30. *On the Stability of Continental Crust.*

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(Read April 17, 1934.—Received June 19, 1934.)

It is generally believed that the earth was initially covered with a hydrosphere of nearly uniform depth, i. e. panthalassa, and subsequently the superficial layer of the lithosphere was fractured here and there, and gradually underwent areal contraction and folding, until the present division of continents and oceans was produced. As to the mechanism by which the contraction, folding and horizontal drift of the continental mass was effected, different hypotheses have already been proposed by previous investigators. Among the mechanical agencies which may act upon the detached piece of the earth's crust the tidal force and the so-called "Polflucht Kraft" have often been quoted as important factors to be taken into account, especially in connection with the land-drift theory of Wegener. It seems, however, difficult to explain by these agencies the enormous horizontal force which must have been expended in building up the principal mountain ranges of the world. On the other hand, it seems to the author that an important factor has been hitherto overlooked which must in any case be subjected to a serious consideration with regards to the stability of the continental mass, though at first sight it may appear rather trivial. In the following, a brief sketch will be given of the consideration which led the author to infer that a uniform concentric spherical crust shell on the earth cannot be absolutely stable but may tend to crumple up into detached pieces due to its own gravitation, provided that a certain necessary condition be fulfilled.

To begin with, let us take a highly idealized case in which the earth consists of a rigid core covered with a uniform surface layer of an ideal crust mass. Assume first that this crust is made of an ideal gravitating substance utterly devoid of elasticity, i. e. with very small rigidity and indefinite compressibility. If such a crust be compressed laterally in such a manner that its thickness is kept constant while its area is decreased, the height of the crust mass above the core remains unaltered, so that the gravitational potential energy of the crust due to the core remains constant, whereas the potential energy due to itself is considerable decreased by this compression because the mass elements are approached mutually. Thus it will be seen that, in absence of elasticity, such a crust will contract indefinitely into a local nucleus, or a number of nuclei, lying on the naked surface of the rigid core.

In the case when the crust is elastic, the tendency of the crust mass to decrease its own potential energy by contraction and concentration will be checked on one hand by the increase of elastic potential energy and on the other hand by the increase of the gravitational potential energy due to the rise of the centre of gavity of thus contracted mass. The question still remains, however, whether it will not be possible that the decrease of the self potential energy due to the local concentration of the crust mass becomes of prevailing influence under some favourable circumstance which may counteract or relieve the increase of potential energy accompanying the process of local accumulation of the crust mass.

Before proceeding further, it will be convenient to get an idea of the relative order of magnitude of the potential energy for the different kinds of configulation of the crust mass. A rough estimation will be made as follows.

Let R (Fig. 1) be the radius of the rigid spherical core with the density ρ. The crust is of a concentric spherical shell with thickness $2d$ and density ρ_c. Assume that this uniform crust is gathered up and moulded into a sphere with the radius a, the volume being kept constant during this transformation. Considering d small in comparison with R, we have

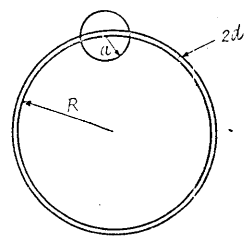

Fig. 1.

$$a^3=6(R+d)^2d.$$

The potential energy of the crust mass due to itself decreases by this change of configuration from its initial value P_d for the spherical shell to its final value P_a for the contracted small sphere with radius a, where

$$\frac{P_a}{\gamma\pi^2\rho_0^2R^5}=-32\left(\frac{a}{R}\right)^5,$$

$$\frac{P_d}{\gamma\pi^2\rho_0R^5}=-64\left(\frac{d}{R}\right)^2=-\frac{16}{9}\left(\frac{a}{R}\right)^6.$$

Thus, assuming a/R small P_a is negligible in comparison with P_a so that, if we denote the decrease of potential energy due to the local concentration by D, we have

$$D=32\left(\frac{a}{R}\right)^5\cdot\gamma\pi^2\rho_0^2R^5.$$

On the other hand, the potential energy of the crust mass due to the attraction of the core is increased by the rise of the centre of gravity of the crust mass thus gathered up, unless this mass does not sink into the core. Assume now that the part of the core just below the contracted crust mass above supposed is liquefied so that the spherical crust mass may sink down until the hydrostatical equilibrium is attained. For the simplicity's sake, assume that the density of the core is just twice that of the crust, i. e., $\rho=2\rho_0$, in which case the centre of the floating sphere will be on the same level with the surface of the core. The core mass displaced by the sinking crust sphere must flow out on the surface of the core. The increase of the potential energy due to this displacement is equivalent to that due to the rise of the displaced core mass by the height equal to the height of the core surface above the centre of volume of the displaced hemisphere. Denoting this increase of potential energy by I, we have for rough approximation

$$I=\frac{1}{3}\left(\frac{a}{R}\right)^4\cdot\gamma\pi^2\rho^2R^5.$$

Thus, the increase of potential energy, I, is decidedly greater than the decrease of potential energy, D, due to the local accumulation of the crust mass, as might have been expected from the beginning. Thus, it is evident that the crustal shell cannot crumple up of its own accord.

Here the question arises whether the stability of the uniform crust holds also good when the core is gradually contracting due to the secular cooling of the earth. If the liquefied core mass which is to be displaced by the sinking spherical crust mass in the above hypothetical process be drained into some cavity which is formed by the thermal contraction of the superficial layer of the core, instead of flowing out over the noncontracting core, the centre of mass of the displaced core material will be saved from rising against the attraction of the core, or even may be made to fall down, provided that the rate of cooling is suitably adjusted.

The necessary amount of cooling required for relieving the upheaval of the displaced core mass may be estimated as follows. Suppose that the core is composed of solid and that the contraction of the superficial layers results in opening a circular hole in each elementary layer such that there is formed a hemispherical cavity on the surface of the core, which receives the sphere moulded with the contracted crust mass above mentioned, as if the sphere has sunk down into the liquid core. The amount of cooling required for such a mode of contraction may be calculated. If the depth below the surface of the core be denoted by z, the areal contraction required of the layer at z will be

$$A(z) = \pi(a^2 - z^2).$$

If the fall of temperature as the function of z be $T(z)$ and the coefficient of areal expansion be α, we have

$$A(z) = A\pi R^2\left(1 - \frac{2z}{R}\right)\alpha\, T(z).$$

Thus

$$\alpha\, T(z) \fallingdotseq \left(\frac{a}{R}\right)^2\left(1 - \frac{z^2}{a^2}\right).$$

Even if we assume a so large that $a/R = 0{\cdot}2$, we will have for $z = 0$, i. e. at the surface

$$T_0 = 5°C,$$

assuming $\alpha = 8 \times 10^{-3}$. It will be seen that only a small amount of cooling suffices to prevent the rise of the centre of gravity of the displaced core mass.

The above estimation was made on the assumption that the core is of solid merely for the sake of convenience. The order of magnitude of the cooling wanted for the superficial layers of the core will not be altered if the core be of fluid nature though the matter may be complicated by the consideration of convection.

The above hypothetical mechanism may be compared with the following model in some essential respects.

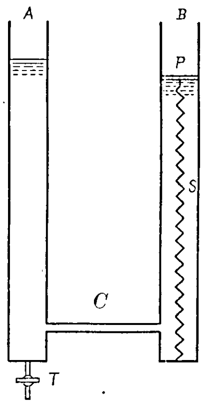

Fig. 2.

Fig. 2 shows a system consisting of two water tank A and B communicating with each other by means of a tube C. P is a piston smoothly fitting into the cylindrical wall of B. A long weak spring S is stretched between the piston and the bottom of the tank B. Suppose that P is in equilibrium under the force due to the spring and the upward pressure due to the level difference of water in A and B. In this case the spring cannot contract further because the decrease of the elastic potential energy by contraction is overbalanced by the increase of the gravitational potential energy. The matter becomes different, however, when the tap T at the bottom of A is opened and the water is made to leak out. Then P will fall and S will contract.

Thus far in the above consideration, the elasticity of the crust mass has entirely been put out of account. In the problem, however, of such a kind as is here concerned where the gravitational equilibrium of a large mass with the size of a continent is dealt with, the elasticity will entirely fall into background, as the mass here in question is indeed of the same order of magnitude as our moon, or some of the largest asteroids. The asteroid Eros of which the mean diameter is of the order of some 30 km., is believed to be of a decidely irregular form far from being nearly spherical. Here, it seems, the elastic force is still predominant over gravity. On the other hand, Ceres and Pallas which are of a few hundred km. in radii seem to be of a nearly spherical form. Hence, in the case of a rocky mass with a volume comparable with a continental crust, it will not be absurd to assume that such a mass will crumple down under the gravitational stress and assume a nearly spherical shape, if left to itself in the free space.

Again, in the above consideration, the cooling of the substratum underlying the crust mass has been supposed to take place entirely independent of the gravitational contraction or accumulation of the crust mass. The circumstance will be become more favourable for the said contraction, if the cooling of the substratum be physically connected in some way or others with the process of contraction. Such a connection is in fact well conceivable as may be seen from the following consider-tion.

If in the crust mass, say "sial" in Wegener's theory, originally covering the highly viscous fluid substratum, say "sima," with a uniform thickness, a gap or chasm be opened owing to its fracture by contraction, the sima will rise and fill up the gap to a height determined by the condition of hydrostatical equilibrium. This is not, however, the only consequence due to the opening of the chasm. The equipotential surface of gravity will be disturbed on account of the change in the configuration of the gravitating matter. The gravity under the chasm will become less than the gravity underneath the sial cover at the same distance from the centre of the earth. The result is that a kind of convection current is brought forth. The sima will rise under the gap and the resulting circulation will produce a horizontal current which is centrifugal in the upper layer and centripetal in the lower. The diverging current of the upper layer will exert a horizontal dragging force upon the margin of the sial crust, namely in such a sense as to widen the gap already opened. Similarly, if we consider a sial continent floating upon the fluid sima, the gravity under the continent will be greater than under sima at the same distance from the earth's centre, so that the convection produced will act in the sense such as to drag the margin of the continent horizontally towards the inside of the land area.

In the sima substratum the density will certainly increase with the depth so that the circulation will be stopped when the equipotential surface will become parallel to the surface of equal density. If, however, there may take place some kind of change of density of the sima material due to the change of pressure, such that its density increases with the pressure, either due to its compressibility or due to some chemical transformation, the circulation may go on still further. A further discussion on this line cannot be made at present without introducing fortuitous assumptions. Still, it will be seen that the circulation of the kind above supposed may at least become a not quite unimportant factor in determining the life history of a continental mass.

The horizontal dragging force acting upon the margin of continent will tend on one hand to cause the folding of the sial crust. The folding will especially be conspicuous at the marginal region. This consideration seems to be applicable for the explanation of the formation of Circum-Pacific mountain ranges. It is believed by many that the bed of the Atlantic Ocean is partially covered with a remnant sial crust, while the Pacific Ocean bed is completely covered with naked sima. These states of matter taken as admitted, we may expect that the marginal dragging force will be less along the Atlantic coast than along the Pacific coast and this may explain the less conspicuous development of folded mountain ranges along the former coast.

If there exist such a flow of the fluid sima from the ocean bed towards the base of the continent, it may well happen that the skin layer of sima near the continental margin is dragged down such as to form a depressed zone just in front of the coast line. It is not improbable that the numerous deeps in front of the Circum-Pacific mountain ranges have been produced in this manner.

If the horizontal dragging force at the margin of a continent be larger on one side than on the other, the result may appear as a drift of the continent as a whole. Such a possibility must, it seems, also be taken into consideration when we are dealing with the question of land-drift, though the other kinds of force may in some case be predominant in determining the sense of drifting motion. Indeed, the difference of dragging force between the Pacific and Atlantic side as above explained will result in moving American continent towards Europe and Africa, so that we must admit the existence of other kind of force playing an important role in determining the relative motion of continental masses, if we assume that the Atlantic Ocean has been widened at all in some past age.

The circulation of sima substratum as above supposed will be accompanied with a vertical exchange of heat by the sinking of the upper cooler mass and the welling up of the lower warmer mass. This enhanced cooling of the sima layer physically connected with the deformation of the continental sial mass is just what is favourable for the gravitational contraction as explained above.

A detailed calculation of the pressure difference which may give rise to the circulation of the sima substratum when a gap is opened in the sial crust, or when a sial continent is floated upon the sima layer, is not easy. We may, however, make a rough estimation of the order of magnitude of the maximum pressure difference produced when a circular cylindrical hole is opened in the sial crust in the following manner.

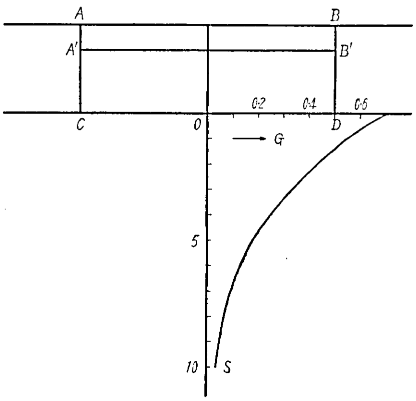

Fig. 3

Let AB in Fig. 3 represent the earth's surface and CD the boundary between the crustal layer with the density ρ_0 and the substratum with the density ρ. Suppose that a circular disc $ABCD$ is cut out from the crust and the gap thus formed is partly filled up by the subcrustal denser mass up to the level $A'B'$ such that

$$\rho_0 AC = \rho A'C,$$

corresponding to the requirement of isostasy. The change in the value of gravity at S on the axis of the circular disc is given by Δg, where

$$\frac{\Delta g}{2\gamma\pi}=(\rho-\rho_0)\overline{SC}+\rho_0\overline{SA}-\rho\overline{SA'}=G.$$

The value of G was calculated and plotted in Fig. 3 for the special case $AB=10$ cm., $AC=BD=3{\cdot}5$ cm. and $A'C=B'D=2{\cdot}5$ cm., corresponding to the assumption $\rho_0=2{\cdot}5$ and $\rho=3{\cdot}5$. The mean value of G down to the depth of 10 cm. is estimated at $G_m=0{\cdot}24$. If the dimension of the figure be magnified such that $AC=BD$ represents the actual thickness of the earth's crust, say 100 km., the corresponding mean value of G which we denote by G_m' will be

$$G_m'=0{\cdot}686\times10^6,$$

and the corresponding difference in the value of gravity is

$$\Delta g_m'=0{\cdot}286 \text{ c. g. s.}$$

The depth of S at 10 cm. below CD in the above figure corresponds to a depth of 286 km. in the case of the earth. Assuming the uniform density of 3·5 down to this depth, the pressure difference at this depth between the point S and the other point on the same level with S widely apart from it, is

$$\Delta P=2{\cdot}86\times10^7 \text{ dyne/cm}^2.$$

If this ΔP be compared with the pressure of a column of rock with $\rho_0=2{\cdot}5$ and the height H cm., we have

$$H\rho_0 g=\Delta P.$$

Taking for g the normal value on the earth suface, we obtain

$$H=1{\cdot}17\times10^4 \text{ cm.,} \quad \text{or} \quad 117 \text{ m.}$$

Considering that a mountain range with a height of 1000 m. seems to be nearly in isostatic compensation on the present earth's surface, it will not be too far fetched to assume that the pressure difference above calculated which is equivalent to that due to a mountain of 117 m. height, may induce the flow of the subcrustal sima stratum during a long time comparable with geological ages.

The present note is written with the mere purpose of drawing the attention of geophysicist towards a problem which seems to have not yet been duly investigated in spite of its important bearing upon the many fundamental problems of geophysics and geology.

30. 大陸地殼の安定度に就て

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ウエーゲナーの假説で假想せられるやうにシアル大陸がシマの海の上に浮んで居ると考へた場合に，板狀の大陸塊が重力的に絕對に安定であるかどうかと考へて見ると，必しもさうでない場合があり得るのである．それは次のやうに考へて見ればわかる．

もし板狀の大陸塊が宇宙間に浮んで居るとすると，それが小さい場合には剛性の爲にその形を保持するが，小遊星中の大きなもの位の容積になると重力による歪力が剛性の抵抗に打勝つて崩壞し自己重力による位置のエネルギーを最小にする樣に球形に近づかうとするのである．地球上に於ける大陸板でも若し地球內核の引力がなければ大陸は集積して球狀にならうとする譯であるが，併し集積する爲にはその大陸の重心が地心に對して高まりそれによる位置エネルギーの增加が集積による陸塊自身の位置エネルギーの減少よりも大きくなるから，さういふ集積作用は阻止され，從つて板狀大陸は安定であり得る譯である．然るに，もしも，地球自身或は表面に近いシマ層の冷却の爲にその表面水準が降下しつゝある場合には事柄はさう簡單でなくなる．卽ち，若し，大陸塊が自己重力で集積する爲に必要な重心の上昇と同じだけ，或はそれ以上の大さだけシマの表面が落下しつゝあるといふ特別の場合には，陸塊の自巳集積は必しも阻止されなくてもすむ譯である．

以上は陸塊の集積と地球の冷却とを全然無關係と考へても云はれることであるが，若し陸塊の集積作用と冷却とが物理的に連關してゐる場合には集積は一層必然的になる譯である．ところがかういふ連關は必しも考へられなくはない．卽ち，陸塊が浮んで居るとすると，その下底の或る深さに於ける重力は，陸塊のない部分の同じ深さの重力よりも大きいからその爲に陸塊の下で對流を生じ得る場合が考へられる．もしこのやうな對流が起るとすれば，その結果は第1には地球表層の冷却を助け，第2には大陸の緣邊を水平に壓してその面積を縮小させやうとする力を生じる．卽ち造山力の如きものを生ずる．同時に又大陸緣邊の海溝の如きをものも作り得ると考へられる．同じ力は又それが大陸塊に對して非對稱的に働けば大陸の水平移動をも生じ得る．

從來の色々な學說では，造山力や又大陸移動の原動力の大さを說明するに困難を感じる場合が多かつたやうであるが．現在の考からするとすべての力は重力そのものゝ有限な分數と考へられるので量の大さに關する困難が除去される見込が多分にあると考へられる．併し正式な數量的取扱は數理物理學者の手を煩はさなければならない．

玆では，具體的な不安定の機構を假定するのが目的ではなくて，唯エネルギー的に考へて陸塊は必しも安定でなく，次第にその面積を縮少するといふことが可能であるといふことを指摘して地球物理學者の注意を促がしたいと思つただけである．