある液體のやうな性質を示すのみならず，短い間比較的急激に作用する力に對しては剛性ある固體のやうな性質を示す事は一般に認められてゐる．近時日本に行はれた水準測量の結果から各地に漫性運動が行はれてゐる事や，嘗ては平坦に海底に沈澱した物質が褶曲する事等は前者の事實を裏書きするものであり，地震波傳播等の現象は後者の事實を裏書きするものである．かくの如く地殼物質には，力の作用時間の長短によつて全く異つた二つの性質がある．筆者はこの種の問題を，手近にて容易に得られるピツチ，パラフィン，砂岩，水ガラス等を用ひて實驗する事によつて確めやうと思ひ，靜力的及動力的の二方法によつて物體の捻れによる變化狀態を實驗してみたのである．以上擧げた物質に關する研究も古くから澤山あるが，何れもその粘性か彈性かの何れかの性質についてのみしか論ぜられてゐない．筆者はこの二性質を同時に考慮して研究したのである．先づ第1報として靜力的に求め得られた主な結果を擧げて見れば次の如くである．

(1) ピツチ，パラフィン，砂岩等は長時間作用する力に對しては粘性高き液體の性質が有力に現れ剛性ある固體のやうな性質が比較的とぼしい．ピツチは最も粘性液體の性質に富み，砂岩は最も剛性ある固體の性質に富んでゐる．

(2) これらの物體に廻轉角に比例する歪力を加へた場合，それによつて生ずる廻轉變形角は

$$\theta=\frac{c}{c+k}\varphi(1-e^{-\frac{k+c}{\varepsilon}t})$$

なる式によつて表す事が出來る．

この式から物質の粘性及彈性性質が別々に求め得られる．

(3) 是等の物體の剛性率及び粘性係數は溫度の上昇と共に減少し，剛性率は溶融點に達すれば消失する．又パラフィンの粘性係數には固體狀態と液體狀態とで非常な差があるが，ピツチには斯の如き現象が見られない．

(4) これらの物質は以上のやうな粘性及び彈性の二性質を有する事から，これらの物質は海綿構造をなしてゐて，その骨骼は彈性體からなり，その間隙は粘性液體で充されてゐると考へる事によつてそれらの性質が容易に說明される．

(5) 地殼物質がこれらの物質と同じ性質を有するならば，歪力の蓄積もあまりに顯著でなく，從つて山脈の褶曲及び其他の同樣な現象が容易に說明し得られる．

動力的方法によつて求められた結果に關しては次の機會に報告する筈である．