33. *On the Magnetic Damper.*

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1. Introduction.

When a metal swings in a magnetic field that is non-uniform or discontinuous, the damping force (the retarding force proportional to the velocity) is generated by Foucault currents induced in the metal. Walterhoffen[1] was the first to devise an apparatus showing skilfully the damping effect of Foucault currents, while Galitzin[2] applied the damping to his Zöllner pendulum seismometer, in which a horizontal copper plate moves in the field produced by a pair of strong horse-shoe magnets fixed above and below it. Heretofore magnetic damper of the Galitzin type have been used for pendulums of long period. The magnetic damper has the advantage over other dampers in having a damping force that is strictly proportional to the velocity, but as its damping force is comparatively small, it has only a limited application and is consequently not used except in the case of pendulums of comparatively large period. By using an electromagnet a strong field can be obtained and the damping force becomes strong, but difficulties arise in the case of continuous observations. The study of magnets, however, has advanced remarkably of late, and materials with large coercive force have been made availabe, so that by their use stronger magnetic fields can be obtained, which enables the magnetic damper to be used in various ways.

In designing a magnetic damper, we must predetermine the magnitude of the damping force that is to be produced by a given magnetic field and a given size of metallic plate. Galitzin investigated the damping force of his magnetic damper, changing the distance of the poles and the thickness of the plate, but without touching on the

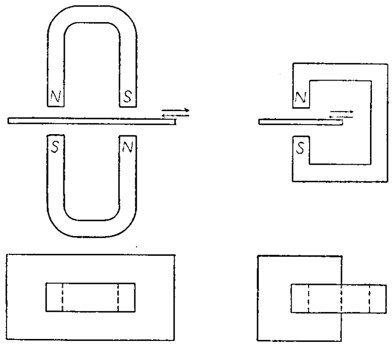
1) A. v. WALTERHOFFEN, *Wied. Ann.*, **19** (1883), 928.
2) B. GALITZIN, *Acad. Sci., St. Petersburg*, **9** (1908), 673.

general problems. Suda[3] experimented with a magnetic damper of Galitzin type and wrote on some subjects. In designing a magnetic damper the effect of the area of the metallic plate must be known. The problem is discussed in this paper.

2. The Mathematics of a Simple Case.

The magnetic damper of a seismometer generally has the form shown in Fig. 1; *a*, *b*. Since type *a* is equal to two of type *b* arranged side by side, we need consider only type *b*. We have the following general electro-magnetic equation:

$$\mathrm{rot}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \ldots.(1)$$



a *b*

Fig. 1. Two types of magnetic damper.

true everywhere in conductive and non-conductive media, where $\boldsymbol{E}$[4] is the electric potential gradient, $\boldsymbol{B}$ the magnetic flux density, and t the time (all in c. g. s. elctromagnetic units). If the change of phenomena with respect to time is not very rapid, the following relation holds in a conductive medium:—

$$\boldsymbol{i} = \sigma \boldsymbol{E}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

where $\boldsymbol{i}$ is the electric current density, and σ the specific electric conductivity of the medium. We have therefore the equation

$$\mathrm{rot}\,\boldsymbol{i} = -\sigma \frac{\partial \boldsymbol{B}}{\partial t} \qquad \ldots\ldots\ \ldots\ldots\ldots\ldots.(3)$$

true for any point in the conductive medium, the meaning of which is that rotation of electric current is induced in the conductor at the place where the magnetic flux density changes with time. We shall now consider the case in which a metallic plate moves in the field that is produced between the poles of a magnet of rectangular cross section, as shown in Fig. 1, *b*. For simplicity we shall assume that the magnetic flux density between the poles is everywhere uniform and zero in the space outside it, and also that the area of the metallic plate is infinitely

3) K. SUDA, *Umi to Sora* (海と空), 2 (1922).

4) The letter of gothic type represents a vector quantity.

large We can then easily calculate the retarding force that acts on the metallic plate. We assume that the metallic plate is crossed by the magnetic flux in the area $ABCD$ shown in Fig. 2, that the sense of the magnetic flux in the figure, points upwards, and that the plate moves toward the left with velocity v. The changes in the flux density $\boldsymbol{B}$ crossing the plate then occur at the two margins AB and CD, but in the space inside $ABCD$, $\boldsymbol{B}=\text{constant}=\boldsymbol{B}_0$, and outside it, $\boldsymbol{B}=0$. If we denote the line element of AB by da, then we have

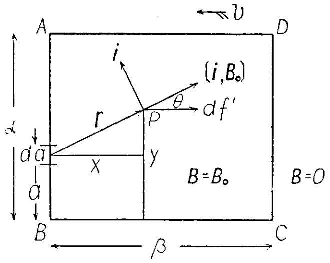


Fig. 2.

$$\frac{\partial \boldsymbol{B}}{\partial t} da = -\boldsymbol{B}_0 v\, da, \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

that is,

$$\operatorname{rot} \boldsymbol{i}\, da = \sigma \boldsymbol{B}_0 v\, da, \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

at the margin AB. And

$$\operatorname{rot} \boldsymbol{i} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

except on lines AB and CD. Hence the current density $\boldsymbol{i}$ produced in the plate at distance $\boldsymbol{r}$ from da is

$$\boldsymbol{i} = \frac{[\operatorname{rot} \boldsymbol{i}, \boldsymbol{r}]\, da}{2\pi r^2}$$

$$= \frac{[\boldsymbol{B}_0, \boldsymbol{r}]\, v\sigma\, da}{2\pi r^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

When the electric currents flow in a magnetic field the mechanical force acts on the conductor. This is expressed by $\boldsymbol{f}=[\boldsymbol{i}, \boldsymbol{B}]$, where $\boldsymbol{f}$ is the mechanical force. The retarding force acting on the elemental volume of the metallic plate $Ddxdy$ (D is the thickness of the plate) therefore is

$$df' = \frac{B_0^2 v\sigma D \cos\theta\, da\, dx\, dy}{2\pi r}$$

$$= \frac{B_0^2 v\sigma D x\, da\, dx\, dy}{2\pi\{x^2+(y-a)^2\}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

So that the retarding force f arising from currents induced by the magnetic flux changes at line AB is

$$f' = \frac{B_0^2 v\sigma D}{2\pi} \int_0^{\alpha} da \int_0^{\beta} dx \int_0^{\alpha} \frac{x\, dy}{x^2+(y-a)^2}, \quad \ldots\ldots (9)$$

where α and β are respectively the longitudinal and transversal lengths of the rectangular field. The retarding force is produced by the magnetic change at line CD in the same manner, and as a result the total retarding force f becomes $2f'$. Hence

$$f = CB_0^2 v\sigma D\alpha\beta = \frac{C\phi^2 v\sigma D}{S}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

and

$$C = \frac{1}{\pi}\left\{2\tan^{-1}\frac{\alpha}{\beta} + \frac{1}{2}\frac{\alpha}{\beta}\log_e\left(1+\frac{\beta^2}{\alpha^2}\right) - \frac{1}{2}\frac{\beta}{\alpha}\log_e\left(1+\frac{\alpha^2}{\beta^2}\right)\right\}, \qquad (11)$$

where ϕ is the total magnetic flux and S is the area of the field. As will be seen from the expression, the retarding force is proportional to the velocity of the plate, to the square of the total flux of the field, to the thickness of the plate, and to the specific conductivity of the plate, but inversely proportional to the area of the field. The numerical value of the dimensionless quantity C depends on the ratio of α and β, that is, the geometrical size of the pole of the magnet.

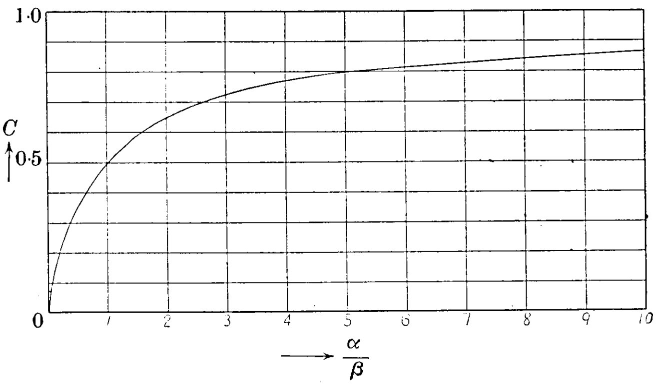


Fig. 3. Value of C when the size of the metallic plate is infinitely large. (α and β are respectively longitudinal and transversal lengths of the field.)

The numerical values of C corresponding to various values of α/β are shown in Fig. 3. The value of C increases with that of α/β, and $C=0{\cdot}5$ when $\alpha/\beta=1$; that is, the sectional size of the pole is a square, and it tends to 1 when α/β becomes infinite. For this reason, in order to obtain larger damping force it is necessary that the magnetic field shall be contracted and that the pole of the magnet shall be of a shape with its longer side in the direction perpendicular to the motion of the metallic plate.

In the above we calculated the damping force when the area of the plate is infinitely large. However, even should the area of the plate be finite, the forms of the obtained equation (10) will be maintained, only the value of C becoming smaller. In this case the value of C is determined not only by the geometrical size of the pole but also by that of metallic plates as well. We examined by experiment the value of C with respect to various sizes of poles and metallic plates. They will be dealt with in the next section.

3. Experiments.

We constructed an experimental apparatus as shown in Fig. 4. A brass pendulum (M) is suspended from a steel knife edge (K). Under the pendulum mass a copper plate of (T) a certain specified area is attached by means of screw (S). The pole pieces of an electromagnet are fixed by bolts (B), which enable us to alter the size of the pole. On the knife edge a lens-mirror (L) is fixed for optically recording the decay curves of the pendulum. A rolled plate of electrolized copper 1·0 mm. thick, was used for the metallic plate. The areas of the plate are from $4^{cm}\times 5^{cm}$ to $10^{cm}\times 10^{cm}$. The area of the pole is of three kinds, $3^{cm}\times 4^{cm}$, $4^{cm}\times 3^{cm}$, and $4^{cm}\times 4^{cm}$. The distance between the poles was in all cases about 4^{mm}. The total flux of the electromagnet was measured by a ballistic method, using a search coil having an area of $6^{cm}\times 6^{cm}$.

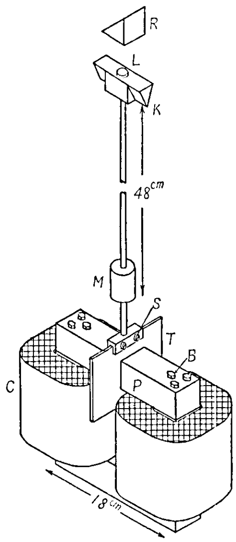


Fig. 4. Experimental apparatus. C coil for electromagnet, P pole piece, B bolts, T copper plate, S screws, M mass of the pendulum, K knife edge, L lens-mirror, R rectangular prism.

The damping force acting on the copper plate can be calculated from the decay curves of the free oscillation of the pendulum, that is,

$$\left.\begin{aligned} f &= 9{\cdot}2103\times\frac{I\Lambda}{T'h^2} \text{ per unit velocity,} \\ T' &= T\sqrt{1+0{\cdot}53720\Lambda^2}, \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where I is the moment of inertia of the pendulum system, h the distance from the axis of rotation of the pendulum to the centre of the plate, Λ the logarithmic decrement (the common logarithm is used), T' the apparent period of the pendulum, and T the natural period of the pendulum. In this case the resistance due to friction of air and of the knife edge ought to be taken into account, but in the present experiment they were found to be negligible.

4. Experimental Results.

First we tried to see if the damping force depends on the amplitude of the plate or not. For this purpose the logarithm of the double-amplitude was plotted as ordinate and the time as abscissa, an example of which is shown in Fig. 5. It will be seen that the points all lie on one straight line. From this we may conclude that the damping force does not depend on the amplitude, even though of fairly large amplitude.

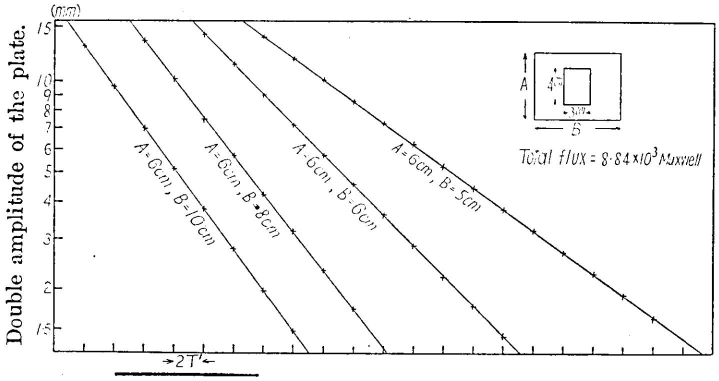


Fig. 5. Showing that the logarithmic decrement does not depend on the amplitude of the metallic plate. T' is the apparent period of the pendulum.

Next we changed the total flux of the electromagnet by controlling the electric current of the coil and measured the damping force. The result is shown in Fig. 6. The logarithmic decrements are in linear relation to the square of the total flux; that is to say, the damping force is proportional to the square of the total flux.

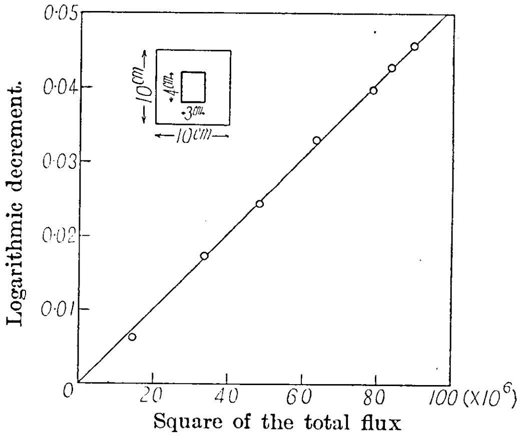


Fig. 6. Showing that the logarithmic decrement is proportional to the square of the total magnetic flux of the field (in Maxwell).

As already stated in the second paragraph, the damping force acting on the metallic plate is expressed by the equation (all c. g. s. elctromagnetic units)

$$f=\frac{C\phi^2\sigma D}{S} \quad \text{per unit velocity,}$$

where ϕ is the total flux, σ the specific conductivity, D the thickness of the plate, S the area of the pole, and C the dimensionless quantity determined by the geometrical size of the pole and the plate. We used plates of various size and determined the values of C by using the above expression. Throughout the experiment the thickness of the plate was constant (1·0 mm.). The experiments were as follows:

Experiment.	No. 1	No. 2	No. 3	No. 4
Sectional size of the pole.	rectangle $4^{cm} \times 4^{cm}$	rectangle $4^{cm} \times 3^{cm}$	rectangle $3^{cm} \times 4^{cm}$	circle dia. $= 4^{cm}$
Distance between the poles.	$4{\cdot}3^{mm}$	$4{\cdot}2^{mm}$	$4{\cdot}2^{mm}$	4.2^{mm}
Total magneitc flux.	Maxwell $9{\cdot}08 \times 10^3$	Maxwell $8{\cdot}84 \times 10^3$	Maxwell $7{\cdot}66 \times 10^3$	Maxwell $9{\cdot}08 \times 10^3$

The results are shown in graph (Fig. 7, 8 and 9), in which A and B represent the longitudinal and the transveral lengths of the plate respectively. In the case in which the size of the pole is a rectangle, when B is constant when A becomes more than twice α, the increment of the value of C is assymptotic (Fig. 7, a, b, c). But when A is constant and B increases, the value of C slowly approaches the assymptotic value (Fig. 8, a, b, c). We see from this that, so far as the size of the

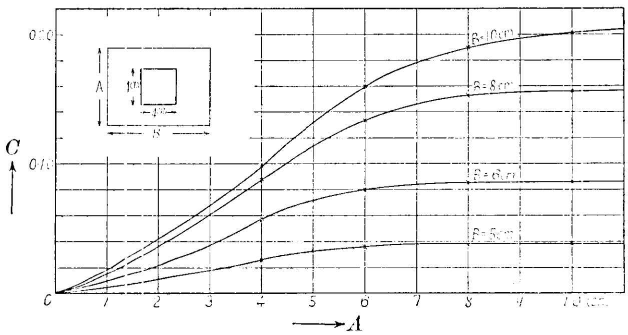


Fig. 7, a.

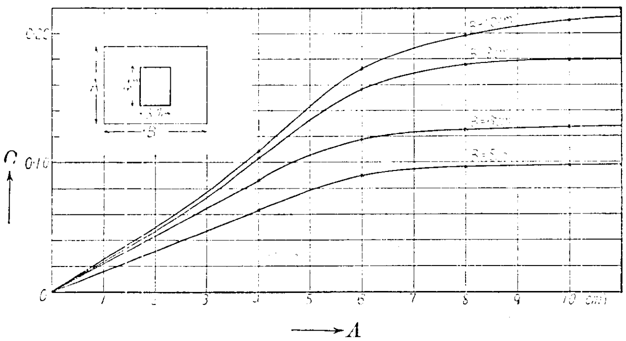


Fig. 7, b.

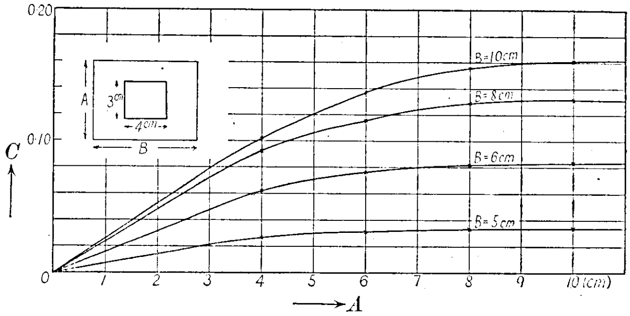


Fig. 7, *c*.

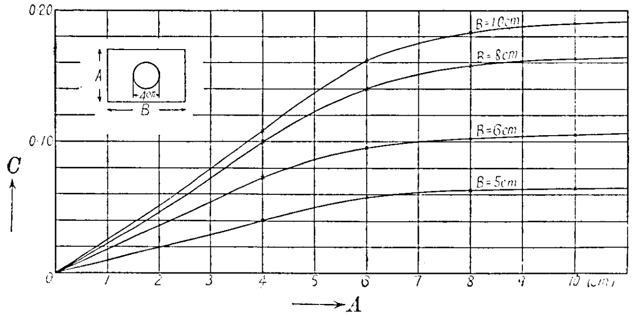


Fig. 7, *d*.

plate is concerned, in order to obtain a large damping force, the length of A need only be twice α, but the length of B should be at least more than three times β. In short, the plate must have a large length as possible in the direction of swing of the plate. But in practice, to make the metallic plate so large is difficult, since the seismograph is liable to be affected by physical disturbances, such for example as the vibration of the plate itself in an earthquake. The value of C of the most suitable size of metallic plate therefore seems to be about 0·2.

5. Remarks.

It is essential that the metallic plate of the magnetic damper shall contain no magnetic substances. If it does, no matter in how small a quantity, the retarding force cannot be completely proportional to the velocity of the plate owing to attraction or repulsion between the plate and the poles. The disturbance will be evident especially when the period of the pendulum is comparatively large. We tried to examine this disturbance by a torsion pendulum. To both ends of an aluminium pipe, two copper plates were attached horizontally and this was suspended by means of phosphorbronze wire from the middle of the pipe; the area of each copper plate being $4^{cm} \times 5^{cm}$ (thickness 1·0 mm.) and the period of the pendulum 20 sec. To the copper plate on one side an electromagnet (the size of the pole is a circle with diametre 2 cm.) was attached. A lens-mirror was attached vertically to the middle of the pipe, the mirror reflecting the rays from a light source and forming its image on a glass scale 1 metre distant from the mirror. Electric currents were then sent to the coil of the electromagnet and the deviations of the equilibrium position of the pendulum noted. The

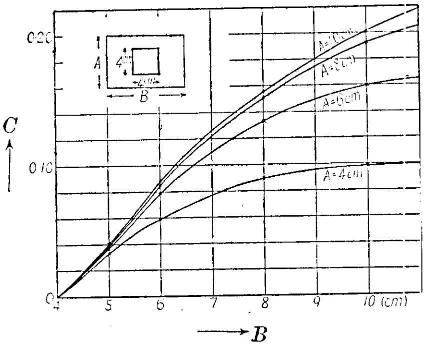


Fig. 8, *a*.

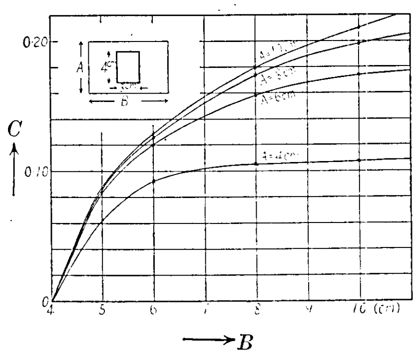


Fig. 8, *b*.

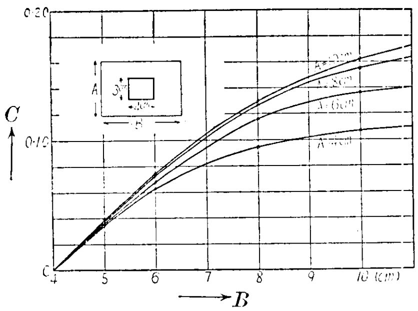


Fig. 8, *c*.

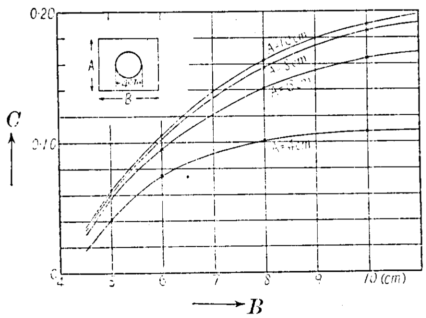


Fig. 8, *d*.

moment of inertia of the pendulum being $7{\cdot}4\times10^2$ c. g. s. (the distance between the axis of rotation and the centre of the plate is 15 cm.) and the period of the pendulum being 20 sec.; calculation showed that a force of 0·05 dynes acting on the plate causes a deviation in the equilibrium position of 1 mm. on the glass scale. But in this test the deflexion of equilibrium position was not perceived on the scale, even

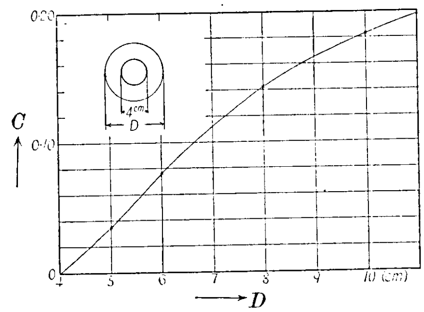


Fig. 9.

when the magnetic field had been made so strong that the pendulum was in a fair state of over damping.

In conclusion, the present writer wishes to express his thanks to Professors M. Ishimoto and T. Matuzawa for their kind advices and criticisms in connection with these studies.

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33. 磁氣制振器に就て

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近年大きい頑磁力を有する特種鋼が製作せられるに至り，永久磁石に依り可成りの强磁場を得ることが可能となつたので，地震計制振器としての磁氣制振器も使用範圍を擴め得る樣になつた．本文は磁氣制抵器の設計を試みる場合に必要な一事項に就き調査したものである．

磁氣制振器の單位速度に對する制振力 f は次式で與へられる．

$$f=\frac{C\phi^2\sigma D}{S}.$$

此處に ϕ は全磁束，σ は金屬板の比電氣傳導率，D は金屬板の厚さ．S は磁場の面積である(總べて *c.g.s.* 電磁單位)． C はディメンションを持たない量であつて磁場及金屬板の幾何學的形狀に依り決定せられる．此處では C の値を種々の場合に就き理論的或は實驗的に求めたのである．結果の概略は次の如くである．

1) 磁場の形狀が矩形をなし金屬板の面積が限り無く大きい場合．

此の場合は比較的簡單な計算から C の値を求め得る．磁場の矩形の各邊を α, β とし是等は夫々金屬板の運動に直角及平行の方向に向くものとすると，$\frac{\alpha}{\beta}=0$ のとき C=0 であり，$\frac{\alpha}{\beta}$ の値が增加すると共にCの値は增加し，$\frac{\alpha}{\beta}=1$ (卽ち磁場が正方形)のとき C=0·5 であり，$\frac{\alpha}{\beta}\to\infty$ に對し C→1 となる(第3圖)．卽ち磁場の形狀は金屬板の運動に垂直な方向に細長くした方が C の値を大にすることが出來て有利である．

2) 金屬板の面積が有限の場合．

此の場合は理論的の計算が極めて複雜になるので此處では實驗的に C の値を決定した．金屬板を矩形とし各邊の長さを A, B とし A, B は夫々金屬板の運動に垂直及平行の方向に向くものとすると，磁場が矩形の場合に B を一定にし A を增加せしむる時は C の値は比較的速かに漸近値に接近するが，A を一定とし B を增加せしむる時は C の値は容易に漸近値に接近しない(第7及8圖)．卽ち C の値を大とするためには金屬板は其の運動に平行な方向に出來るだけ長くすべきである．磁場が圓形の場合にも定性的には矩形の場合と同樣のことが言へる(第7, 8, 9圖)．
