32. *The Most Suitable Formula for the Japanese Gravity Values.*

By Chûji TSUBOI,

Earthquake Research Institute.

(Read Sept. 19, 1933.—Received Sept. 20, 1933.)

Recently, Captain K. Atumi[1] of the Japanese Military Land Survey, after studying the plumb-line deviations at Japanese 68 stations, determined the Geoid undulations throughout the country. As most of the astronomical observations used in his calculations were not made with his precise studies in view, the original observations and, as a consequence the results of the calculations, do not seem to be very accurate. Nevertheless, the conclusions arrived at by him are so remarkable that they must be regarded as indicating something near the truth. His chief conclusions are :—

1. The components of the plumb-line deviation at the Tokyo Astronomical Observatory (Azabu), which is the base station of the Japanese triangulation, are

$$\xi = -9''.46,$$

$$\lambda = +9''.35.$$

2. The semi-major axis of the ellipsoid of revolution that best fits in with Japanese astronomical observations is 6,376,918 m, which is 480 m shorter than that of the Bessel's ellipsoid.

3. The ellipticity of the newly determined ellipsoid is 1/311.

This last conclusion is of special interest, since the large difference in this ellipticity from the normal one of 1/298 seems to indicate certain abnormal subterranean conditions prevailing in the neighbourhood of Japan, noted for its seismic and volcanic activities.

Seeing that it is generally conceded that the ellipticity of the terrestrial ellipsoid can be determined more accurately from gravity observations than from geodetic measurements, the writer intends here to determine the ellipticity of the earth solely from the Japanese gravity values. Gravity measurements in Japan have been and still are

1) A. ATUMI, *Jap. Journ. Astr. Geophys.*, 10 (1933), 307.

made under the supervision of the Imperial Japanese Geodetic Commission. The stations occupied in Hokkaidô, Honsyû, Sikoku, and Kyûsyû, which amount to 122 in all, extend from $\varphi=45°25'$ in the north down to $\varphi=31°28'$ in the south. The Bouguer anomalies at these stations, which have already been published[2], were obtained by comparing the g_0'' at the stations with the values expected from Helmert's formula of 1901, namely,

$$\gamma_0=978{\cdot}030(1+0{\cdot}005302\sin^2\varphi-0{\cdot}000007\sin^2 2\varphi).$$

The anomalies in Japan are generally very large, and range from $+0{\cdot}231$ to $-0{\cdot}043$. The mean value of the anomalies with regard to algebraic sign is $+0{\cdot}049$, while that without it is $0{\cdot}054$. Such large Bouguer anomalies show that the distribution of gravitating masses near Japan differs considerably from what may be expected from the formulas of Helmert and others.

Let us now determine the numerical values of the constants in the equation,

$$\gamma_0=g_\varepsilon(1+B\sin^2\varphi),$$

that best fits in with the Japanese gravity values. The term $\sin^2 2\varphi$ is omitted because no great accuracy is necessary for the purpose of the present calculation, which is only to get an expression for the general gravity distribution in Japan. Now we have 122 observation equations of the form

$$g_0''=g_\varepsilon(1+B\sin^2\varphi)$$

in which g_0'' and φ are observed values, while g_ε and B are unknowns to be determined by calculation. To simplify the calculation, the equation was changed into the form

$$\begin{aligned}(g_0''-979{\cdot}000)&=(g_\varepsilon-979{\cdot}000)+g_\varepsilon B\sin^2\varphi\\&=\Delta g+A\sin^2\varphi,\end{aligned}$$

and the two constants,

$$\begin{cases}\Delta g=g_\varepsilon-979{\cdot}000,\\A=g_\varepsilon B,\end{cases}$$

were determined by the method of least squares. The normal equations

1) *Comptes Rendus d. Séances.* 16me *Conf. Gén. d. l'Assoc. Géod. Intern.*, (1911), 252; "Rikwa-Nenpyô," (1930), 311.

by which the constants are to be determined came out as

$$\begin{cases} 122\Delta g + 44{\cdot}170A = 115{\cdot}949, \\ 44{\cdot}170\Delta g + 16{\cdot}388A = 44{\cdot}140. \end{cases}$$

As a solution of this set of equations, we have

$$\begin{cases} \Delta g = -1{\cdot}019, \\ A = 5{\cdot}4392. \end{cases}$$

We have therefore

$$\begin{cases} g_{\varepsilon} = 979{\cdot}000 - 1{\cdot}019 = 977{\cdot}981, \\ B = \dfrac{5{\cdot}4392}{977{\cdot}981} = 0{\cdot}005562, \end{cases}$$

whence the final result

$$\gamma_0 = 977{\cdot}981(1 + 0{\cdot}005562 \sin^2\varphi).$$

The difference between this and Helmert's formulas is

$$\Delta\gamma = 0{\cdot}049 - 0{\cdot}254 \sin^2\varphi$$

which, as shown in Fig. 1, become zero at latitude $\varphi = 26°3'$. While the difference for the Japanese gravity stations at lower latitudes is not large, it becomes quite appreciable for stations at higher latitudes. The Bouguer anomalies calculated from the new formula became compara-

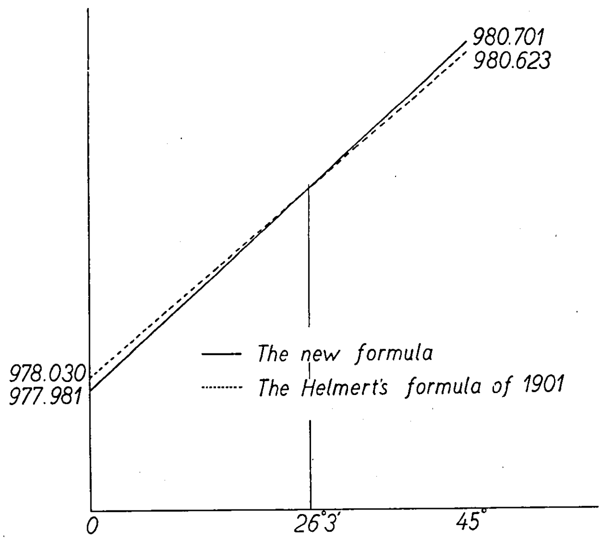

Fig. 1.

tively small and range from $+0{\cdot}153$ to $-0{\cdot}088$. The mean of the anomalies with regard to sign is 0, while that without regard to it is $0{\cdot}035$. The geographical distribution of Bouguer anomalies as calculated from the new formula is shown in Fig. 2.

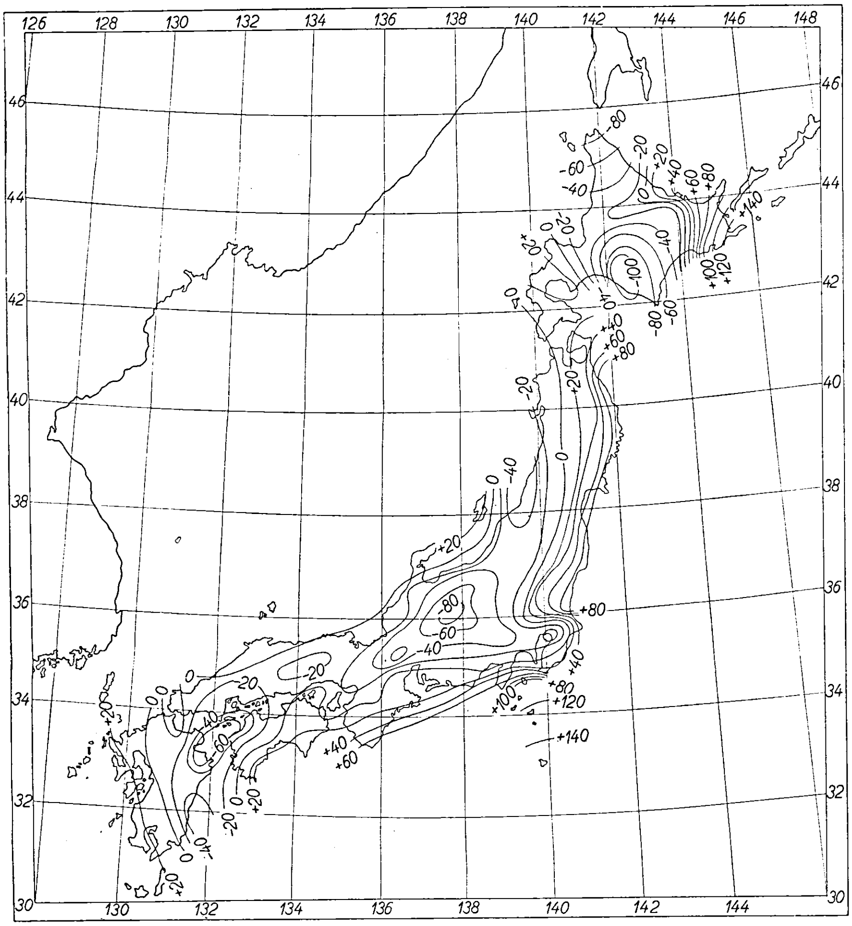

Fig. 2. Bouguer Anomalies as Calculated from the New Formula.

From the well-known relation between coefficient B of $\sin^2\varphi$ in the formula and ellipticity e of the terrestrical ellipsoid

$$B=\frac{5}{2}\frac{\omega^2 a}{g_{\varepsilon}}-e$$

we get $e=1/322$. a and ω are the length of the equatorial radius and the angular velocity of the earth respectively.

Though there is not a small difference between the ellipticity value obtained by Atumi and that obtained by the writer, both are decidedly smaller than the value obtained from gravity measurements made in various parts of the world. Since, as already stated, the difference between Helmert's formula and the present formula is not large in the lower latitudes of Japan, the conclusion is that the discrepancy in the two formulas is chiefly due to the abnormally large gravity value for the higher latitudes of Japan, especially the northern half of Honsyû.

If in a certain region, the deviation of gravity there from the normal that has been established as the result of gravity measurements made throughout the world implies crustal instability in that region, then the northern half of Honsyû is an excellent example of such instability. The existence of numerous volcanoes and the great frequency with which earthquakes occur there certainly seem to bear out the implication of crustal instability in that part of Japan, which is believed to be a tectonic unit, quite distinct from the rest of Japan. It is the writer's opinion that, as revealed by gravity measurements, the geological and tectonic characteristics of northern Honsyû are closely correlated with its particular physical structure.

Table I. Bouguer Anomalies in Japan as Calculated by the New Formula.

Station	φ	λ	$g_0''-\gamma_0$	Station	φ	λ	$g_0''-\gamma_0$
1. Sibusi	31° 28′	131° 5′	+0.009	13. Kurume	33 19	130 32	− 2
2. Kagosima	31 36	130 32	+ 20	14. Karatu	33 27	129 59	+ 8
3. Miyazaki	31 55	131 24	− 56	15. Kôti	33 34	133 34	0
4. Hitoyosi	32 12	130 46	+ 6	16. Nakatu	33 36	131 11	+ 3
5. Yatusiro	32 31	130 36	+ 11	17. Singû	33 43	136 0	+ 68
6. Nobeoka	32 34	131 39	− 38	18. Matuyama	33 50	132 45	− 56
7. Nagasaki	32 45	129 52	+ 27	19. Orio	33 53	130 42	− 3
8. Kumamoto	32 48	130 43	− 9	20. Tyôhu	34 0	131 0	+ 10
9. Nakamura	32 59	132 55	+ 25	21. Tokuyama	34 4	131 44	− 19
10. Hatizyô-zima	33 6	139 50	+ 154	22. Tokusima	34 5	134 35	− 3
11. Uwazima	33 13	132 35	− 15	23. Yamaguti	34 11	131 29	− 22
12. Ooita	33 15	131 36	− 65	24. Wakayama	34 14	135 11	+ 2

(to be continued.)

Table I. (*continued.*)

Station	φ	λ	$g_0''-\gamma_0$	Station	φ	λ	$g_0''-\gamma_0$
25. Edazima	34 15	132 30	− 32	59. Tyôsi	35 44	140 51	+ 50
26. Marugame	34 18	133 49	+ 9	60. Kawagoe	35 55	139 30	+ 4
27. Hirosima	34 23	132 27	− 37	61. Kamisuwa	36 2	138 8	− 76
28. Yamada	34 30	136 43	+ 2	62. Hukui	36 3	136 15	− 3
29. Hukuyama	34 30	133 23	− 14	63. Tutiura	36 6	140 12	+ 97
30. Okayama	34 39	133 56	− 16	64. Takayama	36 9	137 16	− 64
31. Nara	34 41	135 51	− 6	65. Tukuba	36 13	140 6	+ 60
32. Rendaizi	34 42	138 57	+ 67	66. Matumoto	36 14	137 59	− 88
33. Mikage	34 43	135 15	− 31	67. Matuida	36 19	138 48	− 25
34. Hamamatu	34 43	137 43	+ 12	68. Mito	36 23	140 33	+ 80
35. Oosima	34 45	139 22	+ 112	69. Maebasi	36 24	139 4	− 28
36. Ueno	34 46	136 8	+ 2	70. Ueda	36 24	138 16	− 50
37. Himedi	34 50	134 42	+ 1	71. Kanazawa	36 33	136 42	− 27
38. Hamada	34 54	132 6	+ 7	72. Utunomiya	36 33	139 53	+ 54
39. Okazaki	34 57	137 10	+ 3	73. Nagano	36 40	138 11	− 63
40. Siduoka	34 58	138 23	− 9	74. Toyama	36 40	137 13	− 37
41. Kyôto	35 2	135 47	− 40	75. Nikkô	36 44	139 38	− 18
42. Wada	35 2	140 1	+ 15	76. Ootahara	36 52	140 1	+ 7
43. Tuyama	35 5	134 1	− 23	77. Nanao	37 3	136 58	+ 16
44. Numadu	35 5	138 52	+ 10	78. Taira	37 4	140 53	+ 85
45. Nagoya	35 10	136 53	− 27	79. Takata	37 7	138 16	− 12
46. Oohara	35 15	140 24	+ 27	80. Kuwano	37 23	140 20	+ 27
47. Odawara	35 15	139 9	− 4	81. Nagaoka	37 27	138 53	− 22
48. Hikone	35 16	136 15	− 70	82. Wakamatu	37 30	139 57	− 25
49. Hukutiyama	35 18	135 9	− 15	83. Hukusima	37 45	140 27	+ 16
50. Kamakura	35 19	139 34	− 17	84. Nakamura	37 47	140 55	+ 77
51. Gihu	35 26	136 46	− 48	85. Yonezawa	37 54	140 8	− 8
52. Nakatugawa	35 29	137 32	− 44	86. Niigata	37 55	139 1	− 40
53. Matue	35 30	133 3	+ 2	87. Kawarada	38 1	138 18	+ 31
54. Tottori	35 30	134 14	− 1	88. Yamagata	38 15	140 16	− 7
55. Tiba	35 36	140 9	− 45	89. Sendai	38 15	140 52	+ 50
56. Kôhu	35 39	138 35	− 55	90. Sinzyô	38 45	140 18	− 16
57. Turuga	35 39	136 3	− 49	91. Kesennuma	38 55	141 36	+ 84
58. Tokyo	35 43	139 46	− 30	92. Itinoseki	38 55	141 6	+ 55

(*to be continued.*)

Tabel I. (*continued.*)

Station	φ		λ		$g_0''-\gamma_0$	Station	φ		λ		$g_0''-\gamma_0$
93. Sakata	38	55	139	50	− 41	108. Imoppe	42	34	141	57	− 106
94. Midusawa	39	8	141	8	+ 25	109. Suttu	42	48	140	13	+ 26
95. Yuzawa	39	9	140	30	+ 10	110. Obihiro	42	55	143	12	− 63
96. Tôno	39	18	141	31	+ 60	111. Kusiro	42	58	144	23	+ 103
97. Miyako	39	38	141	58	+ 85	112. Sapporo	43	5	141	20	− 31
98. Akita	39	42	140	7	− 11	113. Nemuro	43	21	145	30	+ 153
99. Marioka	39	42	141	10	+ 32	114. Simohurano	43	21	142	30	− 40
100. Oodate	40	16	140	34	+ 3	115. Asahigawa	43	46	142	22	− 16
101. Hukuoka	40	16	141	19	+ 38	116. Rubesibe	43	47	143	38	− 36
102. Hatinohe	40	31	141	30	+ 87	117. Rumoe	43	56	141	39	− 22
103. Hirosaki	40	36	140	28	− 3	118. Abasiri	44	1	144	1	+ 58
104. Aomori	40	49	140	45	+ 20	119. Monbetu	44	21	143	21	+ 28
105. Hakodate	41	47	140	46	+ 14	120. Nayori	44	22	142	24	− 37
106. Urakawa	42	9	142	45	− 89	121. Esasi	44	57	142	33	− 59
107. Muroran	42	19	140	58	+ 36	122. Wakkanai	45	25	141	41	− 81

32. 日本の重力に最もよく適合する式

地震研究所 坪井忠二

日本に於ける 122 點の重力に最もよく適合する式を定めた. 其の結果は

$$\gamma_0=977{\cdot}981(1+0{\cdot}005562\sin^2\varphi)$$

となつた. 之から地球の扁平率を求めると 1/322 となり. 嘗て熱海大尉が日本の鉛直線偏倚から求めた 1/311 と云ふ値と同様に大變小さい. ヘルメルトの式では扁平率は 1/298 となるが. 此の著しい差は日本の近傍で質量分布の有様に特徴がある事を示す. 新しい式とヘルメルトの式との差は日本の南半分では餘り著しくないから, 上記の質量の異常は日本の北半分で特に著しいであらう. 地震や火山が日本の北半分に多いのも之と關係がある事と思ふ.