

25. *On Stresses in the Interior and in the Vicinity  
of a Horizontal Cylindrical Inclusion of Circular  
Section in a Gravitating Semi-infinite  
Elastic Solid. (II)*

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For the same object of investigation as stated in an earlier paper,<sup>1)</sup> we shall solve the problem of a cylindrical heterogeneous matter included in a gravitating semi-infinite elastic solid in a state of plane stress, and study the difference in stress distribution of two cases, the state of the semi-infinite medium of one being a plane strain and the other plane stress.

In the present study we take the following expressions<sup>2)</sup> of  $\Delta$ ,  $2\omega$ ,  $\Delta'$  and  $2\omega'$  as particular solutions of the equilibrium equations of the gravitating semi-infinite solid and the cylindrical inclusion:

$$\left. \begin{aligned} \Delta &= 0, \\ 2\omega &= -\frac{\rho g r}{\mu} \sin \theta, \end{aligned} \right\} \dots\dots\dots (1)$$

$$\left. \begin{aligned} \Delta' &= \frac{\rho' g r}{2(\lambda' + \mu')} \cos \theta, \\ 2\omega' &= \frac{\lambda'}{2\mu'} \frac{\rho' g r}{(\lambda' + \mu')} \sin \theta, \end{aligned} \right\} \dots\dots\dots (2)$$

which differ from the expressions used in problems relating to plane

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1) G. NISHIMURA and T. TAKAYAMA, "On Stresses in the Interior and in the Vicinity of a Spherical Inclusion in a gravitating Semi-infinite Elastic Solid," *Bull. Earthq. Res. Inst.*, **11** (1933).

G. NISHIMURA and T. TAKAYAMA, "On Stresses in the Interior and in the Vicinity of a Horizontal Cylindrical Inclusion of Circular Section in a Gravitating Semi-infinite Elastic Solid," *Bull. Earthq. Res. Inst.*, **11** (1933).

2) The notations of all physical quantities used in this paper have the same meanings as those in the previous paper.

strain.<sup>3)</sup>

The respective displacements  $u, v$  and  $u', v'$  corresponding to (1) and (2) are obtained by the method used in the preceding papers. They are expressed by

$$\left. \begin{aligned} u &= \frac{\rho g}{8\mu} r^2 \cos \theta, \\ v &= -\frac{3\rho g}{8\mu} r^2 \sin \theta, \end{aligned} \right\} \dots\dots\dots (3)$$

$$\left. \begin{aligned} u' &= \frac{(\lambda' + 3\mu')}{16\mu'(\lambda' + \mu')} \rho' g r^2 \cos \theta, \\ v' &= -\frac{(3\lambda' + \mu')}{16\mu'(\lambda' + \mu')} \rho' g r^2 \sin \theta. \end{aligned} \right\} \dots\dots\dots (4)$$

The stresses corresponding to (1), (2) and (3), (4), respectively, are given by

$$\widehat{rr} = \frac{1}{2} \rho g r \cos \theta, \quad \widehat{\theta\theta} = -\frac{1}{2} \rho g r \cos \theta, \quad \widehat{r\theta} = -\frac{1}{2} \rho g r \sin \theta, \quad \dots (5)$$

$$\widehat{rr}' = \frac{3}{4} \rho' g r \cos \theta, \quad \widehat{\theta\theta}' = \frac{1}{4} \rho' g r \cos \theta, \quad \widehat{r\theta}' = -\frac{1}{4} \rho' g r \sin \theta. \quad (6)$$

As complementary solutions of  $\Delta, 2\sigma$  and  $\Delta', 2\sigma'$  of the equilibrium equations of two solids, we use the same expressions which were used in the preceding paper,<sup>4)</sup> so that the displacement expressions  $u, v, u', v'$  and the stress expressions  $\widehat{rr}, \widehat{\theta\theta}, \widehat{zz}, \widehat{r\theta}, \widehat{r\theta}', \widehat{\theta\theta}', \widehat{zz}', \widehat{r\theta}'$  corresponding to these dilatations and rotations are the same as those of the previous paper.

Now the stress components of a gravitating semi-infinite elastic solid, the state of which is one of plane stress, having no heterogeneous inclusion in its interior are expressed by

$$\left. \begin{aligned} \widehat{rr} &= -\frac{1}{2} \rho g \xi + \frac{3}{4} \rho g r \cos \theta - \frac{1}{2} \rho g \xi \cos 2\theta + \frac{1}{4} \rho g r \cos 3\theta, \\ \widehat{\theta\theta} &= -\frac{1}{2} \rho g \xi + \frac{1}{4} \rho g r \cos \theta + \frac{1}{2} \rho g \xi \cos 2\theta - \frac{1}{4} \rho g r \cos 3\theta, \\ \widehat{r\theta} &= -\frac{1}{4} \rho g r \sin \theta + \frac{1}{2} \rho g \xi \sin 2\theta - \frac{1}{4} \rho g r \sin 3\theta. \end{aligned} \right\} \dots (7)$$

3) In the plane strain problem we took the following expressions as particular solutions of the equilibrium equations:

$$\Delta = 0, \quad 2\sigma = -\frac{\rho g}{\mu} r \sin \theta, \quad \Delta' = \frac{\rho' g r}{(\lambda' + 2\mu')} \cos \theta, \quad 2\sigma' = 0.$$

4) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

These expressions are obtained by the following conditions: The normal and the shearing components of stress vanish at the horizontal surface  $x=\xi$ , and the stress at any point in the solid increases linearly with increase in its position from the upper surface of the solid. This semi-infinite solid is in a state of plane stress. Expressions (7) differ from those in the case of a plane strain problem.

Now the boundary conditions of the present study are as follows: the heterogeneous inclusion and the outer medium are perfectly cemented at the contact surface, so that they cannot slide upon each other at the boundary. In all the spaces in the medium distant from the inclusion the stress distributions are equal to those expressed by (7).

Adjusting the arbitrary constants of the general expressions of displacement and stress to satisfy these conditions, we obtain the final results of the stresses in the gravitating semi-infinite medium and in the inclusion.

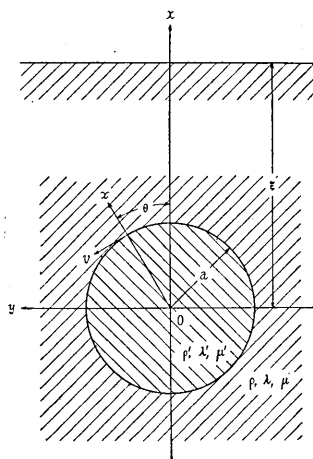


Fig. 1.

$$\begin{aligned}
 \widehat{rr} = & -\frac{\rho g \xi}{2} - \frac{\mu(\lambda' + \mu' - \lambda - \mu)}{2(\lambda + \mu)(\lambda' + \mu' + \mu)} \left(\frac{a}{r}\right)^2 \rho g \xi \\
 & + \left[ \frac{3}{4} \rho g r + \frac{(\rho' g a - \rho g a)(2\lambda + 3\mu)}{2(\lambda + 2\mu)} \left(\frac{a}{r}\right) \right. \\
 & \quad - \frac{[\lambda\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\} + (\lambda + 2\mu)\{\mu'(\lambda' + \mu') + \mu(-\lambda' + \mu')\}]}{4(\lambda + 2\mu)\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\}} \left. \frac{\rho' g a^4}{r^3} \right. \\
 & \quad \left. + \frac{[\lambda(\lambda + \mu)\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\} + 2\mu\mu'(\lambda' + \mu')(\lambda + 2\mu)]}{4(\lambda + \mu)(\lambda + 2\mu)\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\}} \frac{\rho g a^4}{r^3} \right] \cos \theta \\
 & + \frac{\rho g \xi}{2\mu} \left[ -\mu + \frac{2(\lambda + \mu)\vartheta c_2}{\vartheta_2} \left(\frac{a}{r}\right)^2 - \frac{6\mu\vartheta c_2''}{\vartheta_2} \left(\frac{a}{r}\right)^4 \right] \cos 2\theta \\
 & + \frac{\rho g r}{2\mu} \left[ \frac{\mu}{2} - \frac{5}{4} \frac{(\lambda + \mu)\vartheta c_3}{\vartheta_3} \left(\frac{a}{r}\right)^4 + \frac{4\mu\vartheta c_3''}{\vartheta_3} \left(\frac{a}{r}\right)^6 \right] \cos 3\theta, \dots\dots\dots (8) \\
 \widehat{\theta\theta} = & -\frac{\rho g \xi}{2} + \frac{\mu(\lambda' + \mu' - \lambda - \mu)}{2(\lambda + \mu)(\lambda' + \mu' + \mu)} \rho g \xi \left(\frac{a}{r}\right)^2
 \end{aligned}$$

$$\begin{aligned}
 & + \left[ \frac{1}{4} \rho g r - \frac{\mu(\rho' g a - \rho g a)}{2(\lambda + 2\mu)} \left( \frac{a}{r} \right) \right. \\
 & \quad + \frac{[\lambda \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \} + (\lambda + 2\mu) \{ \mu'(\lambda' + \mu') + \mu(-\lambda' + \mu') \}]}{4(\lambda + 2\mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho' g a \left( \frac{a}{r} \right)^3 \\
 & \quad \left. - \frac{[\lambda(\lambda + \mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \} + 2\mu\mu'(\lambda' + \mu')(\lambda + 2\mu)]}{4(\lambda + \mu)(\lambda + 2\mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho g a \left( \frac{a}{r} \right)^3 \right] \cos \theta \\
 & + \frac{\rho g \xi}{2\mu} \left[ \mu + \frac{6\mu \vartheta_{c_2''}}{\vartheta_2} \left( \frac{a}{r} \right)^4 \right] \cos 2\theta \\
 & + \frac{\rho g r}{2\mu} \left[ -\frac{\mu}{2} + \frac{(\lambda + \mu) \vartheta_{c_3}}{\vartheta_3} \left( \frac{a}{r} \right)^4 - \frac{4\mu \vartheta_{c_3''}}{\vartheta_3} \left( \frac{a}{r} \right)^6 \right] \cos 3\theta, \dots \dots \dots (9)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r\theta} = & \left[ \frac{1}{4} \rho g r + \frac{\mu(\rho' g a - \rho g a)}{2(\lambda + 2\mu)} \left( \frac{a}{r} \right) \right. \\
 & \quad + \frac{[\lambda \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \} + (\lambda + 2\mu) \{ \mu'(\lambda' + \mu') + \mu(-\lambda' + \mu') \}]}{4(\lambda + 2\mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho' g a \left( \frac{a}{r} \right)^3 \\
 & \quad \left. - \frac{[\lambda(\lambda + \mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \} + 2\mu\mu'(\lambda' + \mu')(\lambda + 2\mu)]}{4(\lambda + \mu)(\lambda + 2\mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho g a \left( \frac{a}{r} \right)^3 \right] \\
 & \hspace{15em} \times (-\sin \theta)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{\rho g \xi}{2\mu} \left[ \mu + \frac{(\lambda + \mu) \vartheta_{c_2}}{\vartheta_2} \left( \frac{a}{r} \right)^2 - \frac{6\mu \vartheta_{c_2''}}{\vartheta_2} \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \\
 & + \frac{\rho g \xi}{2\mu} \left[ -\frac{\mu}{2} - \frac{3(\lambda + \mu) \vartheta_{c_3}}{\vartheta_3} \left( \frac{a}{r} \right)^4 + \frac{4\mu \vartheta_{c_3''}}{\vartheta_3} \left( \frac{a}{r} \right)^6 \right] \sin 3\theta, \dots \dots \dots (10)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r'r'} = & -\frac{(\lambda + 2\mu)(\lambda' + \mu')}{2(\lambda + \mu)(\mu + \lambda' + \mu')} \rho g \xi \\
 & + \left[ \frac{3}{4} \rho' g r - \frac{\{ \mu'(\lambda' + \mu') + \mu(-\lambda' + \mu') \}}{4 \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho' g r \right. \\
 & \quad \left. + \frac{\mu\mu'(\lambda' + \mu') \rho g r}{2(\lambda + \mu) \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \right] \cos \theta \\
 & + \frac{\mu' \rho g \xi \vartheta_{D_2''}}{\mu \vartheta_2} \cos 2\theta \\
 & + \frac{\rho g a}{2\mu} \left[ \frac{(\lambda' + \mu') \vartheta_{D_3}}{4 \vartheta_3} \left( \frac{r}{a} \right)^3 - \frac{2\mu' \vartheta_{D_3''}}{\vartheta_3} \left( \frac{r}{a} \right) \right] \cos 3\theta, \dots \dots \dots (11)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{\theta\theta'} = & -\frac{(\lambda + 2\mu)(\lambda' + \mu')}{2(\lambda + \mu)(\mu + \lambda' + \mu')} \rho g \xi \\
 & + \left[ \frac{1}{4} \rho' g r - \frac{3 \{ \mu'(\lambda' + \mu') + \mu(-\lambda' + \mu') \}}{4 \{ \mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu') \}} \rho' g r \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{3\mu\mu'(\lambda' + \mu')\rho gr}{2(\lambda + \mu)\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\}} \Big] \cos \theta \\
 & + \frac{\rho g \xi}{2\mu} \left[ -\frac{2\mu' \vartheta_{D_2''}}{\vartheta_2} + \frac{2(\lambda' + \mu') \vartheta_{D_2}}{\vartheta_2} \left(\frac{r}{a}\right)^2 \right] \cos 2\theta \\
 & + \frac{\rho gr}{2\mu} \left[ -\frac{5(\lambda' + \mu') \vartheta_{D_3}}{4 \vartheta_3} \left(\frac{r}{a_3}\right)^3 + \frac{2\mu' \vartheta_{D_3''}}{\vartheta_3} \left(\frac{r}{a}\right) \right] \cos 3\theta, \dots\dots\dots(12)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r}\theta' = & \left[ \frac{1}{4} \rho' gr + \frac{\{\mu'(\lambda' + \mu') + \mu(-\lambda' + \mu')\} \rho' gr}{4\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\}} \right. \\
 & \left. - \frac{\mu\mu'(\lambda' + \mu')\rho gr}{2(\lambda + \mu)\{\mu(\lambda' + 3\mu') + \mu'(\lambda' + \mu')\}} \right] (-\sin \theta) \\
 & + \frac{\rho g \xi}{2\mu} \left[ -\frac{2\mu' \vartheta_{D_2''}}{\vartheta_2} + \frac{(\lambda' + \mu') \vartheta_{D_2}}{\vartheta_2} \left(\frac{r}{a}\right)^2 \right] \sin 2\theta \\
 & + \frac{\rho gr}{2\mu} \left[ \frac{2\mu' \vartheta_{D_3''}}{\vartheta_3} - \frac{3(\lambda' + \mu') \vartheta_{D_3}}{\vartheta_3} \left(\frac{r}{a}\right)^3 \right] \sin 3\theta, \dots\dots\dots(13)
 \end{aligned}$$

where  $\vartheta_2, \vartheta_{c_2}, \vartheta_{c_2''}, \vartheta_{D_2}, \vartheta_{D_2''}$  and  $\vartheta_3, \vartheta_{c_3}, \vartheta_{c_3''}, \vartheta_{D_3}, \vartheta_{D_3''}$  are expressed by the following determinants :

$$\left. \begin{aligned}
 \vartheta_2 = & \begin{vmatrix} 2(\lambda + \mu), & -6\mu, & -2\mu' & 0 \\ -(\lambda + \mu), & 6\mu, & -2\mu' & (\lambda' + \mu') \\ -\frac{(\lambda + 2\mu)}{2\mu}, & 1, & -1, & \frac{\lambda'}{6\mu'} \\ -\frac{1}{2}, & -1, & -1, & \frac{(2\lambda' + 3\mu')}{6\mu'} \end{vmatrix}, \\
 \vartheta_{c_2} = & \begin{vmatrix} \mu, & -6\mu, & -2\mu' & 0 \\ \mu, & 6\mu, & -2\mu', & (\lambda' + \mu') \\ \frac{1}{2}, & 1, & -1, & \frac{\lambda'}{6\mu'} \\ \frac{1}{2}, & -1, & -1, & \frac{(2\lambda' + 3\mu)}{6\mu'} \end{vmatrix}, \\
 \vartheta_{c_2''} = & \begin{vmatrix} 2(\lambda + \mu), & \mu, & -2\mu', & 0 \\ -(\lambda + \mu), & \mu, & -2\mu', & (\lambda' + \mu') \\ -\frac{(\lambda + 2\mu)}{2\mu}, & \frac{1}{2}, & -1, & \frac{\lambda'}{6\mu'} \\ -\frac{1}{2}, & \frac{1}{2}, & -1, & \frac{(2\lambda' + 3\mu')}{6\mu'} \end{vmatrix}, \dots\dots(14)
 \end{aligned} \right\}$$

$$\begin{aligned}
 \vartheta_{D_2} &= \begin{vmatrix} 2(\lambda + \mu), & -6\mu, & -2\mu', & \mu \\ -(\lambda + \mu), & 6\mu, & -2\mu', & \mu \\ -\frac{(\lambda + 2\mu)}{2\mu}, & 1, & -1, & \frac{1}{2} \\ -\frac{1}{2}, & -1, & -1, & \frac{1}{2} \end{vmatrix}, \\
 \vartheta_{D_2''} &= \begin{vmatrix} 2(\lambda + \mu), & -6\mu, & \mu, & 0 \\ -(\lambda + \mu), & 6\mu, & \mu, & (\lambda' + \mu') \\ -\frac{(\lambda + 2\mu)}{2\mu}, & 1, & \frac{1}{2}, & \frac{\lambda'}{6\mu'} \\ -\frac{1}{2}, & -1, & \frac{1}{2}, & \frac{(2\lambda' + 3\mu')}{6\mu'} \end{vmatrix}. \\
 \vartheta_3 &= \begin{vmatrix} \frac{5}{2}(\lambda + \mu), & -8\mu, & \frac{(\lambda' + \mu')}{2}, & -4\mu' \\ -\frac{3}{2}(\lambda + \mu), & 8\mu, & \frac{3}{2}(\lambda' + \mu'), & -4\mu' \\ -\frac{(3\lambda + 5\mu)}{8\mu}, & 1, & \frac{(3\lambda' + \mu')}{16\mu'}, & -1 \\ \frac{(\lambda - \mu)}{8\mu}, & -1, & \frac{(5\lambda' + 7\mu')}{16\mu'}, & -1 \end{vmatrix}, \\
 \vartheta_{C_3} &= \begin{vmatrix} \mu, & -8\mu, & \frac{(\lambda' + \mu')}{2}, & -4\mu' \\ \mu, & 8\mu, & \frac{3}{2}(\lambda' + \mu'), & -4\mu' \\ \frac{1}{4}, & 1, & \frac{(3\lambda' + \mu')}{16\mu'}, & -1 \\ \frac{1}{4}, & -1, & \frac{(3\lambda' + 7\mu')}{16\mu'}, & -1 \end{vmatrix}, \\
 \vartheta_{C_3''} &= \begin{vmatrix} \frac{5}{2}(\lambda + \mu), & \mu, & \frac{(\lambda' + \mu')}{2}, & -4\mu' \\ -\frac{3}{2}(\lambda + \mu), & \mu, & \frac{3}{2}(\lambda' + \mu'), & -4\mu' \\ -\frac{(3\lambda + 5\mu)}{8\mu}, & \frac{1}{4}, & \frac{(3\lambda' + \mu')}{16\mu'}, & -1 \\ \frac{(\lambda - \mu)}{8\mu}, & \frac{1}{4}, & \frac{(5\lambda' + 7\mu')}{16\mu'}, & -1 \end{vmatrix}, \dots (15)
 \end{aligned}$$

$$\vartheta_{D_3} = \begin{pmatrix} \frac{5}{2}(\lambda + \mu), & -8\mu, & \mu, & -4\mu' \\ -\frac{3}{2}(\lambda + \mu), & 8\mu, & \mu, & -4\mu' \\ -\frac{(3\lambda + 5\mu)}{8\mu}, & 1, & \frac{1}{4}, & -1 \\ \frac{(\lambda - \mu)}{8\mu}, & -1, & \frac{1}{4}, & -1 \end{pmatrix},$$

$$\vartheta_{D_3'} = \begin{pmatrix} \frac{5}{2}(\lambda + \mu), & -8\mu, & \frac{(\lambda' + \mu')}{2}, & \mu \\ -\frac{3}{2}(\lambda + \mu), & 8\mu, & \frac{3}{2}(\lambda' + \mu'), & \mu \\ -\frac{(3\lambda + 5\mu)}{8\mu}, & 1, & \frac{(3\lambda' + \mu')}{16\mu'}, & \frac{1}{4} \\ \frac{(\lambda - \mu)}{8\mu}, & -1, & \frac{(5\lambda' + 7\mu')}{16\mu'}, & \frac{1}{4} \end{pmatrix}.$$

The foregoing results bring to light many interesting facts, some of which we have summarised as follows:

1. The stress distributions in the medium and in the inclusion vary, in general, with variation of radial distance from the center of inclusion and with variation of azimuthal direction. The magnitudes of all stresses at all points in and around the inclusion vary with the magnitudes of both densities  $\rho$ ,  $\rho'$ , the elastic constants of both solids, the distance  $\xi$  between the free surface of the semi-infinite solid and the center of inclusion, and with the radius  $a$  of the cylindrical inclusion.

2. All components of stresses in the interior of both the gravitating semi-infinite solid and the gravitating inclusion are expressed by three terms related to  $\rho g \xi$ ,  $\rho g a$ , and  $\rho' g a$  respectively.

3. The respective magnitudes related to the terms  $\rho g \xi$  of  $\widehat{rr}$ ,  $\widehat{\theta\theta}$ , and  $\widehat{r\theta}$  become constant along the radial direction when the position in the medium is far from the inclusion.

4. The respective magnitudes related to the terms  $\rho g a$  of  $\widehat{rr}$ ,  $\widehat{\theta\theta}$ , and  $\widehat{r\theta}$  tend to increase linearly along the radial direction when the positions in the medium are far from the inclusion.

5. The respective magnitudes related to the term  $\rho' g a$  of  $\widehat{rr}$ ,  $\widehat{\theta\theta}$  and  $\widehat{r\theta}$  tend to decrease and finally vanish along the radial directions,

becoming inversely proportional to the radial distance from the center of the inclusion when the positions are far from the inclusion.

6. The magnitudes of the terms related to  $\rho g \xi$  of all components of stress  $\widehat{rr'}$ ,  $\widehat{\theta\theta'}$  and  $\widehat{r\theta'}$  in the cylindrical inclusion, although constant along the radial direction, are not so along the azimuthal direction.

7. The magnitudes of the terms related to  $\rho g a$  and  $\rho' g a$  of the respective components of stresses increase linearly along the radial direction but not along the azimuthal direction.

8. The stress distributions in the medium, especially around the inclusion, are much affected by the presence of inclusion, and the effect of inclusion upon the stress distributions in the medium, especially in the vicinity of the inclusion, decrease also when the distance between the position in the medium and the center of the inclusion become large.

9. The magnitudes of all terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$  of stresses in the inclusion vary with variation in magnitude of the elastic constants of both solids.

10. The distribution of stresses and the magnitudes of stresses all differ according as to whether the state of the medium is one of plane stress or whether the solids are in plane strain.

To find the general properties above mentioned we shall study more closely the stress distributions in and around the inclusion.

When the Poisson ratio of each of the solids are  $\frac{1}{4}$ , the stress distributions in and around the inclusion are expressed by

$$\begin{aligned} \widehat{rr'} = \rho g \xi & \left[ -\frac{1}{2} - \frac{(\mu' - \mu)}{2(\mu + 2\mu')} \left(\frac{a}{r}\right)^2 \right. \\ & \left. + \left\{ -\frac{1}{2} + \frac{2(\mu - \mu')}{(\mu + 2\mu')} \left(\frac{a}{r}\right)^2 - \frac{3(\mu - \mu')}{2(\mu + 2\mu')} \left(\frac{a}{r}\right)^4 \right\} \cos 2\theta \right] \\ & + \rho g a \left[ \left\{ \frac{3}{4} \left(\frac{r}{a}\right) - \frac{5}{6} \left(\frac{a}{r}\right) + \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left(\frac{a}{r}\right)^3 \right\} \cos \theta \right. \\ & \left. + \left\{ \frac{1}{4} \left(\frac{r}{a}\right) + \frac{5(\mu' - \mu)}{4(\mu + 2\mu')} \left(\frac{a}{r}\right)^3 + \frac{(\mu - \mu')}{(\mu + 2\mu')} \left(\frac{a}{r}\right)^5 \right\} \cos 3\theta \right] \\ & + \rho' g a \left[ \frac{5}{6} \left(\frac{a}{r}\right) - \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left(\frac{a}{r}\right)^3 \right] \cos \theta, \dots\dots\dots(16) \\ \widehat{\theta\theta'} = \rho g \xi & \left[ -\frac{1}{2} - \frac{(\mu - \mu')}{2(\mu + 2\mu')} \left(\frac{a}{r}\right)^2 + \left\{ \frac{1}{2} + \frac{3(\mu - \mu')}{2(\mu + 2\mu')} \left(\frac{a}{r}\right)^4 \right\} \cos 2\theta \right] \end{aligned}$$



$$\begin{aligned}
 & + \rho g a \left[ \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) - \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\
 & \quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{(\mu - \mu')}{4(\mu + 2\mu')} \left( \frac{a}{r} \right)^3 - \frac{(\mu - \mu')}{(\mu + 2\mu')} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\
 & + \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (17)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r\theta} & = \rho g \xi \left[ \frac{1}{2} + \frac{(\mu - \mu')}{(\mu + 2\mu')} \left( \frac{a}{r} \right)^2 - \frac{3(\mu - \mu')}{2(\mu + 2\mu')} \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \\
 & + \rho g a \left[ \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) + \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left( \frac{a}{r} \right)^3 \right\} \sin \theta \right. \\
 & \quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{3(\mu - \mu')}{4(\mu + 2\mu')} \left( \frac{a}{r} \right)^3 + \frac{(\mu - \mu')}{(\mu + 2\mu')} \left( \frac{a}{r} \right)^5 \right\} \sin 3\theta \right] \\
 & + \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) - \frac{(\mu + 2\mu')}{6(2\mu + \mu')} \left( \frac{a}{r} \right)^3 \right] \sin 3\theta, \dots\dots\dots (18)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r r'} & = \rho g \xi \left[ -\frac{3\mu'}{2(\mu + 2\mu')} - \frac{3\mu'}{2(\mu + 2\mu')} \cos 2\theta \right] \\
 & + \rho g a \left[ \frac{\mu'}{4(2\mu + \mu')} \frac{r}{a} \cos \theta + \frac{3\mu'}{4(\mu + 2\mu')} \frac{r}{a} \cos 3\theta \right] \\
 & + \rho' g a \left[ \frac{3}{4} \frac{r}{a} - \frac{\mu'}{4(2\mu + \mu')} \frac{r}{a} \right] \cos \theta, \dots\dots\dots (19)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{\theta\theta'} & = \rho g \xi \left[ -\frac{3\mu'}{2(\mu + 2\mu')} + \frac{3\mu'}{2(\mu + 2\mu')} \cos 2\theta \right] \\
 & + \rho g a \left[ \frac{3\mu'}{4(2\mu + \mu')} \frac{r}{a} \cos \theta - \frac{3\mu'}{4(\mu + 2\mu')} \frac{r}{a} \cos 3\theta \right] \\
 & + \rho' g a \left[ \frac{1}{4} \frac{r}{a} - \frac{3\mu'}{4(2\mu + \mu')} \frac{r}{a} \right] \cos \theta, \dots\dots\dots (20)
 \end{aligned}$$

$$\begin{aligned}
 \widehat{r\theta'} & = \rho g \xi \left[ \frac{3}{2} \frac{\mu'}{(\mu + 2\mu')} \sin 2\theta \right] \\
 & + \rho g a \left[ \frac{\mu'}{4(2\mu + \mu')} \frac{r}{a} \sin \theta - \frac{3\mu'}{4(\mu + 2\mu')} \frac{r}{a} \sin 3\theta \right] \\
 & + \rho' g a \left[ -\frac{1}{4} \frac{r}{a} - \frac{\mu'}{4(2\mu + \mu')} \frac{r}{a} \right] \sin \theta, \dots\dots\dots (21)
 \end{aligned}$$

A few examples in which different values of  $\frac{\mu'}{\mu}$  (or  $\frac{\lambda'}{\lambda}$ ) are given

are shown below :

$$\frac{\mu'}{\mu} = \frac{1}{5}, \lambda = \mu, \lambda' = \mu' : -$$

$$\begin{aligned} \widehat{r'r} = & \rho g \xi \left[ \left\{ -\frac{1}{2} + \frac{2}{7} \left( \frac{a}{r} \right)^2 \right\} + \left\{ -\frac{1}{2} + \frac{8}{7} \left( \frac{a}{r} \right)^2 - \frac{6}{7} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ & + \rho g a \left[ \left\{ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{5}{6} \left( \frac{a}{r} \right) + \frac{7}{66} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ & \quad \left. + \left\{ \frac{1}{4} \left( \frac{r}{a} \right) - \frac{5}{7} \left( \frac{a}{r} \right)^3 + \frac{4}{7} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ & + \rho' g a \left[ \frac{5}{6} \left( \frac{a}{r} \right) - \frac{7}{66} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (22) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta} = & \rho g \xi \left[ - \left\{ \frac{1}{2} + \frac{2}{7} \left( \frac{a}{r} \right)^2 \right\} + \left\{ \frac{1}{2} + \frac{6}{7} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ & + \rho g a \left[ \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) - \frac{7}{66} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ & \quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{7} \left( \frac{a}{r} \right)^3 - \frac{4}{7} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ & + \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{7}{66} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (23) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta} = & \rho g \xi \left[ \frac{1}{2} + \frac{4}{7} \left( \frac{a}{r} \right)^2 - \frac{6}{7} \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \\ & + \rho g a \left[ \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) + \frac{7}{66} \left( \frac{a}{r} \right)^3 \right\} \sin \theta \right. \\ & \quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{3}{7} \left( \frac{a}{r} \right)^3 + \frac{4}{7} \left( \frac{a}{r} \right)^5 \right\} \sin 3\theta \right] \\ & + \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) - \frac{7}{66} \left( \frac{a}{r} \right)^3 \right] \sin \theta, \dots\dots\dots (24) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta'} = & \rho g \xi \left[ -\frac{3}{14} - \frac{3}{14} \cos 2\theta \right] \\ & + \rho g a \left[ \frac{1}{44} \left( \frac{r}{a} \right) \cos \theta + \frac{3}{28} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ & + \rho' g a \left[ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{1}{44} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (25) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta'} &= \rho g \xi \left[ -\frac{3}{14} + \frac{3}{14} \cos 2\theta \right] + \rho g a \left[ \frac{3}{44} \left( \frac{r}{a} \right) \cos \theta - \frac{3}{28} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{1}{4} \left( \frac{r}{a} \right) - \frac{3}{44} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (26) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta'} &= \rho g \xi \left[ \frac{3}{14} \sin 2\theta \right] + \rho g a \left[ \frac{1}{44} \left( \frac{r}{a} \right) \sin \theta - \frac{3}{28} \left( \frac{r}{a} \right) \sin 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{1}{44} \left( \frac{r}{a} \right) \right] \sin \theta, \dots\dots\dots (27) \end{aligned}$$

$$\frac{\mu'}{\mu} = 1, \lambda = \mu, \lambda' = \mu' :-$$

$$\begin{aligned} \widehat{rr} &= \rho g \xi \left[ -\frac{1}{2} - \frac{1}{2} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{5}{6} \left( \frac{a}{r} \right) + \frac{1}{6} \left( \frac{a}{r} \right)^3 \right\} \cos \theta + \frac{1}{4} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{5}{6} \left( \frac{a}{r} \right) - \frac{1}{6} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (28) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta} &= \rho g \xi \left[ -\frac{1}{2} + \frac{1}{2} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) - \frac{1}{6} \left( \frac{a}{r} \right)^3 \right\} \cos \theta - \frac{1}{4} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{1}{6} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (29) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta} &= \rho g \xi \frac{1}{2} \sin 2\theta \\ &+ \rho g a \left[ \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) + \frac{1}{6} \left( \frac{a}{r} \right)^3 \right\} \sin \theta - \frac{1}{4} \left( \frac{r}{a} \right) \sin 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) - \frac{1}{6} \left( \frac{a}{r} \right)^3 \right] \sin \theta, \dots\dots\dots (30) \end{aligned}$$

$$\begin{aligned} \widehat{r'r'} &= \rho g \xi \left[ -\frac{1}{2} - \frac{1}{2} \cos 2\theta \right] \\ &+ \rho g a \left[ \frac{1}{12} \left( \frac{r}{a} \right) \cos \theta + \frac{1}{4} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{1}{12} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (31) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta}' &= \rho g \xi \left[ -\frac{1}{2} + \frac{1}{2} \cos 2\theta \right] + \rho g a \left[ \frac{1}{4} \left( \frac{r}{a} \right) \cos \theta - \frac{1}{4} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{1}{6} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (32) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta}' &= \rho g \xi \left[ \frac{1}{2} \right] \sin 2\theta + \rho g a \left[ \frac{1}{12} \left( \frac{r}{a} \right) \sin \theta - \frac{1}{4} \left( \frac{r}{a} \right) \sin 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{1}{12} \left( \frac{r}{a} \right) \right] \sin \theta, \dots\dots\dots (33) \end{aligned}$$

$\frac{\mu'}{\mu} = 5, \lambda = \mu, \lambda' = \mu' :-$

$$\begin{aligned} \widehat{rr} &= \rho g \xi \left[ -\left\{ \frac{1}{2} + \frac{2}{11} \left( \frac{a}{r} \right)^2 \right\} + \left\{ -\frac{1}{2} - \frac{8}{11} \left( \frac{a}{r} \right)^2 + \frac{6}{11} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{5}{6} \left( \frac{a}{r} \right) + \frac{11}{42} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ &\quad \left. + \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{5}{11} \left( \frac{a}{r} \right)^3 - \frac{4}{11} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{5}{6} \left( \frac{a}{r} \right) - \frac{11}{42} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (34) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta} &= \rho g \xi \left[ \left\{ -\frac{1}{2} + \frac{2}{11} \left( \frac{a}{r} \right)^2 \right\} + \left\{ \frac{1}{2} - \frac{6}{11} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) - \frac{11}{42} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ &\quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{1}{11} \left( \frac{a}{r} \right)^3 + \frac{4}{11} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{11}{42} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (35) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta} &= \rho g \xi \left[ \frac{1}{2} - \frac{4}{11} \left( \frac{a}{r} \right)^2 + \frac{6}{11} \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \\ &+ \rho g a \left[ \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) + \frac{11}{42} \left( \frac{a}{r} \right)^3 \right\} \sin \theta \right. \\ &\quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{3}{11} \left( \frac{a}{r} \right)^3 - \frac{4}{11} \left( \frac{a}{r} \right)^5 \right\} \sin 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) - \frac{11}{42} \left( \frac{a}{r} \right)^3 \right] \sin \theta, \dots\dots\dots (36) \end{aligned}$$

$$\begin{aligned} \widehat{r r'} &= \rho g \xi \left[ -\frac{15}{22} - \frac{15}{22} \cos 2\theta \right] + \rho g a \left[ \frac{5}{28} \left( \frac{r}{a} \right) \cos \theta + \frac{15}{44} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{5}{28} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (37) \end{aligned}$$

$$\begin{aligned} \widehat{\theta \theta'} &= \rho g \xi \left[ -\frac{15}{22} + \frac{15}{22} \cos 2\theta \right] + \rho g a \left[ \frac{15}{28} \left( \frac{r}{a} \right) \cos \theta - \frac{15}{44} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{11}{42} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (38) \end{aligned}$$

$$\begin{aligned} \widehat{r \theta'} &= \rho g \xi \left[ \frac{15}{22} \right] \sin 2\theta + \rho g a \left[ \frac{5}{28} \left( \frac{r}{a} \right) \sin \theta - \frac{15}{44} \left( \frac{r}{a} \right) \sin 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{5}{28} \left( \frac{r}{a} \right) \right] \sin \theta, \dots\dots\dots (39) \end{aligned}$$

$\frac{\mu'}{\mu} = \infty, \lambda = \mu, \lambda' = \mu' :$

$$\begin{aligned} \widehat{r r} &= \rho g \xi \left[ -\left\{ \frac{1}{2} + \frac{1}{4} \left( \frac{a}{r} \right)^2 \right\} + \left\{ -\frac{1}{2} - \left( \frac{a}{r} \right)^2 + \frac{3}{4} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{5}{6} \left( \frac{a}{r} \right) + \frac{1}{3} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ &\quad \left. + \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{5}{8} \left( \frac{a}{r} \right)^3 - \frac{1}{2} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ &+ \rho' g a \left[ \frac{5}{6} \left( \frac{a}{r} \right) - \frac{1}{3} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (40) \end{aligned}$$

$$\begin{aligned} \widehat{\theta \theta} &= \rho g \xi \left[ -\frac{1}{2} + \frac{1}{4} \left( \frac{a}{r} \right)^2 + \left\{ \frac{1}{2} - \frac{3}{4} \left( \frac{a}{r} \right)^4 \right\} \cos 2\theta \right] \\ &+ \rho g a \left[ \left\{ \frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) - \frac{1}{3} \left( \frac{a}{r} \right)^3 \right\} \cos \theta \right. \\ &\quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{1}{8} \left( \frac{a}{r} \right)^3 + \frac{1}{2} \left( \frac{a}{r} \right)^5 \right\} \cos 3\theta \right] \\ &+ \rho' g a \left[ -\frac{1}{6} \left( \frac{a}{r} \right) + \frac{1}{3} \left( \frac{a}{r} \right)^3 \right] \cos \theta, \dots\dots\dots (41) \end{aligned}$$

$$\begin{aligned} \widehat{r \theta} &= \rho g \xi \left[ \frac{1}{2} - \frac{1}{2} \left( \frac{a}{r} \right)^2 + \frac{3}{4} \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \\ &+ \rho g a \left[ \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{1}{6} \left( \frac{a}{r} \right) + \frac{1}{3} \left( \frac{a}{r} \right)^3 \right\} \sin \theta \right. \\ &\quad \left. + \left\{ -\frac{1}{4} \left( \frac{r}{a} \right) + \frac{3}{8} \left( \frac{a}{r} \right)^3 - \frac{1}{2} \left( \frac{a}{r} \right)^5 \right\} \sin 3\theta \right] \end{aligned}$$

$$+ \rho'ga \left[ -\frac{1}{6} \left( \frac{a}{r} \right) - \frac{1}{3} \left( \frac{a}{r} \right)^3 \right] \sin \theta, \dots\dots\dots (42)$$

$$\begin{aligned} \widehat{rr}' &= \rho g \xi \left[ -\frac{3}{4} - \frac{3}{4} \cos 2\theta \right] + \rho ga \left[ \frac{1}{4} \left( \frac{r}{a} \right) \cos \theta + \frac{3}{8} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho'ga \left[ \frac{3}{4} \left( \frac{r}{a} \right) - \frac{1}{4} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (43) \end{aligned}$$

$$\begin{aligned} \widehat{\theta\theta}' &= \rho g \xi \left[ -\frac{3}{4} + \frac{3}{4} \cos 2\theta \right] + \rho ga \left[ \frac{3}{4} \left( \frac{r}{a} \right) \cos \theta - \frac{3}{8} \left( \frac{r}{a} \right) \cos 3\theta \right] \\ &+ \rho'ga \left[ \frac{1}{4} \left( \frac{r}{a} \right) - \frac{3}{4} \left( \frac{r}{a} \right) \right] \cos \theta, \dots\dots\dots (44) \end{aligned}$$

$$\begin{aligned} \widehat{r\theta}' &= \rho g \xi \left[ \frac{3}{4} \sin 2\theta \right] + \rho ga \left[ \frac{1}{4} \left( \frac{r}{a} \right) \sin \theta - \frac{3}{8} \left( \frac{r}{a} \right) \sin 3\theta \right] \\ &+ \rho'ga \left[ -\frac{1}{4} \left( \frac{r}{a} \right) - \frac{1}{4} \left( \frac{r}{a} \right) \right] \sin \theta. \dots\dots\dots (45) \end{aligned}$$

These results are numerically calculated and tabulated in the annexed tables and shewn in the annexed figures. These figures shew the properties of stress distributions in and around the inclusion already discussed.

When the cylindrical inclusion is more rigid than the surrounding medium, the magnitudes of the respective terms with respect to  $\rho ga$  and  $\rho g \xi$  of  $\widehat{rr}_{r=a}$ ,  $\widehat{r\theta}_{r=a}$ ,  $\widehat{rr}'_{r=a}$ ,  $\widehat{r\theta}'_{r=a}$  are large, as are those of the terms related to  $\rho'ga$  of  $\widehat{r\theta}_{r=a}$  and  $\widehat{r\theta}'_{r=a}$ . The magnitudes of the terms related to  $\rho'ga$  of  $\widehat{rr}_{r=a}$  and  $\widehat{rr}'_{r=a}$ , however, are small in this case.

When the cylindrical inclusion is more rigid than the surrounding medium, the magnitudes of the respective terms related to  $\rho g \xi$  and  $\rho ga$  of  $\widehat{\theta\theta}_{r=a}$  are generally small, while those of the terms related to  $\widehat{\theta\theta}'_{r=a}$ , on the contrary, are generally large. The terms related to  $\rho'ga$  of the components of stresses  $\widehat{\theta\theta}_{r=a}$  and  $\widehat{\theta\theta}'_{r=a}$  have special distributions, different from those of all other stress components, when the magnitude of  $\frac{\mu'}{\mu}$  is large.

When the Poisson's ratio of each of the two solids are  $\frac{1}{4}$ , the effective radius of the existence of inclusion upon the stress distributions in the medium is about 2 times the radius of the inclusion. The rigidity ratio  $\frac{\mu'}{\mu}$  has little effect upon this effective radius.

Even when  $\frac{\mu'}{\mu} = \infty$ , the magnitudes of all terms related to  $\rho ga$ ,  $\rho g\xi$  and  $\rho' ga$  of stresses in the inclusion are finite.

For the differences in magnitude of stresses in and around the inclusion for the case when the medium is in plane stress and when it is in plane strain, the following table shews the magnitudes of stresses at special points in the two cases when  $\lambda = \mu$ ,  $\lambda' = \mu'$  and  $\frac{\mu'}{\mu} = \frac{1}{5}$  and  $\infty$ .

Table I.  
(Ordinary figures = plane stress)  
(Figures in italics = plane strain)

$\frac{\mu'}{\mu}$	Stress components	$\frac{r}{a}$	$\theta$	$\rho ga$	$\rho g\xi$	$\rho' ga$
$\frac{1}{5}$	$\widehat{rr}$	1	0°	0.130 <i>0.116</i>	-0.428 <i>-0.428</i>	0.727 <i>0.727</i>
		1	60°	0.833 <i>0.809</i>		
	$\widehat{\theta\theta}$	1	90°		-2.143 <i>-1.95</i>	
		1	0°			-0.06 <i>-0.06</i>
		1	90°			
	$\widehat{r\theta}$	1	90°	0.130 <i>0.117</i>		
		1	45°		0.214 <i>0.143</i>	
		1	90°			-0.273 <i>-0.273</i>
	$\widehat{rr'}$	1	0°	0.130 <i>0.116</i>	-0.428 <i>-0.428</i>	0.727 <i>0.727</i>
		1	60°	0.141 <i>0.141</i>		
		1	90°		-0.428 <i>-0.428</i>	
	$\widehat{\theta\theta'}$	1	0°			0.1818 <i>0.1818</i>
		1	90°			
		1	60°			
	$\widehat{r\theta'}$	1	90°	0.130 <i>0.117</i>		
		1	45°		0.214 <i>0.143</i>	
		1	90°			-0.273 <i>-0.273</i>

(to be continued.)

Table I. (*continued*).

$\frac{\mu'}{\mu}$	Stress components	$\frac{r}{a}$	$\theta$	$\rho ga$	$\rho g\xi$	$\rho'ga$	
$\infty$	$\widehat{rr}$	1	0°	0.625 0.750	-1.500 -1.500	0.500 0.500	
	$\widehat{\theta\theta}$	1	0°	0.209 0.250	0 -0.17	0.167 0.167	
		1	90°				
		1	0°				
	$\widehat{r\theta}$	1	90°	0.625 0.750	0.750 0.500	-0.500 -0.500	
		1	45°				
		1	90°				
	$\infty$	$\widehat{rr'}$	1	0°	0.625 0.750	-1.500 -1.500	0.50 0.50
		$\widehat{\theta\theta'}$	1	45°	0.795 1.237	-1.50 -1.50	-0.50 -0.50
1			90°				
1			0°				
$\widehat{r\theta'}$		1	90°	0.625 0.750	0.750 0.500	-0.500 -0.500	
		1	45°				
		1	90°				

This table shews that the magnitudes related to  $\rho ga$  and  $\rho g\xi$  of all stress components in and around the inclusion are generally different in the two cases, but the magnitude related to  $\rho'ga$  of all stress components is exactly the same in both cases.

When the state of the medium is one of plane stress, the stress distribution in the medium far from the inclusion are not affected by the elasticity constants of the medium, but in the case of plane strain they are affected by the elasticity constants of the medium.



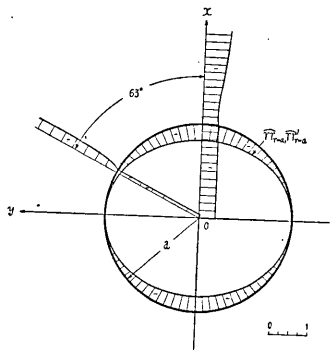


Fig. 2a. Distribution of the terms related to  $\rho g \xi$  of  $\hat{r}r$  and  $\hat{r}r'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

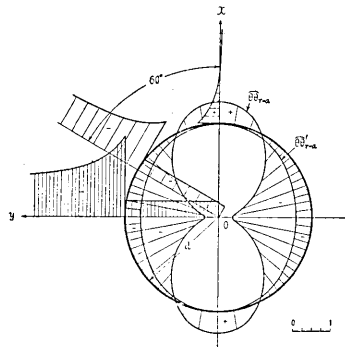


Fig. 3a. Distribution of the terms related to  $\rho g \xi$  of  $\hat{\theta}\theta$  and  $\hat{\theta}\theta'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

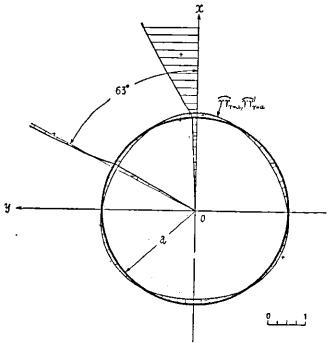


Fig. 2b. Distribution of the terms related to  $\rho g a$  of  $\hat{r}r$  and  $\hat{r}r'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

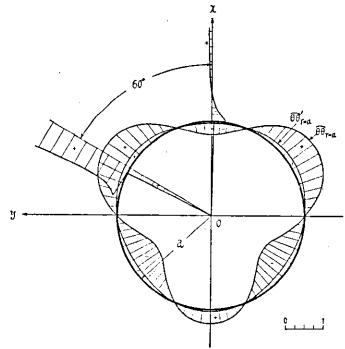


Fig. 3b. Distribution of the terms related to  $\rho g a$  of  $\hat{\theta}\theta$  and  $\hat{\theta}\theta'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

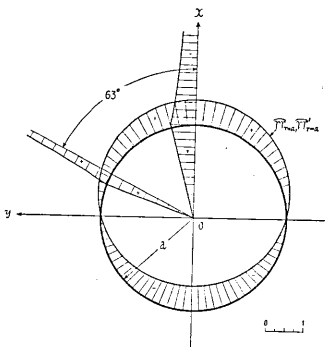


Fig. 2c. Distribution of the terms related to  $\rho' g a$  of  $\hat{r}r$  and  $\hat{r}r'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

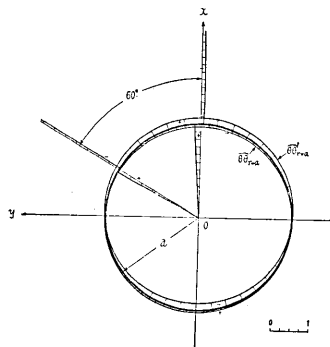


Fig. 3c. Distribution of the terms related to  $\rho' g a$  of  $\hat{\theta}\theta$  and  $\hat{\theta}\theta'$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

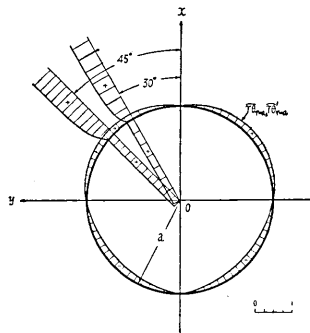


Fig. 4a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r\theta}$  and  $\widehat{r\theta'}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

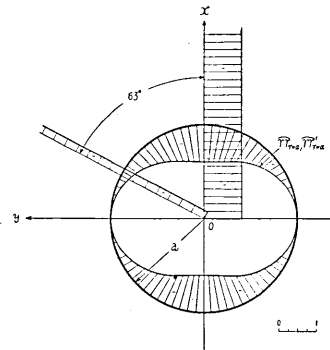


Fig. 5a. [Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r r}$  and  $\widehat{r r'}$  when  $\frac{\mu'}{\mu} = 1$ .

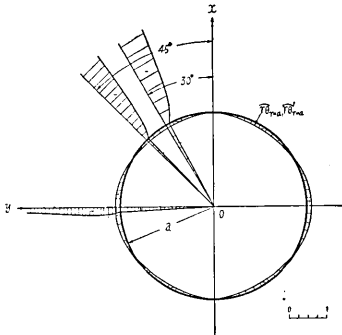


Fig. 4b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta'}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

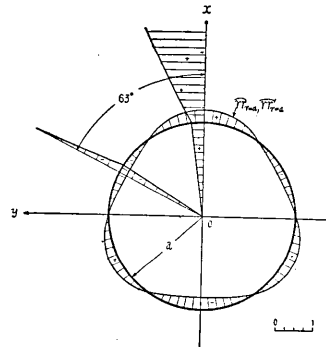


Fig. 5b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r r}$  and  $\widehat{r r'}$  when  $\frac{\mu'}{\mu} = 1$ .

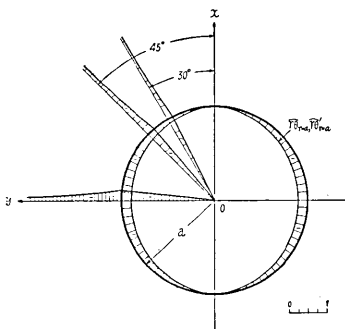


Fig. 4c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta'}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

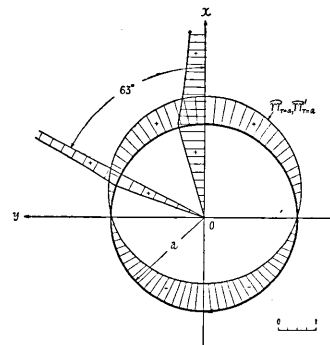


Fig. 5c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{r r}$  and  $\widehat{r r'}$  when  $\frac{\mu'}{\mu} = 1$ .

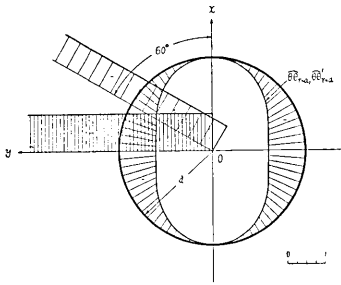


Fig. 6a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{\theta\theta}$  and  $\widehat{\theta\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

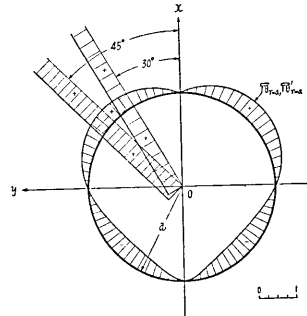


Fig. 7a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

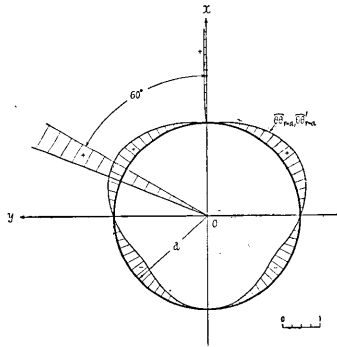


Fig. 6b. Distribution of the terms related to  $\rho g a$  of  $\widehat{\theta\theta}$  and  $\widehat{\theta\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

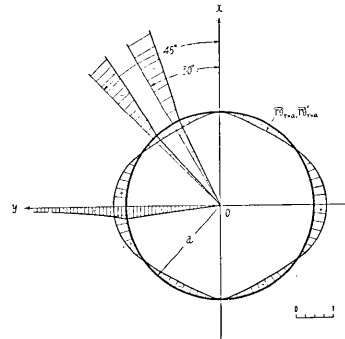


Fig. 7b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

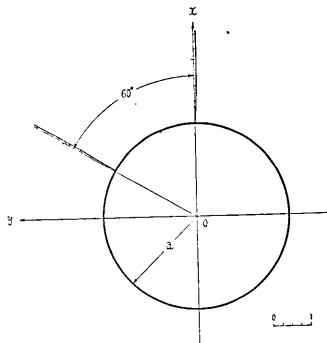


Fig. 6c. Distribution of the terms related to  $p' g a$  of  $\widehat{\theta\theta}$  and  $\widehat{\theta\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

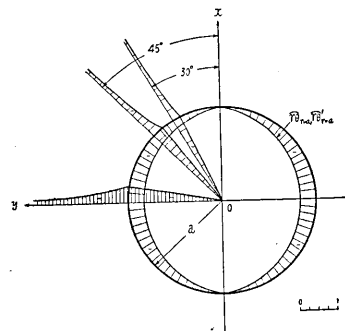


Fig. 7c. Distribution of the terms related to  $p' g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu}=1$ .

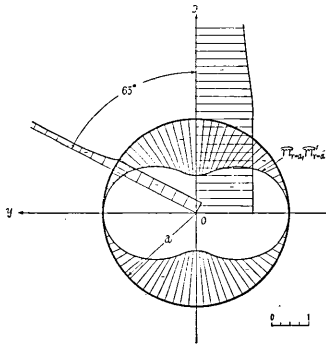


Fig. 8a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r r}$  and  $\widehat{r r}'$  when  $\frac{\mu'}{\mu} = 5$ .

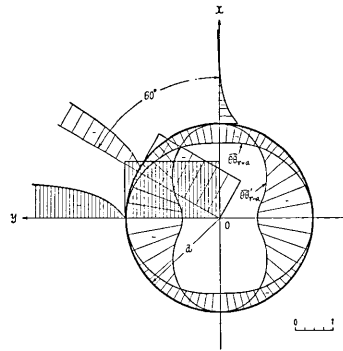


Fig. 9a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{\theta \theta}$  and  $\widehat{\theta \theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

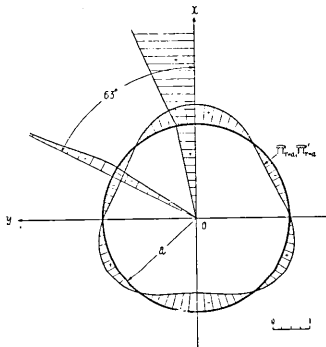


Fig. 8b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r r}$  and  $\widehat{r r}'$  when  $\frac{\mu'}{\mu} = 5$ .

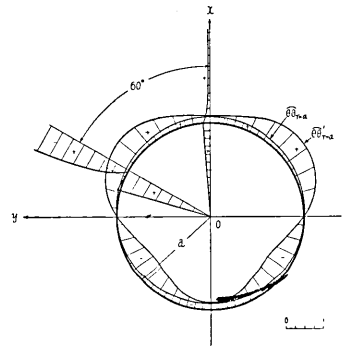


Fig. 9b. Distribution of the terms related to  $\rho g a$  of  $\widehat{\theta \theta}$  and  $\widehat{\theta \theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

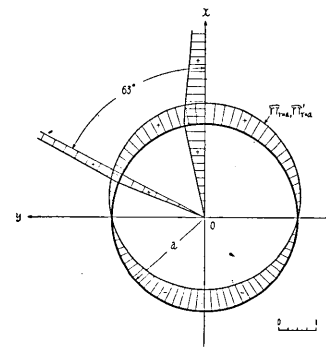


Fig. 8c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{r r}$  and  $\widehat{r r}'$  when  $\frac{\mu'}{\mu} = 5$ .

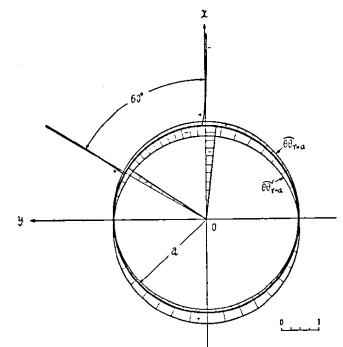


Fig. 9c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{\theta \theta}$  and  $\widehat{\theta \theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

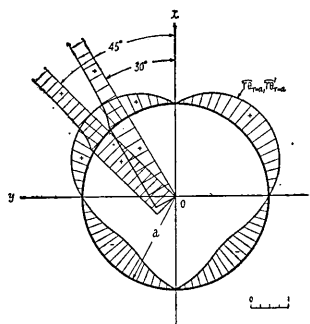


Fig. 10a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

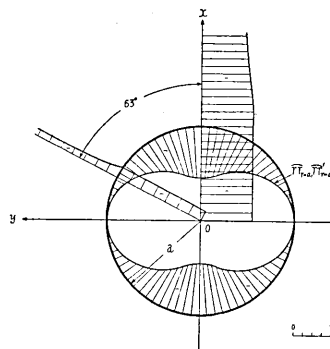


Fig. 11a. Distribution of the terms related to  $\rho g \xi$  of  $\widehat{r\bar{r}}$  and  $\widehat{r\bar{r}'}$  when  $\frac{\mu'}{\mu} = \infty$ .

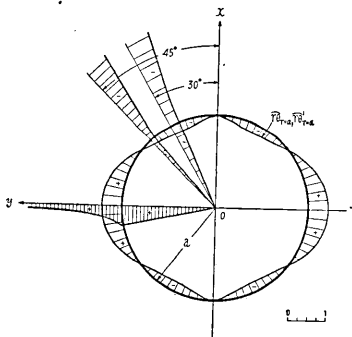


Fig. 10b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

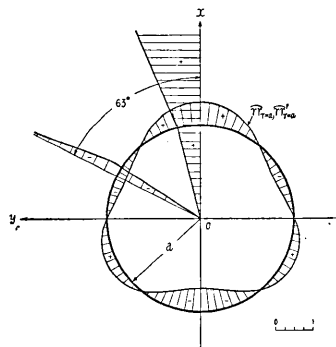


Fig. 11b. Distribution of the terms related to  $\rho g a$  of  $\widehat{r\bar{r}}$  and  $\widehat{r\bar{r}'}$  when  $\frac{\mu'}{\mu} = \infty$ .

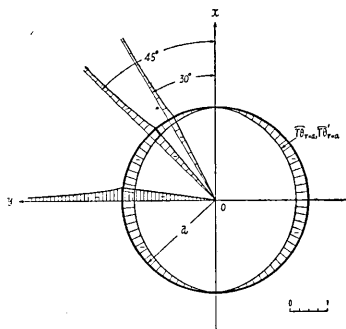


Fig. 10c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{r\theta}$  and  $\widehat{r\theta}'$  when  $\frac{\mu'}{\mu} = 5$ .

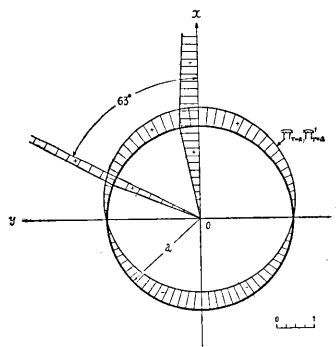


Fig. 11c. Distribution of the terms related to  $\rho' g a$  of  $\widehat{r\bar{r}}$  and  $\widehat{r\bar{r}'}$  when  $\frac{\mu'}{\mu} = \infty$ .

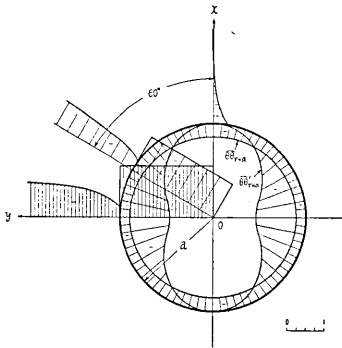


Fig. 12a. Distribution of the terms related to  $\rho g \bar{\xi}$  of  $\hat{\theta}\hat{\theta}$  and  $\hat{\theta}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

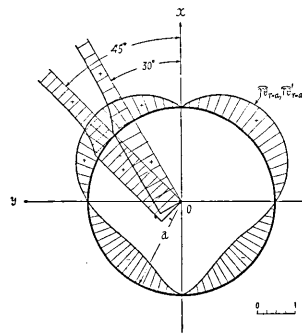


Fig. 13a. Distribution of the terms related to  $\rho g \bar{\xi}$  of  $\hat{r}\hat{\theta}$  and  $\hat{r}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

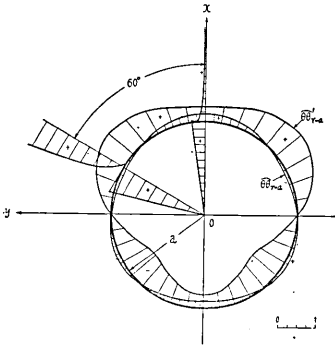


Fig. 12b. Distribution of the terms related to  $\rho g a$  of  $\hat{\theta}\hat{\theta}$  and  $\hat{\theta}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

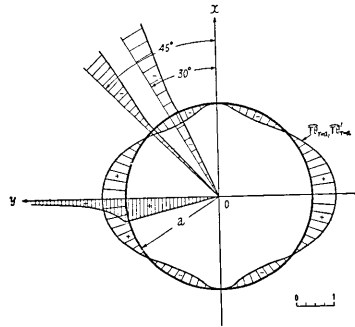


Fig. 13b. Distribution of the terms related to  $\rho g a$  of  $\hat{r}\hat{\theta}$  and  $\hat{r}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

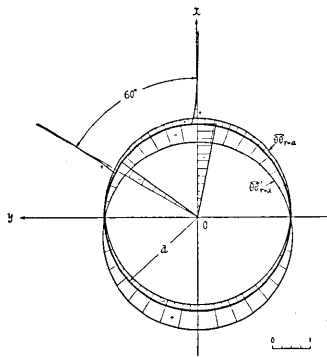


Fig. 12c. Distribution of the terms related to  $\rho' g a$  of  $\hat{\theta}\hat{\theta}$  and  $\hat{\theta}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

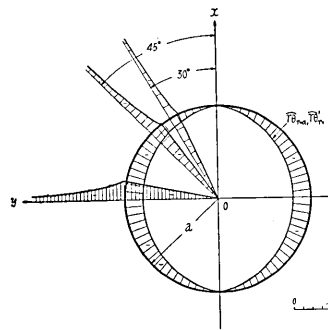


Fig. 13c. Distribution of the terms related to  $\rho' g a$  of  $\hat{r}\hat{\theta}$  and  $\hat{r}\hat{\theta}'$  when  $\frac{\mu'}{\mu} = \infty$ .

Table IIa.

Magnitudes of the respective terms related to  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $rr_{r=a}$  ( $=rr'_{r=a}$ ) when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g\xi$	-0.428	-0.415	-0.378	-0.321	-0.214	-0.107	-0.050	-0.013	0
$\rho ga$	0.130	0.115	0.075	0.020	-0.060	-0.096	-0.085	-0.050	0
$\rho'ga$	0.727	0.716	0.683	0.630	0.514	0.363	0.248	0.126	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g\xi$	-0.013	-0.050	-0.107	-0.214	-0.321	-0.378	-0.415	-0.428	
$\rho ga$	0.050	0.085	0.096	0.060	-0.020	-0.075	-0.115	-0.130	
$\rho'ga$	-0.126	-0.248	-0.363	-0.514	0.630	-0.683	-0.716	-0.727	

Table IIb.

Magnitudes of the respective terms related to  $\rho q\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{\theta\theta}_{r=a}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho q\xi$	0.5713	0.4893	0.2543	-0.1077	-0.7857	-1.4637	-1.8260	-2.0607	-2.1427
$\rho ga$	-0.368	-0.281	-0.047	0.269	0.700	0.833	0.693	0.393	0
$\rho'ga$	-0.0600	-0.0590	-0.0563	-0.0520	-0.0423	-0.030	-0.0205	-0.0104	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho q\xi$	-2.0607	-1.8260	-1.4637	-0.7857	-0.1077	0.2543	0.4893	0.5713	
$\rho ga$	-0.393	-0.693	-0.833	-0.700	-0.269	0.047	0.281	0.368	
$\rho'ga$	0.0104	0.0205	0.0300	0.0423	0.0520	0.0563	0.0590	0.0600	

Table IIc.

Magnitudes of the respective terms related to  $\rho q\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{\theta\theta}'_{r=a}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho q\xi$	0	-0.013	-0.050	-0.107	-0.214	-0.321	-0.378	-0.415	-0.428
$\rho ga$	-0.0390	-0.0254	0.0106	0.0591	0.1238	0.1411	0.1159	0.0653	0
$\rho'ga$	0.1818	0.1790	0.1710	0.1570	0.1280	0.0906	0.0620	0.0316	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho q\xi$	-0.415	-0.378	-0.321	-0.214	-0.107	-0.050	-0.013	0	
$\rho ga$	-0.0653	-0.1159	-0.1411	-0.1238	-0.0591	-0.0106	0.0254	0.0390	
$\rho'ga$	-0.0316	-0.0620	-0.0906	-0.1280	-0.1570	-0.1710	-0.1790	-0.1818	





Table IIIc.

Magnitudes of the respective terms related to  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{r\theta}_{r=a}$  ( $=\widehat{r\theta}'_{r=a}$ ) when  $\frac{\mu'}{\mu}=1$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g\xi$	0	0.171	0.321	0.433	0.500	0.433	0.321	0.171	0
$\rho ga$	0	-0.110	-0.189	-0.208	-0.117	0.072	0.203	0.299	0.333
$\rho'ga$	0	-0.058	-0.114	-0.167	-0.236	-0.288	-0.313	-0.328	-0.333
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g\xi$	-0.171	-0.321	-0.433	-0.500	-0.433	-0.321	-0.171	0	
$\rho ga$	0.299	0.203	0.072	-0.117	-0.208	-0.189	-0.110	0	
$\rho'ga$	-0.328	-0.313	-0.288	-0.236	-0.167	-0.114	-0.058	0	

Table IVa.

Magnitudes of the respective terms related to  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{rr}_{r=a}$  ( $=\widehat{rr}'_{r=a}$ ) when  $\frac{\mu'}{\mu}=5$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g\xi$	-1.364	-1.324	-1.205	-1.024	-0.683	-0.342	-0.161	-0.042	0
$\rho ga$	0.519	0.471	0.338	0.155	-0.115	-0.252	-0.234	-0.139	0
$\rho'ga$	0.572	0.563	0.537	0.495	0.404	0.286	0.195	0.099	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g\xi$	-0.042	-0.161	-0.342	-0.683	-1.024	-1.205	-1.324	-1.366	
$\rho ga$	0.139	0.234	0.252	0.115	-0.155	-0.338	-0.471	-0.519	
$\rho'ga$	-0.099	-0.195	-0.286	-0.404	-0.495	-0.537	-0.563	-0.572	

Table IVb.

Magnitudes of the respective terms related to  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{\theta\theta}_{r=a}$  when  $\frac{\mu'}{\mu}=5$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g\xi$	-0.363	-0.361	-0.352	-0.341	-0.318	-0.295	-0.284	-0.275	-0.273
$\rho ga$	0.178	0.172	0.156	0.134	0.083	0.054	0.033	0.016	0
$\rho'ga$	0.095	0.094	0.090	0.083	0.067	0.048	0.033	0.016	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g\xi$	-0.275	-0.284	-0.295	-0.318	-0.341	-0.352	-0.361	-0.363	
$\rho ga$	-0.016	-0.033	-0.054	-0.083	-0.134	-0.0156	-0.172	-0.178	
$\rho'ga$	-0.016	-0.033	-0.048	-0.067	-0.083	-0.090	-0.094	-0.095	

Table IVc.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$   
of  $\widehat{\theta\theta'}_{r=a}$  when  $\frac{\mu'}{\mu} = 5$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	0	-0.041	-0.159	-0.341	-0.682	-1.023	-1.205	-1.323	-1.364
$\rho g a$	0.195	0.233	0.334	0.465	0.620	0.609	0.478	0.263	0
$\rho' g a$	-0.286	-0.282	-0.268	-0.248	-0.202	-0.143	-0.098	-0.050	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-1.323	-1.205	-1.023	-0.682	-0.341	-0.159	-0.041	0	
$\rho g a$	-0.263	-0.478	-0.609	-0.620	-0.465	-0.334	-0.233	-0.195	
$\rho' g a$	0.050	0.098	0.143	0.202	0.248	0.268	0.282	0.286	

Table IVd.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$   
of  $\widehat{r\theta}_{r=a}$  ( $=\widehat{r\theta'}_{r=a}$ ) when  $\frac{\mu'}{\mu} = 5$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	0	0.233	0.438	0.591	0.682	0.591	0.438	0.233	0
$\rho g a$	0	-0.139	-0.234	-0.252	-0.115	0.155	0.338	0.471	0.519
$\rho' g a$	0	-0.075	-0.146	-0.214	-0.313	-0.372	-0.403	-0.422	-0.429
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-0.233	-0.438	-0.591	-0.682	-0.591	-0.438	-0.233	0	
$\rho g a$	0.471	0.338	0.155	-0.115	-0.252	-0.234	-0.139	0	
$\rho' g a$	-0.422	-0.403	-0.372	-0.303	-0.214	-0.146	-0.075	0	

Table Va.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$   
of  $\widehat{rr}_{r=a}$  ( $=\widehat{rr'}_{r=a}$ ) when  $\frac{\mu'}{\mu} = \infty$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	-1.500	-1.455	-1.325	-1.125	-0.750	-0.375	-0.175	-0.045	0
$\rho g a$	0.625	0.571	0.422	0.216	-0.088	-0.250	-0.239	-0.143	0
$\rho' g a$	0.500	0.492	0.470	0.433	0.353	0.250	0.171	0.087	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-0.045	-0.175	-0.375	-0.750	-1.125	-1.325	-1.455	-1.500	
$\rho g a$	0.143	0.239	0.250	0.088	-0.216	-0.422	-0.571	-0.425	
$\rho' g a$	-0.087	-0.171	-0.250	-0.353	-0.433	-0.470	-0.492	-0.500	

Table Vb.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$  of  $\widehat{\theta\theta}_{r=a}$  when  $\frac{\mu'}{\mu} = \infty$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	-0.5000	-0.4850	-0.4410	-0.3750	-0.2500	-0.1250	-0.0585	-0.0150	0
$\rho g a$	0.2085	0.1905	0.1410	0.0723	-0.0294	-0.0833	-0.0797	-0.0480	0
$\rho' g a$	0.1670	0.1645	0.1570	0.1445	0.1180	0.0834	0.0571	0.0290	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-0.0150	-0.0585	-0.1250	-0.2500	-0.3750	-0.4410	-0.4850	-0.5000	
$\rho g a$	0.0480	0.0797	0.0833	0.0294	-0.0723	-0.1410	-0.1905	-0.2085	
$\rho' g a$	-0.0290	-0.0571	-0.0834	-0.1180	-0.1445	-0.1570	-0.1645	-0.1670	

Table Vc.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$  of  $\widehat{\theta\theta}'_{r=a}$  when  $\frac{\mu'}{\mu} = \infty$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	0	-0.045	-0.175	-0.375	-0.750	-1.125	-1.325	-1.455	-1.500
$\rho g a$	0.375	0.414	0.518	0.650	0.795	0.750	0.575	0.3175	0
$\rho' g a$	-0.50	-0.493	-0.470	-0.433	-0.353	-0.250	-0.171	-0.087	0
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-1.455	-1.325	-1.125	-0.750	-0.375	-0.175	-0.045	0	
$\rho g a$	-0.3175	-0.575	-0.750	-0.795	-0.650	-0.518	-0.414	-0.375	
$\rho' g a$	0.087	0.171	0.250	0.353	0.433	0.470	0.493	0.500	

Table Vd.

Magnitudes of the respective terms related to  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$  of  $\widehat{r\theta}_{r=a}$  ( $=\widehat{r\theta}'_{r=a}$ ) when  $\frac{\mu'}{\mu} = \infty$ .

$\theta$	0°	10°	20°	30°	45°	60°	70°	80°	90°
$\rho g \xi$	0	0.256	0.482	0.650	0.750	0.650	0.482	0.256	0
$\rho g a$	0	-0.143	-0.239	-0.250	-0.088	0.216	0.422	0.571	0.625
$\rho' g a$	0	-0.087	-0.171	-0.250	-0.353	-0.433	-0.470	-0.492	-0.500
$\theta$	100°	110°	120°	135°	150°	160°	170°	180°	
$\rho g \xi$	-0.256	-0.482	-0.650	-0.750	-0.650	-0.482	-0.256	0	
$\rho g a$	0.571	0.422	0.216	-0.088	-0.250	-0.239	-0.143	0	
$\rho' g a$	-0.492	-0.470	-0.433	-0.353	-0.250	-0.171	-0.087	0	



Table VIc.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$   
of  $\widehat{r\theta}_{\theta=30^\circ}$  and  $\widehat{r\theta}'_{\theta=30^\circ}$  when  $\frac{\mu'}{\mu} = \frac{1}{5}$ .

$\frac{r}{a}$		$\widehat{r\theta}$			$\widehat{r\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta = 30^\circ$	$\rho g \xi$	0.1855	0.1855	0.1855	0.1855	0.3325	0.4177	0.5067	0.5103	0.4789	0.4611
	$\rho g a$	0	-0.0478	-0.0957	-0.0957	-0.2635	-0.3675	-0.5431	-0.7374	-1.1088	-1.4845
	$\rho' g a$	0	-0.0683	-0.1365	-0.1365	-0.1156	-0.1002	-0.0712	-0.0483	-0.0297	-0.0216
$\theta = 45^\circ$	$\rho g \xi$	0.2140	0.214	0.2140	0.2140	0.3841	0.4820	0.5844	0.5892	0.5528	0.5324
	$\rho g a$	0	-0.0299	-0.0597	-0.0597	-0.2025	-0.2950	-0.4660	-0.6640	-1.0270	-1.3900
	$\rho' g a$	0	-0.0965	-0.1930	-0.1930	-0.1636	-0.1415	-0.1007	-0.0682	-0.0420	-0.0306
$\theta = 90^\circ$	$\rho g \xi$	0	0	0	0	0	0	0	0	0	0
	$\rho g a$	0	0.0649	0.1297	0.1297	0.1984	0.2184	0.1944	0.1323	0.0728	0.0494
	$\rho' g a$	0	-0.1363	-0.2725	-0.2725	-0.2311	-0.2005	-0.1424	-0.0966	-0.0593	-0.0434

Table VIIa.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$   
of  $\widehat{rr}_{\theta=0^\circ}$  and  $\widehat{rr}'_{\theta=0^\circ}$  when  $\frac{\mu'}{\mu} = 1$ .

$\frac{r}{a}$		$\widehat{rr}$			$\widehat{rr}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta = 0^\circ$	$\rho g \xi$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
	$\rho g a$	0	0.1667	0.3333	0.3333	0.4684	0.6006	0.9944	1.6043	2.7292	3.7941
	$\rho' g a$	0	0.3333	0.6667	0.6667	0.6316	0.5994	0.5056	0.3957	0.2708	0.2059
$\theta = 63^\circ$	$\rho g \xi$	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061	-0.2061
	$\rho g a$	0	-0.1046	-0.2091	-0.2091	-0.1841	-0.1599	-0.0891	0.0073	0.1578	0.2806
	$\rho' g a$	0	0.1513	0.3027	0.3027	0.2870	0.2721	0.2296	0.1797	0.1232	0.0934

Table VIIb.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$

of  $\widehat{\theta\theta}_{\theta=0^\circ}$  and  $\widehat{\theta\theta}'_{\theta=0^\circ}$  when  $\frac{\mu'}{\mu}=1$ .

$\frac{r}{a}$		$\widehat{\theta\theta}$			$\widehat{\theta\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta=0^\circ$	$\rho g \xi$	0	0	0	0	0	0	0	0	0	0
	$\rho g a$	0	0	0	0	0.0261	0.0424	0.0617	0.0624	0.0492	0.0391
	$\rho' g a$	0	0	0	0	-0.0261	-0.0424	-0.0617	-0.0624	-0.0492	-0.0391
$\theta=60^\circ$	$\rho g \xi$	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75
	$\rho g a$	0	0.1875	0.3750	0.3750	0.4256	0.4711	0.5934	0.7813	1.1496	1.5195
	$\rho' g a$	0	0	0	0	-0.0130	-0.0211	-0.0309	-0.0313	-0.0246	-0.0195
$\theta=90^\circ$	$\rho g \xi$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
	$\rho g a$	0	0	0	0	0	0	0	0	0	0
	$\rho' g a$	0	0	0	0	0	0	0	0	0	0

Table VIIc.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$

of  $\widehat{r\theta}_{\theta=30^\circ}$  and  $\widehat{r\theta}'_{\theta=30^\circ}$  when  $\frac{\mu'}{\mu}=1$ .

$\frac{r}{a}$		$\widehat{r\theta}$			$\widehat{r\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta=30^\circ$	$\rho g \xi$	0.433	0.433	0.433	0.433	0.433	0.433	0.433	0.433	0.433	0.433
	$\rho g a$	0	-0.1042	-0.2083	-0.2083	-0.2742	-0.3323	-0.4824	-0.6979	-1.0942	-1.4779
	$\rho' g a$	0	-0.0834	-0.1667	-0.1668	-0.1383	-0.1177	-0.0801	-0.0521	-0.0308	-0.0221
$\theta=45^\circ$	$\rho g \xi$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	$\rho g a$	0	-0.0589	-0.1178	-0.1178	-0.1935	-0.2575	-0.4165	-0.6330	-1.0180	-1.3850
	$\rho' g a$	0	-0.1178	-0.2357	-0.2357	-0.1960	-0.1668	-0.1133	-0.0737	-0.0436	-0.0314
$\theta=90^\circ$	$\rho g \xi$	0	0	0	0	0	0	0	0	0	0
	$\rho g a$	0	0.1667	0.3333	0.3333	0.2767	0.2356	0.1603	0.1042	0.0616	0.0443
	$\rho' g a$	0	-0.1667	-0.3333	-0.3333	-0.2767	-0.2356	-0.1603	-0.1042	-0.0616	-0.0443



Table VIIIc.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$

of  $\widehat{r\theta}_{\theta=30^\circ}$  and  $\widehat{r\theta}'_{\theta=30^\circ}$  when  $\frac{\mu'}{\mu} = 5$ .  
 $\theta=45^\circ$        $\theta=45^\circ$   
 $\theta=90^\circ$        $\theta=90^\circ$

$\frac{r}{a}$		$\widehat{r\theta}$			$\widehat{r\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta = 30^\circ$	$\rho g \xi$	0.5905	0.5905	0.5905	0.5975	0.4960	0.4440	0.3860	0.3840	0.4040	0.4150
	$\rho g a$	0	-0.1258	-0.2516	-0.2516	-0.2593	-0.2936	-0.4352	-0.6693	-1.0839	-1.4733
	$\rho' g a$	0	-0.1070	-0.2143	-0.2143	-0.1742	-0.1455	-0.0944	-0.0580	-0.0326	-0.0229
$\theta = 45^\circ$	$\rho g \xi$	0.6820	0.6820	0.6820	0.6820	0.5738	0.5117	0.4461	0.4431	0.4664	0.4794
	$\rho g a$	0	-0.0575	-0.1150	-0.1150	-0.1575	-0.2110	-0.3740	-0.6800	-1.0100	-1.3800
	$\rho' g a$	0	-0.1515	-0.3030	-0.3030	-0.2460	-0.2060	-0.1340	-0.0820	-0.0461	-0.0325
$\theta = 90^\circ$	$\rho g \xi$	0	0	0	0	0	0	0	0	0	0
	$\rho g a$	0	0.2599	0.5197	0.5197	0.3697	0.2800	0.1557	0.0934	0.0565	0.0419
	$\rho' g a$	0	-0.2143	-0.4286	-0.4286	-0.3434	-0.2910	-0.1888	-0.1160	-0.0652	-0.0458

Table IXa.

Magnitudes of the respective terms of  $\rho g \xi$ ,  $\rho g a$  and  $\rho' g a$

of  $\widehat{rr}_{\theta=0^\circ}$  and  $\widehat{rr}'_{\theta=0^\circ}$  when  $\frac{\mu'}{\mu} = \infty$ .  
 $\theta=63^\circ$        $\theta=63^\circ$

$\frac{r}{a}$		$\widehat{rr}$			$\widehat{rr}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta = 0^\circ$	$\rho g \xi$	-1.5000	-1.5000	-1.5000	-1.5000	-1.5195	-1.5060	-1.4062	-1.2656	-1.1295	-1.0753
	$\rho g a$	0	0.3125	0.6250	0.6250	0.7520	0.8570	1.1632	1.6878	2.7563	3.8060
	$\rho' g a$	0	0.25	0.5	0.5	0.5063	0.5027	0.4563	0.3748	0.2647	0.2033
$\theta = 63^\circ$	$\rho g \xi$	-0.3092	-0.3092	-0.3092	-0.3092	-0.2286	-0.1843	-0.1429	-0.1492	-0.1740	-0.1867
	$\rho g a$	0	-0.1284	-0.2569	-0.2569	-0.2831	-0.2738	-0.1845	-0.0449	0.1398	0.2726
	$\rho' g a$	0	0.1135	0.2270	0.2270	0.2306	0.2285	0.2072	0.1703	0.1204	0.0923



Table IXb.

Magnitudes of the respective terms of  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{\theta\theta}_{\theta=0^\circ}$  and  $\widehat{\theta\theta}_{\theta=0^\circ}$  when  $\frac{\mu'}{\mu} = \infty$ .  
 $\theta=60^\circ$   $\theta=60^\circ$   
 $\theta=90^\circ$   $\theta=90^\circ$

$\frac{r}{a}$		$\widehat{\theta\theta}$			$\widehat{\theta\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta=0^\circ$	$\rho g\xi$	0	0	0	-0.5000	-0.3065	-0.1892	-0.0368	0.0156	0.0185	0.0127
	$\rho ga$	0	0.1875	0.3750	0.2090	0.1176	0.0755	0.0410	0.0416	0.0405	0.0350
	$\rho'ga$	0	-0.2500	-0.5000	0.1666	0.0993	0.0542	-0.0123	-0.0416	-0.0431	0.0365
$\theta=60^\circ$	$\rho g\xi$	-1.1250	-1.1250	-1.1250	-0.1250	-0.2870	-0.3947	-0.5651	-0.6641	-0.7176	-0.7329
	$\rho ga$	0	0.3750	0.7500	-0.0833	0.1459	0.2933	0.5399	0.7708	1.1491	1.5197
	$\rho'ga$	0	-0.1250	-0.2500	0.0834	0.0496	0.0271	-0.0063	-0.0208	-0.0215	-0.0182
$\theta=90^\circ$	$\rho g\xi$	-1.5000	-1.5000	-1.5000	0	-0.2805	-0.4632	-0.7412	-0.8906	-0.9630	-0.9814
	$\rho ga$	0	0	0	0	0	0	0	0	0	0
	$\rho'ga$	0	0	0	0	0	0	0	0	0	0

Table IXc.

Magnitudes of the respective terms of  $\rho g\xi$ ,  $\rho ga$  and  $\rho'ga$   
of  $\widehat{r\theta}_{\theta=30^\circ}$  and  $\widehat{r\theta}'_{\theta=30^\circ}$  when  $\frac{\mu'}{\mu} = \infty$ .  
 $\theta=45^\circ$   $\theta=45^\circ$   
 $\theta=90^\circ$   $\theta=90^\circ$

$\frac{r}{a}$		$\widehat{r\theta}$			$\widehat{r\theta}'$						
		0	0.5	1.0	1.0	1.1	1.2	1.5	2.0	3.0	4.0
$\theta=30^\circ$	$\rho g\xi$	0.6495	0.6495	0.6495	0.6495	0.5195	0.4460	0.3690	0.3650	0.3930	0.4080
	$\rho ga$	0	-0.1250	-0.2500	-0.2500	-0.2408	-0.2686	-0.4125	-0.6563	-1.0794	-1.4712
	$\rho'ga$	0	-0.1250	-0.2500	-0.2500	-0.2010	-0.1661	-0.1049	-0.0625	-0.0339	-0.0235
$\theta=45^\circ$	$\rho g\xi$	0.7500	0.7500	0.7500	0.7500	0.6000	0.5155	0.4258	0.4219	0.4537	0.4717
	$\rho ga$	0	-0.0442	-0.0884	-0.0884	-0.1258	-0.1788	-0.3500	-0.5985	-1.0050	-1.3800
	$\rho'ga$	0	-0.1768	-0.3535	-0.3535	-0.2850	-0.2350	-0.1483	-0.0885	-0.0480	-0.0332
$\theta=90^\circ$	$\rho g\xi$	0	0	0	0	0	0	0	0	0	0
	$\rho ga$	0	0.3125	0.6250	0.6250	0.4311	0.3170	0.1645	0.0938	0.0559	0.0415
	$\rho'ga$	0	-0.2500	-0.5000	-0.5000	-0.4021	-0.3324	-0.2097	-0.1250	-0.0677	-0.0469

25. 重力の働ける半無限弾性体内に存在する  
圓形填充物附近の應力 (II)

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前論文と全く同じ目的で填充物内外の應力釣合を論じてあるが、重力の作用してゐる半無限弾性體は前論文と異ひ plane stress の状態に保たれてゐると考へて計算を運んでみた。Plane strain 問題として取扱つた時の結果と詳しく定性定量的の比較をする爲めに數値計算を行ひその量を表と圖に示めた。