52. *A Note on the Tilting Motion of the Earth's Crust observed at Zinsen (Chemulpo).*

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1. Introduction.

What is written in this paper is the revision of Sekiguchi's pioneer observation[1], but not the observation made by the present author himself. In 1916, Sekiguchi, while he was in the Meteorological Observatory of Zinsen, noticed the existence of the tilting motion of the earth's crust from the periodic variation of line-intervals of the records obtained by the Omori horizontal pendulum seismograph, installed in the observatory. This periodic tilting motion of the earth's crust was naturally attributed to the load of tides in the port of Zinsen and its neighbouring seas, where the tidal range is as large as 10 metres.

Sekiguchi calculated then the rigidity of the earth's crust, by comparing the observed amount of tilt with that theoretically deduced. In his treatment, however, there are assumptions, especially in tidal materials, which seem to the present author to be rather rough.

The present author felt much anxious about that the results obtained under such assumptions will deviate much from the truth. In these anxieties, the author made a revision of Sekiguchi's observation, making use of the co-tidal and co-range charts of the Yellow Sea and its adjacent bays, which he has compiled in his previous research,[2] and also of a method of analysis different from that used by Sekiguchi. The author has thus obtained something new, which will be described in the following paragraphs.

2. Sekiguchi's observation and the instrument used.

The instrument used in Sekiguchi's observation is the Omori horizontal pendulum, installed in the cellar of the Zinsen Meteorological

1) R. SEKIGUCHI, *Mem. Imp. Marine Obs.*, **1** (1922), 1.
2) *Earthq. Res. Inst.*, **10** (1932), 531.

Observatory (Lat. 37° 28′ 30″ N ; Long. 126° 37′ 36″ E) in the azimuth as to record EW component of earthquake vibrations. The magnification coefficient of the seismograph was 150, the period of the free oscillation of the pendulum being 28 sec. The sensibility of the seismograph for the tilt of the ground in EW direction is given therefore by

$$V=\frac{150\,g\,T_0^2\sin 1''}{4\pi^2}\ \text{cm./sec. of arc},$$

where g and T_0 are the acceleration of gravity and the period of the free oscillation of the pendulum respectively. Giving numerical values to g, T_0 and π in the above expression, we have

$$V=14{\cdot}1.$$

The recording drum of the seismograph revolved once in 30 minuits and it slided towards east with the revolution. The recording pen of the seismograph described therefore a series of straight lines, the interval of these lines varying with the velocity of tilting of the ground.

The air temperature in the cellar of the observatory was very steady during the period of the observation, diurnal variations being scarcely recognized. The humidity of the air was constantly at the point of saturation.

The location of the observatory is to be seen in Fig. 5, below.

The period of the observation extended from Jan. 1 to Mar. 27, 1916, that is, over three complete lunations. It was observed that there occurre dusually two maximum and two minimum line-intervals of the seismogram in a day, synchronously to the ebb and flood of tides in the port of Zinsen.

3. Reduction of the observed data.

Supposing for a while that the diurnal inequality of tilt is negligible, let the tilt towards *west* of the ground be expressed by

$$\psi=\psi_m\cos(2T+2h-2s-\varepsilon_m)+\psi_s\cos(2T-\varepsilon_s),$$

where ψ_m is the amplitude of M_2 component-tilt, ψ_s that of S_2 component-tilt, T the hour angle of the mean sun, h the mean longitude of the sun, s the mean longitude of the moon and ε_m and ε_s the phase lags. If we put again

$$2T+2h-2s=\sigma_m t,$$

where σ_m is the angular velocity of M_2 component, then t is approximately equal to the mean solar time reckoned from the upper culmination of the moon, and we have

$$\psi=\psi_m \cos(\sigma_m t-\varepsilon_m)+\psi_s \cos\{(\sigma_m t-\varepsilon_m)+(\zeta-a)\},$$

in which $\zeta=2(s-h)$ and $a=\varepsilon_s-\varepsilon_m$.
This expression can be written

$$\psi=R\cos(\sigma_m t-\varepsilon_m+\theta),$$

if

$$R^2=\psi_m^2+\psi_s^2+2\psi_m\psi_s\cos(\zeta-a),$$

and

$$\theta=\tan^{-1}\frac{\psi_s\sin(\zeta-a)}{\psi_m+\psi_s\cos(\zeta-a)}.$$

On the other hand the line-interval of the seismogram is given by

$$W=I-V\frac{d\psi}{dt}\Delta t,$$

where I is the pitch of the screw attached to the recording drum of the seismograph, Δt the time of one complete revolution of the recording drum.
We have therefore

$$W_{\text{max.}}-W_{\text{min.}}=\frac{2\pi}{180}\sigma_m V\Delta t\cdot R.$$

The times of occurrence of the maximum and minimum widenings of the line-interval that follow the upper culmination of the moon are given respectively by

$$t_{\text{max.}}=\frac{\varepsilon_m-\theta+90^\circ}{\sigma_m},$$

$$t_{\text{min.}}=\frac{\varepsilon_m-\theta+270^\circ}{\sigma_m}.$$

The time of occurrence of the maximum *eastward* tilt t_1 is $\frac{\varepsilon_m-\theta+180^\circ}{\sigma_m}$; hence we have

$$t_1=\frac{t_{\text{max.}}+t_{\text{min.}}}{2}.$$

In Sekiguchi's paper, values of $W_{\text{max.}}-W_{\text{min.}}$ and t_1 are given for each day. We will cite them in the next, together with the time, referred to the meridan 135° E, of the moon's culmination over the meridian of Zinsen.

Table I.

Table for $W_{max.}-W_{min.}$, t_1 and the time of moon's transit over the meridian of Zinsen, the time used being referred to the meridian 135° E.

Date	☾'s Transit		t_1		$W_{max.}-W_{min.}$	Date	☾'s Transit		t_1		$W_{max.}-W_{min.}$
	h	m	h	m	mm		h	m	h	m	mm
Dec. 31	19	54	12	06	3·5	Jan. 15	20	51	12	08	1·5
Jan. 1	8	20	12	09	2·7	16	9	16	12	59	2·0
1	20	39	11	41	3·2	16	21	41	13	19	2·5
2	9	19	12	41	3·2	17	10	06	13	13	1·5
2	21	50	12	39	4·4	17	22	32	12	28	4·1
3	10	22	11	52	3·5	18	10	57	12	48	4·3
3	22	55	11	34	5·0	18	23	22	11	53	5·8
4	11	29	12	00	4·4	19	11	47	12	43	5·2
4	—		—		—	19	—		—		—
5	0	02	11	26	4·6	20	0	12	11	48	7·0
5	12	36	12	10	6·1	20	12	35	13	39	5·0
6	1	07	11	37	6·8	21	0	59	12	01	5·0
6	13	39	11	06	5·6	21	13	22	11	53	3·2
7	2	07	10	56	4·0	22	1	45	11	45	1·2
7	14	36	10	38	4·0	22	14	08	11	53	2·0
8	3	02	10	42	4·2	23	2	28	12	32	0·7
8	15	29	10	46	2·5	23	14	49	12	25	1·0
9	3	53	10	51	4.0	24	3	11	13	04	1·2
9	16	17	11	58	3·2	24	15	32	11	58	1·0
10	4	40	10	50	3·6	25	3	54	13	06	0·3
10	17	03	11	27	4·5	25	16	15	12	00	0·5
11	5	25	10	20	2·8	26	4	37	13	08	0·1
11	17	47	11	43	3·0	26	17	00	11	45	0·4
12	6	09	11	21	1·7	27	5	23	13	06	0·3
12	18	31	10	29	2·0	27	17	48	11	42	0·0
13	6	53	9	21	2·8	28	6	13	12	32	1·5
13	19	16	10	14	3·0	28	18	40	11	51	0·1
14	7	39	11	36	1·3	29	7	07	12	38	1·0
14	20	03	12	27	2·0	29	19	36	11	54	0·0
15	8	27	13	19	1·5	30	8	06	11	24	0·6

(to be continued.)

Table I (*continued.*)

Date	☾'s Transit		t_1		$W_{max}-W_{min.}$	Date	☾'s Transit		t_1		$W_{max.}-W_{min.}$
	h	m	h	m	mm		h	m	h	m	mm
Jan. 30	20	35	12	37	1·8	Feb. 16	22	54	12	51	0·4
31	9	09	12	20	1·5	17	11	17	12	27	0·2
31	21	41	12	33	3·7	17	23	40	12	49	0·2
Feb. 1	10	14	12	31	2·8	18	—		—		—
1	22	45	12	13	5·0	18	12	02	12	27	0·5
2	11	17	12	42	4·1	19	0	25	13	05	2·8
2	23	47	12	11	6·5	19	12	47	11	43	3·5
3	—		—		—	20	1	19	11	51	4·6
3	12	17	12	27	4·5	20	13	30	11	44	3·0
4	0	45	12	14	6·0	21	1	52	11	38	4·0
4	13	13	(11	28)	(8·2)	21	14	14	13	16	4·5
5	1	38	11	56	5·0	22	2	36	11	54	5 3
5	14	04	11	26	6·0	22	14	59	11	31	7·0
6	2	28	11	31	7·1	23	3	22	12	C8	4·0
6	14	52	11	38	6·2	23	15	46	11	14	5·0
7	3	15	11	29	3·3	24	4	10	11	35	5·4
7	15	38	12	21	0·5	24	16	36	11	54	5·0
8	4	01	11	29	3·5	25	5	02	11	57	1·6
8	16	24	12	06	5·2	25	17	30	11	29	3·2
9	4	47	10	58	5·0	26	5	59	11	01	2·8
9	17	10	11	35	2·8	26	18	29	11	01	1·8
10	5	33	11	57	2·8	27	6	59	10	46	1·8
10	17	57	13	03	1·0	27	19	30	10	14	2·6
11	6	21	11	09	0·5	28	8	10	11	13	1·7
11	18	45	12	45	0·4	28	20	33	11	42	1·8
12	7	10	10	50	1·0	29	9	03	(12	31)	(2·4)
12	19	35	12	40	0·0	29	21	33	(12	22)	(3·8)
13	8	00	11	45	0·8	Mar. 1	10	03	12	12	2·8
13	20	25	12	04	0·0	1	22	31	11	43	2·2
14	8	50	11	39	1·0	2	10	59	11	46	3·8
14	21	16	12	44	0·3	2	23	25	11	50	4·9
15	9	41	12	18	0·3	3	11	51	11	39	3·8
15	22	06	12	54	0·1	3	—		—		—
16	10	30	12	15	0·3	4	0	15	11	44	4·0

(*to be continued.*)

Table I. (*continued.*)

Date	☾'s Transit	t_1	$W_{max.}-W_{min.}$	Date	☾'s Transit	t_1	$W_{max.}-W_{min.}$
	h m	h m	mm		h m	h m	mm
Mar. 4	12 40	11 35	4·7	Mar. 16	22 19	(12 53)	(1·8)
5	1 03	11 26	4·0	17	10 41	12 43	3·8
5	13 27	11 17	4·6	17	23 03	11 27	4·9
6	1 48	11 09	3·0	18	11 25	12 45	3·5
6	14 14	11 31	3·5	18	23 47	11 43	5·0
7	2 37	11 38	2·8	19	12 09	11 51	4·5
7	15 00	12 15	2·0	19	—	—	—
8	3 23	10 51	2·8	20	0 31	11 28	4·7
8	15 47	12 27	2·0	20	12 54	11 51	4·6
9	4 11	11 03	2·0	21	1 17	11 28	5·0
9	16 36	11 24	2·0	21	13 41	11 18	5·2
10	5 01	(11 02)	(3·6)	22	2 06	11 09	5·2
10	17 26	11 34	1·0	22	14 32	11 28	4·1
11	5 51	10 23	1·0	23	2 58	11 02	4·6
11	18 17	11 43	0·0	23	15 26	10 49	5·2
12	6 42	10 17	2·0	24	3 54	10 36	3·0
12	19 08	10 52	1·3	24	16 23	10 36	2·5
13	7 33	9 57	0·8	25	4 53	11 36	2·9
13	19 58	11 32	1·1	25	17 24	10 35	1·3
14	8 23	9 37	0·8	26	5 55	(11 55)	(1·5)
14	20 47	12 13	2·3	26	18 26	(11 26)	(1·0)
15	9 10	12 50	2·0	27	6 57	11 03	1·0
15	21 34	12 26	2·0	27	19 26	10 33	0·0
16	9 56	12 32	2·1				

The quantities to be computed are ψ_m and ε_m. We will calculate them in the next by making use of the above data and mathematical relations.

We cannot, however, treat at once those data given in Table I. Being as read off from seismograms, they involve, besides variations to be found, many undesired disturbances such as diurnal inequality, phase inequality and meteorological ones. Consequently, we must, at the first step of the computation, eliminate all inequalities other than the phase inequality, by grouping the observed data according to the time of moon's transit. As the period of the observation is short, being only three months, complete elimination cannot be expected, but the greater part of the inequalities may be got rid of, if $W_{max.}-W_{min.}$ and t_1 are arranged according to such a schedule as is given by the Hydrographic Department of the British Admiralty for the computation of non-harmonic tidal constants.

Grouping and averaging of the data written in Table I give the following results.

☾'s Transit	$W_{max.}-W_{min.}$	t_1		Note
h	mm	h	m	
0	4·3	12	04	The time of ☾'s transit is referred to 135° E. meridian.
1	5·2	11	50	
2	4·2	11	36	
3	3·7	11	37	
4	2·9	11	37	
5	2·6	11	29	
6	1·4	11	37	
7	1·4	10	58	
8	1·4	11	40	
9	1·9	12	20	
10	2·5	12	35	
11	3·7	12	12	

The values of $W_{max.}-W_{min.}$ and t_1, obtained here, must now be free from all inequalities other than the phase inequality. $W_{max.}-W_{min.}$ and t_1 are not of course independent but are related to each other. As can be seen from the expressions of R and θ, points that represent the values of $W_{max.}-W_{min.}$ and the corresponding values of $\sigma_m t_1$, in polar co-ordinates, must all fall on a circle, having its centre at $\left(\frac{2\pi}{180}\sigma_m\psi_m V\Delta t,\ \varepsilon_m+180^\circ\right)$ and having for its radius $\frac{2\pi}{180}\sigma_m\psi_s V\Delta t$. These relations can be easily understood by the aid of Fig. 1.

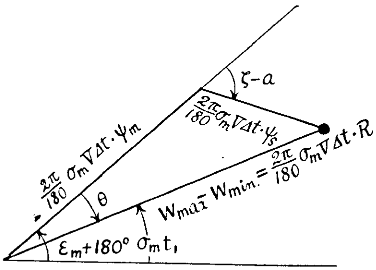

Fig. 1.

We can find, therefore, ψ_m and ε_m from the co-ordinates of the centre of the circle, ψ_s from the magnitude of the radius of the circle, and ε_s from the positions of the points on the circle.

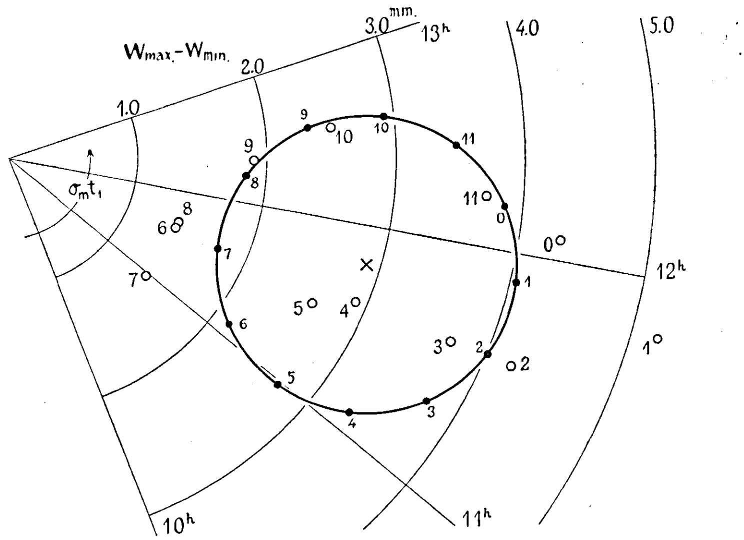

Fig. 2. O....not adjusted ●....adjusted
Numerals represent the time of moon's transit.

Actually, however, the points do not fall always on a circle, but are distributed irregularly, as are shown in Fig. 2 and 3, because of the

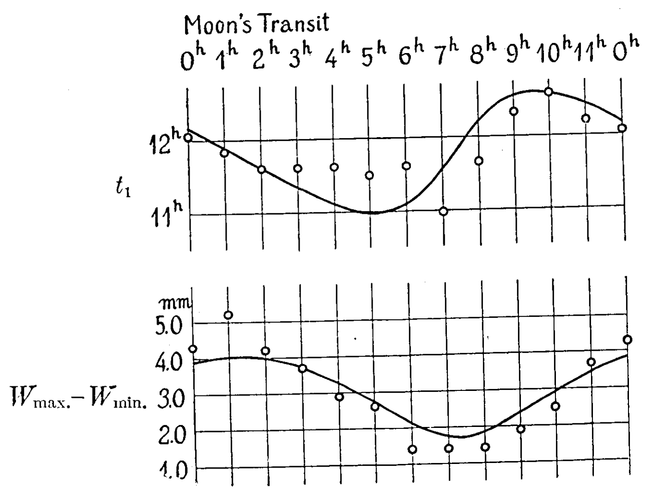

Fig. 3.

disturbances and inequalities not yet completely eliminated. We must therefore determine the circle of minimum errors by the method of least squares and obtain therefrom the values of ψ_m and ε_m.

Putting that

$$(W_{max.}-W_{min.})\cos\sigma_m t_1 = A+H\cos(\zeta-\zeta_0) = A+C\cos\zeta+D\sin\zeta,$$
$$(W_{max.}-W_{min.})\sin\sigma_m t_1 = B-H\sin(\zeta-\zeta_0) = B-C\sin\zeta+D\cos\zeta,$$

where

$$A=\frac{2\pi}{180}\sigma_m V\Delta t\cdot\cos(\varepsilon_m+180°),$$

$$B=\frac{2\pi}{180}\sigma_m V\Delta t\cdot\sin(\varepsilon_m+180°),$$

$$H=\frac{2\pi}{180}\sigma_m V\Delta t\cdot\psi_s,$$

$$\zeta_0=a+\varepsilon_m+180°=\varepsilon_s+180°,$$
$$C=H\cos\zeta_0,$$
$$D=H\sin\zeta_0,$$

we have as the observation equations:

A	B	C	D	
1	0	1·000	0·000	= 4·2,
1	0	0·866	0·500	= 4·9,
1	0	0·500	0·866	= 3·8,
1	0	0·000	1·000	= 3·4,
1	0	−0·500	0·866	= 2·6,
1	0	−0.866	0·500	= 2·3,
1	0	−1·000	0·000	= 1·3,
1	0	−0·866	−0·500	= 1·0,
1	0	−0·500	−0·866	= 1·3,
1	0	0·000	−1·000	= 1·9,
1	0	0·500	−0·866	= 2·5,
1	0	0·866	−0·500	= 3·7,
0	1	0·000	1·000	= −0·8,
0	1	−0·500	0·866	= −1·5,
0	1	−0·866	0·500	= −1·7,
0	1	−1·000	0·000	= −1·5,
0	1	−0·866	−0·500	= −1·2,
0	1	−0·500	−0·866	= −1·2,
0	1	0·000	−1·000	= −0·6,
0	1	0·500	−0·866	= −0·9,
0	1	0·866	−0·500	= −0·5,
0	1	1·000	0·000	= −0·1,
0	1	0·866	0·500	= 0·2,
0	1	0·500	0·866	= −0·4,

from which we obtain

$$A=2{\cdot}740, \quad B=-0{\cdot}850, \quad C=1{\cdot}086 \quad \text{and} \quad D=0{\cdot}430,$$

that is

$$\sqrt{A^2+B^2}=2{\cdot}869, \quad \frac{B}{A}=-0{\cdot}310, \quad \sqrt{C^2+D^2}=1{\cdot}168, \quad \frac{D}{C}=0{\cdot}396.$$

We have therefore as the observed tilt of the ground

$$\psi_m=0{\cdot}''0411, \quad \varepsilon_m=163^\circ\,; \quad \psi_s=0{\cdot}''0167, \quad \varepsilon_s=202^\circ,$$

or

$$\psi_m=0{\cdot}''0411, \quad \zeta_m=180^\circ.$$

The observed tilt is therefore

$$0{\cdot}''0411 \cos 2t \qquad \text{towards east.}$$

4. The tide-generating potential of the moon.

The most important causes that produce the deflection of a horizontal pendulum will be enumerated as, firstly, the direct tide-generating potential of the moon and the sun ; secondly, the earth-tides ; thirdly, the potential variation due to the deformation of the earth caused by the earth-tides ; fourthly ; the gravitational attraction of the oceanic tides ; fifthly and finally, the deformation of the earth's crust due to tidal loading. There are of course many other causes which can also produce the deflection of a horizontal pendulum, but their contributions to the deflection are very small, compared with the causes just referred to.

Each of the causes enumerated above consists of many components of different periods, of which M_2 is the largest. Therefore in this and the following paragraphs, we will confine our attention, for the sake of simplicity, only to M_2 component respectively of the tide-generating potential and of other causes.

As is stated in every text-book of geophysics, the tilt of the earth's surface due to the earth-tides and the change in the direction of the plumb-line which is due both to the direct tide-generating potential and to the deformation of the earth as the result of the earth-tides, are of the same period and phase. They come, therefore, into the observed amount of apparent tilt as total, namely, as $(1-h+k)W$, if we denote the change in the direction of the plumb-line due to the direct tide-generating potential of the moon by W. The quantity $(1-h+k)$ is what is generally called the diminishing factor, and is denoted by D.

W will be calculated easily by the formula

$$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3\left(\frac{1}{2}-\frac{5}{4}e^2\right)\cos^4\frac{I}{2}\cos\lambda\sin 2t \quad \text{towards east,}$$

where M=mass of moon,

E=mass of earth,

a=radius of earth,

c=mean distance of moon,

λ=latitude of the place,

e=eccentricity of moon's orbit,

and I=inclination of moon's orbit to equator.

For the central day of the period of the observation we have

$$I=26\cdot^\circ 89$$

so the calculated value of W is

$$-0\cdot''01221 \sin 2t \quad \text{towards east,}$$

in which t is the local lunar time. For the sake of convenience of the comparison which will be made later, we will here convert t to the lunar time referred to the meridian 135°E. Then the above expression becomes

$$-0\cdot''01221 \sin(2t-17) = -0\cdot''01169 \sin 2t + 0\cdot''00353 \cos 2t.$$

5. Attraction of the oceanic tides.

In the calculation of the deflection of a horizontal pendulum due to the attraction of the oceanic tides in the Yellow sea and its adjacent bays, Sekiguchi divided these seas into three parts, according to their tides. In A section, which is the area east-side of the meridian of 120° E and north-side of the parallel of latitude 37°20′ N, tidal constants were assumed to be the same everywhere, namely, $H_m=9\cdot 6$ ft., $\zeta_m=149°$. In B section, which is the area including the portion lying on the east side of the straight line connecting Amminto ($\varphi=36°35'$N, $\lambda=126°13'$ E) and Coppeki point ($\varphi=38°07'$ N, $\lambda=124°39'$ E) and excluding the section A, tidal constants were assumed to be $H_m=8\cdot 2$ ft., $\zeta_m=149°$ everywhere. As for the section C, which includes the remaining part of the Yellow Sea and the Gulf of Pechili, he assumed that $H_m=3\cdot 5$ ft. everywhere and that ζ_m increases uniformly with the latitude and is independent with the longitude.

Actually, however, the distribution of tides in the Yellow Sea is such that is shown in Fig. 4. This is the co-tidal and co-range chart of the Yellow Sea, as determined from the tidal constants of the ports situated on the coast of the sea under question, with due regards to the depth and other hydrodynamical conditions of the sea. The present author calculated, by the method of mechanical integration, and using this tidal chart, the change in the direction of the plumb-line due to the attraction of the tides in the Yellow Sea by the aid of the following formulae:

$$-\frac{\gamma\rho h}{g \sin 1''}(\log r_2 - \log r_1)(\cos\theta_2 - \cos\theta_1) \text{ towards east,}$$

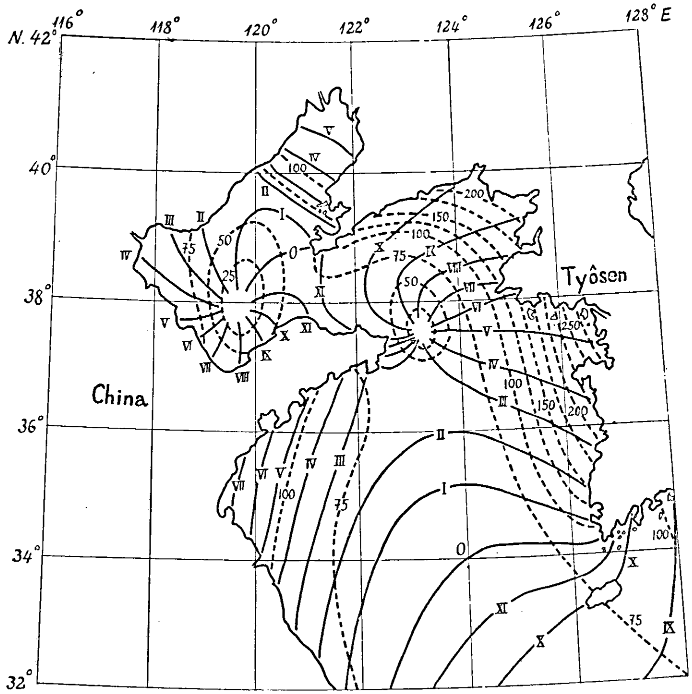

Fig. 4.

M_2 co-tidal and co-range chart.
Full lines—Co-tidals referred to 135° E. meridian.
Dotted lines—Co-ranges in cm.

where r = the distance from the observatory,

θ = the azimuth measured from the north in the clock-wise sense,

γ = the gravitational constant,

ρ = the density of the sea water,

and h = the height of tides above the mean sea level.

In the above expression, h is the function of the place, that is, both of r and θ and can be calculated easily by the data given in Fig. 4 as follows:

$$h = H_m \cos\zeta_m \cos 2t - H_m \sin\zeta_m \sin 2t.$$

Results of the calculation are given in the next paragraph.

6. The correction for the slope of the sea bottom.

As can be seen in Fig. 5, there are vast areas that appear above the sea surface at the time of low water, in the neighbourhood of the observatory. Actual coast line varies therefore with the height of the sea surface. We must take into consideration these effect in the calculation of the attraction of the sea water.

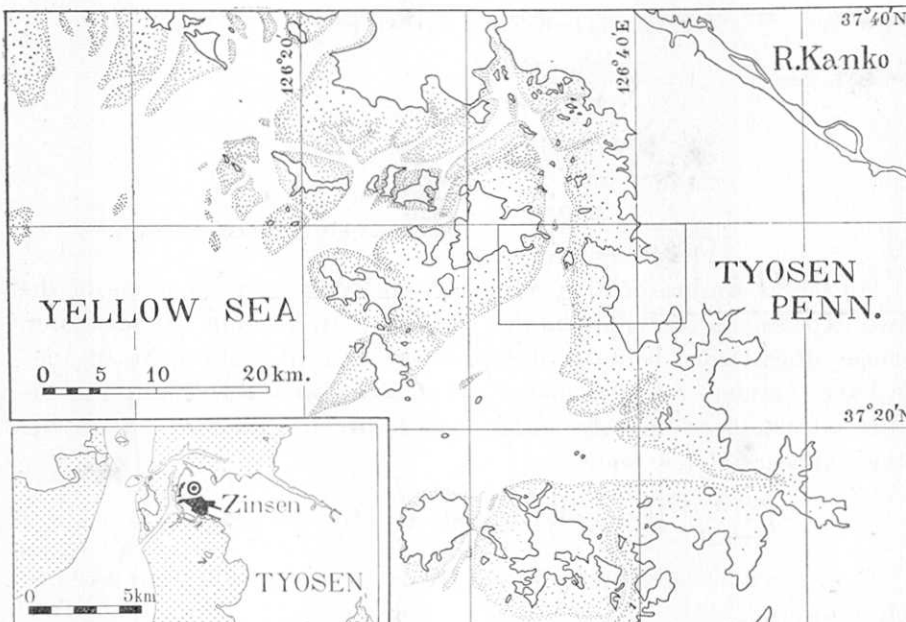

Fig. 5. The location of the observatory is indicated by ⊙. Dotted areas indicate sand-banks that appear at low water.

Sekiguchi assumed the form of the sea near the observatory to be a rectangle, with the depth uniformly increasing towards west, and the observatory being situated on the extension of the meadian line of the sea. The distance of the observatory from the nearest coast line was assumed to be 5 km. On these assumptions, he calculated mathematically the effect of the slope of the sea bottom.

In the present paper, the effect of the slope of the sea bottom was treated in a different manner, which seems to be more true to the actual condition of the sea.

Let us consider, for the sake of simplicity, only M_2 tide, and assume that the height of the sea bottom above the mean sea level be kh at a place, where h is the range of tides at the place under question and $1>k>-1$, then the variation of the water height at the place is given by

$$hf(k,t),$$

where

$$f(k,t)=0, \qquad \text{if } t_1<t<2\pi-t_1,$$

$$=\cos t-k, \qquad \text{if } -t_1<t<t_1,$$

and

$$\cos t_1=k.$$

If we put

$$f(k,t)=A_0+A_1\cos t+A_2\cos 2+\ldots.$$

then we have

$$A_0=-\frac{\sin t_1}{\pi}+\frac{t_1\cos t}{\pi},$$

$$A_1=\frac{\cos t_1\sin t_1}{\pi}+\left(1-\frac{t_1}{\pi}\right),$$

$$A_n=\frac{2}{\pi(n-1)\,n(n+1)}(n\cos nt_1\sin t_1-\cos t_1\sin nt_1).$$

As far as we are dealing with only M_2 component, A_1 alone of the above expressions contribute to the calculated attraction of the sea water, because other terms, being of different periods, contribute to M_4, M_6, etc. We have therefore for the change in the direction of the plumb-line due to the attraction of the sea water distributed in the area where the bottom appears at low water,

$$-\frac{\gamma\rho hA_1(k)}{g\sin 1''}(\log r_2-\log r_1)(\cos\theta_2-\cos\theta_1) \quad \text{toward east.}$$

In the actual calculation, successive steps of r and θ were taken in such a way as

$$(\log r_{n+1}-\log r_n)=0{\cdot}1, \quad (\cos\theta_{n+1}-\cos\theta_n)=0{\cdot}1,$$

for all values of n, so we have

$$\frac{0{\cdot}01\,\gamma\rho}{g\sin 1''}\sum A_1(k)\cdot h$$

$$=\frac{0{\cdot}01\,\gamma\rho}{g\sin 1''}[\sum A_1(k)H_m\cos\zeta_m\cos 2t+\sum A_1(k)H_m\sin\zeta_m\sin 2t],$$

which will hold good not only for the areas under question but also to the remaining part of the ocean, if we consider

$$A_1(k)=1, \quad \text{if} \quad k<-1.$$

The results of the calculation are given in Table II for the successive steps of r, and is poltted in Fig. 6.

Table II.

Attraction of oceanic tides (Slope correction inclusive).

r No.	Distance km.	cos $2t$ $\times 10^{-7}$	sin $2t$ $\times 10^{-7}$	r No.	Distance km.	cos $2t$ $\times 10^{-7}$	sin $2t$ $\times 10^{-7}$
1	0·64	850″	− 472″	17	25	73002″	−39993″
2	0·80	3907	− 2168	18	32	78840	−42953
3	1.00	8517	− 4725	19	40	84591	−45706
4	1·3	13236	− 7343	20	50	88825	−48171
5	1·5	17148	− 9514	21	64	92628	−50515
6	2·0	21498	−11930	22	80	95861	−52670
7	2·5	27266	−15131	23	100	98319	−54854
8	3	33520	−18602	24	128	100034	−56814
9	4	39673	−22044	25	160	101353	−58147
10	5	44764	−24895	26	200	102246	−58464
11	6	48932	−27221	27	256	102126	−58171
12	8	52587	−29249	28	320	101193	−58127
13	10	56203	−31219	29	400	100549	−58663
14	13	60938	−33628	30	512	100367	−59378
15	16	64791	−35770	31	640	100281	−59709
16	20	68319	−37597	32	800	100148	−59616

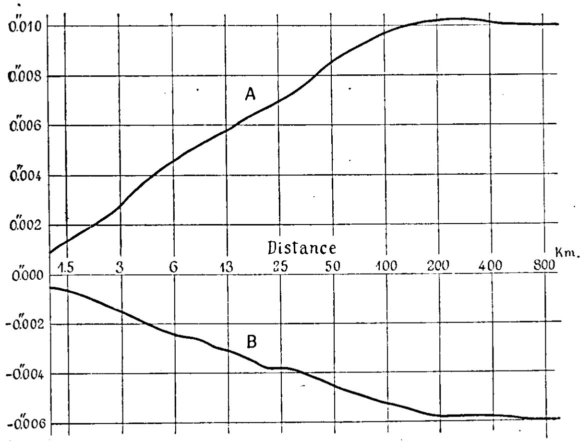

Fig. 6.

7. Tilt of the earth's surface due to tidal loads.

We have already calculated above the amount of contributions of the first four causes to the deflection of a pendulum, out of the five important causes enumerated in **3**. The last cause now to be considered is the tilt of the earth's surface due to tidal loads distributed in the neighbouring seas of the observatory; we will calculate this contribution in this paragraph.

According to Boussinesq's solution of the problem of surface loading, tilt of the surface of an semi-infinite elastic body due to a surface load are equivalent to the direction change of the vertical due to the attraction of the load, multiplied by

$$\Theta=\frac{(\lambda+2\mu)}{4\pi(\lambda+\mu)\mu}\frac{g^2}{\gamma}=\frac{172}{\mu},$$

if we take $\lambda=\mu$ and express μ in unit of 10^{10} c. g. s.

In the case of the actual earth's crust, however, Boussinesq's solution does not hold good because of the irregularities and heterogenieties of the structure existing in the crust. It is nearly impossible to treat mathematically the problem of the tilt of the earth's surface at a station due to a surface loading. The present author introduced in his previous paper, the idea of the *effective rigidity* as a method of approximate solution of the problem. He calls μ in Θ as the effective rigidity of the earth's crust when it is considered to be the function of the distance between the load and the station, and assumes that the effective rigidity has a certain uniform value—say μ_0—within some distance d from the station, and beyond that distance it becomes infinitely large.

Under these assumptions, the calculated value of the tilt of the earth's crust at Zinsen due to the tidal loads distributed in the neighbouring seas of the station, can be expressed as

$$\frac{172}{\mu_0}(A\cos 2t+B\sin 2\mathrm{t}),$$

where

$$A\cos 2t+B\sin 2t$$

expresses the deflection of a horizontal pendulum due to the gravitational attraction of tides in the neighbouring seas of the station, when the calculation is extended to the successive steps of distance. A and B in the above expression are therefore functions of the distance from the station, and are plotted in Fig. 6.

Comparing the coefficients of the cosine terms and sine terms of the observed tilt with those of the calculated one, we obtain the following two equations:

Observed values		Attraction of the moon and the earth-tides	Attraction of the sea water	Tilt due to tidal load
$0.''0411$	$=$	$0.''0035\,D$	$+0.''0100$	$+172\,A/\mu_0$,
0	$=$	$-0.''0117\,D$	$-0.''0060$	$+172\,B/\mu_0$,

or, to be the same thing,

$$+0.''0311 = +0.''0035\,D + 172\,A/\mu_0,$$
$$+0.''0060 = -0.''0117\,D + 172\,B/\mu_0.$$

As can be seen in Fig. 6, the functions A and B are approximately proportional to each other, the ratio of the value of A to that of B being the same for all values of distance. We can therefore eliminate from the above two equations the terms which include the effective rigidity, the most vague quantity in the equation, making use of this relation existing between A and B.

Then we have

$$D = -2.6.$$

The generally accepted value of D is 0·8, which gives for the rigidity of the earth 13×10^{11} c. g. s. or so. Negative values of D or the values of it greater than the unity are theoretically meaningless. This absurdity in the value of D is probably due to the fact that there is phase-lag between the actual and the calculated values of the tilt of the earth's crust due to tidal loading. These features will be clearly seen in Fig. 7, a vector diagram like that used in the theory of the of the alternating electric current.

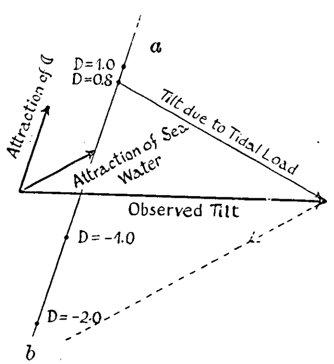

Fig. 7. Vector diagram of the observed and calculated tilts.

In the figure, the resultant vector of the attraction of the moon and the earth-tide is taken on the line a b, which is parallel to the attraction of the moon. Then, if we assume that the tilt due to tidal loading is in phase (dotted line in the figure) with the attraction of the sea water, the value of D must be -2.6 in order to give the observed tilt. On the contrary, if we assume that $D=0.8$, then the tilt due to tidal loading becomes out of phase with the attraction of sea water, the phase-lag being about 2 hours.

When we disregard this phase-lag, and assume that the value of D is 0·8, and also that the effective rigidity of the crust is the same for all distances from the station, we obtain as the rigidity of the crust $6{\cdot}2 \times 10^{11}$ c. g. s., which value does not vary much even if we assume different values for D. This value of the rigidity of the earth's crust is very near to the value of the crust assumed by Sekiguchi.

In conclusion, the writer wishes to express his most cordial thanks to Dr. R. Sekiguchi for his generosity of permitting the writer to make this revision, using the results of his observation.

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52. 仁川に於て觀測された地殼の傾斜運動に就いて

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此の論文に記載せるものは、著者自身に依つて爲された觀測ではなく、1916 年に關口鯉吉氏に依つて爲された觀測を計算し直したものである。關口氏の計算に於ては潮汐及び仁川附近の海岸に非常に多く分布する干出の取扱に就いて稍〻粗末ではないかと思はれる假定が二三存在する。著者は此の點に關して多少の變更を試み、且つ關口氏と異つた計算方法を用ひて計算をやり直して見たのである。其の結果は次の如く、大體に於て關口氏の得られたものと同一であるが、多少の異なる點も無いでもない樣である。

計算の結果に依れば、仁川の地殼は、潮差二米突余の太陰半日潮と、其の起潮力に對して振幅 0.″041 の傾斜運動を爲してゐるが、潮汐と其れの荷重に因つて生ずる地殼の撓曲との間には約二時間の位相差が存在するのである。地殼潮汐の減少率を 0·8 とし、且つ此の位相差を考へぬ事にすれば、地殼の剛性として $6{\cdot}2 \times 10^{11}$ c. g. s. なる値を得る事になる.
