26. *Vertical Crustal Displacement in the Seismic Region of Itô, on the East Coast af the Idu Peninsula.*

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The town of Itô and neighbourhood on the east coast of the Idu peninsula were visited in 1930 by an unusually large number of small earthquakes, the sensible ones of which were reported to have been more than 4321 in number.[1] In order to ascertain whether the earth's crust of this seismic region was deformed or not in connection with these seismic activities, precise levels were run three times in 1930 in the region along the route shown in Fig. 1. These surveys showed that the earth's crust here upheaved in the form of an inverted V since the earlier surveys were made in 1923–1924. At the end of 1930 the largest upheaval amounted to as much as 220 mm., or more, with no sign of abatement. These crustal deformations were already discussed in the writer's earlier papers.[2]

It being now two years since the last survey of 1930, urgent need was felt for running a line of levels once more over the same ground in order to measure the amount of vertical crustal displacement that must have occurred in the interval. Thanks to the gene-

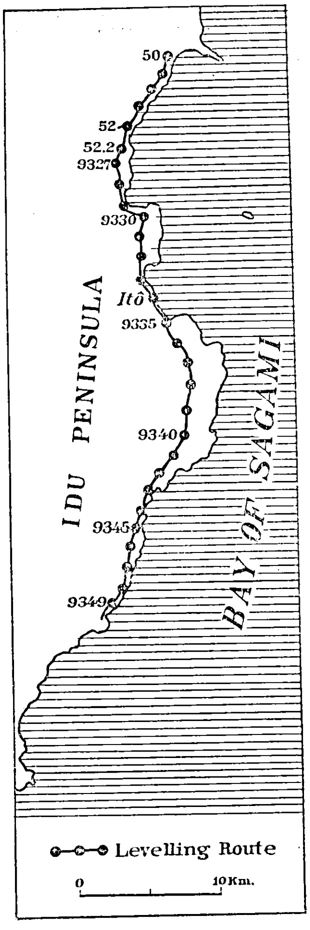

Fig. 1. Levelling Route over the Itô Seismic Region.

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[1] N. NASU, F. KISHINOUYE and T. KODAIRA, *Bull. Earthq. Res. Inst.*, **9** (1931), 22.

[2] C. TSUBOI, *Bull. Earthq. Res. Inst.*, **9** (1931), 151; **10** (1932), 570; *Proc. Imp. Acad.*, **7** (1931), 153; *Jap. J. Astr. Geophys.*, **10** (1933), 93.

rous grant given the writer by the Hattori Hôkô-kwai Foundation, he was enabled to arrange with the Military Land Survey for the work of levelling. The survey, which was begun on Dec. 25, 1932, was finished on March 16, 1933. The changes of height of the bench marks since the earlier survey are given in Table I, together with those during the former intervals.

Table I. Changes of Height of Bench Marks in the Itô Seismic Region.

I....1923-1924.
II....March 15—April 14, 1930.
III....Nov. 9—Dec. 3, 1930.
IV....Dec. 19, 1930—Jan. 3, 1931.
V....Dec. 25, 1932—March 16, 1933.

B.M.	II—I	III—II	IV—III	IV—I	V—IV	B. M.	II—I	III—II	IV—III	IV—I	V—IV
50				− 4·3	− 6·3	9336	+93·8	+118·1	+ 6·8		+127·2
50.1				+ 6·3	− 6·7	9337	+96·9	+122·1	+ 4·3		+116·3
51				+15·4	− 9·5	9338	+74·8	+ 88·1	− 2·5		+105·8
51.1				+22·5	− 15·4	9339	+47·5	+ 56·1	− 0·7		+ 93·4
52				+23·1	− 17·1	9340	+21·5	+ 31·8	− 4·5		+ 77·5
52.2				+ 3·7	− 13·8	9341	+ 5·7	+ 6·0	− 8·0		+ 63·9
9327				−11·0	− 13·4	9342	− 7·3	− 11·5	− 8·0		+ 50·6
9328	mm ± 0·0	± 0·0	± 0·0	± 0·0	± 0·0	9343	−11·8	− 17·8	−10·1		+ 39·4
.9329	− 3·6	+ 6·3	−21·6		+ 19·6	9344	− 26·5	− 21·4	−14·5		+ 33·7
9330	− 4·9	+ 5·6	+ 9·1		+ 27·2	9345				−38·8	+ 22·6
9331	−10·5	+ 21·5	+17·4		+ 61·6	9346				−44·3	+ 19·4
9332	+ 1·7	+ 35·6	+26·9		+ 96·7	9347				−52·1	+ 17·2
9333	+10·6	+ 59·4	+27·2		+123·4	9348				−51·8	+ 11·2
9334	+33·9	+ 81·0	+25·0		+137·0	9349				−52·2	+ 10·3
9335	+72·6	+102·3	+13·6		+135·8						

The values in Table I are graphically shown in Fig. 2. It will be seen from the figure that although the seismicity in the Itô region, in the two years that have elapsed since the last survey of 1930, has been quite inactive compared with that in 1930, the crustal deformation still goes on as before. It should be noted therefore that in the present case there is no apparent connection between the seismic activity and the rate of crustal deformation.

Seeing that we have now the results of four series of precise levelling, all executed in the course of three years, it is possible by simple interpolation to find the approximate height of any bench mark in the region at any time during the interval. In making this calculation, however, the following considerations should be borne in mind.

First of all, it must be emphasised that what we are measuring by means of precise levelling is the difference in the heights of two consecutive bench marks and not their individual heights. Consequently the levelling is nothing but an aid to determine the form of the curve of a land profile by integrating the differential equation

$$\frac{dh}{dx}=f(x) \quad \ldots(1)$$

into the form

$$h=\int_0^x f(x)\,dx,$$

where h is the relative height of a bench mark at $x=x$ with respect to the height of the reference bench mark at $x=0$. If $\frac{dh}{dx}$ is independent of time t as is generally assumed, the above method of integration is quite rigid. Strictly speaking however it is a function of t and instead of the differential equation (1), we should have

$$\frac{\partial h}{\partial x}=F(x,t). \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

It is not possible to integrate this equation from $x=0$ to $x=x$ unless it is known how t is involved in $F(x,t)$, but this is beyond the scope of a levelling survey. What is observed as the curve of land profile by the ordinary method is, therefore, in some cases only an apparent one which may be sensibly different from the true profile at a parti-

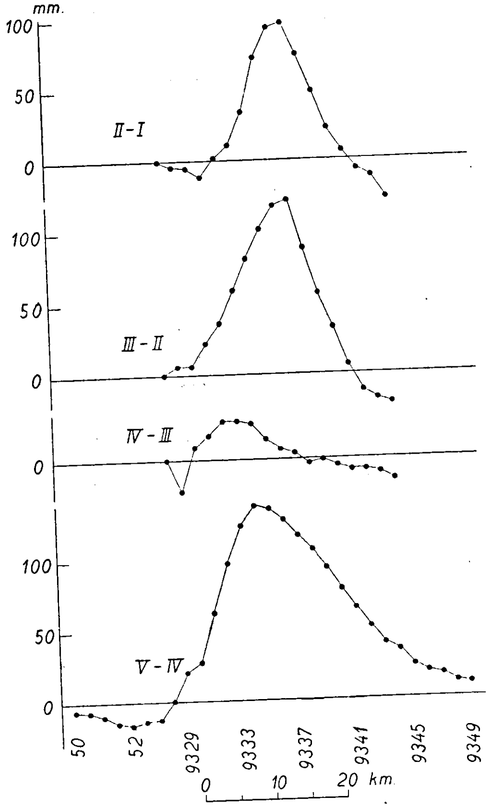

Fig. 2. Changes of height of bench marks in the Itô seismic region.

cular time.

By the following examples, it will be seen how, in some cases, false results are obtained if we neglect the change of $\frac{\partial h}{\partial x}$ with time. Suppose first that a straight line of length S is levelled, one end of which is fixed and the other is uniformly upheaving according to the relation

$$y=at. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If T is the time required for the levelling to cover the whole length of S, we have

$$\frac{x}{S}=\frac{t}{T}.$$

The differential equation of the profile to be determined is therefore

$$\frac{dh}{dx}=\frac{at}{S}=\frac{aT}{S^2}x,$$

and integrating this equation with respect to x, we have

$$h=\frac{1}{2}\frac{aT}{S^2}x^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The line S will appear, therefore, as if it were parabolic instead of straight. At $t=T$, $x=S$, we have according to (4)

$$y=\frac{1}{2}aT,$$

but the true y, according to (3), is $y=aT$, which is twice as large as the apparent y.

Suppose next that a line in levelled, one end of which is fixed and the other is subjected to a motion

$$y=\sin\frac{\pi}{T}t. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Then we have as before

$$\frac{dh}{dx}=\frac{1}{S}\sin\frac{\pi}{S}x,$$

and integrating this equation with respect to x, we have

$$h=\frac{1}{\pi}\left(1-\cos\frac{\pi}{S}x\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The line will appear as if it were of the form shown in Fig. 3 instead of straight. At $t=T$, $x=S$, we have according to (6)

$$y=\frac{2}{\pi}$$

but the true y is 0 according to (5).

For the purpose of investigating the vertical crustal displacement, we calculate the difference of two series of old and new levellings along the same route. Our next problem then is to examine how the difference is affected if we neglect the change of $\frac{\partial h}{\partial x}$ with time during each of the surveys. Suppose a straight line is levelled, one end of which is fixed and the other is subjected to a motion

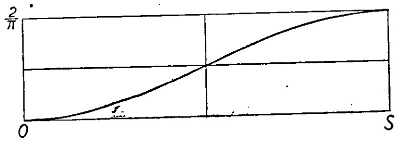

Fig. 3.

$$y=at$$

during the first levelling, and then to

$$y=a(T-t)$$

during the second, which is supposed to have begun the moment the first was finished. Then, for the former survey, integrating the equation

$$\frac{dh}{dx}=\frac{aT}{S^2}x,$$

we have
$$h=\frac{1}{2}\frac{aT}{S^2}x^2,$$

while for the latter, integrating the equation

$$\frac{dh}{dx}=\frac{a(T-t)}{S}=\frac{aT}{S}-\frac{aT}{S^2}x,$$

we have
$$h=\frac{aT}{S}x-\frac{1}{2}\frac{aT}{S^2}x^2.$$

The difference of these two curves is

$$\Delta h=\frac{aT}{S}x-\frac{aT}{S^2}x^2,$$

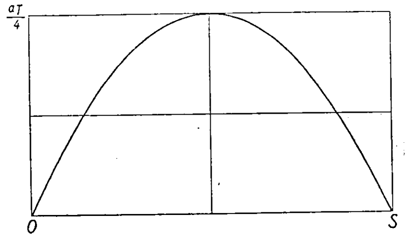

Fig. 4.

the form of which is shown in Fig. 4. Although the true crustal displacement is merely an alternate tilting, the apparent one is an upheaval with the form of an inverted U with its maximum $\frac{aT}{4}$.

Suppose in the second example, a line is levelled, one end of which is fixed and the other is subject to a motion

$$y=\sin\frac{\pi}{T}t$$

during the former survey and to a motion

$$y=-\sin\frac{\pi}{T}t$$

during the latter. Then, for the former survey, integrating the equation

$$\frac{dh}{dx}=\frac{1}{S}\sin\frac{\pi}{T}x,$$

we have

$$h=\frac{1}{\pi}\left(1-\cos\frac{\pi}{S}x\right),\ \cdots\cdot$$

while for the latter, integrating the equation

$$\frac{dh}{dx}=-\frac{1}{S}\sin\frac{\pi}{S}x,$$

we have

$$h=-\frac{1}{\pi}\left(1-\cos\frac{\pi}{S}x\right).$$

The difference of these two curves is

$$\Delta h=-\frac{2}{\pi}\left(1-\cos\frac{\pi}{S}x\right),$$

the form of which is shown is Fig. 5.

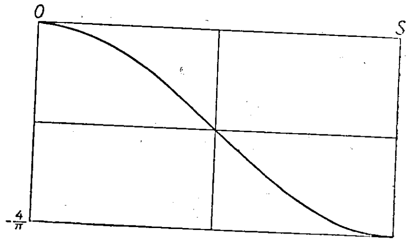

Fig. 5.

As is apparent in the preceding examples, we are in some cases led to false result if we do not take the change of $\frac{\partial h}{\partial x}$ with time into consideration. For the purpose of eliminating these effects, the following procedure is necessary.

Suppose we are making a levelling survey along a route on which lie bench marks $b_1, b_2, \ldots . b_n$, and we find by the survey that the height difference of b_p and b_{p+1} at $t_{p,p+1}$ is $h_{p.\,p+1}(t_{p,p+1})$. From these data alone, we cannot obtain the value of $h_{1,p}(T)$ because $t_{1,2}, t_{2,3} \ldots . t_{n-1,n}$ are all different. By expanding $h_{p,p+1}(t)$ into Taylor's series, we have,

$$h_{1,2}(t)=h_{1,2}(t_{1,2})+(t-t_{1,2})\frac{dh_{1,2}(t_{1,2})}{dt}+\ldots.$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$h_{p,p+1}(t)=h_{p,p+1}(t_{p,p+1})+(t-t_{p,p+1})\frac{dh_{p,p+1}(t_{p,p+1})}{dt}+\cdots\cdot$$

....

Therefore, we have

$$h_{1,n}(t)=\sum_{p=1}^{n-1}h_{p,p+1}(t_{p,p+1})+\sum_{p=1}^{n-1}(t-t_{p,p+1})\frac{dh_{p,p+1}(t_{p,p+1})}{dt}+\cdots\cdot\cdot$$

If by another series of levelling, we find $h_{1,2}(t'_{1,2})$, $h_{2,3}(t'_{2,3})$, $\ldots\,.h_{n-1,n}(t'_{n-1,n})$, then in the first approximation, we have

$$h_{1,2}(t)=h_{1,2}(t_{1,2})+\frac{t-t_{1,2}}{t'_{1,2}-t_{1,2}}\{h_{1,2}(t'_{1,2})-h_{1,2}(t_{1,2})\}$$

....

$$h_{p,p+1}(t)=h_{p,p+1}(t_{p,p+1})+\frac{t-t_{p,p+1}}{t'_{p,p+1}-t_{p,p+1}}\{h_{p,p+1}(t'_{p,p+1})-h_{p,p+1}(t_{p,p+1})\}$$

....

Therefore we have,

$$h_{1,n}(t)=\sum_{p=1}^{n-1}h_{p,p+1}(t_{p,p+1})+\sum_{p=1}^{n-1}\frac{t-t_{p,p+1}}{t'_{p,p+1}-t_{p,p+1}}\{h_{p,p+1}(t'_{p,p+1})-h_{p,p+1}(t_{p\ p+1})\}.$$

These are the expressions that give the heights of the bench marks at any particular time. The effect of the second term, which is generally assumed negligible, becomes sensible if $(t-t_{p,p+1})$ is comparable in length with $(t'_{p,p+1}-t_{p,p+1})$, and if $h_{p,p+1}(t_{p,p+1})$ and $h_{p,p+1}(t'_{p,p+1})$ are different to some extent.

This is found to be the case with the levelling over the Itô region. The second levelling in 1930 was begun on Nov. 9 and finished on Dec. 3, while the third was begun on Dec. 19 and finished on Jan. 15 of the year following. The interval between the two surveys is only 16 days—shorter than the time required for the surveys themselves. In calculating the successive heights of the bench marks in the region, therefore, the necessary reductions should be made for the effect of the second term in the above expressions.

From the values given in Table I, we know that the changes of height difference of two consecutive bench marks between two consecutive levellings were as follows;

Table II. Changes of Height Differences of Consecutive Bench Marks in the Itô Seismic Region.

B. M.	II—I	III—II	IV—III	V—IV
9328	mm			
9329	− 3·6	+ 6.3	−21·6	+19·6
9330	− 1·3	− 0·7	+30·7	+ 7·6
8331	− 5·6	+15·9	+ 8·3	+34·4
9332	+12·2	+14·1	+ 9·5	+35·1
9333	+ 8·9	+23·8	+ 0·3	+26·7
9334	+23·3	+21·6	− 2·2	+13.6
9335	+38·7	+21.3	−11·4	− 1·2
9336	+21·2	+15·8	− 6·8	− 8·6
9337	+ 3·1	+ 4·0	− 2·5	−10·9
9338	−22·1	−14·0	− 6·8	−10·5
9339	−27·3	−32·0	+ 1·8	−12·4
9340	−26·0	−24·3	− 3·8	−15·9
9341	−15·8	−25·8	− 3·5	−13·6
9342	−13·0	−17·5	0	−13·3
9343	− 4·5	− 6·3	− 2·1	−11·2
9344	−14·7	− 3·6	− 4·4	− 5·7

The successive height differences of two consecutive bench marks are therefore as follows;

Table III. Successive Height Differences of Consecutive Bench Makrs in the Itô Seismic Region.

B. M.	II—I	III—I	IV—I	V—I
9328	mm			
9329	− 3·6 (93)	+ 2·7 (314)	−18·9 (354)	+ 0·7 (1116)
9330	− 1·3 (93)	− 2·0 (315)	+28.7 (354)	+36·3 (1120)
9331	− 5·6 (81)	+10·3 (318)	+18·6 (357)	+53·0 (1120)
9332	+12·2 (76)	+26·3 (318)	+35·8 (357)	+70·9 (1120)
9333	+ 8·9 (76)	+32·7 (322)	+33·0 (361)	+59·7 (1115)
9334	+23·3 (78)	+44·9 (322)	+42·7 (361)	+56·3 (1115)
9335	+38·7 (82)	+60·0 (322)	+48·6 (361)	+47·4 (1114)
9336	+21·2 (83)	+37·0 (325)	+30·2 (365)	+21·6 (1113)
9337	+ 3·1 (83)	+ 7·1 (325)	+ 4·6 (364)	− 6·3 (1112)
9338	−22·1 (86)	−36·1 (326)	−42·9 (365)	−53·4 (1111)
9339	−27·3 (88)	−59·3 (330)	−57·5 (374)	−69·9 (1110)
9340	−26·0 (89)	−50·3 (330)	−54·1 (374)	−70·0 (1109)
9341	−15.8 (97)	−41·6 (331)	−45·1 (375)	−58·7 (1090)
9342	−13·0 (97)	−30·5 (335)	−30·5 (374)	−43·8 (1094)
9343	− 4·5 (100)	−10·8 (335)	−12·9 (374)	−24·1 (1093)
9344	−14·7 (104)	−18·3 (335)	−22·7 (376)	−28·4 (1097)

The numbers in brackets are the number of days that elapsed since Jan. 1, 1930 to the day on which the survey was made.

From these values we can, by linear interpolation, find the height differences of two consecutive bench marks at any particular time, and by summing them up into $\sum_{p=1}^{p} h_{p-1,p}(T)$, we obtain $h_{1,p}(T)$, which is the relative height of the bench mark b_p at time T with respect to the referred bench mark. In Table IV are given the relative heights of the bench marks on specified days with respect to the reference bench mark B. M. 9328. The successive land profiles which were obtained in this manner are shown in Fig. 6.

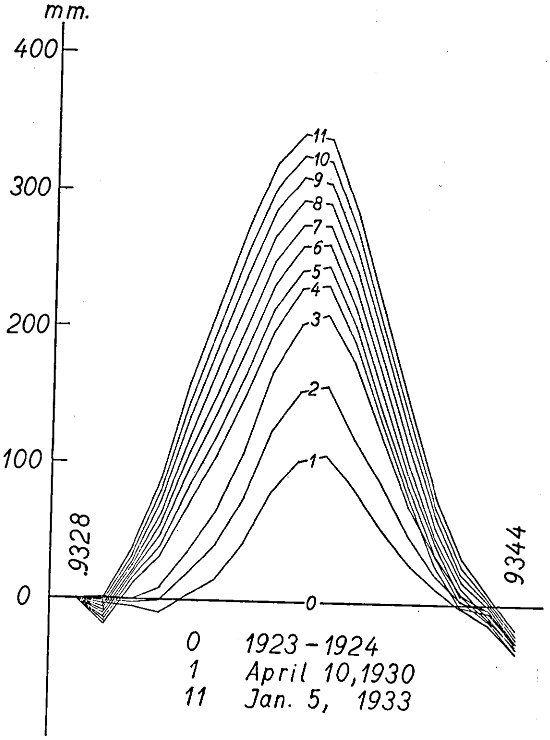

Fig. 6. Successive Heights of Bench Marks in the Itô Seismic Region at Every 100 Days Beginning with April 10, 1930.

The mean vertical velocities of the bench marks can be obtained from their successive heights given in Table IV and are given in Table V.

If the velocities are set down in parallel columns, arranged side

Table IV. Heights of Bench Marks in the Itô Seismic Region as Referred to B. M. 9328.

B. M.	100	200	300	350	400	500	600	700	800	900	1000	1100
9328	mm 0	0	0	0	0	0	0	0	0	0	0	0
9329	− 3·4	− 0·6	+ 2·2	− 16·6	− 17·7	− 15·1	− 12·5	− 9·9	− 7·4	− 4·8	− 2·3	+ 0.3
9330	− 4·8	− 2·3	+ 0·2	+ 8.9	+ 11·4	+ 15·0	+ 18·6	+ 22·3	+ 25·7	+ 29·3	+ 32·8	+ 36·4
9331	− 9·2	+ 0·1	+ 9·3	+ 22·6	+ 31·9	+ 40·0	+ 48·1	+ 56·3	+ 64·2	+ 72·3	+ 80·4	+ 88·5
9332	+ 4·3	+ 19·5	+ 34·5	+ 59·9	+ 69·7	+ 82·4	+ 95·1	+106·8	+120·3	+133·1	+145·8	+158·5
9333	+ 15·5	+ 40·4	+ 65·0	+ 92·8	+106·1	+120·3	+136·6	+151·8	+168·9	+185·2	+201·4	+215·6
9334	+ 40·8	+ 75·0	+108·8	+136·4	+149·5	+165·5	+183·6	+200·6	+219·5	+237·6	+255.6	+273·6
9335	+ 81·1	+124·6	+167·7	+188·5	+198·0	+213·9	+231·9	+248·8	+267·5	+285·5	+303·3	+321·1
9336	+103·3	+153·4	+203·0	+221·3	+227·8	+242·6	+259·4	+275·2	+292·7	+309·6	+326·2	+343·1
9337	+106·7	+158·4	+209·7	+226·8	+231·9	+245·2	+260·6	+274·9	+291·0	+306·4	+321·6	+356·8
9338	+ 83·8	+129·7	+175·2	+186·6	+188·5	+200·5	+214·5	+227·5	+242·2	+256·2	+270·1	+284·0
9339	+ 54·8	+ 87·6	+119·9	+128·1	+130 5	+140·9	+153·2	+164·5	+177·6	+189·6	+202·1	+214·3
9340	+ 28·5	+ 50·4	+ 72·6	+ 76·1	+ 75·9	+ 84·2	+ 94·3	+103·4	+114·3	+124·5	+134·5	+144·5
9341	+ 12·3	+ 23·3	+ 34·4	+ 33·0	+ 30·0	+ 35·1	+ 41·9	+ 47·7	+ 55·3	+ 62·2	+ 68·8	+ 75·4
9342	− 1·0	+ 2·8	+ 6·6	+ 2·5	− 1·0	+ 2·3	+ 7·3	+ 11·2	+ 17·0	+ 22·0	+ 26·8	+ 31·6
9343	− 5·5	− 4·4	− 3·3	− 9·1	− 14·3	− 12·5	− 9·0	− 6·6	− 2·4	+ 1·1	+ 4·4	+ 7·7
9344	− 20·1	− 20·6	− 21·1	− 29·0	− 37·2	− 36·2	− 33·4	− 31·8	− 28·4	− 25·6	− 23·2	− 20.8

Table V. Mean Daily Vertical Velocities of Bench Marks in the Itô Seismic Region in 0·001 mm.

B. M.	100	250	325	350	375	450
9328	0	0	0	0	0	0
9329	28	28	−376	−199	− 22	26
9330	25	25	+174	112	+ 50	36
9331	93	92	266	226	186	81
9332	152	150	508	352	196	127
9333	249	246	556	411	266	142
9334	342	338	552	407	262	160
9335	435	431	416	303	190	159
9336	501	496	366	248	130	148
9337	517	513	342	222	102	133
9338	459	455	228	133	38	120
9339	328	323	164	106	48	104
9340	219	222	70	33	− 4	83
9341	110	112	− 30	− 45	− 60	51
9342	38	38	− 82	− 76	− 70	33
9343	11	11	−116	−110	−104	18
9344	− 5	− 5	−158	−161	−164	10

uniform afterwards.

by side in the order of time, and the same values in the different columns are joined by contour lines, we get a system of curves as shown in Fig. 7. From this figure, it is apparent that the point of largest upheaval velocity has not remained always in the same place, but has migrated northward with a monthly velocity of 1 km. until the occurrence of the North Idu destructive earthquake of Nov. 26, 1930. After that, the point suddenly moved down southward to a new position where it remained up to the time of the last survey. After the Idu earthquake, the upheaval velocity reduced to one-half of the former values. These facts may be regarded as evidences that the equilibrium of the earth's crust at a certain place is sensibly affected even by an earthquake whose epicentre may be situated some distance away from it.

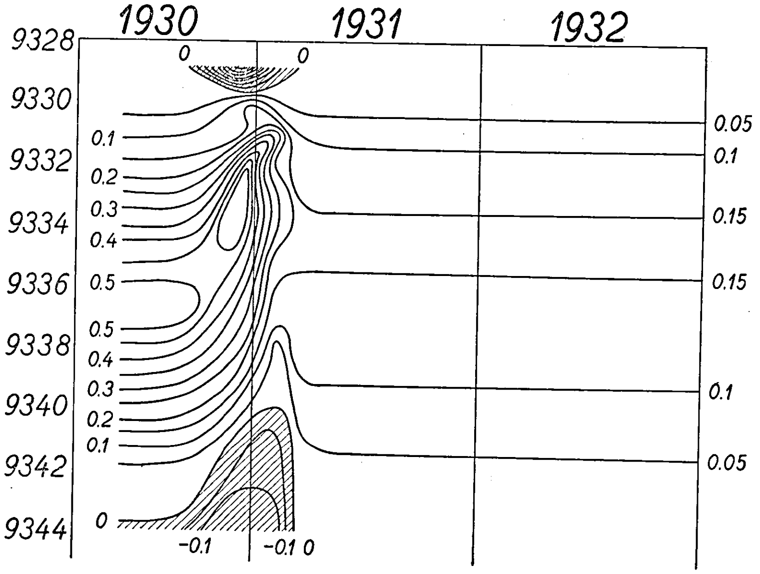

Fig. 7. Daily Vertical Velocities of Bench Marks in the Itô Seismic Region in mm.

The writer wishes to express his sincere thanks to Professor Torahiko Terada for his interest in this study and for his kindness in recommending the writer to the Hattori Hôkô-kwai for the grant. He wishes also to express his gratitude to the council of the Hattori Hôkô-kwai for the generous aid by means of which the survey was made possible.

26. 伊豆伊東地方に於ける地殼の垂直移動

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昭和五年の伊東地震群に關する研究の一つとして，同地方に水準測量が三回行はれたが，其の結果に就いては前論文に於いて二三論じておいた。昭和七年の十二月二十五日から翌八年三月十六日にかけて更に新測量が行はれたので，此の論文は其の結果について若干の考察を行つたものである。昭和六年七年を通じて伊東地方の地震は殆んど終熄してゐたのであるが，地殼の變動は尙相當盛んで二年間に最大 137mm. の隆起をなした。玆に於いて地震活動と地殼變動の速さとが一見無關係の樣に見えるのは注意すべき事である。次に此の樣に變動の盛んな地方では，測量をしてゐる間に起つてゐる變動をも考慮に入れなければ正しい垂直移動の値が得られない事を指摘し，伊東の例に就いて之を應用して，百日每の地表の形を算出した。更に其の結果から平均の一日の垂直速度を求めた。速度の最大な點は一定でなく，昭和五年中は伊東附近から始つて徐々に北上して行つたが伊豆地震後に突然南下した。

最後に此の測量は服部報公會の補助金によるものなる事を記して，深甚の謝意を表する。