54. *Supplemental Note and Corrigendum to my "Study on the Propagation of Seismic Waves. (The second paper)."*[1]

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In Chapter II of the above paper, I have been too hasty in identifying the solution of the equation of motion in isotropic elastic medium which is derived from the vector potential $\boldsymbol{\psi}=\boldsymbol{B}\dfrac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}P^m_n(\cos\theta)\times\cos(m\varphi+\varepsilon)$ by means of the relation $\boldsymbol{\sigma}=\operatorname{rot}\operatorname{rot}\boldsymbol{\psi}e^{ipt}$ with the transverse wave of first kind named by Prof. K. Sezawa. But a little consideration shows the inadequacy of the inference. By means of the Helmholtz's theorem the displacement vector must be expressed by

$$\boldsymbol{\sigma}=(\operatorname{grad}\phi+\operatorname{rot}\boldsymbol{A}),$$

provided $\boldsymbol{A}$ is solenoidal, i. e. $\boldsymbol{A}$ is curl of some vector, so if we take the vector as $\boldsymbol{\psi}e^{ipt}$ ($\boldsymbol{A}=\operatorname{rot}\boldsymbol{\psi}e^{ipt}$), the displacement

$$\boldsymbol{\sigma}=(\operatorname{grad}\phi+\operatorname{rot}\operatorname{rot}\boldsymbol{\psi})\,e^{ipt}$$

obtained in Chapter II must be general solution of equation of motion in isotropic elastic medium. In fact, if we calculate the rotation of displacement

$$\operatorname{rot}\boldsymbol{\sigma}=\operatorname{rot}\operatorname{rot}\operatorname{rot}\boldsymbol{\psi}e^{ipt}=k^2\operatorname{rot}\boldsymbol{\psi}e^{ipt},$$

where

$$\operatorname{rot}_r\boldsymbol{\psi}=B\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\Big[\frac{\sin\theta_0}{2}\{(n+m)(n-m+1)P^{m-1}_n(\cos\theta)\sin(\overline{m-1}\varphi+\varphi_0+\varepsilon)$$
$$-P^{m+1}_n(\cos\theta)\sin(\overline{m+1}\varphi-\varphi_0+\varepsilon)\}-m\cos\theta_0P^m_n(\cos\theta)\sin(m\varphi+\varepsilon)\Big],$$

$$\operatorname{rot}_\theta\boldsymbol{\psi}=\frac{kB}{2n+1}\Big[\frac{\sin\theta_0}{2}\Big\{-\frac{(n+m)(n+m-1)(m-1)}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m-1}_{n-1}(\cos\theta)}{\sin\theta}$$
$$\times\sin(\overline{m-1}\varphi+\varphi_0+\varepsilon)$$

1) H. KAWASUMI, *Bull. Earthq. Res. Inst.*, **11** (1933), 403~453.

$$+\frac{m+1}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m+1}_{n-1}(\cos\theta)}{\sin\theta}\sin(\overline{m+1}\varphi-\varphi_0+\varepsilon)$$

$$+\frac{(n-m+1)(n-m+2)(m-1)}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m-1}_{n+1}(\cos\theta)}{\sin\theta}\sin(\overline{m-1}\varphi+\varphi_0+\varepsilon)$$

$$-\frac{m+1}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m+1}_{n+1}(\cos\theta)}{\sin\theta}\sin(\overline{m+1}\varphi-\varphi_0+\varepsilon)\Big\}$$

$$-m\cos\theta_0\Big\{\frac{n+m}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m}_{n-1}(\cos\theta)}{\sin\theta}\sin(m\varphi+\varepsilon)$$

$$+\frac{n-m+1}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^{m}_{n+1}(\cos\theta)}{\sin\theta}\sin(m\varphi+\varepsilon)\Big\}\Big]$$

$$+\frac{B}{n(n+1)}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}H^{(2)}_{n+\frac{1}{2}}(kr)\}\Big[\frac{\sin\theta_0}{2}\Big\{(n+m)(n-m+1)\frac{dP^{m-1}_{n}(\cos\theta)}{d\theta}$$

$$\times\sin(\overline{m-1}\varphi+\varphi_0+\varepsilon)-\frac{dP^{m+1}_{n}(\cos\theta)}{d\theta}\sin(\overline{m+1}\varphi-\varphi_0+\varepsilon)\Big\}$$

$$-m\cos\theta_0\frac{dP^{m}_{n}(\cos\theta)}{d\theta}\sin(m\varphi+\varepsilon)\Big],$$

$$\mathrm{rot}_\varphi\psi=\frac{kB}{2n+1}\Big[\frac{\sin\theta_0}{2}\Big\{-\frac{(n+m)(n+m-1)}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m-1}_{n-1}(\cos\theta)}{d\theta}$$

$$\times\cos(\overline{m-1}\varphi+\varphi_0+\varepsilon)$$

$$+\frac{1}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m+1}_{n-1}(\cos\theta)}{d\theta}\cos(\overline{m+1}\varphi-\varphi_0+\varepsilon)$$

$$+\frac{(n-m+1)(n-m+2)}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m-1}_{n+1}(\cos\theta)}{d\theta}\cos(\overline{m-1}\varphi+\varphi_0+\varepsilon)$$

$$-\frac{1}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m+1}_{n+1}(\cos\theta)}{d\theta}\cos(\overline{m+1}\varphi-\varphi_0+\varepsilon)\Big\}$$

$$-\cos\theta_0\Big\{\frac{n+m}{n}\frac{H^{(2)}_{n-\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m}_{n-1}(\cos\theta)}{d\theta}\cos(m\varphi+\varepsilon)$$

$$+\frac{n-m+1}{n+1}\frac{H^{(2)}_{n+1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP^{m}_{n+1}(\cos\theta)}{d\theta}\cos(m\varphi+\varepsilon)\Big\}\Big]$$

$$+\frac{B}{n(n+1)}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}H^{(2)}_{n+\frac{1}{2}}(kr)\}\Bigg[\frac{\sin\theta_0}{2}\Big\{(n+m)(n-m+1)(m-1)\frac{P_n^{m-1}(\cos\theta)}{\sin\theta}$$

$$\times\cos(\overline{m-1}\varphi+\varphi_0+\varepsilon)-(m+1)\frac{P_n^{m+1}(\cos\theta)}{\sin\theta}\cos(\overline{m+1}\varphi-\varphi_0+\varepsilon)\Big\}$$

$$-m^2\cos\theta_0\frac{P_n^m(\cos\theta_0)}{\sin\theta}\cos(m\varphi+\varepsilon)\Bigg],$$

we find the existence of r-component, and the displacement related with $\mathrm{rot}_r\,\sigma$ and the terms in the second clotchets of $\mathrm{rot}_\theta\,\sigma$ and $\mathrm{rot}_\varphi\,\sigma$ are the transverse waves of second kind, and the terms in the first clotchets of $\mathrm{rot}_\theta\,\sigma$ and $\mathrm{rot}_\varphi\,\sigma$ correspond to the transverse wave of first kind of Prof. K. Sezawa.[2] Thus we see the solution in chapter II is a general solution.

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54. 地震波の傳播（第 2 報）補遺及び訂正

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上記論文第 II 章に於て筆者の求めた等方彈性體を傳はる彈性横波の解を妹澤教授の第 1 種の横波のみに當ると述べたが調べて見ると第 2 種の横波も含む一般解である事が解つた．尚筆者の解と妹澤教授の解との關係をも明にした．

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2) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **10** (1932), 299～334.
K. SEZAWA, "Sindô-gaku," (1932), 653～657.