24. *Study on the Propagation of Seismic Waves.*
(*The second paper*)
Amplitude of Seismic Waves with the Structure of the Earth's Crust and Mechanisms of their Origin.[1]

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Introduction.

The foretelling the earthquake occurrence, which is one of the greatest aim of seismolgical investigation in the "Land of Earthquake" Japan, can not be realized without the exact knowledge of the chronological and inter-regional relations of the activity of earthquake generating forces. The late Prof. F. Omori[2] was the first to discuss the mechanism of earthquake occurrence from the results of seismometrical observations. In case of the earthquake of Feb. 6, 1907, a very peculiar seismograms showing no preliminary tremors were obtained at Osaka and its vicinity, while ordinary seismograms having distinct P and S phases were obtained at other places of Japan. Prof. Omori had to consider the mecanism of occurrence of the earthquake to explain these conspicuous phenomena. Since then he paid attention to the directions of the initial motions of the earthquakes accompanying the Asamayama explosions.[3] But Prof. T. Shida[4] was the first who could find the way actually to the purpose

1) Already published in Japanese in the *Disin* or the *Journ. Seismol. Soc. Japan*, September and November of 1932 and May of 1933.

2) F. OMORI, *Bull. Earthq. Invest. Commit.*, **1** (1907), 145-154.

3) F. OMORI, *ibid.*, **6** (1912), 11; **7** (1914), 1; and *Publ. Earthq. Invest. Commit.*, **21** (1905).

4) T. SHIDA, "Disingaku no Saikin" (Modern Topics in Seismology), (in Japanese). The manuscript of the lecture delivered at the Kamigamo Seismological Observatory on July 19, 1921. On page 31 we find the following passage. "Since about ten years I have been aware of the fact that we can judge from the records of an earthquake motion at neibouring stations whether the earthquake began with sudden cracking or depression at hypocentre, and we are able to know the direction of the crack as well in the former case. And I have now obtained considerable results. etc."

See also

Torahiko TERADA and Takeo MATUZAWA, A Historical Sketch of the Developement of Seismology in Japan. (1926).

M. HASEGAWA, *Beitr. z. Geophys.*, **27** (1930), 102-125.

M. MATUYAMA, Bankin no Disingaku—i.e., Modern Seismology, (in Japanese), (1925).

in the seismometrical investigations. He found very good examples, for which he had been seeking since several years, in the Tenryûgawa Earthquake of May 18, 1917 and the Awazi Earthquake of Nov. 26, 1916, and he delivered the results at the meetings of the Physico-mathematical Society of Japan and the Earthquake Investigation Committee in 1917 and 1918. In the former earthquake the distribution of dilatation and compression (or 'pull' and 'push' according to Prof. Shida) of the initial motion were divided into quadrants by two nodal lines crossing at right angles at the epicentre. In the latter earthquake the "pulls" (or dilatations) were confined within a circle near the epicentre. And Prof. Shida explained the former as "crack earthquake" and the latter as "depression earthquake." Since then ardent interrest of Japanese seismologists centred upon this interesting problem. From investigations of several earthquakes[5] Prof. S. T. Nakamura found intimate relation between the positions of nodal lines of initial motions of an earthquake and geological techtonic line near the epicentral region, and explained the mechanism of the earthquake which had been called the crack earthquake by Prof. Shida as sliding along one of the nodal planes. The "fault earthquake" theory thus introduced found its supporters among the members of the Central Meteorological Observatory and was applied to almost all earthquakes up to the close of the year 1931, when Mr. K. Tanahasi[6] found an example which could not be explained by this category. It was the deep-seated Central Japan Earthquake of June 2, 1931, in which we cannot find nodal planes consequential to the fault earthquake theory. But we find nodal lines resembling hyperbola, from which we can only conclude the existence of nodal cone surface. The theory for this model is due to Prof. M. Matuyama. Prof. M. Ishimoto[7] greatly interested upon this example and found another example of this type in the Ise-bay Earthquake of June 3, 1929. In case of shallow earthquake in which two dimensional relation can only be observed at the earth's surface near the epicentre we cannot find any differrence from the mere quadrant distributions of "pull and push" between the two mechanisms of the Prof. Matuyama's

5) The Omati earthquake of 1918 (*Journ. Met. Soc., Japan*, **37** (1918), 390-401.); the Miyosi earthquake of 1919 (*ibid.*, **38** (1919), 395-404); the Simabara earthquake of 1922 (*ibid.*, [ii], **1** (1923), 1-13), etc.

6) K. TANAHASI, *Umi to Sora*, **11** (1931), 277-288.

7) M. ISHIMOTO, Lecture delivered at the colloquium of the seismological Institute on Jan. 20, 1932.

M. ISHIMOTO, *Proc. Imp. Acad.*, **8** (1932), 36-39.

model and that of the fault earthquake theory. (We shall for convenience call in the following the former as Model A and the latter Model B.) But in case of deep-seated earthquake the spherical waves are observed as they are and the difference of the mechanisms are revealed at a glance. From the above consideration Prof. Ishimoto adovacated strongly the necessity of considering the problems in three dimensions. And in March of 1932 he reexamined the problems from this point of view and could explain almost all earthquakes by the model A, and he thought this as the evidence of his 'magma intrusion theory' or 'plutonic earthquake theory' of earthquake occurrence.[8] And this gave a new question of the validity between this hypothesis and the former 'fault earthquake theory.'

On the other hand Mr. H. Honda[9] studied in June of 1931 the magnitudes of initial motions of the North Idu and Ito Earthquakes quantitatively, and explained the result by Dr. H. Nakano's theoretical investigation[10] in homogenious isotropic semi-infinite solid. But the present writer[11] advocated in the next month the necessity of considering the effect of heterogeneity of the crust showing that the result of Mr. Honda could be enumerated by the method of K. Zoeppritz, E. Wiechert and Dr. H. Jeffreys. Since that time the writer has been intending the quantitative study of seismic waves, as he has stated in his first paper with the same title. He was encouraged by Mr. Tanahasi's result and Prof. Ishimoto's opinion and studied the amplitudes of seismic waves quantitatively taking the structure of the earth's crust into consideration for the purpose of verifying the mechanisms of earthquake occurrence. And a part of the result will be contributed here.

Before entering the main subject historical review of the former investigations, practical and theoretical, will be given for the indication of the way of future study.

Chapter I. Historical review of the former Investigations.

i) *Theoretical sides.*

In early part of last century Cauchy and Poisson determined the

8) M. Ishimoto, lectured on the meeting of the Earthquake Research Institute on Mar. 15, 1932. *Bull. Earthq. Res. Inst.*, 10 (1932), 449-471.

9) H. Honda, *Geophys. Mag.*, 4 (1931), 185-213.

10) H. Nakano, *Geophys. Mag.*, 2 (1930), 189-348.

11) H. Kawasumi, On the initial motion of earthquake, read at the meeting of the Earthquake Research Institute on July 7, 1931.

equations of motion in elastic solid. Poisson[12] used it to investigate the propagation of waves through an isotropic solid and found two types of waves which, at great distance from the source of disturbance, are practically " longitudinal " and " transverse." Cauchy[13] applied his equations to the questions of propagation of light in crystalline as well as in isotropic medium. Thus the theory of elastic waves drew attentions of many authrities on account of its applicability to the theory of light. Green[14] investigated the reflexion and refraction of an elastic waves, and waves in an crystalline medium. In 1849 Stokes solved most fundamental problems of elastic waves in his " Dynamical Theory of Diffraction.[15] " He found that the Poisson's two waves are waves of irrotational dilatation and equivoluminal distortion, the latter involving the rotation of elements of the medium. The Poisson's formula,[16] expressing the displacement at any time in terms of initial distribution of displacement and velocity in a medium in which only one kind of wave can exist was extended by him to the problems in elastic medium in which two kind of waves are possible. From the application of this formula Stokes obtained a solution of elastic waves due to a force opperative at a limmited portion in the medium. In 1871 Lord Rayleigh[17] obtained the solution of waves due to a force opperative at a point in a simpler fashion than that of Stokes in his investigation of the " Light from the Sky." Lord Kelvin[18] showed in 1884 that the solution of wave equation satisfies the original equation after it has subjected to multiple differentiation with respect to coordinates x, y and z. The well-known Rayleigh[19] wave was find in 1885. In 1904 H. Lamb[20] showed an elegant procedure in the investigation of " the Propagation of Tremors over the Surface of an Elastic Solid " which were followed by many later investigators, and A. E. H. Love[21] treated general problem of waves in isotropic

12) S. D. POISSON, *Paris Mèm. de l'Acad.*, 8 (1829).

13) A. L. CAUCHY, *Exercices de Mathematique*, (1830).

14) George GREEN, *Phil. Trans., Cambridge*, 7 (1939), or Math. Papers (1871), 245.

15) G. G. STOKES, *Phil. Trans., Cambridge*, 9 (1849) and *Math. and Phys. Papers*, 2, 243-328.

16) S. D. POISSON, *Mèm. de l'Institute* 3, (1820), 121.
Lord RAYLEIGH, "Theory of Sound," §273 and Note to §273.

17) Lord RAYLEIGH, *Phil. Mag.*, 61 (1871), 107, 274 and 447, or Scientific Papers, 1, 87-103.

18) Lord KELVIN, Bartimore Lecture 1884, and *Phil. Mag.*, [v], 47 (1899), 480-493; 48 (1899), 227-236.

19) Lord RAYLEIGH, *Proc. Math. Soc., London*, 17 (1885), or Scientific Papers, 2, 441.

20) H. LAMB, *Phil. Trans. Roy. Soc., London*, A 203 (1904), 1.

21) A. E. H. LOVE, *Proc. Math. Soc., London*, [ii], 1 (1904), 291-344.

medium in which he made corrections to the formula obtained by Stokes. Kirchhoff's formula is also extended to the waves in elastic solid. The waves due to body forces were solved by the application of Lord Kelvin's procedure in the solution of the corresponding statical problem. The effect of various nuclei of strain were also considered by the superpositon of the waves due to a force opperative at a point.

Few investigations have since appeared except the discovery of the Love wave[22] untill Prof. Shida discovered in actual earthquake of the fact already stated which had also been expected by G. W. Walker[23] on Stokes' dynamical theory of diffraction. New epoch was then made in Japan. In 1922 Prof. S. T. Nakamura[24] explained the mechanisms of earthquake occurrence by the analogy of the field due to a system of electric charges. He could explain sufficiently the direction of initial motion by this analogy, but the magnitudes were not explained. He should have to follow Lord Kelvin and use a wave function of the form $\frac{1}{r}e^{i(pt-hr)}$ instead of $\frac{1}{r}$. The paper of Dr. Nakano[25] on this problem published in 1923 was unfortunately burnt to ashes by the disastrous fire which ruined the City of Tokyo. In 1925 Mr. K. Suda[26] showed various models by the combination of Rayleigh's solution for a force at a point. But the mathematical solution for each of them were not given. In the treaties of Prof. Matuyama[27] on seismolgy published in the same year we find a few models obtained by the method of Lord Kelvin by the superposition of $\frac{1}{r}e^{i(pt-hr)}$. Prof. T. Matuzawa[28] calculated in the next year actually the models considered by Love in Cartesian coordinates.

All the investigations stated above belong to the so called "method of singularities[29] and the model of Mr. M. Hasegawa[30] which appeared in 1930 also belong to this category. The "method of series[31]" which Stokes already used in other branch of science were introduced by

22) A. E. H. Love, "Some Problems of Geodynamics," (1911), 160-165.

23) G. W. Walker, "Modern Seismology," (1913), 64.

24) Saem. Nakamura, *Journ. Met. Soc. Japan*, 41 (1922), e. 1.

25) H. Nakano, *Seismol. Bull. Centr. Met. Obs. Japan*, 1 (1923), No. 3.

26) K. Suda, *Umi to Sora*, 5 (1925), 36-39; 46-51; 64-69; 80-88; and *ibid.*, 6 (1926), 24-31; 50-60.

27) M. Matuyama, *loc. cit.*, 4).

28) T. Matuzawa, *Jap. Journ. Astro. Geophs.*, 4 (1926), 1-33.

29) A. E. H. Love, Mathematical Theory of Elasticity, 4th Ed., 15.

30) M. Hasegawa, *loc. cit.*, 4).

31) *loc. cit.*, 29).

Prof. K. Sezawa and Dr. Nakano into studies of the waves in and on elastic solid. The problems treated by these authorities cover so wide a range that cannot the condenced in this narrow space that I will state here only the results concerning spherical waves.

In 1927 Prof. Sezawa obtained a waves expressed in zonal harmonics and discussed the distribution of energy among the dilatational and distortional waves,[32] and this solution was applied to the solution of the Rayleigh waves over a spherical surface.[33] The waves from an elliptical or ellipsoidal origin[34] were obtained to show the effect of the dimension of origin may be negligible at a distance from the origin. Applications of these waves were made to the scattering[35] of elastic waves by the obstacles of various forms. The waves in visco-elastic solid body[36] were also discussed but the solution of spherical waves in this medium was left behind. Prof. K. Sezawa investigated in 1928 the reflection of spherical waves[37] from an internal point of a sphere, and in the next 1929 he obtained most general spherical standing waves in course of the studies of Rayleigh-[38] and Love[39]-waves having azimuthal distributions. He also obtained waves in definite integrals answering to internal multiplet[40] and sheet[41] of internal sources in course of studies of genrations of Rayleigh waves. In July 1931 Prof. K. Sezawa[42] made an analysis of general spherical diverging waves but it was not published. But in February of last year he obtained general spherical waves in visco-elastic medium.[43] In the same month the present writer[44] obtained some spherical waves in a simpler formal fashion and applied them to the present investigation. Mr. Y. Kodaira[45] also read a paper in the next month on the same spherical waves at the meeting of Meteorological

32) K. SEZAWA, *Bull. Earthq. Res. Inst.*, 2 (1927), 13-20.
33) K. SEZAWA, *ibid.*, 2 (1927), 21-28.
34) K. SEZAWA, *do.*, 2 (1927), 29-48.
35) K. SEZAWA, *do.*, 3 (1927), 19-41.
36) K. SEZAWA, *do.*, 3 (1927), 43-53.
37) K. SEZAWA, *do.*, 4 (1928), 123-130.
38) K. SEZAWA, *do.*, 6 (1929), 1-18.
39) K. SEZAWA, *do.*, 7 (1929), 41-64.
40) K. SEZAWA, and G. NISIMURA, *do.*, 7 (1929), 41-64.
41) K. SEZAWA, *do.*, 7 (1929), 417-435.
42) K. SEZAWA, lectured at the Meeting of the Earthq. Res. Inst. on Ju'y 7, 1931.
43) K. SEZAWA, *Bull. Earthq. Res. Inst.*, 10 (1932), 299-334.
44) H. KAWASUMI, Mechanisms of Earthquake Occurrence and Abnormal Felt. Area, read at the colloquium of the Seismological Institute on Feb. 17, 1932.
45) Y. KODAIRA, *Journ. Met. Soc. Japan*, [ii], 10 (1932), 236-251.

Society of Japan.

I must also add here the investigation of Mr. R. Yosiyama[46] who is making investigations of waves in heterogeneous sphere and have already obtained some remarkable results.

ii) *Practical side.*

It was from the Ewing's observation[47] of the earthquake near Tokyo of Mar. 8, 1881 that the two waves in elastic body were distinctly registered. Ewing's tentative explanation of these two waves as normal (longitudinal) and transverse waves were confirmed by the experimental study of J. Milne[48] and T. Gray. But owing to the uncertainty of our knowledge of the nature of the material of the earth's crust several other tentative explanations were proposed by C. G. Knott[49] and S. Kusakabe.[50] And it was quite recent that the first part of preliminary and principal portion of near earthquake were believed to be longitudinal and transverse waves, being hindered also by the deficiency of our knowledge of surface waves. The direction of first motion of an earthquake, which had been used for the determination of the direction of epicentre and hypocentre, were investigated statistically by Mr. M. Kawazoe[51] and recently by Mr. F. Kishinouye.[52] Quite recently Mr. S. I. Kunitomi[53] studied systematically the deviation of this direction from that of epicentre.

But as to the fact found theoretically in 1849 by Stokes that the magnitude and sense of motion—away from (compression) and toward (dilatation) the origin—vary with the direction from the origin, the former once noticed by Prof. Omori in 1907[54] was again dismissed and the latter was left unnoticed untill in 1917 Prof. T. Shida,[55] who had been aware of the possibility contemporaneously with or earlier than the notion of G. W. Walker in his "Moder Seismology," made a new epoch

46) R. Yosiyama, *Bull. Earthq. Res. Inst.*, **11** (1933), 1–12.

47) J. A. Ewing, *Trans. Seismol. Soc. Japan*, **3** (1881), 121–128.

48) J. Milne and T. Gray, *Phil. Trans.*, (1882), 863–883.

J. Milne, *Trans. Seismol. Soc. Japan*, **8** (1885), 1–82.

J. Milne, "The Earthquake and other Earth movements," (1886), 57–66.

49) C. G. Knott, *Trans. Seismol. Soc. Japan*, **12** (1888), 115.

50) S. Kusakabe, *Pull. Imp. Earthq. Invest. Commit.*, **14** (1903), esp. 52.

51) M. Kawazoe, *Journ. Met. Soc. Japan*, **37** (1918), 53–57.

52) F. Kishinouye, *Journ. Seismol. Soc. Japan*, **4** (1932), 18–35.

53) S. J. Kunitomi, lectured at the meeting of Meterological Society of Japan on Feb. 19, 1932.

54) *loc. cit.*, 2).

55) *loc. cit.*, 4).

in seismometrical investigation finding actually the mechanism of occurrence of the Tenryû-gawa earthquake from the systematic observation of "pull" and "push" of initial motion of P-wave. The examination of the geographical distribution of the directions of initial motion, for the purpose of inducing the mechanism of earthquake occurrence, then became the centre of unceasing interest of many seismologists. Prof. S. T. Nakamura and his colleagues succeeded the method and applied it to the Ômati earthquake[56] of 1918; the Miyosi earthquake[57] of 1919; the earthquake[58] of May 21, 1920 originated in the western part of the Inland Sea of Japan; the great Kan-sou earthquake[59] of 1921; the strong earthquake[60] near Tokyo of April 6th, 1922; the earthquake[61] of June 18th, 1922 in the southern coast of Sima province; a small destructive earthquake[62] near the lake Imbanuma of Dec. 8, 1922; and the Simabara[63] and the great Kwanto earthquakes of 1923. It was revealed by these investigations the existence of other types of "pull-push" distribution than those which had been found by Prof. T. Shida. Some earthquakes showed the "pull-push" distribution devided by a straight line, and the other showed curved nodal lines intersecting each other. Prof. S. T. Nakamura[65] explained them by the specical situation and direction of of multiplets opperative at hypocentre at some depth from the earth's surface. (He only considered the quadruple source of type B.)

Mr. K. Suda[66] made studies not only the Simabara earthquake but also of theoretical problems[67] already mensioned and applied them to the earthquakes originating at various parts in Japan and showed the distribution of the direction of the earthquake generating forces in this country. Prof. Matuzawa[68] illustrated the results of his theoretical calculation by actual examples. The discovery of existence of the

56) S. T. NAKAMURA, *loc. cit.*, 5).
57) *do.*
58) S. AOKI, *Journ. Met. Soc. Japan*, **39** (1920), 192-194.
59) T. USIYAMA, *Journ. Met. Soc. Japan*, **40** (1921), 135-138.
60) S. T. NAKAMURA, *ibid.*, **41** (1922), 139-156.
61) T. USIYAMA, *ibid.*, **41** (1922), 222-229.
62) T. USIYAMA, *ibid.*, **41** (1922), 4-13.
63) S. T. NAKAMURA, *ibid.*, [ii], **1** (1922), 1-12.
64) S. T. NAKAMURA, *Rep. Earthq. Inv. Comit.*, **100** (1825) *A*, 67-140.
65) S. T. NAKAMURA, *Journ. Met. Soc. Japan*, **41** (1922), e. 1.
66) K. SUDA, *Umi to Sora*, **3** (1923), 14.
67) *loc. cit.*, 26).
68) T. MATUZAWA, *loc. cit.*, 28).

Mohorovičić's layer in Japan was made by Dr. K. Wadati[69] from the study of the Tazima earthquake, in which a circle, deviding pull and push and called by him an inflexion circle, appeared at a epicentral distance where normal P-wave arrives at the same time with individual $\overline{P}$-wave. The effect of this superficial crustal layer on the distribution of initial motions was taken into consideration by Mr. S. I. Kunitomi to the studies of the Tazima, Tango,[70] Middle Etigo,[71] the Great Kwanto,[72] Ito, North Idu,[73] and Saitama[74] earthquakes, and all these earthquakes were explained by the model B, in which two nodal planes intersect at right angles at the hypocentre. He called these earthquakes "fault earthquakes" and he considered that the earthquake motions are caused by the actual motion of the mass near the hypocentre, and he thought that the amplitude of the seismic P-wave is largest in the direction of the fault, which he termed as a "fault plane" and vanishes at the perpendiculer direction which he called a "nodal plane."

But we have had no example in which the fault and nodal planes are distinguished, and also no theoretical support to such hypothesis. This type of mechanism (model B) was applied by Messrs. Sagisaka,[75] Satô, Isikawa,[76] Hayata,[77] and Oka[78] of the Central Meteorological Observatory to almost all the earthquakes. Moreover Mr. Sagisaka[79] renoticed the relation between the regional distribution of seismogram-types and the mechanism of earthquake occurrence once found by Prof. Omori. Mesrrs. Taguti[80] and Tanahasi[81] also considered the same mechnism in the study of the earthquakes of Kii Province. Mr. K. Fukutomi[82] studied statistically the earthquakes originated in the Kwantô plane, and explained the "pull-push" distributions by one nodal straight

69) K. Wadati, *Geophys. Mag.*, 1 (1927), 89-86.
70) S. I. Kunitomi, *Geophys. Mag.*, 2 (1929), 65-89.
71) S. I. Kunitomi, *Journ. Met. Soc. Japan*, [ii], 3 (1928), 59-85.
72) S. I. Kunitomi, *Geophys. Mag.*, 3 (1930), 149-164.
73) S. I. Kunitomi, *ibid.*, 4 (1931), 73-102,
74) S. I. Kunitomi, read at the meating of the Met. Soc. of Japan, and Earthquake Research Institute, not yet published.
75) K. Sagisaka and H. Sato, *Journ. Met. Soc. Japan*, [ii], 4 (1926), 301-307
76) T. Isikawa, *ibid.*, [ii], 10 (1932), 152-216.
77) K. Hayata, *ibid.*, [ii], 7 (1929), 303-310.
78) Y. Oka, *Geophys. Mag.*, 6 (1932), 213-221.
79) K. Sagisaka, *Geophys. Mag.*, 3 (1930), 165-176.
80) K. Taguti, *Umi to Sora*, 4 (1924), 199-203.
81) K. Tanahasi, *Umi to Sora*, 9 (1929), 197-202; 10 (1930), 13-18 and 213-222.
82) K. Hukutomi, *Journ. Seismol. Soc. Japan*, 3 (1931), 592-616.

line or two nodal lines crossing at right angles which support also mechanism of model B, and found some favourable relations with the secondary causes of the earthquake occurence as the gradient of atmospheric and tidal pressures.[83] I must also add here Mr. Honda's investigation in which quantitative elucidation of Dr. Nakano's theory is made from the study of the North Idu and Ito earthquakes.

Thus it has been considered that almost all the earthquakes can be explained by the mechanism of model B, when Mr. Tanahasi discovered an example of the earthquake which cannot be explained by the mechanism of model B and is well explainable by that of model A. Mr. Tanahasi also discussed the mechanism of occurrence of the S-wave by the theory of Prof. Matuyama, but he mistook it a little and overlooked a conspicuous phenomenon, on which I will return later. Of the following investigations of Prof. Ishimoto and others are stated in the introduction.

Reviewing again the notions of authorities on this problem, we see that J. A. Ewing was the first to identify the preliminary tremor and the principal portion of the earthquake and the Poisson's two waves, longitudinal and transverse, but several questions were raised on this point so long as our knowledge on the nature of the materials consisting the earth's crust were limited, and it was quite recent that the Ewings opinion was generally accepted. The variation of relative magnitudes of these waves with bearings from the hypocentre theoretically expected by Stokes was found by Prof. Omori in 1906, and the corresponding variation of sense of vibration was discovered by Prof. Shida in 1917. Since that time the "pull-push" distribution was solely noticed, and the progress of our knowledge on the crustal structures introduced new features in this field, but the relation of the relative magnitude of P- and S-waves with the mechanism of their origin theoretically studied by Mr. K. Suda and Prof. Matuzawa was renoticed to be seen as seismogram-types by Mr. Sagisaka. But owing to the lack of our exact knowledge on the crustal structure and the reliability of seismographs prevented us to touch on the quantitative examination of the theoretical relations on the amplitudes of seismic waves. But recent rapid reformation of the net of seismic observations in Japan enabled Mr. Honda to enter into

83) Recently after this paper has been written Mr. Fukutomi reexamined these earthquakes and showed that the "pull-push" distributions are better explained by the mechanism of the model A.

84) H. HONDA, *loc. cit.*, 9).

this field in 1931. But all these studies above mensioned are the studies of waves in an isotropic homogeneous solid and their applications, and the effect of the structure of the crust was not taken into consideration. But the present writer noticed that the quantitative study can not be made without taking the actual structure into consideration, and the result of his study in these lines will be communicated in the followings. New epoch is still promissed with the accomplishment of the theory of waves in heterogeneous medium now being commenced by Mr. R. Yosiyama.

Chapter II. General Solution of Spherical Waves Propagated in an Isotropic Elastic Medium.

The solution was already obtained by Prof. K. Sezawa,[85] but I obtained in a different and easier way as his solution had not yet been published.[86]

Equation of motion in an isotropic medium

$$\rho\frac{\partial^2 \boldsymbol{\sigma}}{\partial t^2}=(\lambda+2\mu)\ \mathrm{grad\ div}\ \boldsymbol{\sigma}-\mu\ \mathrm{rot\ rot}\ \boldsymbol{\sigma}, \quad \ldots\ldots(1)$$

(in which σ is displacement and ρ, λ and μ are dencity and Lamé's constants respectively) is satisfied by

$$\boldsymbol{\sigma}_1=\mathrm{grad}\ \phi e^{ipt}, \quad \ldots\ldots(2)$$

$$\boldsymbol{\sigma}_2=\{\mathrm{grad\ div}\ \boldsymbol{\psi}+k^2\boldsymbol{\psi}\}e^{ipt}, \quad \ldots\ldots(3)$$

provided the scalar and vector potentials ϕ and ψ, is a solution of the wave equation

$$(\nabla^2+h^2)\phi=0, \quad \ldots\ldots(4)$$

$$(\nabla^2+k^2)\boldsymbol{\psi}=0, \quad \ldots\ldots(5)$$

in which

$$h^2=\frac{\rho p^2}{\lambda+2\mu}, \quad k^2=\frac{\rho p^2}{\mu}. \quad \ldots\ldots(6)$$

From these we have evidently

$$\boldsymbol{\sigma}_2=\mathrm{rot\ rot}\ \boldsymbol{\psi}e^{ipt}. \quad \ldots\ldots(3')$$

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85) K. Sezawa, On the Amplitudes and Periods of the P- and S-phases in the Earthquake Motion, Read on July 7, 1931, unpublished.

86) H. Kawasumi, The Mechnisms of Earthquake Occurrence and Abnormal Felt Area. Read at the Colloquium of Seismological Institute on Feb. 17, 1932. K. Sezawa's paper was published in June 1932.

$$\left.\begin{aligned} &\operatorname{div} \boldsymbol{\sigma}_1 = -h^2 \phi e^{ipt}, \\ &\operatorname{div} \boldsymbol{\sigma}_2 = \operatorname{div} (\operatorname{rot} \operatorname{rot} \boldsymbol{\psi}) e^{ipt} = 0, \\ &\operatorname{rot} \boldsymbol{\sigma}_1 = \operatorname{rotgrad} \phi e^{ipt} = 0, \\ &\operatorname{rot} \boldsymbol{\sigma}_2 = \operatorname{rot} (\operatorname{rot} \operatorname{rot} \boldsymbol{\psi}) e^{ipt} = k^2 \operatorname{rot} \boldsymbol{\psi} e^{ipt}. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (7)$$

Thus $\boldsymbol{\sigma}_1$ is irrotational and $\boldsymbol{\sigma}_2$ is equivoluminal.

General solutions of (4) and (5) in polar coordinates are, as are well known,

$$\phi = \sum_{n=0}^{\infty} \frac{A_n}{\sqrt{r}} C_{n+\frac{1}{2}}(hr) S_n, \quad \cdots\cdots\cdots\cdots (8)$$

$$= \sum_{n=0}^{\infty} \frac{\boldsymbol{B}_n}{\sqrt{r}} C_{n+\frac{1}{2}}(kr) S_n, \quad \cdots\cdots\cdots\cdots (9)$$

where C_n and S_n are cylindrical function and surface harmonics of order n. As we are now concerned to the diverging waves only, we have only take Hankel's cylindrical function of second kind $H_n^{(2)}$ for C_n and $P_n^m(\cos\theta) \left.\begin{matrix}\sin m\varphi \\ \cos m\varphi\end{matrix}\right\}$ for S_n. Thus relevant solutions of (8) and (9) are

$$\phi = \sum_{n=0}^{\infty} \sum_{m=0}^{n} A_{nm} \frac{H_{n+\frac{1}{2}}^{(2)}(hr)}{\sqrt{r}} P_n^m(\cos\theta) \sin(m\varphi + \varepsilon), \quad \cdots (10)$$

$$\boldsymbol{\psi} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \boldsymbol{B}_{nm} \frac{H_{n+\frac{1}{2}}^{(2)}(kr)}{\sqrt{r}} P_n^m(\cos\theta) \sin(m\varphi + \varepsilon), \quad \cdots\cdots (11)$$

respectively. Displacements $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ can be culculated mechnically by the formula (2) and (3). We shall denote the components of displacement in the directions r, θ and φ by u, v and w respectively. A term in the series gives

$$\left.\begin{aligned} u_1 &= \frac{\partial\phi}{\partial r} e^{ipt} = A_{nm} \frac{d}{dr} \left\{ \frac{H_{n+\frac{1}{2}}^{(2)}(hr)}{\sqrt{r}} \right\} P_n^m(\cos\theta) \sin(m\varphi + \varepsilon) e^{ipt}, \\ v_1 &= \frac{1}{r} \frac{\partial\phi}{\partial\theta} e^{ipt} = A_{nm} \frac{H_{n+\frac{1}{2}}^{(2)}(hr)}{r^{\frac{3}{2}}} \frac{dP_n^m(\cos\theta)}{d\theta} \sin(m\varphi + \varepsilon) e^{ipt}, \\ w_1 &= \frac{1}{r\sin\theta} \frac{\partial\phi}{\partial\varphi} e^{ipt} = mA_{nm} \frac{H_{n+\frac{1}{2}}^{(2)}(hr)}{r^{\frac{3}{2}}} \frac{P_n^m(\cos\theta)}{\sin\theta} \cos(m\varphi + \varepsilon) e^{ipt}. \end{aligned}\right\} \quad (12)$$

$$\left.\begin{aligned}
u_2=&\Big[\Big\{\frac{n(n+1)}{r^{\frac{5}{2}}}H^{(2)}_{n+\frac{1}{2}}(kr)-\frac{2}{r^{\frac{3}{2}}}\frac{dH^{(2)}_{n+\frac{1}{2}}(kr)}{dr}\Big\}B_rP_n^m(\cos\theta)\sin(m\varphi+\varepsilon)\\
&+\frac{d}{dr}\Big(\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\Big)\Big\{B_\theta\frac{dP_n^m(\cos\theta)}{d\theta}\sin(m\varphi+\varepsilon)\\
&-B_\varphi\frac{mP_n^m(\cos\theta)}{\sin\theta}\cos(m\varphi+\varepsilon)\Big\}\Big]e^{ipt},\\
v_2=&\Big\{\frac{1}{r}\frac{\partial}{\partial\theta}(\operatorname{div}\boldsymbol{\psi})+k^2B_v\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}P_n^m(\cos\theta)\sin(m\varphi+\varepsilon)\Big\}e^{ipt},\\
w_2=&\Big\{\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\operatorname{div}\boldsymbol{\psi})+k^2B_\varphi\frac{1}{\sqrt{r}}H^{(2)}_{n+\frac{1}{2}}(kr)P_n^m(\cos\theta)\sin(m\varphi+\varepsilon)\Big\}e^{ipt}.
\end{aligned}\right\}(13)$$

In which $\boldsymbol{B}$ is assumed generally to be in the direction (θ_0, φ_0) and

$$\begin{aligned}
\operatorname{div}\boldsymbol{\psi}=&B_r\frac{d}{dr}\Big\{\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\Big\}P_n^m(\cos\theta)\sin(m\varphi+\varepsilon)\\
&+\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\Big[\frac{d\{B_\theta P_n^m(\cos\theta)\}}{d\theta}\sin(m\varphi+\varepsilon)\\
&-\frac{P_n^m(\cos\theta)}{\sin\theta}\frac{d\{B_\varphi\sin(m\varphi+\varepsilon)\}}{d\varphi}\Big].\quad\ldots\ldots\ldots\ldots(14)
\end{aligned}$$

$$\left.\begin{aligned}
&B_x=B\sin\theta_0\cos\varphi_0,\quad B_y=B\sin\theta_0\sin\varphi_0,\quad B_z=B\cos\theta_0,\\
&B_r=B_x\sin\theta\cos\varphi+B_y\sin\theta\sin\varphi+B_z\cos\theta,\\
&B_\theta=B_x\cos\theta\cos\varphi+B_y\cos\theta\sin\varphi-B_z\sin\theta,\\
&B_\varphi=-B_x\sin\varphi+B_y\cos\varphi.
\end{aligned}\right\}\ldots(15)$$

It is worthy of notice that the components v_1, w_1 of $\boldsymbol{\sigma}_1$, transverse to the direction of propagation, is of higher order with respect to $\frac{1}{r}$ than u_1, and conversely the radial component of $\boldsymbol{\sigma}_2$ is of higher order with respect to $\frac{1}{r}$ than the other components, and they approach the usual idea of longitudinal and transverse waves as they recede further from the origin.

The transverse wave above considered is that of first kind named by Prof. K. Sezawa.[87] He also found another type of transverse wave

87) K. SEZAWA, *loc. cit.*, 43).

and called it transverse wave of the second kind. It is purely distortional wave and has no radial component of displacement.

$$\left.\begin{aligned} u_3&=0,\\ v_3&=\frac{m}{n(n+1)}D\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P^m_n(\cos\theta)}{\sin\theta}\cos(m\varphi+\varepsilon)e^{ipt},\\ w_3&=\frac{-1}{n(n+1)}D\frac{1}{\sqrt{r}}H^{(2)}_{n+\frac{1}{2}}(kr)\frac{dP^m_n(\cos\theta)}{d\theta}\sin(m\varphi+\varepsilon)e^{ipt}.\end{aligned}\right\}\quad\ldots\ldots(16)$$

Any condition near the origin for the displacement or the traction is fulfilled by suitable combination of these three kinds of waves.

Now I will pick out some simple examples from the above solutions and correlate it with the solutions already obtained by the application of the Stokes' formula, and reveal the relation with the models of the mechanisms of earthquake occurrence.

i) $n=0$.

$$\left.\begin{aligned} u_1&=A_0\frac{d}{dr}\left\{\frac{1}{\sqrt{r}}H^{(2)}_{\frac{1}{2}}(hr)\right\}e^{ipt}=-hA_0\frac{H^{(2)}_{1+\frac{1}{2}}(hr)}{\sqrt{r}}e^{ipt},\\ v_1&=w_1=0.\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots(17)$$

This is the simple source, and corresponds to the compression or dilatation centre of A. E. H. Love.[88] The forces which give rise to this wave are, as Love already has shown, three pairs of doublets without moment perpendicular to each other. The transverse waves are not generated by these forces. This type of mechanism is considered as that of the volcanic eruption or explosion experiment. But, as Prof. Omori[89] observed in the earthquakes near the Mt. Asama, not only "push" or compression but also "pull" or dilatation, and in addition to this, transverse waves were clearly observed. So we cannot conclude that the volcanic earthquake is confined to that given rise by dilatational centre.

ii) $n=1$.

a) $m=0$,

$$\left.\begin{aligned} u_1&=A_1\frac{d}{dr}\left\{\frac{H^{(2)}_{1+\frac{1}{2}}(hr)}{\sqrt{r}}\right\}\cos\theta\, e^{ipt}=A_1\sqrt{\frac{2h}{\pi}}\sqrt{1+\left(\frac{2}{(hr)^2}\right)^2}\frac{e^{i(pt-hr+\delta_u)}}{r}\cos\theta,\\ v_1&=-A_1\frac{1}{r^{\frac{3}{2}}}H^{(2)}_{1+\frac{1}{2}}(hr)\sin\theta\, e^{ipt}=-A_1\sqrt{\frac{2h}{\pi}}\sqrt{1+\frac{1}{(hr)^2}}\frac{e^{i(pt-hr-\delta_v)}}{r^2}\sin\theta,\end{aligned}\right\}$$

88) A. E. H. LOVE, *loc. cit.*, 21) and 29).

89) F. OMORI, *loc. cit.*, 3).

$$w_1=0, \quad \tan\delta_u=\frac{h^2r^2-2}{2hr}, \quad \tan\delta_v=\frac{-1}{hr}. \qquad \Bigg\} \qquad \ldots\ldots\ldots\ldots (18)$$

The force which give rise to the above wave is, as well known since the time of Stokes, a siglet in the direction of $\theta=0$.

$$F=A_1 4(\lambda+2\mu)\sqrt{4\pi h}\, e^{i\left(pt+\frac{\pi}{2}\right)}. \quad \ldots\ldots\ldots\ldots (19)$$

And the transverse wave due to this force is obtained by the transformation of the solutions, for example, obtained by Prof. Matuzawa[90] from Cartesian to polar coordinates.

$$\left.\begin{aligned} u_2&=A_1\left(\frac{k}{h}\right)^{\frac{3}{2}}\frac{2}{r^{\frac{3}{2}}}H^{(2)}_{1+\frac{1}{2}}(kr)P_1(\cos\theta)e^{ipt},\\ v_2&=A_1\left(\frac{k}{h}\right)^{\frac{3}{2}}\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}H^{(2)}_{1+\frac{1}{2}}(kr)\right\}\frac{dP_1(\cos\theta)}{d\theta}e^{ipt},\\ w_2&=0,\end{aligned}\right\} \quad \ldots (20)$$

which is identical with that obtained by (13) substituting

$$\cdot' = B_z\frac{H^{(2)}_{\frac{1}{2}}(kr)}{\sqrt{r}},$$

where
$$B_z=A_1\frac{k^{\frac{1}{2}}}{h^{\frac{3}{2}}}.$$

It is also to be remarked that the longitudinal wave (18) is equivalent to a pair of simple sourse situated at ($r=0$) and ($r=\delta z$, $\theta=0$) with different sign and of the same strength $\frac{A_1}{h\delta z}$, that is a doublet

$$\phi=-\frac{A_1}{h}\frac{d}{dz}\left\{\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{r}}\right\} \quad \ldots\ldots\ldots\ldots (21)$$

and transverse wave (20) is the transverse wave of the first kind obtained by Prof. Sezawa. These waves were already shown by Lord Kelvin[91] to be generated by the vibration of a rigid sphere in the direction of $\theta=0$. Professors T. Shida and M. Matuyama considered them as the model of "depression earthquake or Abstürzbeben," though the existence

90) T. MATUZAWA, *loc. cit.*, 28).
91) Lord KELVIN, *loc. cit.*, 18).

of this type of earthquake was suspected by Prof. S. T. Nakamura[92] and Prof. M. Ishimoto from the impossibility of vacant spaces under such high pressure within the earth and conservation of momentum.

b) $m=1$.

There will be no need to say that the waves generated by the force in the direction of $x\left(\theta=\frac{\pi}{2},\ \varphi=0\right)\left\{\text{or } y\left(\theta=\varphi=\frac{\pi}{2}\right)\right\}$ are

$$\left.\begin{aligned}
&\phi=A_1\frac{H^{(2)}_{1+\frac{1}{2}}(hr)}{\sqrt{r}}\sin\theta\cos\varphi, \quad \left\{\phi=A_1\frac{H^{(2)}_{1+\frac{1}{2}}(hr)}{\sqrt{r}}\sin\theta\sin\varphi,\right\}\\
&\psi=B_x\frac{H^{(2)}_{\frac{1}{2}}(kr)}{\sqrt{r}}. \quad \left\{\psi=B_y\frac{H^{(2)}_{\frac{1}{2}}(kr)}{\sqrt{r}}.\right\}
\end{aligned}\right\} \tag{22}$$

iii) $n=2$.

a) $m=0$,

$$\left.\begin{aligned}
&\phi=A_{20}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}P_2(\cos\theta),\\
&u_1=A_{20}\left\{\frac{h}{\sqrt{r}}H^{(2)}_{1+\frac{1}{2}}(hr)-\frac{3}{r^{\frac{3}{2}}}H^{(2)}_{2+\frac{1}{2}}(hr)\right\}P_2(\cos\theta)e^{ipt}\\
&\quad\approx A_{20}\sqrt{\frac{2h}{\pi}}\frac{e^{i(pt-hr)}}{r}\left(-1+\frac{4i}{hr}\right)P_2(\cos\theta),\\
&v_1=A_{20}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{3}{2}}}\frac{dP_2(\cos\theta)}{d\theta}e^{ipt}\approx iA_{20}\sqrt{\frac{2}{\pi h}}\frac{e^{i(pt-hr)}}{r^2}3\sin\theta\cos\theta.\\
&w_1=0
\end{aligned}\right\} \tag{23}$$

This is composed of a quadruple source and a simple source

$$\phi=\frac{A_{20}}{2}\left[\frac{3}{h^2}\frac{d^2}{dz^2}\left\{\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{r}}\right\}+\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{r}}\right] \tag{24}$$

as Lord Rayleigh has already remarked. On the other hand double source is generated by a single force, so we can perceive that the waves here concerned is generated by doublet force and single source at the origin. Really as we have supposed the waves generated by a doublet without moment are[93]

92) S. T. NAKAMURA, *loc. cit.*, 24).

93) Prof. MATUZAWA has already obtained the solution in Cartesian Coordinates, with errors in the transverse wave. *loc. cit.*, 28).

$$\left.\begin{aligned}
u_1&=\frac{-2iZ}{3\cdot 4\pi(\lambda+2\mu)}\sqrt{\frac{\pi h}{2}}\frac{d}{dr}\left\{\frac{1}{2}\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{r}}-\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}P_2(\cos\theta)\right\}e^{ipt}\\
&\approx\frac{-ihZ\,e^{i(pt-hr)}}{4\pi(\lambda+2\mu)r}\left\{\cos^2\theta+i\frac{1}{hr}(4\cos^2\theta-1)\right\},\\
v_1&=\frac{-2iZ}{3\cdot 4\pi(\lambda+2\mu)}\sqrt{\frac{\pi h}{2}}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{3}{2}}}\frac{dP_2(\cos\theta)}{d\theta}e^{ipt}\\
&\approx\frac{-iZ}{4\pi(\lambda+2\mu)}\frac{e^{i(pt-hr)}}{r^2}\sin 2\theta,\\
w_1&=0,\\
u_2&=\frac{2iZ}{4\pi\mu}\sqrt{\frac{\pi k}{2}}\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}P_2(\cos\theta)e^{ipt}\approx\frac{-2Z}{4\pi\mu}\frac{e^{i(pt-kr)}}{r^2}P_2(\cos\theta),\\
v_2&=\frac{iZ}{3\cdot 4\pi\mu}\sqrt{\frac{\pi k}{2}}\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}H^{(2)}_{2+\frac{1}{2}}(kr)\right\}\sin\theta\cos\theta e^{ipt}\\
&\approx\frac{iZk}{4\pi\mu}\frac{e^{i(pt-kr)}}{r}\cos\theta\sin\theta\left(1-i\frac{3}{kr}\right),\\
w_2&=0.
\end{aligned}\right\}\quad(25)$$

The transverse wave of (25) is equivalent to that given by

$$\psi=B_z\frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}}\cos\theta,\quad\ldots\ldots\ldots\ldots\ldots(26)$$

where
$$B_z=\frac{iZ}{4\mu\sqrt{2\pi k}},$$

or Prof. Sezawa's transverse wave of the first kind when $n=2$, $m=0$.

The nodal surface of P-wave in (25) is a plane $\theta=\frac{\pi}{2}$, but in this case no difference in the " pull " or " push " of initial motion is observed on both sides of the nodal plane.

If we superpose to (25) simple source of suitable strength, we obtain nodal cone whose axis coincides with the pole $\theta=0$. The wave given by (23) is an example. The model of Proffessors Matuyama and Ishimoto, in which the generating line of the nodal cone meet with the axis at an angle $\frac{\pi}{4}$, is obtained by superposing

$$\phi_0=-\frac{3}{4}\mathfrak{A}\frac{H^{(2)}_{\frac{1}{2}}(kr)}{\sqrt{r}},\quad\left(\mathfrak{A}=\frac{-2\,iZ}{3\cdot 4\pi(\lambda+2\mu)}\sqrt{\frac{\pi h}{2}}\right)$$

to (25).

From these reasoning, we can see that the intensity of simple source to be superposed is arbitrary so that we can obtain nodal cones of any vertical angle. However simple source does not accompany with transverse waves, and consequently the transverse wave accompanying with such mechanism, which give rise to longitudinal wave, having nodal cone, is therefore the same as that in (25). The relative magnitude of P- and S-waves is a function of θ as well as r, but the ratio of the maximum amplitudes at r=const. of the S- and P-waves due to doublet without moment is

$$\frac{1}{2}\left(\frac{k}{h}\right)^3=\frac{1}{2}\left(\frac{a}{b}\right)^3,$$

where a and b are the velocities of longitudinal and transverse waves respectively, that is the maximum amplitudes are inversely proportional to the cube of the velocities. This ratio becomes the larger when the vertical angle of the nodal cone becomes the smaller.

b) $m=1$.

$$\phi=A_{21}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}\frac{3}{2}\sin 2\theta \begin{Bmatrix}\cos\varphi\\ \sin\varphi\end{Bmatrix}.$$

These correspond to the similar case as c), the direction of doublet being in $x\left(\theta=\frac{\pi}{2},\ \varphi=0\right)$ or $y\left(\theta=\varphi=\frac{\pi}{2}\right)$ direction.

c) $m=2$.

$$\phi=A_{22}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}3\sin^2\theta \begin{Bmatrix}\sin 2\varphi\\ \cos 2\varphi\end{Bmatrix} \quad\ldots\ldots\ldots\ldots\ldots(27)$$

correspond to a quadruple source

$$\frac{\partial^2}{\partial x\partial y}\left\{\frac{3}{2}\frac{A_{22}}{h^2}\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{r}}\right\}, \quad\ldots\ldots\ldots\ldots\ldots(28)$$

and is hitherto been called the model B. This is equivalent to a longitudinal wave due to a doublet force with moment in the direction of x, studied already by Prof. Matuzawa.[94] The waves due to a doublet

$$F_x=\frac{X}{\delta y}e^{ipt} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

94) T. MATUZAWA, *loc. cit.*, 28). In his solution we find small unimportant mistake in the higher order of $\frac{1}{r}$.

in polar coordinates are

$$\left.\begin{aligned}
u_1 &= \frac{iX\sqrt{\frac{1}{2}\pi h}}{4\pi(\lambda+2\mu)}\frac{d}{dr}\left\{\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}\right\}\frac{1}{2}\sin^2\theta\sin 2\varphi e^{ipt},\\
v_1 &= \frac{iX\sqrt{\frac{1}{2}\pi h}}{4\pi(\lambda+2\mu)}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{3}{2}}}\sin\theta\cos\theta\sin 2\varphi e^{ipt},\\
w_1 &= \frac{iX\sqrt{\frac{1}{2}\pi h}}{4\pi(\lambda+2\mu)}\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{3}{2}}}\sin\theta\cos 2\varphi e^{ipt},\\
u_2 &= \frac{iX\sqrt{\frac{1}{2}\pi k}}{4\pi\mu}\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\frac{3}{2}\sin^2\theta\sin 2\varphi\, e^{ipt},\\
v_2 &= \frac{iX\sqrt{\frac{1}{2}\pi k}}{4\pi\mu}\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}H^{(2)}_{2+\frac{1}{2}}(kr)\right\}\frac{1}{2}\sin\theta\cos\theta\sin 2\varphi e^{ipt},\\
w_2 &= \frac{-iX\sqrt{\frac{1}{2}\pi k}}{4\pi\mu}\left\{\frac{kH^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}}\sin^2\varphi+\frac{1}{r^{\frac{3}{2}}}H^{(2)}_{2+\frac{1}{2}}(kr)\cos 2\varphi\right\}\sin\theta e^{ipt}.
\end{aligned}\right\} \quad \ldots\ldots(30)$$

d) If we superpose to (30) another doublet with moment in the direction of $y\left(\theta=\varphi=\frac{\pi}{2}\right)$ with different sign and of the same strength, which is obtained by substituting $\varphi+\frac{\pi}{2}$ for φ of (30) and changing signs, then the components u_1, v_1, w_1, u_2, and v_2 become doubled and

$$w_2=\frac{iA\sqrt{\frac{1}{2}\pi k}}{4\pi\mu}\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}H^{(2)}_{2+\frac{1}{2}}(kr)\right\}\sin\theta\cos 2\varphi\, e^{ipt}\,.. \quad (31)$$

(where $A\equiv X=Y$). And this is a mechanism of model B.

The transverse wave (u_2, v_2, w_2) in this case is derivable from

$$\psi=\boldsymbol{B}_x\frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}}\sin\theta\sin\varphi, \quad \left(B_x=\frac{iA\sqrt{\frac{1}{2}\pi k}}{4\pi\mu}\right) \quad \ldots.(32)$$

and corresponds to the Prof. Sezawa's transverse wave of the first kind.

And if we put $\varphi+\frac{\pi}{4}$ for φ in the solutions above obtained we have exactly the same solution obtained by Mr. M. Hasegawa by the combination of two pairs of doublet without moment perpendicular to each other.

Thus we see that the model of Prof. Matuzawa corresponds to a kind of fault earthquake and that of Prof. Hasegawa corresponds to a kind of crack earthquake or Spaltungsbeben. This difference has not hitherto been noticed and sometimes confused.[95] It will be an interesting task of future investigation to distinguish these two mechanisms by observing the azimuthal component of the transverse wave w_2 given by (30) or (31).

iv)

a) If we superpose the two pairs of doublets with moment considered in iii), c) without changing the sign we have, as A. E. H. Love has already pointed out, a rotational centre round z-axis ($\theta=0$). Then the components u_1, v_1, w_1, u_2 and v_2 vanishes and only

$$w_2=-\frac{ikA}{4\pi\mu}\sqrt{\frac{1}{2}\pi k}\,\frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}}\sin\theta\, e^{ipt} \quad \ldots\ldots\ldots(33)$$

remains. This is the simplest transverse wave of the second kind.

b) The distortional centre round z-axis is that obtained by differentiating the solution of the rotational centre (33) with respect to z,

$$w_3=-\frac{ik^2A}{4\pi\mu}\sqrt{\frac{1}{2}\pi k}\,\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{\sqrt{r}}\sin\theta\cos\theta \quad \ldots\ldots\ldots(34)$$

and this also is the transverse wave of the second kind. Thus it will be observed that longitudinal wave cannot be generated by the force which give rise to transverse waves of the second kind.

In what has been stated, the mechanisms which have hitherto been considered (being obtained by the method of singularities) are correlated with the general solutions obtained by the method of series. The remark will be unnecessary that any other case can be obtained by suitable combination of the general solutions given in (12), (13) and (16). But it is to be remarked that the solution of longitudinal wave (12) is identical with that of Prof. Sezawa and the solution of transverse wave (u_2, v_2, w_2) in (13) is not identical with those of the transverse wave of

95) M. HASEGAWA, *loc. cit.*, 4).
H. HONDA, *loc. cit.*, 9).

the second kind obtained by Prof. Sezawa except when $n=0$ and 1, but they are a combination of a few of the solutions of Prof. Sezawa, and it is worthy of note that general feature at a great distance which is given by the second term of (13) is conspicuously simple.

Another word to be added is about the word "force operative at a point." It is not indeed a force operative at a mathematical point. The definition of Prof. Love[96] is

$$X=\rho\int\int\int X'dx'dy'dz'=\rho V\overline{X} \quad \ldots\ldots\ldots\ldots(35)$$

provided the ρ and V are density and volume where the force is in operation, and $\overline{X}$ is mean force per unit mass in the volume V. Thus we may have a clue to the magnitude of hypocentral region from the observation of earthquake waves and some consideration on the relation of the mechnisms and the forces.

Chapter III. The crustal structure and the amplitude of Seismic bodily waves.

The elastic waves discussed in Chapter II is those in an isotropic homogeneous elastic body. But the material from which the earth consists is neither homogeneous nor perfect elastic, but they may have solid viscosity which causes the waves to decay. The influence of viscosity to the seismic waves seems comparatively small, but the reflexon and refraction is so much affected by the incidence angle that the effect of heterogeneity within the crust cannot be neglected. But the theory of elastic waves in such medium has only been began to be studied by Mr. R. Yosiyama and no rigorous theoretical culculation cannot be effected taking the actual structure of the crust into consideration. The effect on the amplitude of the curving of seismic rays due to continual variation of velocity was considered by Dr. H. Jeffreys.[97] He approximated the continual variation by the small discontinuous variation and the effect of the reflexion and refraction was culculated to show that the effect is negligible.

The variation of amplitude due to the divergence of wave front was first estimated by K. Zöppritz[98] from the consideration of surface density

96) A. E. H. Love, *loc. cit.*, 37), p. 184.

97) H. Jeffreys, *M. N. R. A. S. Geophys. Suppl.*, 1 (1926), 321–334.

98) K. Zöppritz, *Nachr. d. kgl. Ges. d. Wiss. zu Göttingen, Math.-phys. Klasse*, (1912), 123–143.

and conservation of energy. The defect of Zöppritz's method was supplemented after his death by E. Wiechert,[99] and the method was applied by L. Geiger and Prof. B. Gutenberg[100] in the investigation of structure of earth's interior. Dr. H. Jeffreys[101] also attacked the same problem, and the present writer[102] also could explain the variation of the amplitude found by Mr. H. Honda from the North Idu earthquake by the same method.

From the theory of spherical waves in homogeneous isotropic medium follows the amplitude of bodily wave is inversely proportional to hypocentral distance, which is identical with the law that follows from the energy density method. In the heterogeneous medium we should have closer approximation if we use the distance (s) along the curved path along which the wave propagated than the radial distance from the hypocentre.

Taking, for example, longitudinal wave ($n=2$, $m=0$) we can make use of

$$u_1 \approx A\sqrt{\frac{2h}{\pi}}\,\frac{1}{s}\cos(pt-hs)P_2(\cos\theta) \qquad \ldots\ldots\ldots\ldots (36)$$

in place of

$$u_1 \approx A\sqrt{\frac{2h}{\pi}}\,\frac{1}{r}\cos(pt-hr)P_2(\cos\theta).$$

Further approximation will be obtained by taking the variation of h along the path into consideration. By the relations

$$\int_0^s h\,ds = \int_0^s \frac{p}{a}\,ds = pt_P$$

(in which a is velocity of longitudinal wave and t_P is travel-time of P-wave), we have

$$\begin{aligned} u_1 &\approx A\sqrt{\frac{2h}{\pi}}\,\frac{1}{\bar{a}\displaystyle\int_0^s \frac{ds}{a}}\cos\left(pt-\int_0^s h\,ds\right)P_2(\cos\theta) \\ &= A\sqrt{\frac{2h}{\pi}}\,\frac{1}{\bar{a}t_P}\cos p(t-t_P)P_2(\cos\theta) \\ &= A\sqrt{\frac{2p}{\pi a^3}}\left(\frac{a}{\bar{a}}\right)\frac{1}{t_P}\cos p(t-t_P)P_2(\cos\theta), \qquad \ldots\ldots\ldots (37) \end{aligned}$$

99) E. WIECHERT, *ibid.* foot note on page 127.
100) L. GEIGER u. B. GUTENBERG, *do.*, 144-206.
101) H. JEFFREYS, *M. N. R. A. S. Geophys. Suppl.*, 1 (1926), 334-348.
102) H. KAWASUMI, On the initial motion of earthquake motion. Read at the meetng of Earthquake Research Institute on July 7, 1931. See appendix.

from which we can perceive that the amplitude is nearly proportional to $a^{-\frac{3}{2}}$. Considering the constancy of the velocity on the earth's surface, we see that the amplitude on the earth's surface is inversely proportional to the travel-time. Now I will compare this method and the method already mensioned in which energy density is used. The variation of amplitude due to the divergence of wave front will be sufficiently given by the consideration of case in which energy is uniformly sent out in all directions.

Let us suppose that the energy emitted per unit solid angle is I. Then the energy within a range of angle de_h at the hypocentre is $2\pi I \cos e_h de_h$, and this energy is sent to a zone within the epicentral distance θ and $\theta+d\theta$, the area of which on the earth's surface is $2\pi R^2 \sin\theta d\theta$, where R is the radius of the earth. The angle between the wave front and the surface is $\frac{1}{2}\pi - e_0$, (where e_0 is angle of emergence at the earth's surface) so that unit surface corresponds to an area $\sin e_0$ of the wave front. Hence the enrgy per unit area of the wave front is

$$\frac{I}{R^2}\frac{\cos e_h de_h}{\sin\theta \sin e_0 d\theta} = -\frac{I}{R^2}\frac{d(\sin e_h)}{\sin e_0 d(\cos\theta)} \quad \ldots\ldots \quad (38)$$

Considering that the amplitude ($\mathfrak{A}$) of the incident wave at the surface is proportional to the energy density on the wave front, we have

$$\mathfrak{A} \propto \sqrt{\frac{-d(\sin e_h)}{\sin e_0 d(\cos\theta)}} \equiv U. \quad \ldots\ldots\ldots\ldots\ldots (39)$$

The writer already obtained the velocity of the seismic wave within the earth, and calculated time distance curve and the constants of seismic ray

$$K = \frac{r}{v}\sin i = \frac{r}{v}\cos e. \quad \ldots\ldots\ldots\ldots\ldots (40)$$

We can therefore easily culculate U by (39) and (40), but for simplicities sake $\frac{d \sin e_h}{d(\cos\theta)}$ is obtained graphically for the case of hypocentral depth of 250 km. The results are compared in Table 1 with those culculated by the "inverse travel-time formula."

We are naturally led to the notion that the distribution of amplitude given by spherical harmonics at the hypocentre is preserved along the seismic ray, and this is proved by the recent theoretical investigation of the waves in heterogeneous medium by Mr. R. Yosiyama.[103)]

103) R. Yosiyama, *loc. cit.*, 46).

Table 1. Comparison of the variations of amplitude of seismic waves with epicentral distance (θ) culculated by the energy dencity formula (39) and inverse travel time formula.

θ	2°16'.6	3°20'.6	4°22'.8	5°24'.0	6°24'.4	7°24'.5	8°24'.4	9°24'.0
$\frac{1}{U}$	0·0687	0·0773	0·0953	0·1123	0·1421	0·1595	0·1786	0·1916
t_P	48·0	57·9	72·1	84·9	97·7	110·5	123·3	135·9
$\left(\frac{1}{U}\Big/t_P\right)\times 10^3$	1·43	1·30	1·32	1·32	1·45	1·45	1·45	1·41

We have hitherto considered the effect of divergence of wave front only, but if there is finite discontinuity in the nature of the medium in which the wave is propagated, the reflextion and refraction take place breaking usually into four kinds of waves. The reflexion and refraction of spherical waves at spherical discontinuity surface is not only too complicated to rigorous calculation, but it becomes of no use if the heterogeneity is not taken into consideration.

Thus the case of reflexion and refraction of plane wave at plane surface has been used practically. This effect at the discontinuities at crustal layers has been calculated by Mr. T. Suzuki and the writer.[104] Thus the amplitude of incident wave is diminished to

$$\frac{1}{\bar{a}t_P}\prod\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right), \quad \dots\dots\dots\dots\dots\dots\dots(41)$$

where the symbol $\prod\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)$ denotes the product of the ratio of the amplitude ($\mathfrak{A}'$) of refracted wave to that of incident wave ($\mathfrak{A}$) at every discontinuity the wave has traversed.

Thus the motion of a particle on the surface modified by the reflexion at the surface is, taking the former example,

$$u_{1H}=-A\sqrt{\frac{2h_0}{\pi}}\frac{1}{\bar{a}t_P}\prod\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)\left(\frac{u}{\mathfrak{A}}\right)_0 P_2(\cos\theta)\cos p(t-t_P), \quad ..(42)$$

$$u_{1z}=-A\sqrt{\frac{2h_0}{\pi}}\frac{1}{\bar{a}t_P}\prod\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)\left(\frac{w}{\mathfrak{A}}\right)_0 P_2(\cos\theta)\cos p(t-t_P), \quad (42)$$

where the factors $\left(\frac{u}{\mathfrak{A}}\right)_0$ and $\left(\frac{w}{\mathfrak{A}}\right)_0$ is the ratios of horizontal and veritcal (the surface being assumed horizontal) components to the amplitude of

104) H. KAWASUMI and T. SUZUKI, *Disin* (*Journ. Seismol. Soc. Japan*), 4 (1932), 277-307, (in Japanese).

incident wave in case of surface reflexion.

In the next chapters the applications are made what has been stated to some actual earthquakes to reveal the mechanism of their occurrences.

Chapter IV. Amplitude of Seismic waves and the mechanism of occurrence of the deep-seated Central Japan earthquake of June 2nd 1931.

This earthquake was studied by Mr. K. Tanahasi[105] and is a conspicuous example which cannot be explained by any other mechanism than the model A. Mr. Tanahasi explained the distribution of pull and push of initial motion of P- and S-wave by means of the theory of Prof. M. Matuyama.[106] So long as the writer is aware of, Mr. Tanahasi is the first to discuss the direction of initial motion of S-wave, but he has a little misinterpreted the theory of Prof. Matuyama, and overlooked the existence of transverse wave of second kind which appear independently of the P-wave. The writer will here state the result of his quantitative study on the magnitude of the waves to reveal the mechanism of occurrence of the earthquake which has qualitatively been discussed by Mr. Tanahasi.

The depth of this earthquake is given in Kisyô Yôran as 250 km. and the result of Mr. Tanahasi is 240 km. The travel-time curve given by Mr. Tanahasi nearly coinsides with the author's calculated curve for the hypocentral depth of 253·3 km. I will therefore adopt the last value for the depth of this earthquake.

Let us first tabulate in Table II the observations of Mr. Tanahasi supplemented from those given in Kisyô Yôran.

From the distribution of the initial motions of P-wave writen on a map (Fig. 1) we see two nodal curves resembling hyperbola as were already found by. Mr. Tanahasi. Thus we percieve the amplitude of P-wave is proportional to zonal harmonics. The shortest distance between these curves is along a line passing through the epicentre and in the azimuth of which lies the polar axis ($\theta=0$). The intersections of this straight line with the nodal lines are $\theta=0{\cdot}^{\circ}5$ and $\theta=2{\cdot}^{\circ}7$ where θ is epicentral distance. Since we know[107] hypocentral depth and constants of seismic ray K as a function of θ, we are able to know the inclination of seismic ray from the zenith at the hypocentre by the formula

105) K. Tanahasi, *loc. cit.*, 6).
106) M. Matuyama, *loc. cit.*, 4).
107) H. Kawasumi, *Bull. Earthq. Res. Inst.*, 10 (1932). 94-129. Table IX.

Table II. Observed magnitude of initial motions of P- and S-waves of the deep-seated Central-Japan earthquake of June 2, 1931. (Mainly due to Mr. Tanahasi.)

Station	P-wave (μ)			S-wave (μ)		
	E	N	Z	E	N	Z
Nagoya	13	54	−87	560	−345	?
Gihu	35	40	−178	363	−250	47
Hikone	22	25	−190	413	−524	−102
Nagano	−5	18	+117	318	698	−163
Numadu	−27	40	−60	−925	263	43
Mishima	−26	17	−74	−720	418	−218
Wazima	−1	1	+4	680	−478	−116
Kumagaya	1	−1	24	−142	433	98
Kyôto	64	34	−69	152	−335	−31
Wakayama	12	16	—	51	−216	—
Kakioka	18	4	—	−33	264	—
Ôsaka	6	7	−129	−175	−565	−8
Tokyo	Time mark			−173	240	indistinct
Yakohama	−5	9	—	−152	216	—
Toyooka	57	21	−149	75	−217	−27
Tukubasan	1	8	+5	−82	247	100
Kôbe	70	18	−77	−106	−273	+23
Sumoto	10	8	−15	112	−224	−76
Sionomisaki	−5	−11	+18	160	−199	−24
Tyôsi	−32	+5	−22	98	149	−20
Hatidyôzima	10	00	+6	−101	+	6
Sendai	39	49	87	−178	257	122
Sakai	58	20	—			
Kôti	−2	−2	4	122	−67	27
Hamada	16	3	—	8	−73	—
Akita	36	47	155	indistinct		
Marioka	79 to NE	14 to NW	—	61 to NE	90 to NW	—
Simidu	Time mark			43	−46	—
Miyasaki	−7	−9	13	43	−32	−15
Kumamoto	−1	−2	4	Time mark		
Nagasaki	−1	+1	2	7	−13	?
Taikyû	+13	+1	—	00?	−36	—
Zinsen	10	−4	5			
Matumoto	−10	−10				
Kôhu	−10	+20				
Miyadu	+72	+38				
Hukusima	+42	+34	+65			
Yamagata	+70	+60				
Tu	+15	+55				
Aomori	+70	+42				
Kusiro	+8	+12				
Niihama	+18	+6				

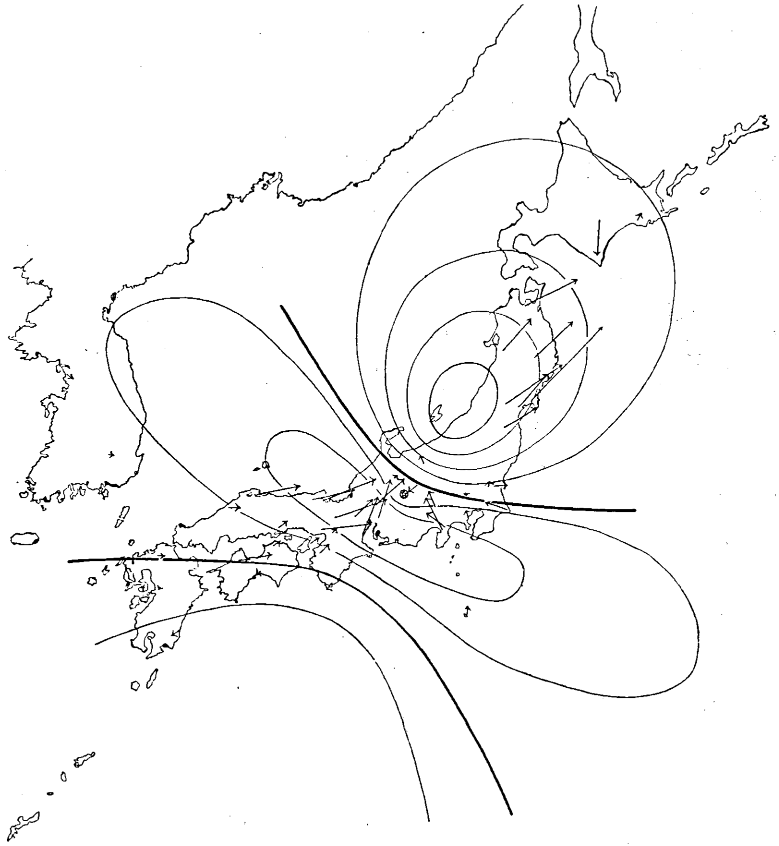

Fig. 1. The distribution of initial motions actually observed (arrows) of P-wave due to the deep-seated earthquake of June 2, 1931, and iso-amplitude lines calculated for the mechanism

$$\phi = A\frac{\overset{(2)}{H}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}P_2(\cos\theta)$$

$$i_h = \sin^{-1}\frac{v_h}{r_h}K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(44)$$

as a function of θ (Table III), where the entities with suffix h are the values at the hypocentre.

Plotting these values on a section paper we obtain $i_h = f(\theta)$ curve from which we obtain $i_h = 12°$ and $58°$ for $\theta = 0^{\circ}\!\cdot5$ and $2^{\circ}\!\cdot7$ respectively. Thus the generating lines of the nodal cone in the plane passing the polar axis intersect at the hypocentre at an angle $12° + 58° = 70°$. The coincidence of this angle with that of $P_2(\cos\theta)$ may be considered as something more than mere coincidence. Thus we are led to believe that the motion on the earth's surface due to this earthquake is given by (23) and (25). If so, the polar axis at the hypocentre inclines from the zenith

$$i_{h0} = 12° + 55° = 67°.$$

The seismic ray emerging at any point on the earth's surface makes an angle θ with the polar axis given by the formula

$$\cos\theta = \cos i_h \cos i_{h0} + \sin i_h \sin i_{h0} \cos \Phi \quad \ldots\ldots\ldots (45)$$

where i_h is zenith distance of the seismic ray at the hypocentre and Φ is the bearing of the ray measured from direction of polar axis.

The angle θ is calculated for the seismic rays emerging at every 1° of epicentral distance and every 15° of azimuth Φ, and corresponding $P_2(\cos\theta)$ is obtained by graphical interpolation from the values given in Jahnke und Emde's Table.

Necessary values $\frac{\mathfrak{A}'}{\mathfrak{A}}$, $\left(\frac{u}{\mathfrak{A}}\right)_0$ and $\left(\frac{w}{\mathfrak{A}}\right)_0$ to calculate (42) and (43) are obtained also graphically from the values calculated by Mr. T. Suzuki and the writer[108] and Dr. H. Jeffreys.[109] The results are tabulated in Table III.

Table III. Angles of emergence and incidence and the values of $\frac{\mathfrak{A}'}{\mathfrak{A}}$, $\left(\frac{u}{\mathfrak{A}}\right)_0$, $\left(\frac{w}{\mathfrak{A}}\right)_0$ etc. for a seismic ray at the epicentral distance θ.

θ	0°	1°9′7	2°16·′6	3°20·′6	4°22·′8	5°24·′0	6°24·′4	7°24·′5	8°24· 4	9°24·′0	10°23·′6
i_h	0	30°32′	51°39′	65°03′	74°11′	80°58′	86°13′	91°29′	94°55′	98°25′	101°34′
e_{50}	90°	64°05′	47°35′	38°45′	34°09′	31°51′	30°53′	30°42′	31°01	31°42′	32°35′
e_{20}	90°	69°01′	56°27′	50°16′	47°18′	55°53′	45°18′	45°12′	45°23′	45°47′	46°19′
e_0	90°	72°59′	63°09′	58°29′	56°21′	55°20′	54°55′	54°51′	54°59′	55°15′	55°39′
$\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)_{50}$	1·143	1·119	1·067	1·022	·987	·965	·954	·952	·955	·963	·972
$\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)_{20}$	1·143	1·123	1·100	1·078	1·066	1·059	1·056	1·055	1·057	1·059	1·061
$\left(\frac{u}{\mathfrak{A}}\right)_0$	0	·668	1·022	1·173	1 235	1·266	1·276	1·277	1·275	1·268	1·256
$\left(\frac{w}{u}\right)_0$	∞	2·84	1·68	1·42	1·31	1·27	1·25	1·25	1·25	1·26	1·28
t_P	34·6	38·5	48·0	5·79	72·1	84·9	97·7	110·5	123·3	135·9	148·5
$\frac{1}{t_P}\Pi\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)\left(\frac{u}{\mathfrak{A}}\right)_0$	0	0·02180	·02499	·02165	·01802	·01524	·01316	·01161	·01044	·00952	·00872

108) The assumption of crustal structure is almost the same as in the first paper (*Bull. Earthq. Res. Inst.*, **10** (1932) 94-129). The surfaces of discontinuity are assumed at the depth of 20 km. and 50 km. from the surface. The ratio of the velocities on both sides of the discontinuity is asumed to be $1 : \sqrt{1\cdot5}$ and the ratio of the dencity is 0·9 : 1.

109) H. JEFFREYS, *loc. cit.* 97).

Using these values the motion on the earth's surface given by (42) and (43) are calculated except the constant factor $\left(-A\sqrt{\frac{2h_0}{\pi}\frac{1}{\bar{a}_i}}\right)$, and tabulated in Table IV and is shown on Fig. 1 and 2.

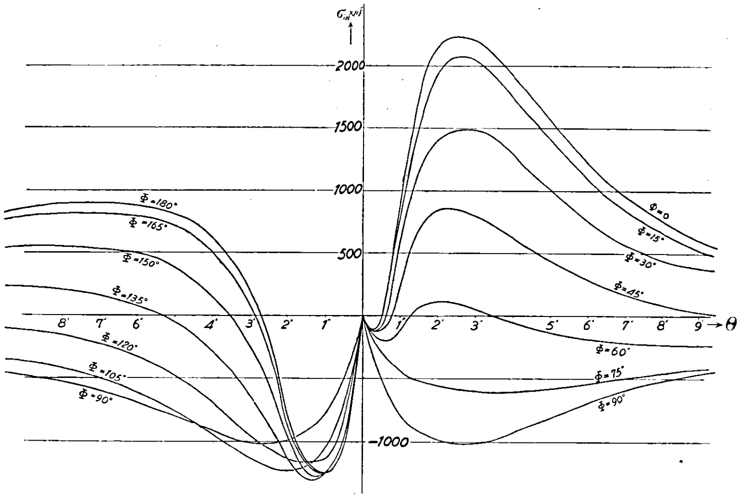

Fig. 2. Variation of Amplitude of P-wave, for the earthquake of June 2, 1931, with epicentral distance for respective azimuths.

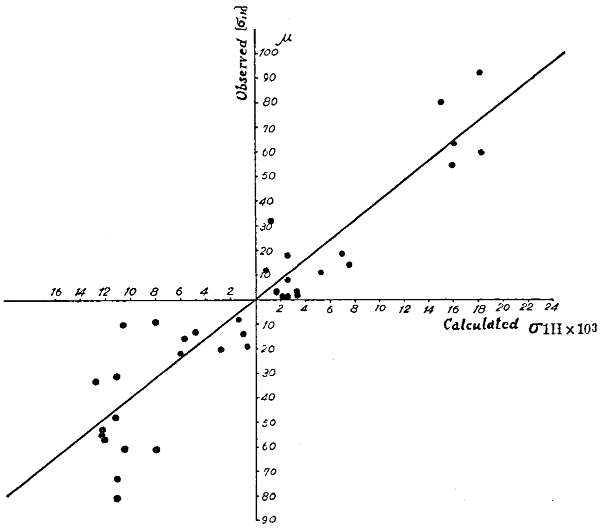

Fig. 3. Comparison of the observed magnitude of the initial motion of P-wave due to the earthquake of June 2, 1931, with that of calculated.

It is clearly observed from Fig. 3 that the distribution of pull and push divided by nodal lines calculated without exception and loci of equal amplitude are in good accord with the magnitude of the initial motions actually observed.

To confirm this point further, the values of u_{1H} for θ and Φ of every observatories are interpolated graphically from Fig. 4 and compared in Table V and Fig. 3 with the values of actually observed. The fairly linear relation seen in Fig. 3 will justify us to grant the

Table IV. Calculated amplitude of P-wave on the earth's surface for the earthquake of June 2, 1931.

$$\left.\begin{aligned} \sigma_{1H} &= \frac{1}{t_P}\Pi\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)\left(\frac{u}{\mathfrak{A}}\right)_0 P_2(\cos\theta)(\text{above}) \\ \sigma_{1Z} &= \frac{1}{t_P}\Pi\left(\frac{\mathfrak{A}'}{\mathfrak{A}}\right)\left(\frac{w}{\mathfrak{A}}\right)_0 P_2(\cos\theta)(\text{below}) \end{aligned}\right\}\times 10^5$$

Φ \ θ	1° 9'·7	2° 16'·6	4° 20'·6	4° 22'·8	5° 24'·0	6° 24'·4	7° 24'·5	8° 24'·4	9° 24'·0	10° 23'·6
0°	1005	2234	2139	1759	1392	1101	924	702	563	452
	2854	3753	3037	2304	1768	1376	1155	878	709	579
15°	918	2061	1955	1598	1255	87	767	615	491	385
	2607	3462	2776	2093	1594	1234	959	769	619	493
30°	709	1752	1457	1159	885	672	551	383	288	209
	2014	2641	2069	1518	1124	840	689	479	363	268
45°	364	862	749	550	369	341	178	70	15	−28
	1034	1465	1064	721	469	301	223	88	19	−36
60°	−28	115	2	−88	−156	−193	−197	−246	−250	−262
	−80	193	3	−115	−198	−241	−246	−308	−315	−335
75°	−405	−560	−619	−595	−561	−521	−475	−457	−430	−405
	−1150	−941	−879	−779	−712	−651	−539	−571	−542	−518
90°	−717	−1030	−1000	−869	−756	−655	−581	−520	−472	−427
	−2036	−1730	−1420	−1138	−960	−918	−726	−650	−595	−547
105°	−942	−1240	−1074	−860	−694	−571	−490	−407	−354	−310
	−2675	−2083	−1525	−1127	−881	−714	−613	−509	−446	−397
120°	−978	−1097	−872	−593	−410	−287	−228	−144	−102	70
	−2778	−1843	−1238	−777	−521	−359	−285	−180	−129	90
135°	−1081	−980	−487	−173	11	111	135	206	244	232
	−3070	−1646	−692	−227	14	139	169	258	307	297
150°	−1075	−700	−61	240	442	509	498	550	545	529
	−3053	−1176	−87	314	561	636	623	688	687	677
165°	−1046	−477	264	613	764	808	825	906	776	745
	−2971	−801	375	803	970	1010	1031	1133	987	951
180°	−1033	−392	385	735	878	916	864	898	860	821
	−2934	−659	547	963	1115	1145	1080	1118	1084	1051

Table V. Comparison of the observed magnitude of the initial motion of P-wave due to the earthquake of June 2, 1931 with that of calculated.

Station	Epicentral distance (θ)	Azimuth Φ	Horizontal Amplitude of P-wave (observed) $\sqrt{E^2+N^2}$	$\sigma_{1//} \times 10^4$
	°		μ	
Nagoya	0·97	−164	−55	−123
Gihu	0·88	−153	−53	−121
Hikone	1·31	−243	−33	−128
Nagano	0·89	6	19	70
Numadu	1·30	103	−48	−112
Misima	1·33	102	−31	−111
Wazima	1·62	− 45	+67	75
Kumagaya	1·48	57	($\angle$R) 1	24
Kyôto	1·80	−144	−73	−111
Wakayama	2·69	−155	−20	− 28
Kakioka	2·20	57	+18	25
Ôsaka	2·18	−149	− 9	− 80
Tôkyô	1·78	73	—	− 47
Yokohama	1·71	81	−10	− 73
Toyooka	2·36	−123	− 61	−105
Tukubasan	2·14	57	+ 8	25
Kôbe	2·40	−145	−72	− 67
Sumoto	2·78	−148	−13	− 48
Sionomisaki	2·92	−172	+12	+ 7
Tyôsi	2·63	70	−32	− 12
Hatidyôzima	2·43	122	−10	−107
Sendai	3·57	24	+63	161
Sakai	3·64	− 119	−61	− 80
Kôti	4·16	− 149	+ 3	+ 17
Hamada	4·70	−117	−16	− 57
Akita	4·16	4	+59	183
Morioka	4·79	13	+80	150
Simidu	5·03	−153	—	+ 47
Miyazaki	6·63	−151	+11	+ 53
Kumamoto	6·61	−142	+ 2	33
Nagasaki	7·28	−138	($\angle$R) 1	+ 22
Taikyû	7·47	−113	−13	− 48
Zinsen	8·90	—	−11	− 45
Matumoto	0.43	14	−14	− 10
Kôbu	0·81	84	−22	− 55
Miyadu	2·07	−125	−81	−111
Hukusima	2·97	27	+54	160
Yamagata	3·24	19	+92	182
Tu	1·55	166	− 57	−120
Aomori	5·60	3	+	133
Kusiro	8·90	12	+14	51
Niihama	4·11	−143	—	− 7

assumption above used, because the observed values are nothing but the amplitude measured on the seismograms divided by the mechanical magnification of seismographs without taking any consideration of the effect of free vibrations due to the start from rest and that of friction more or less impeding seismographs.

If the observation becomes more acculate the deviation from the linear relation with the theoretical value will indicate some effect of geological structure near the observing station beside the deffect of calculation due to the difference of the law of variation of amplitude with hypocentral distance and damping due to viscosity etc. It will be interesting task of future study to know these factors through systematic and statistical observations.

From Fig. 3 we obtain the factor

$$1^{\mu}=\left(A\sqrt{\frac{2h_0}{\pi}\frac{1}{\bar{a}}}\right)2\cdot5\times10^{-4},$$

$$\text{or}\quad A=\frac{p}{2\cdot5}\sqrt{\frac{\pi}{2h_0^3}}\left(\frac{\bar{a}}{a_0}\right)\ \text{C. G. S.},$$

where

$$\bar{a}=\frac{1}{R-r_h}\int_{r_h}^{R}a\,dr,$$

(R: radius of the earth, r_h: distance of hypocentre from the centre of the earth.)

which comes out nearly 1·5 times of a_0 for the velocity obtained in the writer's first paper. And

$$A=\frac{3}{5}p\sqrt{\frac{\pi}{2h_0^3}}\ \text{C. G. S.} \qquad\qquad (46)$$

Now that we know the coefficient of the scalar displacement potential, all the quantities concerning the longitudinal wave are determined, if we know the period of the wave.

There may be a number of mechanisms to give rise P-wave

$$\phi=A\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}P_2(\cos\theta),$$

by suitable conditions of displacement and stress near the origin, but the transverse wave are determined corresponding to each mechanism. For instance Prof. Sezawa obtained waves due to pressure distribution given by $P_2(\cos\theta)e^{ipt}$, on a sphere of radius a, and the ratio R of maximum amplitudes of P- and S-waves becomes 0·24 for $ha=1$ when $\lambda=\mu$. Thus we can percieve the mechanism of occurrence of the earthquake if we know the ratio R. Fortunately we have observations of initial motions of transverse waves obtained by Mr. K. Tanahasi. So

I will determine the constants of spherical transverse waves.

The transverse waves accompanying the longitudinal wave (23) is

$$\psi = \boldsymbol{B}_z \frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}} \cos\theta$$

irrespectively when the P-wave is generated by the pressure distributions as given by Prof. Sezawa or by the doublets without moment in combination with a simple source.

Then by the relation

$$\operatorname{rot}\psi = (\operatorname{rot}\psi)_\varphi = k\boldsymbol{B}_z \frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{\sqrt{r}} \sin\theta\cos\theta,$$

and (3′) we have

$$\left.\begin{aligned} u_2 &= 2kB_z \frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}} P_2(\cos\theta)\, e^{ipt}, \\ v_2 &= -kB_z \frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}\, H^{(2)}_{2+\frac{1}{2}}(kr)\right\} \sin\theta\cos\theta\, e^{ipt}, \\ w_2 &= 0, \end{aligned}\right\} \quad \ldots\ldots(47)$$

and as we are now concerned with a deep-seated earthquake we have only to consider displacement in the direction of θ,

$$v_2 \approx B_z \sqrt{\frac{2}{\pi}}\, k^{\frac{3}{2}} \frac{e^{i(pt-kr)}}{r} \sin\theta\cos\theta. \quad \ldots\ldots\ldots\ldots(48)$$

Now I will draw the vector of initial motions of S-waves on the geographical map. (Fig. 4). Mr. Tanahasi drew a diagram representing the vector of amplitude of S-wave on the earth's surface from Prof. Matuyama's theory which is exactly the same with (47). In his drawing we see a large component (w) of displacement in the direction of φ near the epicentre.

Though, as we have seen, there is no w component in (47), we see conspicuously large w component in the actual observation, shown in Fig. 6 near the nodal line of v_2 i. e. near the equator $\theta = \frac{\pi}{2}$. If we pay special attention to the component in the direction of φ (that is parallel to the loci of θ=const. shown in the Fig. 6) we notice that the motion is of different sign on both sides of the equator and they gradually diminish as we recede from the equator and at last change their signs. On the other hand v_2 component, which is perpendicular to the direction

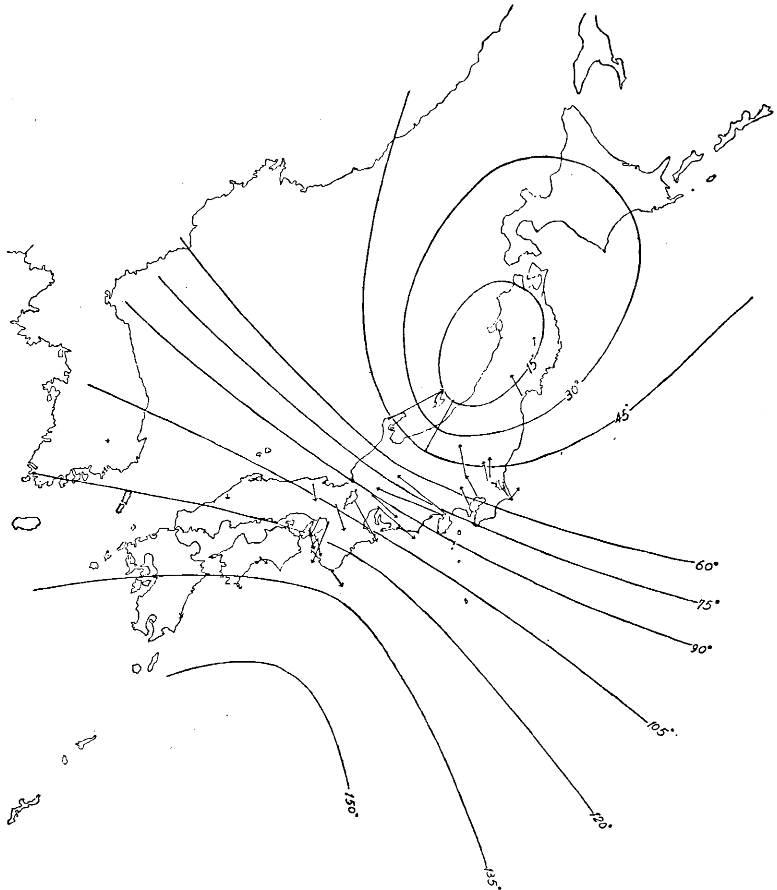

Fig. 4. The distribution of horizontal component of initial motions actually observed of the S-wave due to the earthquake of June 2, 1931 and the loci of constant θ.

of $\theta=$const., is evanescent near the equator and the pole, and is maximum in the intermediate zone.

Thus it is to be considered the w component is due to the transverse wave of the second kind, and it is very interesting to think that the mechanism of occurrence of this wave is independent of the P-wave, and the existence of this makes an interesting contrast to that of simple source which is independent of the transverse waves. We must therefore be careful to pay attention to the existence of these waves in the consideration of the mechanism of earthquake occurrence.

Performing the similar calculation as the case of P-wave I will reveal the mechanisms by which this eanerthquake is generated. It is necessary to calculate separately the effects of reflexion and refraction for each component in the incident plane (σ_i) or perpendicular to it σ_Φ.

Considering a unit sphere near the hypocentre (Fig. 5) and by the cosine formula (42) and

$$\frac{\sin\beta}{\sin i_{h0}}=\frac{\sin\Phi}{\sin\theta}=\frac{\sin(\pi-\varphi)}{\sin i_h}, \quad ..(49)$$

we have

$$\left.\begin{aligned}\sigma_i&=v\cos\beta-w\sin\beta,\\ \sigma_\Phi&=v\sin\beta+w\cos\beta,\end{aligned}\right\} \quad ....(50)$$

or

$$\left.\begin{aligned}u&=\sigma_i\cos\beta+\sigma_\Phi\sin\beta,\\ w&=-\sigma_i\sin\beta+\sigma_\Phi\cos\beta.\end{aligned}\right\} \quad ..(51)$$

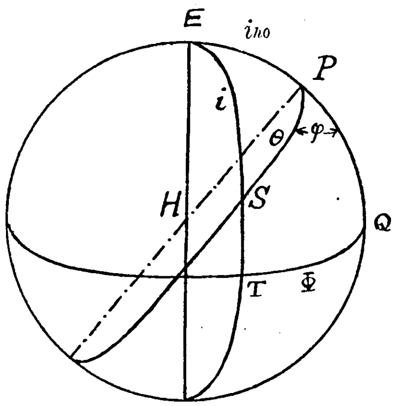

Fig. 5.
H: hypocentre;
HE: vertical line through H;
HP: polar axis; HS: seismic ray;
$ES=i$; $EP=i_{h0}$
$\angle QPS=\varphi$; $QT=\Phi$; $\angle ESP=\beta$.

And we can calculate the motion on earth's surface by

$$\left.\begin{aligned}v_{2iH}&=B_z\sqrt{\frac{2}{\pi}}(k_0)^{\frac{3}{2}}\frac{1}{\bar{a}t_P}e^{ip(t-t_S)}\sin\theta\cos\theta\prod\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)_0\cos\beta,\\ v_{2iZ}&=B_z\sqrt{\frac{2}{\pi}}(k_0)^{\frac{3}{2}}\frac{1}{\bar{a}t_P}e^{ip(t-t_S)}\sin\theta\cos\theta\prod\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{w}{\mathfrak{B}}\right)_0\cos\beta,\\ v_{2\Phi}&=B_z\sqrt{\frac{2}{\pi}}(k_0)^{\frac{3}{2}}\frac{1}{\bar{a}t_P}e^{ip(t-t_S)}\sin\theta\cos\theta\prod\left(\frac{c'}{c}\right)2\sin\beta.\end{aligned}\right\} \quad ..(52)$$

The result of calculation is shown in Table VI, VII, and Fig. 6 and 7.

It is to be observed that the displacement due to the transverse wave (v_2) is nearly perpendicular to the loci of equal-θ and has very small w-component compared with those observed shown in Fig. 4. Thus it is necessary to separate out v_2-component and w_3-component from the actual observations. For this purpose we must first decompose the horizontal displacement vector actually observed into two components, the one in the radial direction (σ_{iH}) and the other perpendicular to the former (σ_Φ). If we divide σ_{iH} and σ_Φ by $\frac{1}{t_P}\prod\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)_0$ and $\frac{1}{t_P}\prod\left(\frac{c'}{c}\right)\cdot 2$ respectively we have the amplitudes σ_i' and σ_Φ' which are free from the effects due to difference of hypocentral distance and the reflexion and refraction.

Table VI. $\frac{\mathfrak{B}'}{\mathfrak{B}}$, $\frac{c'}{c}$, $\left(\frac{u}{\mathfrak{B}}\right)_0$, $\left(\frac{w}{\mathfrak{B}}\right)$, etc. for a seismic ray of transverse wave emerging at epicentral distance θ.

θ	0°	1° 9'·7	2° 16'·6	3° 20'·6	4° 22'·8	5° 24'·0	6° 24'·4	7° 24'·5	8° 24'·4
$\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)_{50}$	1·143	1·112	1·065	1·012	0·973	0·950	0·940	0·937	0·941
$\left(\frac{C'}{C}\right)_{50}$	1·143	1·140	1·082	1·038	1·006	0·988	0·977	0·977	0·980
$\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)_{20}$	1·143	1·115	1·087	1·070	1·065	1·060	1·058	1·058	1·058
$\left(\frac{C'}{C}\right)_{20}$	1·143	1·139	1·116	1·093	1·080	1·075	1·072	1·071	1·073
$\left(\frac{u}{\mathfrak{B}}\right)_0$	2	1·86	1·725	1·775	2·03	2·511	3·23	3·43	3·11
$\left(\frac{w}{\mathfrak{B}}\right)_0$	0	0·65	0·944	1·006	0·950	0·811	0·600	0·370	0·70
$\frac{1}{t}\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{w}{\mathfrak{B}}\right)_0$	0·07512	0·05989	0·04160	0·03302	0·02918	0·02979	0·03289	0·03077	0·02509
$\frac{1}{t}\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{w}{\mathfrak{B}}\right)_0$	0	0·02093	0·02276	0·01872	0·01365	0·00962	0·00611	0·00331	0·00564
$\frac{1}{t}\Pi\left(\frac{C'}{C}\right)2$	0·07512	0·06726	0·05030	0·03801	0·03014	0·02502	0·02144	0·01894	0·01704

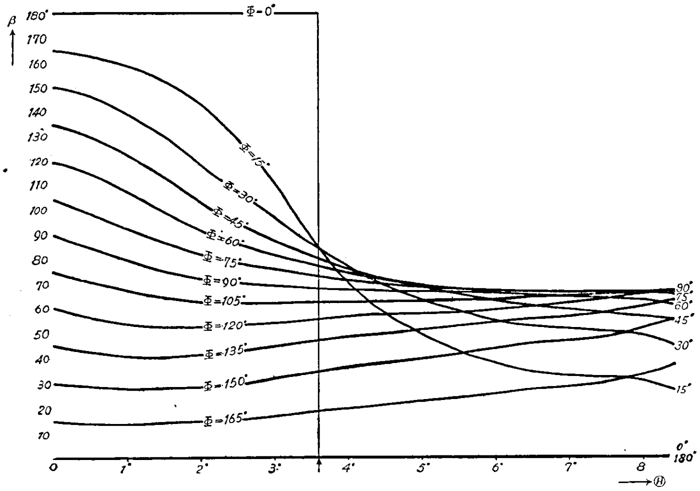

Fig. 6. $\beta-\theta$.

Table VII. Calculated amplitude of S-wave on the earth's surface for the earthquake of June 2, 1931.

$$\left.\begin{aligned} v_2 t_H &= \frac{1}{t}\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)_0 \cos\theta\sin\theta\cos\beta \ldots\text{(above, } H) \\ v_2 t_Z &= \frac{1}{t}\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{w}{\mathfrak{B}}\right)_0 \cos\theta\sin\theta\cos\beta \ldots\text{(middle, } Z) \\ v_{2\Phi} &= \frac{1}{t}\Pi\left(\frac{C'}{C}\right) 2\cos\theta\sin\theta\sin\beta \ldots\ldots\text{(below, } \Phi) \end{aligned}\right\} \times 10^5$$

Φ \ θ		1° 9'.7	2° 16'.6	3° 20'.6	4° 22'.8	5° 24'.0	6° 24'.4	7° 24'.5	8° 24'.4
0°	(H)	+2862	+1063	+ 36	− 362	− 698	−1022	−1055	−1038
	(Z)	−1000	− 581	− 21	+ 169	+ 225	+ 190	+ 113	+ 233
	(Φ)	0	0	0	0	0	0	0	0
15°	(H)	+2679	+ 965	− 7	− 375	− 689	− 995	−1021	− 996
	(Z)	− 936	− 528	− 4	+ 175	+ 223	+ 185	+ 110	+ 224
	(Φ)	1263	1126	879	691	560	466	405	346
30°	(H)	+2168	+ 680	− 6	− 404	− 659	− 911	− 922	− 873
	(Z)	− 758	− 372	− 3	+ 189	+ 213	+ 169	+ 99	+ 196
	(Φ)	2295	2008	1553	1212	977	810	703	597
45°	(H)	+1447	+ 306	− 199	− 424	− 598	− 781	− 807	− 687
	(Z)	− 506	− 168	+ 113	+ 198	+ 193	+ 145	+ 87	+ 154
	(Φ)	2921	2465	1868	1438	1147	942	814	682
60°	(H)	+ 676	− 54	− 308	− 401	− 482	− 574	− 549	− 457
	(Z)	− 236	+ 30	+ 175	+ 189	+ 156	+ 107	+ 59	+ 103
	(Φ)	3058	2419	1764	1320	1029	829	708	577
75°	(H)	+ 7	− 283	− 319	− 305	− 308	− 324	− 293	− 210
	(Z)	− 2	+ 155	+ 181	+ 143	+ 99	+ 60	+ 32	+ 47
	(Φ)	2736	1920	1287	900	660	502	417	309
90°	(H)	− 400	− 309	− 193	− 117	− 71	− 33	− 11	+ 33
	(Z)	+ 140	+ 169	+ 109	+ 55	+ 23	+ 6	+ 1	− 7
	(Φ)	2084	1123	577	296	141	51	16	− 53
105°	(H)	− 521	− 105	+ 77	+ 158	+ 219	+ 283	+ 279	+ 251
	(Z)	+ 182	+ 57	− 46	− 74	− 71	− 53	− 30	− 56
	(Φ)	1289	249	− 173	− 329	− 387	− 404	− 386	− 410
120°	(H)	− 366	+ 292	+ 457	+ 492	+ 540	+ 599	+ 559	+ 432
	(Z)	+ 128	− 160	− 259	− 230	− 117	− 111	− 60	− 97
	(Φ)	551	− 475	− 765	− 808	− 784	− 741	− 680	− 668
135°	(H)	− 27	+ 792	+ 883	+ 823	+ 845	+ 888	+ 801	+ 571
	(Z)	+ 9	− 433	− 501	− 385	− 273	− 165	− 86	− 128
	(Φ)	− 26	− 877	−1052	−1020	− 947	− 870	− 791	− 756
150°	(H)	+ 350	+1275	+1272	+1142	+1103	+1120	+ 992	+ 667
	(Z)	− 122	− 698	− 671	− 534	− 357	− 208	− 107	− 150
	(Φ)	− 212	− 886	− 976	− 933	− 836	− 759	− 687	− 649
165°	(H)	+ 665	+1623	+1545	+1351	+1279	+1271	+1114	+ 722
	(Z)	− 232	− 888	− 876	− 632	− 413	− 236	− 120	− 162
	(Φ)	− 184	− 545	− 581	− 538	− 487	− 440	− 397	− 373
180°	(H)	+ 778	+1750	+1642	+1388	+1339	+1323	+1156	+ 740
	(Z)	− 272	− 958	− 931	− 649	− 433	− 246	− 124	− 166
	(Φ)	0	0	0	0	0	0	0	0

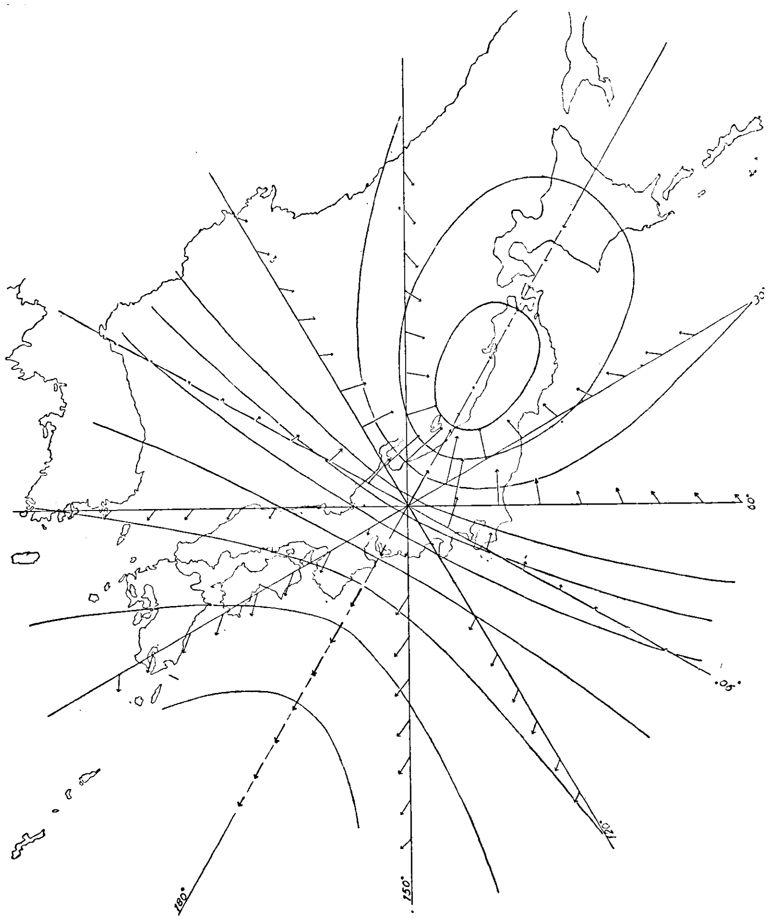

Fig. 7. Calculated amplitude of horizontal component of S-wave (σ_2) on the earth's surface for the earthquake of June 2, 1931.

Then we can calculate the components in the directions of θ and φ by means of the angle β shown in Fig. 6. Denote the values thus obtained with $[v_2]_1$, and $[w_3]_1$. Then $[v_2]_1$ (or $[w_3]_1$) is displacement due to a term inversely proportional to hypocentral distance in v_2 (or w_3) on a sphere at a distance of a second journey of longitudinal wave from the origin, and is proportional to the actual value on the sphere $(v_2)_1$ (or $(w_3)_1$). The results of graphical calculations with the aid of above mensioned values are given in Table VIII.

Now let us first discuss the component v_2 which accompany with

Table VIII. Reduction of observed amplitudes (σ_{lH}, $\sigma_{l\Phi}$) of S-wave of the earthquake of June 2, 1931, to those ($[v_2]_1$, $[w_3]_1$) on a sphere which is a second journey of P-wave from the hypocentre.

Station	σ_{lH}	σ_{Φ}	σ'_l	σ'_{Φ}	β	$[v_2]_1$	$[w_2]_1$	θ	$\sin\theta$
	μ	μ	mm	mm	°	mm	mm	°	
Nagano	764	− 67	11·74	− 0·95	172	−11·76	−0·69	44	996
Wazima	347	742	6·70	12·32	−110	−13·87	−1·92	44	996
Misima	−835	85	−14·75	1·31	68	− 4·31	14·07	76	470
Numadu	−910	366	−15·87	5·15	67	− 1·44	16·69	77	438
Kumagaya	− 60	−453	− 1·11	− 7·25	102	− 6·86	2·59	52	970
Hikone	− 93	−667	− 1·63	−10·22	− 34	4·37	−9·39	95	−174
Gihu	−252	−348	− 3·87	− 4·93	− 25	− 1·41	−6·11	91	−208
Kyôto	40	−361	0·82	− 6·31	− 36	− 4·37	−4·62	103	−438
Nagoya	− 87	−654	− 1·36	− 9·37	− 16	1·27	−9·38	92	−068
Ôsaka	414	−267	9·63	− 5·17	− 30	10·92	0·34	110	−643
Sumoto	36	−243	0·97	− 5·53	− 34	3·89	−4·04	118	−829
Sionomisaki	86	−236	2·39	− 5·56	− 8	3·14	−5·17	124	−927
Kôbe	292	− 52	7·25	− 1·07	− 35	6·55	3·28	112	−695
Toyooka	− 38	−226	− 0·93	− 4·58	− 50	2·90	−3·64	100	−342
Wakayama	103	−196	2·73	− 4·34	− 27	4·40	−2·63	119	−848
Kôti	− 60	−125	− 2·03	− 3·97	− 38	0·84	−4·38	129	−978
Hamada	− 13	− 74	− 0·34	− 2·60	− 60	2·08	−1·59	108	−588
Simidu	− 5	− 63	− 0·17	− 0·33	− 37	1·27	−1·99	136	−999
Miyazaki	− 15	− 51	− 0·46	− 2·45	− 45	1·41	−2·05	142	−970
Nagasaki	0	− 15	0	− 0·48	− 56	0·35	−0·27	134	−999
Taikyû	− 1	− 36	− 0·03	− 1·91	− 65	1·72	−0·84	112	−695
Tyôsi	90	−152	2·35	− 3·32	80	− 2·86	−2·89	62	829
Yokohama	−199	−168	− 3·97	− 2·80	81	− 3·39	3·48	67	719
Tôkyô	−199	−214	− 4·09	− 3·72	86	− 4·00	3·79	62	829
Tukuba	− 40	−257	− 0·92	− 4·92	92	− 4·88	1·17	50	985
Kakioka	14	−267	− 0·33	− 5·19	92	− 5·19	−0·15	51	978
Sendai	36	−312	1·14	− 8·67	85	− 8·44	−1·89	23	719
Morioka	73	− 80	2·54	− 2·09	46	− 0·26	−3·28	16	530

the P-wave. As $[v_2]_1$ is only dependent on $\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$, the comparison are made $[v_2]_1$ and $\sin 2\theta$ in Fig. 8. It is to be noticed that the correlation is not so good as in the case of P-wave, as will be justified from the difficulty of obseration of S-wave owing to the superposition of the longitudinal wave, but it is remarkable that the sign is in good harmony with only one exception. The ratio of $(v_2)_1$ to $[v_2]_1$ is obtained

$$\frac{(v_2)_1}{[v_2]_1}=\left(\frac{a_0}{a_1}\right)^{\frac{5}{2}}\left(\frac{\bar{a}}{a_0}\right)=0{\cdot}42$$

from the ratio of (48) and (52) and the assumption $(\bar{a})_1=a_h=a_1$ between hypocentre and the sphere ($t_P=1$ sec).

Now let us examine the ratio of the maximum amplitudes at the same hypocentral distance of the P- and S-waves due to the wave generated by doublet force without moment and a simple source. Comparing (23), (25) and (44) we have

$$A=\frac{2iZ}{3\cdot 4\pi\rho a^2}\sqrt{\frac{\pi h}{2}}=p\frac{3}{5}\sqrt{\frac{\pi}{2h_0^3}},$$

$$B_z=\frac{iZ}{4\pi\rho b^2}\sqrt{\frac{\pi}{2k}},$$

$$\frac{B_z}{A}=\frac{3}{2}\left(\frac{a}{b}\right)^{\frac{3}{2}}\frac{a}{p}=3\cdot 42\frac{a}{p},$$

$$\therefore\quad B_z=3\cdot 42\frac{a_h}{p}\frac{3}{5}p\sqrt{\frac{\pi}{2h_0^3}},\qquad \ldots .(53)$$

$$\therefore\quad (v_2)_1=0\cdot 48\sin 2\theta \text{ cm. and}$$

$$[v_2]_1=\frac{(v_2)_1}{0\cdot 42}=\frac{0\cdot 48}{0\cdot 42}\sin 2\theta$$

$$=1\cdot 14\sin 2\theta \text{ cm.} \;..(54)$$

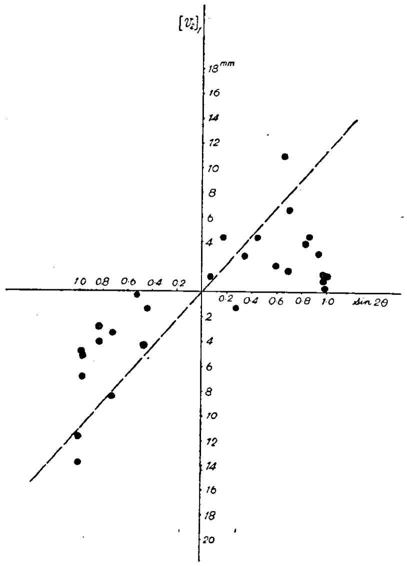

Fig. 8. Comparison of reduced amplitude $[v_2]_1$ from the observed value of the S-wave due to the earthquake of June 2, 1931 with those calculated.

from (46) corresponding to the magnitude of the observed P-wave if the transverse wave v_2 is generated by double force without moment.

This relation is represented by a straight line in Fig. 8. It may therefore be sufficient to conclude a part of S-wave and the P-wave of this earthquake were generated by a doublet without moment in combination with a simple source of strength

$$\phi=\frac{A}{2}\frac{H_{\frac{1}{2}}^{(2)}(hr)}{\sqrt{r}} \;..(55)$$

Let us now consider the remaining component $[w_3]_1$. The variation of $[w_3]_1$ with θ is shown in Fig. 9. The

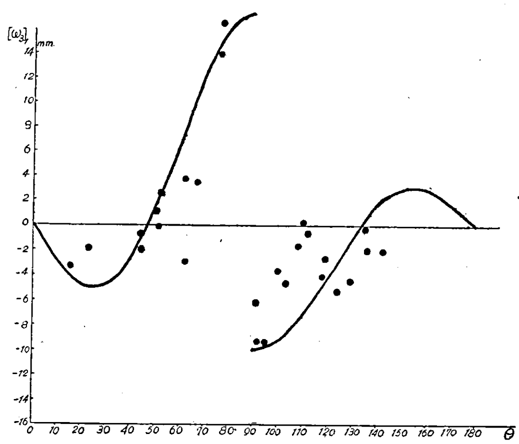

Fig. 9. The relation with θ of the reduced amplitude $[w_3]_1$ of the S-wave due to the earthquake of June 2, 1931.

spherical wave having w-component of transverse wave only is Prof. Sezawa's second kind of transverse wave. And the solutions depending only on θ are those in which

$$\left.\begin{aligned} &m=0, \\ &u_3=v_3=0, \\ &w_3=\frac{1}{n(n+1)}D\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP_n(\cos\theta)}{d\theta}e^{ipt}. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(56)$$

On the other hand it is clearly observed that the sign of $[w_3]_t$ is clearly opposite on both sides of $\theta=\frac{\pi}{2}$, and the solution having such nature is those of even n. But it is necessary to superpose the solutions whose $n=2$, 4 and 6. But for such large values of n as 6 the function $H^{(2)}_{n+\frac{1}{2}}(kr)$ becomes very large at short distance from origin, $(R\{H^{(2)}_{6+\frac{1}{2}}(1)\}=8632)$ and it is impossible to consider that the material at such point withstands failure. It will be more easily understood if the displacement is discontinuous at the equatorial plane $\left(\theta=\frac{\pi}{2}\right)$, because we can obtain amplitude shown by the curves in Fig. 9 by the superposition of the waves of $n=1$ and 3, cutting at the equator and changing the signs on both sides of the equator. This consideration is based on the supposition of the existence of a crack at the equatorial plane, and it harmonizes with the fact that the P- and S-waves observed in this earthquake are that generated by the mechanism called "crack earthquake" by Prof. M. Matuyama, and that the cracks expected in the former mechanism which generates w_3-wave is at the same place with the latter. Then it is easily understood the difference

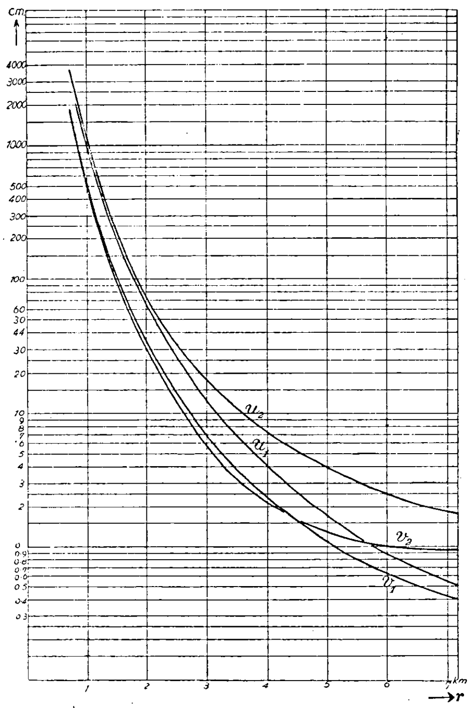

Fig. 10. Variation of maximum amplitudes of longitudinal and transverse waves with hypocentral distance.

of amplitude on both sides of $\theta=\frac{\pi}{2}$, the side of larger amplitude may easily considered to be the side mainly moved. But such wave is only possible when the crack is extending up to the earth's surface, and if it had not actually existed, it must have been impossible to come to exist at once by such an earthquake, and the wave should have been more or less modified when they were propagated to a distance. And the full interpretation is a matter of future when the theoretical investigation will have been completed. It is interesting, however, to remember that the origin of this eathquake is near the "ithmus" or the "rift vally" of the main island of Japan and the equatorial plane is nearly coincident with the line connecting the Wakasa-bay, the Lake Biwa and the Ise-bay, and, moreover, the side in which w_3 is of larger amplitude is the upper side of the equatorial plane which may be considered as the crack plane. It is interesting, if it be a mere chance, that the circumstances of the occurrence of this earthquake is the same with the orogenic movement in the geological ages. And if it is not event of mere chance, it is the more interesting to know that such orogenic force is acting at such a depth as 250 km. from the earth's surface.

Thus it is a matter of mere speculation at present, so I will give here only the formulae of spherical wave which give the displacement shown by the curves in Fig. 9.

We have from (56)

$n=1.$

$$w_3=\frac{1}{2}D_1\frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP_1(\cos\theta)}{d\theta}e^{ipt}\approx\frac{-D_1}{1}\sqrt{\frac{2}{\pi k}}\frac{e^{i(pt-kr)}}{r}\sin\theta. \quad .(57)$$

$m=3.$

$$w_3=\frac{1}{12}D_3\frac{H^{(2)}_{3+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP_3(\cos\theta)}{d\theta}\approx\frac{D_3}{12}\sqrt{\frac{2}{\pi k}}\frac{e^{i(pt-kr)}}{r}\frac{dP_3(\cos\theta)}{d\theta}, \quad .(58)$$

and if we put

$$-3D_1=D_3, \quad \ldots\ldots\ldots\ldots\ldots(59)$$

and superpose these two solutions we have node at about $\theta=47°$. Thus we have by similar consideration as before,

$$(w_{3tII})_0=D_1\sqrt{\frac{2}{\pi k}}\sin\theta\left(1-\frac{15}{8}\sin^2\theta\right)\frac{1}{\bar{a}t_P}\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)_0\sin\beta,$$

$$(w_3)_1 = D_1\sqrt{\frac{2}{\pi k}}\sin\theta\left(1-\frac{15}{8}\sin^2\theta\right)\frac{1}{a_1},$$

$$\frac{(w_3)_1}{(w_{3III})_0} = \left(\frac{a_0}{a_1}\right)^{\frac{1}{2}}\left(\frac{\bar{a}}{a_0}\right)\frac{t_P}{\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)\sin\beta} = 1{\cdot}16\frac{t_P}{\Pi\left(\frac{\mathfrak{B}'}{\mathfrak{B}}\right)\left(\frac{u}{\mathfrak{B}}\right)_0\sin\beta}.$$

So we have $(w_3)_1$ by multiplying 1·16 into $[w_3]_1$ of Table VIII, and from Fig. 9,

$$\left.\begin{aligned} D_1 &= \sqrt{\frac{2}{\pi k_1}}\frac{1}{a_1} = 1{\cdot}16\text{ cm.,} \qquad \text{for} \quad 0<\theta<\frac{\pi}{2}, \\ \text{or} \qquad &= 0{\cdot}66\text{ cm.,} \qquad \text{for} \quad \frac{\pi}{2}<\theta<\pi. \end{aligned}\right\} \qquad (60)$$

Thus we have all the mathematical expressions for the bodily waves of the earthquake of June 2, 1931. And from these expressions we can know all the phenomena so far as the seismic waves are concerned, provided we know the properties of material consisting the earth and the period of the seismic waves. As to the former the velocities of the seismic waves in the region here concerned has been estimated by the writer in his first paper and we have to know the density only. The density within the earth has been estimated by several authorities[110] from geochemical and geophysical point of view. As to the period of the seismic waves I could determine to be about 2·7 seconds in both P- and S-waves from the seismograms obtained at Kobe which are reproduced in Mr. K. Tanahasi's paper. We have thus

$$a_0 = 5\text{ km./sec.,}$$

$$a_h = 8{\cdot}35\text{ km./sec.,}$$

$$a/b = \sqrt{3},$$

$$T = \frac{2\pi}{p} = 2{\cdot}7\text{ sec.,}$$

$\rho = 3{\cdot}6$ approximately at the depth of 250 km. from the surface, and we have

$$Ah^{\frac{5}{2}} = 0{\cdot}81\text{ cm.,} \quad Bk^{\frac{3}{2}} = 5{\cdot}47\text{ cm.}$$

from (46).

Let us now examine the displacement, stress and energy near the hypocentre where we have no need of taking the effect of heterogeneity into consideration.

110) See for example B. GUTENBERG, "Aufbau der Erde", (1925), 45.

We have already seen sufficient evidence to believe, at least, the u_1 and v_2 components of displacements of this earthquake were generated by simple source (55) and a doublet without moment

$$F\delta z = -iA4\pi\rho a^2\sqrt{\frac{2}{\pi h}}e^{ipt} = -iB_z 4\pi\rho b^2\sqrt{\frac{2k}{\pi}}e^{ipt} = 1{\cdot}23\times 10^{25}e^{i(pt+\frac{\pi}{2})} \quad \ldots (61)$$

And the corresponding mathematical expressions of these waves are

$$\left.\begin{aligned}
u_1 &= 0{\cdot}81\frac{d}{d(hr)}\left(\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}\right)P_2(\cos\theta)e^{ipt}\ \text{cm.},\\
v_1 &= -1{\cdot}22\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{(hr)^{\frac{3}{2}}}\sin 2\theta e^{ipt}\ \text{cm.},\\
u_2 &= +21{\cdot}90\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{(kr)^{\frac{3}{2}}}P_2(\cos\theta)e^{ipt}\ \text{cm.},\\
v_2 &= -5{\cdot}47\frac{1}{kr}\frac{d}{d(kr)}\left\{\sqrt{kr}H^{(2)}_{2+\frac{1}{2}}(kr)\right\}\sin 2\theta e^{ipt}\ \text{cm.},\\
w_1 &= w_2 = 0.
\end{aligned}\right\} \quad \ldots (62)$$

The numerical values of these expressions at $r=0{\cdot}5/h=1{\cdot}8$ km. and $r=1/h=3{\cdot}6$.km. are tabulated in Table IX. It is to be remarked that the longitudinal and transverse waves are nearly opposite in phase, and the latter is not particularly larger than the former as at large distances from the hypocentre. These circumstances are clearly visualized from

Table IX. Amplitudes of longitudinal (u_1, v_1) and transverse (u_2, v_2) waves in cm. at the hypocentral distances (I) $r=0{\cdot}5/h=1{\cdot}8$ km. and (II) $r=1/h=3{\cdot}6$ km.

θ (degree)	u_1		v_1		u_2		v_2	
	(I)	(II)	(I)	(II)	(I)	(II)	(I)	(II)
0	86·2	6·10	0	0	−106·8	−5·97	0	0
10	82·3	5·82	+16·7	+1·20	−101·9	−5·70	−15·6	−0·99
20	71·1	5·03	+31·4	+2·26	− 88·0	−4·92	−29·4	−1·87
30	53·9	3·81	+42·3	+3·04	− 66·7	−3·73	−39·6	−2·52
40	32·8	2·32	+48·0	+3·46	− 40.6	−2·29	−45·0	−2·86
50	10·3	0·73	+48·0	+3·46	− 12·8	−0·72	−45·0	−2·86
60	−10·8	−0·76	+42·3	+3·04	+ 13·3	+0·75	−39·6	−2·52
70	−28·0	−1·98	+31·4	+2·26	+ 34·6	+1·94	−29·4	−1·87
80	−39·2	−2·77	+16·7	+1·20	+ 48·6	+2·72	−15·6	−0·99
90	−43·1	−3·05	+ 0	0	+ 53·4	+2·99	0	0

the variation of the maximum amplitudes with hypocentral distance which are shown in Table X and Fig. 10. It also shows so rapid an increase of amplitude as we approach the hypocentre that we can not extrapolate the amplitude at short epicentral distance by the simple law as inverse hypocentral distance. It is also seen from the diagram that the maximum amplitude of the wave reaches as large as 1 metre at about 2 km. from the hypocentre. It seems improbable that at such place the formula of elasticity is applicable, so we shall examine in the following whether the stress calculated from the theory of elasticity do not exceed the ultimate strength of the material.

Table X. Variation of maximum amplitudes of longitudinal and transverse waves with hypocentral distances.

P-wave	hr	0·2	0·4	0·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0
	r (km.)	0·72	1·44	2·15	2·87	3·60	4·20	5·02	5·75	6·46	7·18
	u_1 (cm.)	3643	292	45·7	14·7	6·1	3·0	1·7	1·0	0·7	0·5
	v_1 (cm.)	1822	116	23·8	7·9	3·5	1·8	1·1	0·7	0·5	0·4
S-wave	kr	0·12	0·23	0·35	0·46	0·58	0·69	0·81	0·92	1·04	1·15
	r (km.)	0·41	0·83	1·24	1·66	2·07	2·48	2·90	3·31	3·73	4·14
	u_2 (cm.)	32981	2105	431	144	63·1	33·1	19·7	12·9	9.0	6·6
	r_2 (cm.)	19366	1022	201	62·9	25·4	12·0	7·0	3·8	2·6	2·0

The stress in an isotropic homogeneous medium in polar coordinates are given by the formulae

$$\left.\begin{aligned}
\widehat{rr}&=\lambda\Delta+2\mu\frac{\partial u}{\partial r},\\
\widehat{\theta\theta}&=\lambda\Delta+2\mu\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right),\\
\widehat{\varphi\varphi}&=\lambda\Delta+2\mu\left(\frac{1}{r\sin\theta}\frac{\partial w}{\partial\varphi}+\frac{v}{r}\cot\theta+\frac{u}{r}\right),\\
\widehat{r\theta}&=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right),\\
\widehat{r\varphi}&=\mu\left(\frac{1}{r\sin\theta}\frac{\partial u}{\partial\varphi}+\frac{\partial w}{\partial r}-\frac{w}{r}\right),\\
\widehat{\varphi\theta}&=\mu\left\{\frac{1}{r}\left(\frac{\partial w}{\partial\theta}-w\cot\theta\right)+\frac{1}{r\sin\theta}\frac{\partial v}{\partial\varphi}\right\},
\end{aligned}\right\}\quad\ldots\ldots(63)$$

where

$$\Delta=\frac{1}{r^2\sin\theta}\left\{\frac{\partial}{\partial r}(r^2u\sin\theta)+\frac{\partial}{\partial\theta}(rv\sin\theta)+\frac{\partial}{\partial\varphi}(rw)\right\}.\quad\ldots\ldots(64)$$

And corresponding to (62),

$$\left.\begin{aligned}
\widehat{rr}&=\Bigg[-\lambda h^2A\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}+2\mu\left\{A\frac{d^2}{dr^2}\left(\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}\right)\right.\\
&\quad\left.+2kB_z\frac{d}{dr}\left(\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\right)\right\}\Bigg]P_2(\cos\theta)\,e^{ipt},\\
\widehat{\theta\theta}&=-\left\{\left[A\left\{\lambda h^2\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}+2\mu\frac{1}{r}\frac{d}{dr}\left(\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}\right)\right\}\right.\right.\\
&\quad\left.-4\mu kB_z\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{5}{2}}}\right]P_2(\cos\theta)\\
&\quad\left.+2\mu\left[3A\frac{1}{r^{\frac{5}{2}}}H^{(2)}_{2+\frac{1}{2}}(hr)+kB_z\frac{1}{r^2}\frac{d}{dr}\left(\sqrt{r}H^{(2)}_{2+\frac{1}{2}}(kr)\right)\right]\cos2\theta\right\}e^{ipt},\\
\widehat{\varphi\varphi}&=-\Bigg[\lambda h^2A\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{\sqrt{r}}P_2(\cos\theta)\\
&\quad+2\mu\left\{3A\frac{H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{5}{2}}}+kB_z\frac{1}{r^2}\frac{d}{dr}\left(\sqrt{r}H^{(2)}_{2+\frac{1}{2}}(kr)\right)\right\}\cos^2\theta\Bigg]e^{ipt},\\
\widehat{r\theta}&=\mu\Bigg[6A\left\{\frac{4H^{(2)}_{2+\frac{1}{2}}(hr)}{r^{\frac{5}{2}}}-\frac{hH^{(2)}_{1+\frac{1}{2}}(hr)}{r^{\frac{3}{2}}}\right.\\
&\quad-kB_z\left\{16\frac{H^{(2)}_{2+\frac{1}{2}}(kr)}{r^{\frac{5}{2}}}-\frac{k^2H^{(2)}_{2+\frac{1}{2}}(kr)}{\sqrt{r}}-2k\frac{H^{(2)}_{1+\frac{1}{2}}(kr)}{r^{\frac{3}{2}}}\right\}\Bigg]\sin\theta\cos\theta\,e^{ipt},\\
\widehat{r\varphi}&=\widehat{\varphi\theta}=0.
\end{aligned}\right\}\quad\ldots\ldots(65)$$

And from the constants already mensioned $\mu=8{\cdot}4\times10^{11}$ dyne/cm. and

$$\mu Ah^{\frac{5}{2}}=1{\cdot}90\times10^6\ \text{dyne/cm.},$$

$$\mu B_zk^{\frac{7}{2}}=44{\cdot}4\times10^6\ \text{dyne/cm.},$$

the numerical calculation was made and tabulated in table XI.

Table XI. The stresses in 10^6 dyne/cm^2. at hypocentral distances (I) $r=0\cdot5/h=1\cdot8$ km. and (II) $r=1/h=3\cdot6$ km.

(I) $r=1\cdot8$ km.	$\widehat{rr}$	$(0\cdot17+i1758\cdot5)\,P_2(\cos\theta)$	$(7\cdot89-i3716\cdot93)P_2(\cos\theta)$	$(8\cdot06-i1958\cdot4)P_2(\cos\theta)$
	$\widehat{r\theta}$	$(-0\cdot60+i3565\cdot8)\sin\theta\cos\theta$	$(-12\cdot33-i3668\cdot97)\sin\theta\cos\theta$	$(-12\cdot93-i103\cdot2)\sin\theta\cos\theta$
	$\widehat{\theta\theta}$	$(0\cdot37+i846\cdot7)\,P_2(\cos\theta)$ $(-0\cdot59+i991\cdot8)\cos2\theta)$	$(+8\cdot98+i999\cdot8)P_2(\cos\theta)$ $(-12\cdot92+i857\cdot1)\cos2\theta$	$(9\cdot35+i153\cdot0)\,P_2(\cos\theta)$ $(-13\cdot51-i54\cdot64)\cos2\theta$
	$\widehat{\varphi\varphi}$	$(-0\cdot10-i152\cdot0)\,P_2(\cos\theta)$ $(-0\cdot59-i911\cdot8)\cos^2\theta)$	$(-12\cdot92+i857\cdot1)\cos^2\theta$	$(-0\cdot10-i152\cdot0)\,P_2(\cos\theta)$ $+(-13\cdot51-i54\cdot64)\cos^2\theta$
(II) $r=3\cdot6$ km.	$\widehat{rr}$	$(0\cdot149+i106\cdot39)\,P_2(\cos\theta)$	$(4\cdot170-i144\cdot36)\,P_2(\cos\theta)$	$(4\cdot319-i37.97)\,P_2(\cos\theta)$
	$\widehat{r\theta}$	(0·4[illegible]2 $+i118\cdot60)\sin\theta\cos\theta$	$(-7\cdot396-i130\cdot40)\sin\theta\cos\theta$	$(-6\cdot914-i11\cdot80)\sin\theta\cos\theta$
	$\widehat{\theta\theta}$	$(0\cdot255-i32\cdot17)\,P_2(\cos\theta)$ $(-0\cdot564-i32\cdot79)\cos2\theta$	$(+7\cdot584+i46\cdot62)\,P_2(\cos\theta)$ $(9\cdot667+25\cdot50)\cos2\theta$	$(7\cdot839+i14\cdot45)\,P_2(\cos\theta)$ $(-102\cdot31-i7\cdot29)\cos2\theta$
	$\widehat{\varphi\varphi}$	$(-0\cdot094-i5\cdot465)\,P_2(\cos\theta)$ $(-0\cdot564-i32\cdot79)\cos^2\theta$	$(-9.667+i25\cdot50)\cos^2\theta$	$(-0\cdot094-i5\cdot465)\,P_2(\cos\theta)$ $(-102\cdot31-i7\cdot29)\cos^2\theta$

All the components of stress at $r=1\cdot8$ km. are the order of 10^9 dyne/cm.^{2}, and they are of the order 10^7–10^8 dyne/cm.^{2} at $r=3\cdot6$ km. Let us then consider the strength of material, We have seen the rigidity is nearly the same with that of steel. The region we are now considering is within the so-called sima and ultimate tensile strength of the material must be smaller than that of steel ($5-9\times10^9$ dyne/cm^2.) and will be larger than that of granite ($3-5\times10^7$ dyne/cm^2.). Even though we assume the ultimate strength as large as 10^9 dyne/cm^2. as Dr. H. Jeffreys[111] has already adopted, the stress at $1\cdot8$ km. outstrips this limit. We cannot therefore discuss what happened at the shorter distance from the data of observations of the elastic waves at larger distances. The magnitude of the hypocentral region in this sence may not be smaller than the sphere $r=2$ km. I have hitherto used the word hypocentre, which should of course mean the centre of the hypocentral region.

The energy stored up near the hypocentral region is consumed in the crustal deformation and a part of the energy is transmitted as the seismic waves. We can at present discuss the latter only. The energy flux across a surface due to periodic wave

$$F=\int(\dot{u}X_s+\dot{v}Y_s+\dot{w}Z_s)dS$$

changes its sign periodically, so we will here examine only the energy

111) H. Jeffreys, *M. N. R. A. S. Geophys. Suppl.*, 1 (1927), 483-494.

transmited in one period through a spherical surface r=const.

$$E=\int_0^T dt\int_0^{2\pi} d\varphi\int_0^U (u\widehat{rr}+v\widehat{r\theta}+w\widehat{r\varphi})\,r^2\sin\theta d\theta. \quad \ldots\ldots(66)$$

We must however remember the stress, $\widehat{rr}$, $\widehat{r\theta}$ and $\widehat{r\varphi}$ is the force which the external portion of the sphere excert on the internal portion of the sphere, and E is consequently the energy which flow into the sphere in one period. Taking real values of u, v, $\widehat{rr}$ and $\widehat{r\theta}$ we have at $r=1/h=3\cdot6$ km.

$$u=u_1+u_2=(1\cdot694\cos pt-3\cdot861\sin pt)P_2(\cos\theta)\ \text{cm.},$$

$$v=v_1+v_2=(-1\cdot091\cos pt+0\cdot788\sin pt)2\sin\theta\cos\theta\ \text{cm.},$$

$$\widehat{rr}=(4\cdot319\cos pt+37\cdot97\sin pt)P_2(\cos\theta)\times10^6\ \text{dyne/cm}^2.,$$

$$\widehat{r\theta}=(-6\cdot914\cos pt+11\cdot80\sin pt)\sin\theta\cos\theta\times10^6\ \text{dyne/cm}^2.,$$

and

$$\mathrm{E}=-7\cdot23\times10^{19}\ \text{erg.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(67)$$

At great distance from the hypocentre E becomes independent of r and can be calculated seperately for P- and S-waves.[112)]

$$\mathrm{E}=-0\cdot3\times10^{19}\ \text{erg for P-wave},$$

$$\mathrm{E}=-6\cdot9\times10^{19}\ \text{erg for S-wave}.$$

And the total energy is nearly equal to that emitted through the spherical surface $r=3\cdot6$ km. In the deep-seated earthquakes it is usual that the wave motions are damped after a few vibrations, so that the total energy emitted as seismic waves from the hypocentral region is the order of 10^{20} or 10^{21} erg.

As we have already seen the strength of doublet force was about $1\cdot23\times10^{25}$ dyne/cm., let us now examine the strength of simple sourse given by (58).

$$\phi=\frac{0\cdot81}{2h}\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{hr}}e^{ipt}=2\cdot91\times10^5\frac{H^{(2)}_{\frac{1}{2}}(hr)}{\sqrt{hr}}e^{ipt}\ (\text{cm.})^2.\ldots\ldots(68)$$

And the pressure due to this at $r=1/h=3\cdot6$ km. is

$$\widehat{rr}=-i(1\cdot00+i2\cdot96)e^{ipt}\times10^6\ \text{dyne/cm.}^2, \quad \ldots\ldots(69)$$

whose maximum absolute value is $3\cdot12\times10^6$ dyne/cm^2 which is equal to about 3 atmosperic pressure on the other hand the pressure prevailing at a depfh of 250 km. from the surface is

112) The result is identical with the formula obtained in other way due to B. Galizin (*Eomtes Rendus*, **160** (1915), 810-813) and H. Jeffreys *M. N. R. A. S. Geophys. Suppl.*, **1** (1922), 22-31).

$$3\times 10^3\times 250\times 10^5 \text{ dyne/cm}^2.=7{,}500 \text{ atmospheric pressure,}$$

(by taking simply the density 3 and the acceleration of gravity as 10^3), which is 2,000 times as large as the pressure due to the simple source.

Though still remains the transverse wave of the second kind, I will omit the discussion because it still being subjected to some umbiguities.

But I will state another word on the surface waves. Rayleigh-wave due to multiple sourse within a semi-infinite isotropic homogeneous solid has been solved by Prof. K. Sezawa[113] and G. Nishimura. If we assume for the moment that the earth is homogeneous and isotropic semi-infinite body, then the solution for simple source $\frac{e^{i(pt-hr)}}{r}$ is

$$\left.\begin{aligned} u&=\frac{4ie^{ipt}\pi\beta_1 k^2\kappa^2 e^{-\alpha_1\xi}}{h^2F'(\kappa)}\frac{\partial H_0^{(2)}(\kappa\Delta)}{\partial(\kappa\Delta)}=i7\cdot 60\, e^{-\alpha_1\xi}H_1^{(2)}(\kappa\Delta)e^{ipt},\\ w&=-\frac{2ie^{ipt}\pi k^2(2\kappa^2-k^2)\kappa}{h^2F'(\kappa)}e^{-\alpha_1\xi}H_0^{(2)}(\kappa\Delta)=i11\cdot 15e^{-\alpha_1\xi}H_0^{(2)}(\kappa\Delta)\,e^{ipt},\end{aligned}\right\}(70)$$

which become at $\kappa\Delta=1$

$$\left.\begin{aligned}&[u]=8\cdot 5e^{-\alpha_1\xi},\quad [w]=6\cdot 8e^{-\alpha_1\xi},\\ &e^{-\alpha_1\xi}=1\cdot 85\times 10^{-48},\end{aligned}\right\}\ldots.(80)$$

in which $\kappa=\frac{p}{v_R}$, v_R=velocity of Rayleigh wave, Δ=epicentral distance and ξ=is hypocentral depth which is taken to be 250 km. The amplitude (u_l) of the direct longitudinal wave incident at the surface is incomparablly larger than that of Rayleigh-wave.

$$u_l=\frac{\partial}{\partial r}\left(\frac{e^{i(pt-hr)}}{r}\right)\approx\frac{-ih}{r}e^{i(pt-hr)}=1\cdot 11\times 10^{-23}e^{i\left(pt-hr-\frac{\pi}{2}\right)}.\quad ..(81)$$

Thus, though Prof. Sezawa neglected the effect of direct and reflected bodily waves, we see here the necessity of taking these into consideration. The statement is also varid in case of waves due to quadruple source. On the other hand Rayleigh-wave as well as Love-wave due to transverse wave diminishes its amplitude with hypocentral depth with the factor $e^{-\beta_1\xi}$ and the factor for this earthquake is $e^{-\beta_1\xi}=2\cdot 51\times 10^{-22}$. And though this is conspicuously larger than that due to longitudinal wave, the amplitudes of the surface waves are still smaller than those due to direct bodily waves.

The above relation was the reason for the wide spread knowledge that the surface waves do not occure in case of deep-seated earthquake.

113) K. Sezawa and G. Nishimura, *loc. cit.*, 40).

And from the above values the absence of surface waves in case of deep-seated earthquake seems to be absolute. And indeed Prof. Sezawa[114] could satisfy the boundary conditions in the reflexion of elastic waves from an internal point of an isotropic homogeneous sphere without taking the surface waves into consideration.

(End of Chapter IV), (to be continued).

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24. 地震波の傳播（第二報）

發震機巧，地殼構造と地震波の振幅に就て

地震學教室 河角廣

地震發生の機巧を審かにして其の發震力の地方的並びに時代的活動の有樣を明かにし地震を豫知するは地震國日本に於ける地震研究の最大目的であるが，地震の計測から始めて發震機巧を推定したのは大森博士である．博士は地震記象型の特異性に注意せられたのであつたが，志田博士は地震初動の「押し」，「引き」の地理的分布から裂罅地震及び陷落地震等の區別をせられたが，中村左衞門太郎博士は前者を斷層（地辷）地震と解釋せられた．（此の型の地震の初動の「押し」「引き」分布は直交二平面によつて四つの部分に分けられる．此れを B 型と呼んでおく．）其の後殆ど凡ての地震が此の型で解釋出來る樣に考へられて來たが，棚橋嘉一氏は昭和六年六月二日の日本中部の深發地震が所謂斷層地震說にて說明出來ず，志田博士の裂罅地震說に對する松山博士の型（卽節圓錘によつて「押し」「引き」が分たれるもの．此れを A 型と呼ぶ）でよく說明される事を見出された．石本博士は此れに特に興味を持れた幾多の地震が此の A 型で說明出來る事から岩漿貫入說を出された．筆者は兼々地震動の大きさの定量的研究を企て居たので此處に此の問題を定量的に調査し發震機巧をも確めて見た．其の調査の一部を此處に報告する．

先づ第一章に從來の彈性波としての理論的諸研究及び實際的の地震の諸調査の歷史的概括を說べ，第二章に彈性球面波の一般解を簡單に求め，此れと從來の諸發震機巧に對する解との關係を調査し，第三章には實際に地球が一樣な彈性體でない爲の影響を考へ，第四章には此等の諸理論を實際に應用して昭和六年六月二日の日本中部の深發地震を調査し，此の地震が地下約 250 粁の深にある震原に於て約 N 10° E の方向に 67° 傾いた方向に働く 1.23×10^{25} dyne/cm の强さの二重力及び震原距離約 3·6 粁に於て約 3 氣壓の靜水壓を與へるに相等する震原によつて起された波動と二重力に直角な面の兩側に於て二重力の方向のまはりに起されたと見へる違つた向きの回轉によつて起されたと見られる橫波とよりなる事を確め，其等波動による震原近くの變位及び歪力を計算し，震原距離 1·8 粁に於て振幅約 2 米に達し歪力は約 10^9 dyne/cm^2 のものとなり恐らく物質强度の限界を越すと思はれる．卽此の約 3 粁以內の現象は地震波の觀測からは推知出來ないのではないかと思はれる．此の意味から其處を震原域と呼んだ．尙此の二重力及び單源によつて運ばれる勢力

114) K. SEZAWA, *loc. cit.*, 37).

は平均毎秒約 $2{\cdot}7\times10^{19}$ erg. である. 從つて此の地震の際に波動として運ばれた全勢力は 10^{20}〜10^{21} erg. であつた事が推知出來た. 尙此等の波による表面波の問題も考へて見たが此れは理論的には到底觀測出來ない程の小さいものである事が知られ; 或は此の樣な深發地震では絕對的に表面波は起らないものかも知れない. (續く)