37. *Tilting Motion of the Earth's Crust observed at Ryozyun (Port Arthur).*

By Ryûtarô TAKAHASI,

Earthquake Research Institute.

(Read Mar. 15, 1932.—Received June 20, 1932.)

1. Introduction. One of the most interesting facts which the recent geophysical investigations in this country revealed, may be that the earth's crust has, at least in some part of it, mozaic structures and that it *does* move incesantly, either related or not related with the occurrences of earthquakes. These developments of our knowledge we owe solely to the introduction of geodetic means to the study of earthquakes and other geophysical phenomena. The existence of secular deformations of the earth's crust and the predominance of mozaic structure in certain regions, have already become the facts of common knowledge in our country. Studies on the size and motion of these land blocks, relations among their size, the frequency of earthquakes and the geological structure, and the general behaviours of their movements after remarkable earthquakes, have been made in so through a manner that there now remain only such problems about land blocks as those regarding their characteristics or the statistics of their movements, except for the study on the fundamental phenomena of their behaviours.

The geodetic surveys which have given invaluable data for these studies were mostly executed, however, in regions more or less disturbed by earthquakes, only few of them having been made in regions entirely free from disturbances of earthquakes. It is, therefore, of great importance to ascertain whether or not the secular deformation of the earth's crust is equally existing in regions entirely free from the earthquake disturbances. In these views, we have started to make observations on the secular movement of the earth's crust at Ryozyun (Port Arthur), where no earthquake has been experienced since the middle of the last century. The secular movement of the earth's crust was known by observing the tilt of the ground by horizontal pendulums. According to the results of the observation so far obtained, we may conclude that there is no sensible secular tilt of the earth's crust at Ryozyun. This conslusion, however, does not worth much confidence because the present observation is too short and was interrupted for about two months under an unavoidable circumstance.

We were fortunate enough, however, in having observed as a by-product the tilting motion of the earth's crust due to tidal loads in the neighbouring seas of the observation station. The present writer has already studied in this Bulletin the similar phenomena observed at other places.[1] Investigations of the tilt of the earth's crust due to tidal loads are very important for the analysis of the earth-tides and, at the same time, for the studies of the structure and rigidity of the earth's crust as well as that of the whole earth. Studies in these lines will give, therefore, much clue to the elucidation of the mechanism and nature of the block movements mentioned above.

2. General description of the observation and the results obtained. The present observation was made in the cellar of the wireless telegraph office, belonging to the Ryozyun Engineering College. The office is situated on a hill in the west suburb of the city of Ryozyun, and is ca. 62 m. high above the mean sea level and is 1·2 km. distant from the nearest coast line. The location of the office is to be seen in Fig. 1.

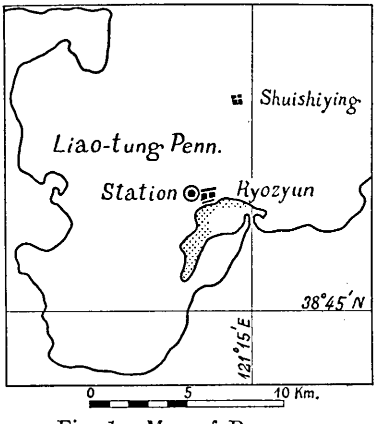

Fig. 1. Map of Ryozyun. Dotted area indicates sand-banks that appear at low water.

The instrument used in the present observation is a pair of the Ishimoto tiltmeters. The tiltmeter is essentially a horizontal pendulum of the Zöllner suspension, the movable horizontal rod, the suspension fibres and the frame work being all made of fused quartz. Suspension fibres are fused to the frame work and to the movable rod. The instrument is therefore scarcely affected by the variation of temperature of the surrounding air and is also entirely free from elastic after-effect. The angular position of the horizontal rod is recorded continuously by projecting a light beam from a light source and receiving the reflected beam from a mirror attached to the rod on a sheet of photographic paper wound on a revolving drum, the drum being placed

1) *Bull. Earthq. Res. Inst.*, 6 (1929), 85; 7 (1929), 95; 10 (1932), 145.

at the distance of 1 m. from the pendulum. The drum makes one revolution per week by a clock-work contained in its inside, the linear speed being 4·2 cm. per day. Time-marks are recorded twice a day on the record, at 6^h and 18^h, by interrupting the electric current which feeds the light source. The sensibility of the tiltimeter was calibrated before its installation so that we can calculate the sensibility of the instrument at any desired time, or adjust it to any predetermined value, by measuring or regulating suitably the period of free oscillation of the pendulum. Details about the instrument will not be described here as they are fully given in Ishimoto's paper.[2]

The cellar of the wireless telegraph office in which the pendulums were installed has been left as it was digged down to the ground, and is not lined by any material. A concrete box, which is 2 m. × 2 m. × 3 m. in inside dimension, was half embedded into the floor of the cellar, and the tiltmeters were placed directly on the bottom of this concrete box. The concrete box is provided with a wooden entrance door and with a roof covered with a thick layer of sand. The pendulums were installed in the azimuths of 75° and 165°, measured in the clock-wise sense from north to east. The general view of the concrete box and of the installation of the tiltmeters will be obtained by Fig. 2. The sensibility of

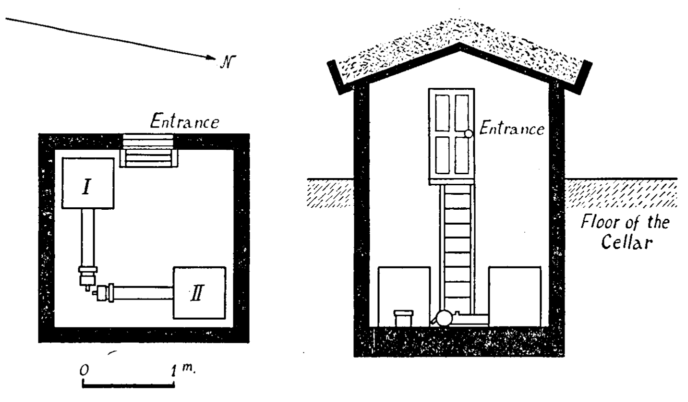

Fig. 2. Observing chamber.

the instrument was adjusted at the beginning of the observation so as it gives the deflection of 2 cm. on the record corresponding to the tilt of 1 second of arc. The sensibility has been calibrated since then many times by measuring the period of free oscillation of the pendulum.

2) M. Ishimoto, *Jap. Journ. Astro. Geophys.*, 6 (1928), 83.

The observation was commenced on Feb. 1, 1931, and has still been continued. In the present paper, however, a part of the observation, extending from April 1, 1931 to Feb. 3, 1932, will only be dealt with. The actual observation, the installation of, and the attendance to the instrument were kindly made by Mr. H. Watanabe, under the guidance of Professor T. Hori, to whom the writer wishes here to express his hearty thanks.

In Fig. 3 (Plate LXXXIV) is given a reproduction of the record obtained by the tiltmeter for the period from Mar. 30 to April 6, 1931. The straight line in the record is the datum-line produced by a fixed mirror placed near the pendulum mirror, and the wavy curve is the record of the tilt of the earth's crust. In this figure as well as in the following, the letters **E**, **W**, **N** and **S** are the directions towards which the ground tilts down. The deflection of the tilt-curve to the side of the letter **E** means, thus, that the crust tilts down to the east side. As can be seen in the figure, the tilt of the earth's crust at Ryozyun is composed of variations respectively with a very long, diurnal and semi-diurnal periods. These features of tilt will be made clearer by making a vector-diagram such as given in Fig. 4. This shows a curve that a rod, planted perpendicularly to the ground, will describe on the celectial sphere, according to the tilting motion of the ground.

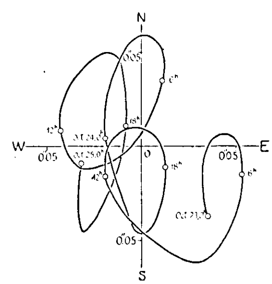

Fig. 4.
Vector diagram of tilt.

Since the deflection of the tiltmeter corresponds to the variation of the angle between the direction of the plumb-line and the surface of the earth, different causes can be enumerated that will result it:

1. The change in the direction of the plumb-line as the direct effect of the tide-generating force of the sun and the moon.
2. Tilt of the earth's surface due to the earth-tides.
3. The change in the direction of the plumb-line due to the deformation of the earth caused by the earth-tides.
4. The change in the direction of the plumb-line due to the attraction of oceanic tides.
5. Tilt due to the load of sea water in oceans.
6. The change in the direction of the plumb-line due to the deformation of the earth produced by tidal loads.
7. The tilt and the change in the direction of the plumb-line due

to the periodic variation of the level of underground water.

8. The atmospheric tides both as load and as attraction.

9. The tilt of the ground due to the thermal stress produced by the variation of the atmospheric temperature and by the sun-shine.

10. Secular or cummulative tilt of land blocks or of the earth's crust.

11. Secular variation of the direction of the plumb-line.

The tiltmeter deflections are thus composed of many defletions which is produced by various causes. It is therefore necessary for the detailed study of the phenomena to analyse the observed deflection into these component deflections. The first three of these items are inseparable in the observation. Some of these causes are very small in amount and need not be taken into consideration. In the following paragraph we will analyse into different components the tilt observed during the period from Oct. 20 to Nov. 17, 1931, in which the tilt due to the annual variation is smallest.

3. Analysis of the observed tilt. We find by further inspection of the record that there is tilt of the period of a few days, besides the tilts already cited. Being due directly or indirectly to meteorological variations, neither these tilts of medium period, nor the long perioded ones can be treated by the method of analysis to be described later. We must therefore eliminate these disturbing tilts in the first step of the analysis. For this purpose, the overrupping mean was taken of every set of 25 successive hourly readings, and the average was assigned to the middle epoch of the set. The average was calculated in this way for every hour throughout the period taken for the analysis, and the values were connected by a smooth curve, this being taken as the datum-line.

Deviation of each hourly reading from this datum-line was analysed by the method of harmonic analysis. Hourly reading of the records used for the analysis are given in Table I and are plotted in Fig. 5. The method of harmonic analysis employed in the present paper is due to A. T. Doodson,[3] details of which will need not be explained here. The results were of course obtained after making all corrections such as those due to the factor of reduction, acceleration in epoch, disturbances among different components, reduction to local time, etc.

Results of the analysis are as follows:

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3) A. T. Doodson, *Phil. Transact.*, 227 A (1927), 223.

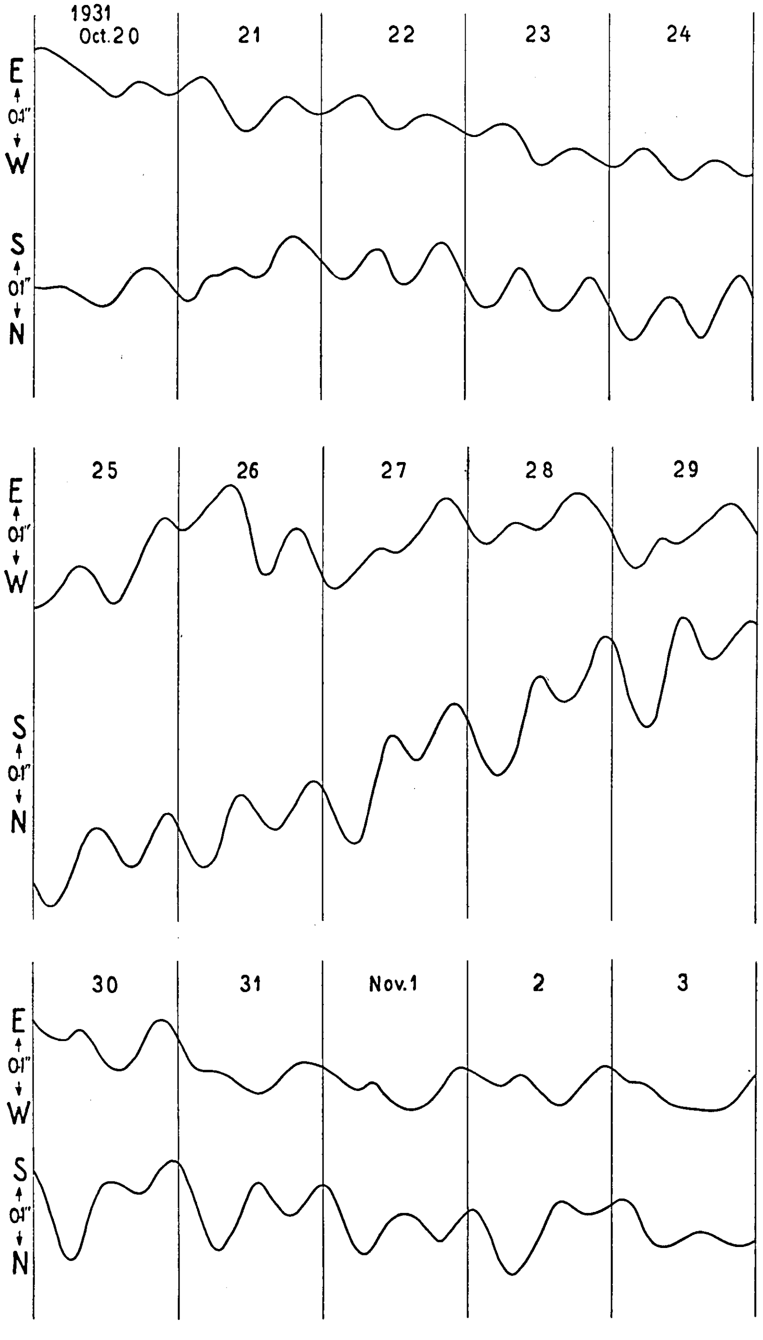
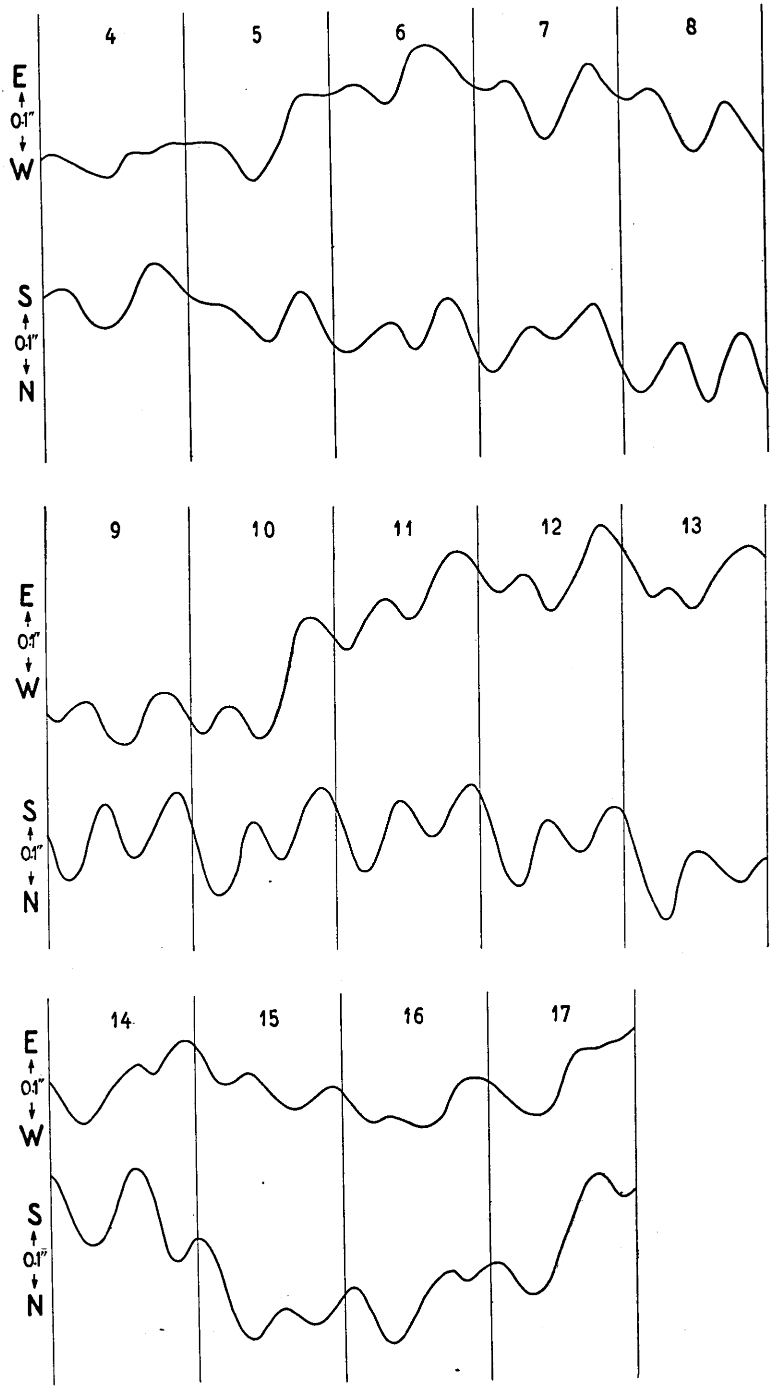

1931
Oct.20
21
22
23
24
E
0.1"
W
S
0.1"
N
25
26
27
28
29
E
0.1"
W
S
0.1"
N
30
31
Nov.1
2
3
E
0.1"
W
S
0.1"
N

Fig. 5.



Observed tilt-curves.

Tilt \ Comp.		M_2	S_2	N_2	[K_2]	K_1	O_1	[P_1]	M_4	MS_4
Towards azimuth 75°	H''	0·01745	0·01098	0·00506	0·00296	0·01861	0·00638	0·00615	0·00101	0·00052
	$\kappa°$	227	247	225	247	299	277	299	311	36
Towards azimuth 165°	H''	0·03323	0·01575	0·00821	0·00423	0·02513	0·00985	0·00832	0·00127	0·00097
	$\kappa°$	304	340	318	340	249	276	249	218	317

These results give the component tilts of the crust in the azimuths 75° and 165°, measured from north to east. When transformed to the tilts in east and south directions, they become as shown below.

Component	M_2		K_1		O_1	
Tilt	cos 2*t*	sin 2*t*	cos *t*	sin *t*	cos *t*	sin *t*
toward east	−0·00669	−0·01946	+0·00638	−0·02179	+0·00102	−0·00865
toward south	+0·01825	0·02331	−0·01104	−0·01844	+0·00079	−0·00782

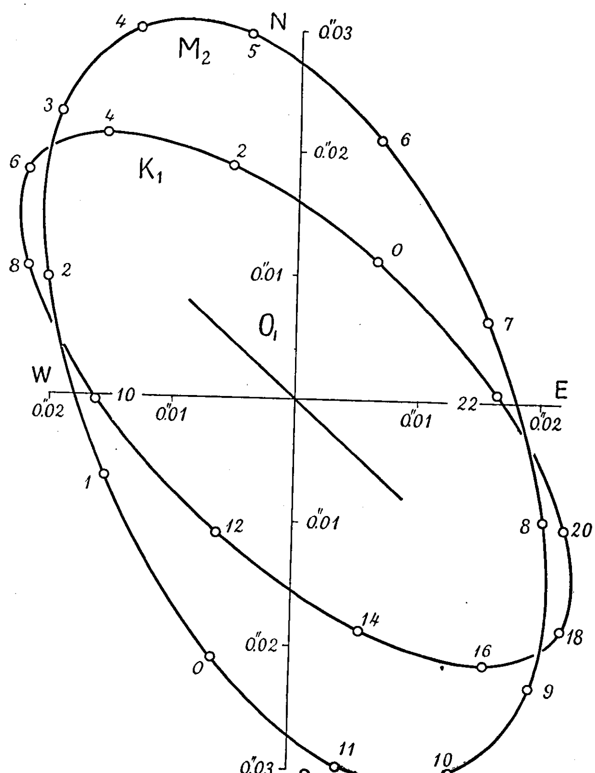


Fig. 6. Vector-diagrams of the observed component-tilts. Numerals represent component-hours.

Vector-diagrams of different component-tilts are given in Fig. 6.

We will in the next describe about the secular tilt of the earth's crust, which was the original aim of the present investigation to observe. In Fig. 7 are plotted the averages of 12 successive hourly read-

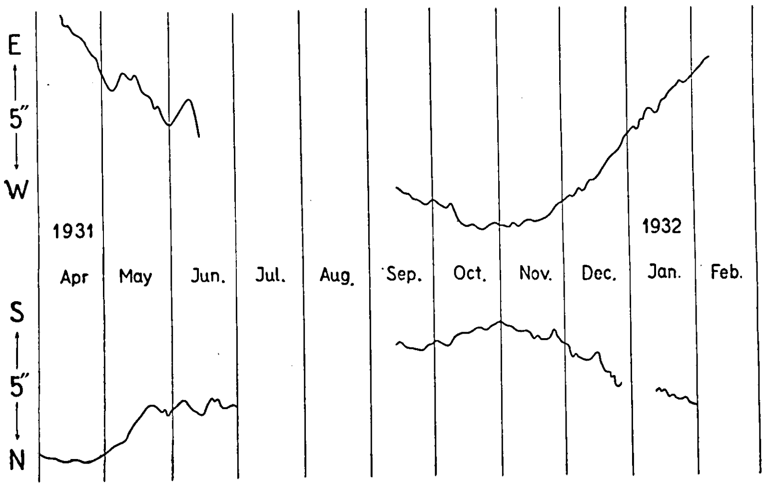


Fig. 7. Annual variation of tilt.

ings. The absolute position of the right half of the tilt curve in the figure is not certain since the observation was missed during July and August. The figure may, however, probably represent the true tilt of the crust without much errors. If this be granted, the observed long perioded variation is an annual one, which may be due to meteorological and astronomical causes, but not a cumulative one. Astronomical and tidal variations with an annual period must have their maximum in July. Since the observed annual variation has its maximum in October, it may mainly be due to meteorological cause or causes.

4. Tide-generating forces of the sun and the moon. The tide-generating potential of a heavenly body, the earth-tides produced by it, and the variation of potential due to the earth's deformation thus produced are of the same period and phase. If the change in the direction of the plumb-line due to the tide-generating potential be W, the tilt of the crust due to the earth-tide, and the change in the direction of the plumb-line due to the deformation of the earth will be $-hW$ and kW respectively. In the observed tilt, they are therefore superposed on each other giving as total, $(1+k-h)\,W$. The term $(1+k-h)$ is called the diminishing factor, which we will hereafter denote by R. W will be easily calculated by the following expressions.[4)]

$\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \cos\lambda \sin\lambda\, G_1 \cos 2t$	towards south	for M_2;
$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \cos\lambda\, G_1 \sin 2t$	towards east	
$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \cos 2\lambda\, G_2 \cos t$	towards south	for O_1;
$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \sin\lambda\, G_2 \sin t$	towards east	
$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \cos 2\lambda\, G_3 \cos t$	towards south	for K_1.
$-\frac{3}{\sin 1''}\frac{M}{E}\left(\frac{a}{c}\right)^3 \sin\lambda\, G_3 \sin t$	towards east	

Results of the calculation are as follows:

Components	M_2		S_2		K_1		O_1	
	cos 2*t*	sin 2*t*	cos 2*t*	sin 2*t*	cos *t*	sin *t*	cos *t*	sin
Towards east	0	−0″·01178	0	−0″·00571	0	−0″·00642	0	−0″·00482
Towards south	0″·00739	0	0″·00358	0	−0″·00220	0	−0″·00165	0

4) Notations: $G_1=\left(\frac{1}{2}-\frac{5}{4}e^2\right)\cos^4\frac{I}{2}$,

$$G_2=\left(\frac{1}{2}-\frac{5}{4}e^2\right)\sin I\cos^2\frac{I}{2},$$

$$G_3=\left[\left(\frac{1}{4}+\frac{3}{8}e^2\right)^2\sin^4 I+\left(\frac{1}{4}+\frac{3}{8}e_1^2\right)^2 G^2\sin^4\omega+2\left(\frac{1}{4}+\frac{3}{8}e^2\right)\left(\frac{1}{4}+\frac{3}{8}e_1^2\right)\times G\sin^2 I\sin^2\omega\cos 2\nu\right]^{\frac{1}{2}},$$

$$G=\frac{\text{mass of }\odot}{\text{mass of }☾}\times\left(\frac{\text{mean distance of }☾}{\text{mean distance of }\odot}\right)^3=0{\cdot}46164,$$

M mass of moon,
E mass of earth,
a radius of earth,
c mean distance of moon,
λ latitude of the place.
t hour angle of component fictious star or component hour,
e eccentricity of moon's orbit,
I inclination of moon's orbit to equator,
ω inclination of earth's equator to ecliptic,
e_1 eccentricity of earth's orbit,
ν R. A. of intersection of moon's orbit with equator.

5. Attraction of the oceanic tides. The distribution configuration of oceanic water varies with tides. Accordingly, its attraction working at a station varies as well. We must therefore calculate for the present analysis the amount of the deflection of a horizontal pendulum due to this cause. In actual calculations, the method of mechanical integration must be employed because of the irregular distribution of tides. Let the height of sea surface above the mean sea level be h in an elementary area limited by radii r_1 and r_2 and azimuths θ_1 and θ_2, the station under question being the centre of the coordinates. Azimuth are measured clockwise from north to east, as usual. Then the amount of change in the direction of the plumb-line due to the attraction of sea water in this area is given by

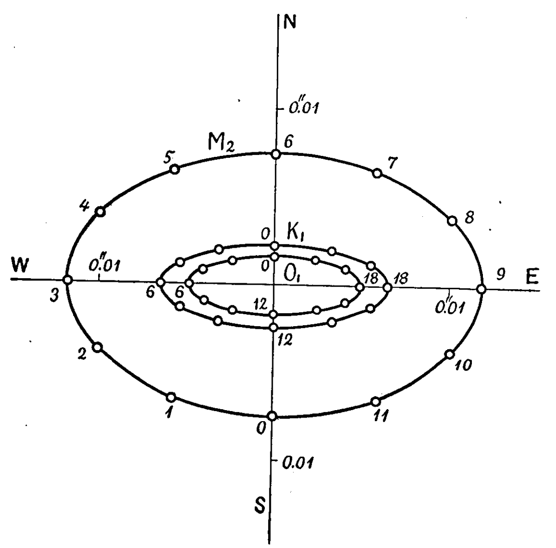


Fig. 8. Component plumb-line deviations due to the attraction of the moon and the sun. Numerals represent component hours.

$$\frac{\gamma\rho h}{g \sin 1''}(\log r_2 - \log r_1)(\cos\theta_2 - \cos\theta_1) \qquad \text{towards east,}$$

$$\frac{\gamma\rho h}{g \sin 1''}(\log r_2 - \log r_1)(\sin\theta_2 - \sin\theta_1) \qquad \text{towards south,}$$

in which γ, ρ and g are the universal gravitation constant, density of sea water and the acceleration of gravity respectively. In the above expressions, h is a function both of time t and the place. If we take, for the sake of brevity, only M_2-tide into consideration,

$$h = H_m \cos(2t - \zeta_m) = H_m \cos\zeta_m \cos 2t + H_m \sin\zeta_m \sin 2t,$$

in which H_m and ζ_m are functions of r and θ.

Now, if we take, in the mechanical integration, the successive steps of r and θ in such manners that

$$\log r_{n+1} - \log r_n = p, \quad \cos\theta_{n+1} - \cos\theta_n = q \quad \text{and} \quad \sin\theta_{n+1} - \sin\theta_n = r,$$

for all values of n, where p, q and r being constants, the above expressions for the plumb-line deflection will be given by

$$\frac{\gamma\rho pq}{g \sin 1''}[(\sum H_m \cos \zeta_m) \cos 2t + (\sum H_m \sin \zeta_m) \sin 2t] \qquad \text{towards east,}$$

$$\frac{\gamma\rho pr}{g \sin 1''}[(\sum H_m \cos \zeta_m) \cos 2t + (\sum H_m \sin \zeta_m) \sin 2t] \qquad \text{towards south.}$$

In the present calculation p, q, r were all taken to be 0·1, and with regard to t, the lunar time referred to the meridian 135° E was used.

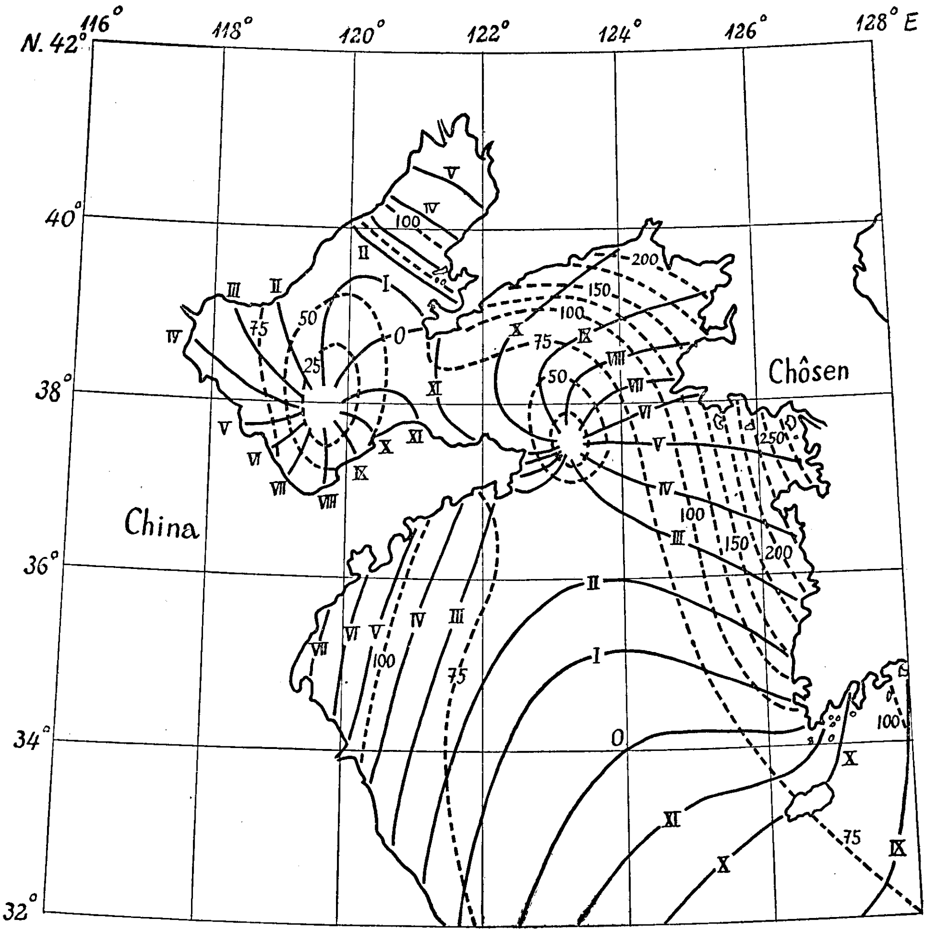


Fig. 9. M_2 Co-tidal and Co-range Chart.
Full lines—Co-tidals referred to 135°E meridian
Dotted lines—Co-ranges in cm.

H_m and ζ_m[5] were determined for all parts of the Yellow Sea and its adjacent bays by the tidal constants of the ports situated on the coast of the sea under question, with due regards on the depths and other hydrodynamical conditions of the ocean. In Fig. 9 are shown the co-tidal and co-range lines thus obtained and in Fig. 10 the chart which gives $H_m \cos \zeta_m$ and $H_m \sin \zeta_m$. Harmonic constants of the ports used

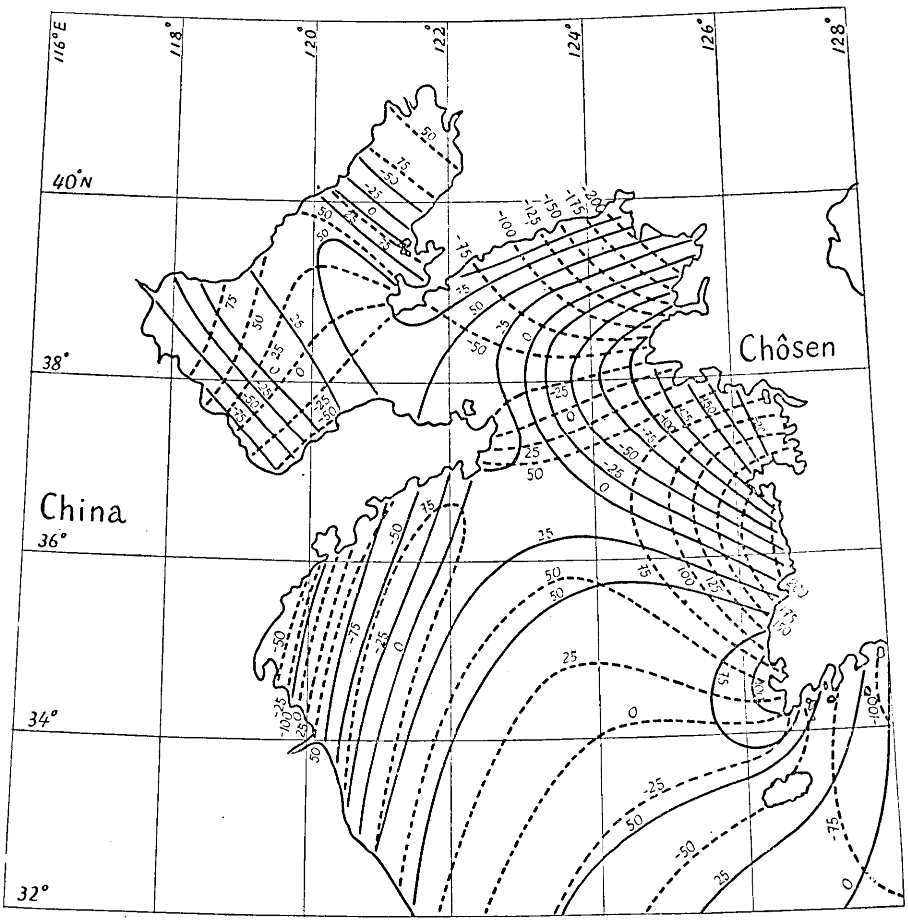


Fig. 10. Height of the sea surface
at $t=0$: $H_m \cos \zeta_m$, full lines;
and at $t=\text{III}$: $H_m \sin \zeta_m$, dotted lines.
unit = cm.

5) $\zeta_m = \kappa_m - 2(135° - L)$, where L is the longitude of the place.

in drawing these charts are given in Table II.[6]

The calculated amount of change in the direction of the plumb-line due to the attraction of tides are given in the following table for successive steps of r. They are already reduced to the local lunar time of Ryozyun.

r in km.	Attraction of M_2-tide			
	toward east		toward south	
	$H'' \times 10^7$	κ°	$H'' \times 10^7$	κ°
1·26	189	311	194	311
1·58	852	311	980	311
2·00	1684	311	1752	311
2·51	2698	311	2815	311
3·16	3577	311	3845	311
3·98	3957	311	4220	311
5·01	4498	311	4790	310
6·31	5250	310	5800	310
7·94	5780	310	6985	310
10·0	6153	308	8285	310
12.6	5850	305	10080	309
15·8	5525	301	12020	309
20·0	5135	296	13480	306
25·1	4915	288	14900	303
31·6	4857	281	16540	300
39·8	5192	276	18080	297
50·1	5872	273	19670	293
63·1	6300	271	21500	290
79·4	7130	266	23470	288
100·	8295	263	25370	286
126·	9772	258	27080	285
158·	11480	256	28560	284
200·	13140	254	29610	283
251·	14852	256	29430	283
316·	16260	258	28350	283
398·	16295	260	27200	284
501·	15230	259	26310	285
631·	14850	260	26130	287
794·	15090	261	26250	288

6. Corrections for the slope of sea bottom. In the above calculation of the attraction of sea water, all the parts of sea bottom were assumed not to appear above the sea surface, even when the tides are at low water. Actually, however, there is wide area which appears above the sea surface at low water and that, too, in the port of Ryozyun which is nearest to the station. We must correct therefore the results obtained in the last paragraph for this effect. If we take

6) For the port where spring range or spring rise are known together with H.W.I. full and change, following relations were employed in obtaining harmonic constants, using approximate values for $(\kappa_m - \kappa_s)$, $\dfrac{H_s}{H_m}$ and $\dfrac{H_0 + H'}{H_m}$:

$$\kappa_m = 29^\circ T - \tan^{-1}\frac{\sin(\kappa_m - \kappa_s)}{\dfrac{H_m}{H_s} + \cos(\kappa_m - \kappa_s)}, \qquad H_m = \frac{\text{spring range}}{2\left(1 + \dfrac{H_s}{H_m}\right)} = \frac{\text{spring rise}}{2\left(1 + \dfrac{H_s}{H_m} + \dfrac{H_0 + H'}{H_m}\right)}.$$

M_2-tide only into consideration for the sake of simplicity, and assume that the diurnal inequality is negligible, and that the height of sea bottom is kH_m above the mean sea level, where $1 \geqq k \geqq -1$, the load acting upon this area can be represented by the shaded portion of Fig. 11, that is,

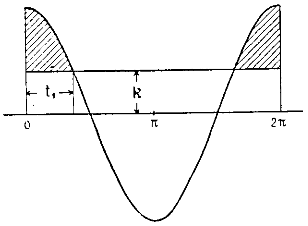


Fig. 11.

$$H_m(\cos t - k) \qquad \text{for} \qquad -t_1 < t < t_1$$
$$\text{and} \qquad 0 \qquad \text{for} \qquad t_1 < t < 2\pi - t_1,$$

$\cot t_1$ being equal to k. We have therefore over-estimated the load in the last paragraph by the amount

$$H_m f(k, t) \equiv H_m k \qquad \text{for} \qquad -t_1 < t < t_1,$$
$$\equiv H_m \cos t \qquad\qquad t_1 < t < 2\pi - t_1.$$

Being of an un-harmonic form, the variation of the over-estimated load involves harmonics of higher order than the first. But as far as

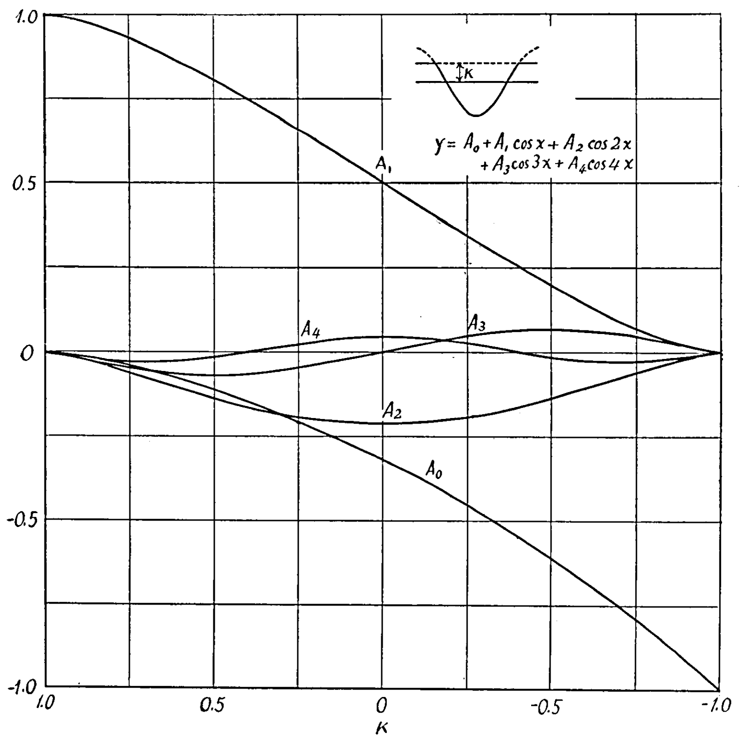


Fig. 12. The graph for the calculation of the correction for the slope of the sea-bed.

we are dealing only with M_2-tide, the first harmonics of the variation of the above load alone gives contribution to the results of caluculation, because the higher harmonics are of different periods and contribute to M_4, M_6, etc. If we put then

$$f(k, t)=A_0+A_1\cos t+A_2\cos 2t+\ldots.$$

we have

$$A_0=-\frac{\sin t_1}{\pi}+\frac{t_1\cos t_1}{\pi},$$

$$A_1=\frac{\cos t_1 \sin t_1}{\pi}+\left(1-\frac{t_1}{\pi}\right),$$

$$A_n=\frac{2}{\pi(n-1)n(n+1)}(n\cos nt_1\sin t_1-\cos t_1\sin nt_1).$$

In Fig. 12, we have given the values of A_0, A_1, etc. against the values of k taken in abscissa.

By these considerations, we obtain as the quantities to be subtracted from the results of the last paragraph

$$\frac{\gamma\rho pq}{g\sin 1''}H_{m0}\cos(2t-\kappa_{m0})\sum A_1,$$

$$\frac{\gamma\rho pr}{g\sin 1''}H_{m0}\cos(2t-\kappa_{m0})\sum A_1,$$

where H_{m0} and κ_{m0} are tidal constants of Ryozyun and are the same for all the parts of the port. Summation is to be extended only for the area where the bottom appear at low water. The amount of correction thus obtained are

$$0''\!\cdot\!000272\cos(2t-311)=0''\!\cdot\!0001784\cos 2t-0''\!\cdot\!0002052\sin 2t$$

towards east,

$$0''\!\cdot\!000343\cos(2t-311)=0''\!\cdot\!0002250\cos 2t-0''\!\cdot\!0002587\sin 2t$$

toward south.

After making this correction, the amount of change in the direction of the plumb-line due to the attraction of oceanic tides becomes

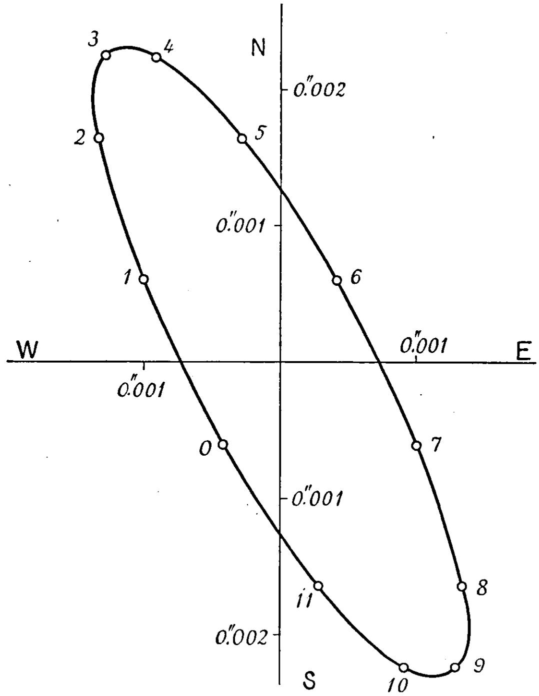


Fig. 13. Attraction of the oceanic M_2-tide.
(Corrected for the slope of sea-bottom)
Numerals represent component hours.

r in km.	Attraction of M_2-tide			
	Towards east		Towards south	
	cos $2t$	sin $2t$	cos $2t$	sin $2t$
3·98	+0"·0000811	−0"·0000931	+0"·0000517	−0"·0000596
5·01	+ 1162	− 1342	+ 0830	− 1083
10·0	+ 2008	− 2798	+ 3073	− 3763
20·0	+ 0468	− 2558	+ 5680	− 8323
50·1	− 1477	− 3808	+ 5430	− 15513
100·	− 2794	− 6178	+ 4749	− 21813
200·	− 5405	− 10558	+ 4420	− 26223
501·	− 4690	− 12888	+ 4565	− 22813
794·	− 4144	− 12852	+ 5861	− 22379

7. Bending of the crust due to tidal loads. The main part of the observed amount of tilt is due to the bending of the earth's crust by tidal loads and those tilts already calculated in the above several paragraphs. There are several other causes, as mentioned above, that can produce the tilt of the crust or the change in the direction of the plumb-line, but their amounts are negligibly small. The tilt of the crust due to tidal loads will therefore be obtained, when the values which were already calculated above are subtracted from the observed. Thus, it will be our next step to consider in what manner the tidal loads produce the tilt of the earth's crust.

Boussinesq[7] solved a problem of surface loading on a semi-infinite homogeneous elastic solid. According to him, the tilt of the surface of the elastic body due to a load is given by

$$\frac{\lambda+2\mu}{4\pi(\lambda+\mu)\mu}g\rho\frac{\partial}{\partial x}\int\frac{hd\sigma}{r},$$

λ and μ being Lamé's constants. On the other hand, the change in the direction of the plumb-line due to the attraction of the same load will be written as

$$\frac{\gamma\rho}{g}\frac{\partial}{\partial x}\int\frac{hd\sigma}{r}.$$

Therefore the tilt of the earth's crust due to the tidal load is equal in amount to the deflection of the plumb-line due to the attraction of the load multiplied by

$$\Theta=\frac{\lambda+2\mu}{4\pi(\lambda+\mu)\mu}\frac{g^2}{\gamma}.$$

If we take $\lambda=\mu$, and express μ in units of 10^{10} c. g. s., we have

$$\Theta=\frac{172}{\mu}.$$

Boussinesq's solution holds good, however, only for a homogeneous non-gravitating elastic body, and it is of much questions whether or not it can be applied equally to the actual earth in its unmodified form. In and on the actual earth's crust, there exist many topographical irregularities as well as geological heterogenieties. The earth's crust is very far, from all points of views, from homogeniety. Even if we imagine that these irregularities or heterogenieties exist only within the upper-

7) LOVE, "*Math. Theory Elasticity*," Cambridge Univ. Press (1927), 193.

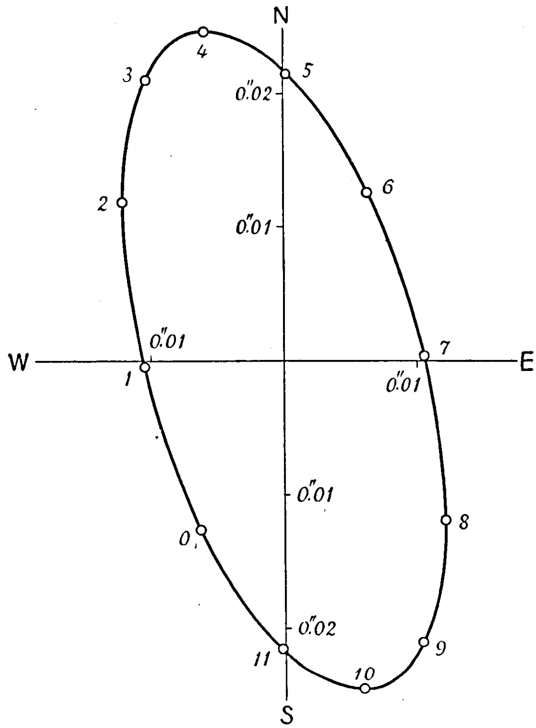


Fig. 14.
The part of the observed tilt which is due to M_2 tidal loads. Numerals represent component-hours.

most part of the earth's crust of several ten kilometers' thickness, and this may probably be near the truth, there exist still in the crust many layers of different rocks, the elasticity of which increasing with depth.

Recently M. Matsumura[8] and G. Nishimura[9] have solved independently, and with interesting results, the problem of surface loading on a one-layered elastic body. Their results, however, are rather complicated, and do not seem to be expressed simply as that of Boussinesq was. With the actual earth, matters are so complicated that it is not far from impossibility to treat it mathematically.

It is, however, probable that in producing tilts at a point by a load, only surface layers of the crust partake in the play when the distance between the load and the point is short, and as this distance increases,

8) M. Matsumura, *Journ. Civil Engin. Soc.*, 17 (1931), 813, 1121, (in Japanese).
9) G. Nishimura, *Bull. Earthq. Res. Inst.*, 10 (1932), 23.

deeper layers come into play more and more. This is equivalent to regard λ and μ in Θ to be functions of distance between the load and the station. If we can know by what functions of distance these effective elastic constants are to be expressed, we will obtain a clue to the elucidation of the subterranean structure near the station. In the following pages, we will call the rigidity in such meanings as *effective rigidity*. T. Shida[10] assumed, by similar considerations, the effective rigidity of the earth's crust to be a linear function of the distance and determined the coefficients of terms of the function. The author assumed, in several papers already published, the effective rigidity to have a uniform value—say μ_0—within a certain distance d from the station, and beyond this distance to be infinitely large. By this assumption, he determined the values of d and μ_0 for several observations. For obtaining the relation of the effective rigidity to the distance, it is a customary process to assume it to be a power series of distance and find the numerical values of the coefficient of each term. For this method, a large number of tiltmeter observations are necessary that are executed at different places, so this cannot be realized for the present case. Though the assumptions for the effective rigidity made by the author and by T. Shida are equally nothing but the first approximation, there exist many experimental facts which seem to be in favour of the author's assumption.

If a solid rectangular block was pressed down upon a layer of sand, surface depression occurs within a short distance from the pressed area, but beyond this distance the sand-pile behaves as if it were perfectly rigid. Similar phenomena can be observed with plastic materials such as rice-jelly, asphalt, etc. T. Terada and N. Miyabe[11] have observed similar phenomena during his experiment in which he pressed a sand-pile laterally. On the other hand, it is a well-known fact that rocks have elastic properties different from those of metals and other common elastic bodies. For most rocks, Hooke's law does not hold good, even in the laboratory experiments with small specimens. It will not be absurd to imagine, then, that the crust which is composed of these rocks has peculier elastic characteristics, especially against such long-perioded forces like those treated in this paper, since it is of large scale and in the strong field of gravity. The earth's crust may, in this respect, be

10) T. SHIDA, *Mem. Coll. Sci., Kyôto Imp. Univ.*, 4 (1910), 1.

11) T. TERADA and N. MIYABE, *Bull. Earthq. Res. Inst.*, 4 (1928), 33; 6 (1929), 109; 7 (1929), 65.

considered to resemble rather a sand-pile. Block structure of the crust may also be cited as in favour of the author's assumption.

In these views we will next treat the tilt of the crust observed at Ryozyun. If we extend the calculation of the change in the direction of the plumb-line due to the attraction of sea water to successive steps of distance from the station, calculated change will be expressed by the forms

$$A \cos 2t + B \sin 2t \qquad \text{towards east,}$$
$$C \cos 2t + D \sin 2t \qquad \text{towards south,}$$

where A, B, C and D are functions of distance. In Fig. 15 are given these quantities against the distance taken in logarithmic scale. We have then the following four equations:

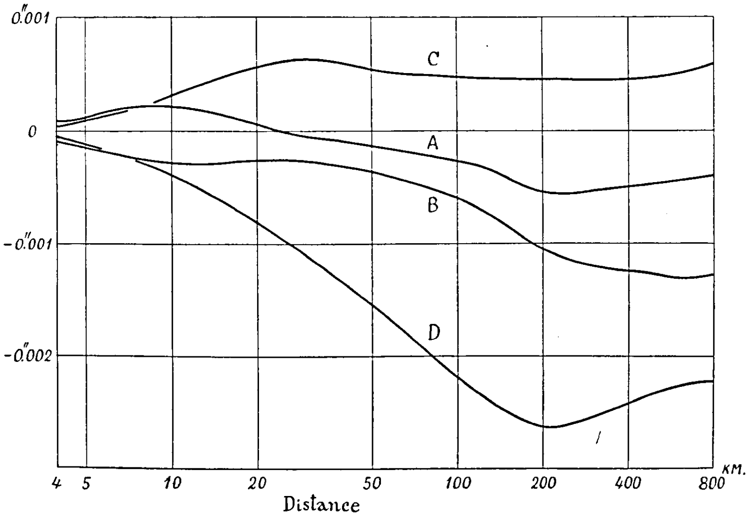


Fig. 15. Attraction of oceanic M_2-tide.

Attraction of oceanic tides	Earth-tides	Tidal loads	Observed values
″			″
−0·00041	″	$+172\,A/\mu_0=$	−0·00669
−0·00129	$-0{\cdot}01178\,R$	$+172\,B/\mu_0=$	−0·01946
+0·00059	$+0{\cdot}00739\,R$	$+172\,C/\mu_0=$	+0·01825
−0·00224		$+172\,D/\mu_0=$	−0·02331,

or

	″	″
$172\,A/\mu_0$		$=-0{\cdot}00628,$
$172\,B/\mu_0$	$-0{\cdot}01178\,R$	$=-0{\cdot}01817$
$172\,C/\mu_0$	$+0{\cdot}00739\,R$	$=+0{\cdot}01766$
$172\,D/\mu_0$		$=-0{\cdot}02107,$

For the purpose of determining the values of μ_0 and R from these equations we have constructed Fig. 16, taking μ_0 and distance as co-ordinates and R as a parameter. Theoretically the four curves A, B, C

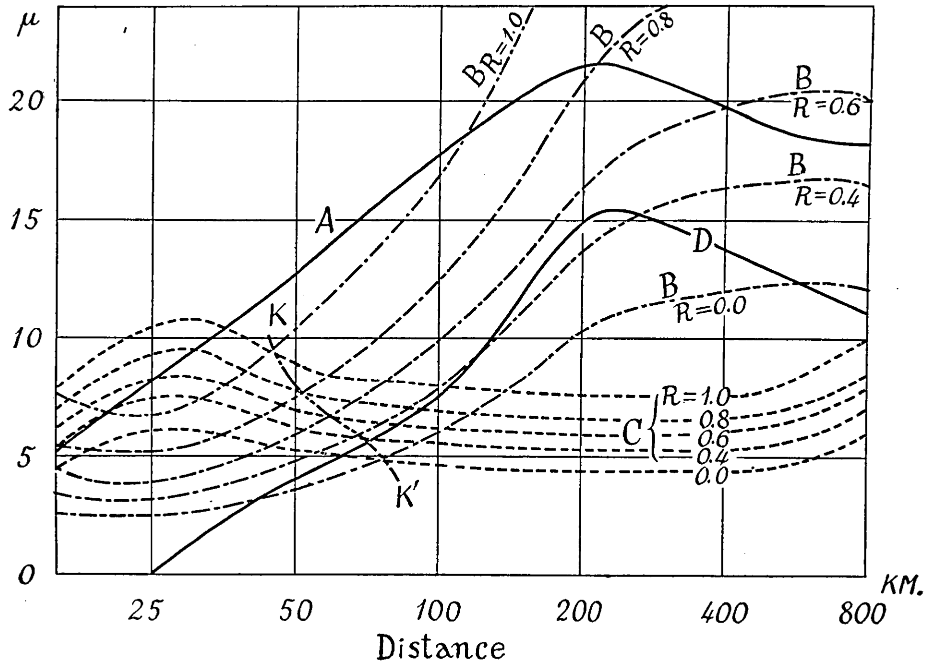


Fig. 16.

and D in the figure must intersect at a point for some value of R, the point of intersection corresponding to the effective rigidity μ_0 and the distance d beyond which the effective rigidity becomes infinitely large. In Fig. 16, we cannot however find such a point; we will therefore determine a point which satisfies the above condition with minimum errors. In the figure, KK' curve is the locus of intersection of B and C curves. We have therefore adopted as the point of minimum errors the point which lies on the curve KK', and is equi-distant from A and D curves, because A and D are nearly parallel. From the co-ordinates of this point, values of μ_0, d and R were obtained as

$$\mu_0 = 6{\cdot}9 \times 10^{10} \quad \text{c. g. s.}$$
$$d = 50 \quad \text{km.}$$
$$R = 0{\cdot}8$$

The value of R, thus obtained, is very near to those which were hitherto determined by many authors. The present value of R gives 16×10^{11} c. g. s. as the rigidity of the earth. The value of μ_0, obtained here, is very small, being nearly equal to the rigidity of andesite. The effective rigidity at a station is a sort of average value of the rigidities of rocks which lie near or under the station, so it must have a value which is near to those of rocks prevailing at the station. Rocks that are predominant at Ryozyun and its neighbourhood, belonging to paleozoic or archean era, have, however, rigidities which are larger than the value of the effective rigidity here obtained.

This discordance of the values of rigidities is partly due to the deficiencies which may exist in the observation and in the treatment of the results of the observation. The main part of this discordance will, however, probably be due to the conditions under which rocks are in the actual earth's crust. Since the earth's crust resembles, as mentioned above, rather a sand-pile in respect of its elastic characteristics in such conditions, its effective rigidity may also be of small value just like the sand is. Then the value of the effective rigidity obtained here is very reasonable, but never absurd.

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37. 旅順に於ける地殼傾斜變化の觀測

地震研究所 高橋龍太郎

地震の殆んど起らない地方に於ても、地塊運動、又は地殼の積年的變形があるか否か、と言ふ事を明にする目的を以て、昭和六年二月以來、旅順に於て地殼の傾斜變化の觀測を行つて居る。此の論文に於て發表したものは其の觀測の一部、即ち觀測當初より昭和七年二月に到る約一年間の期間に對するものである。

觀測の結果に依れば、旅順に於ては今の所積年的の傾斜運動は認められないと言へる樣である。又此の觀測に於て同時に潮汐の負荷に因つて起る地殼の變形が認められた。本論文の主なる部分は此の變形の研究である。

觀測の結果を調和解析によつて分解して、種々の擾亂性分を除去し、其を理論上より得る値と比較して、地殼潮汐の減小率として、0·8 なる値を得た。此の値から計算すると地球の剛性は大凡 16×10^{11} c. g. s. となる。又同時に地殼のみの剛性が $6{\cdot}9 \times 10^{10}$ c. g. s. と求められた。然し乍ら此の値は旅順附近の岩石の其れと比較する時は著しく小さいのである。論文の最後に於て此の不一致の依つて來るべき原因を論じ、併せて地殼の彈性に論及してある。

Table I. Hourly readings of tiltmeter records.

1931 Pendulum I (Azimuth 75°)

	Oct.									
	20	21	22	23	24	25	26	27	28	29
h	″	″	″	″	″	″	″	″	″	″
0	7·1418	7·0725	7·0344	6·9954	6·9409	6·9276	7·0593	6·9798	7·0655	7·0500
1	434	780	344	923	424	308	585	643	523	305
2	372	842	375	939	479	362	655	588	414	118
3	341	905	437	978	557	448	772	643	351	6·9978
4	279	928	538	7·0102	643	565	889	752	398	954
5	208	811	593	118	713	728	7·1006	876	484	7·0032
6	161	655	601	133	705	939	123	985	577	203
7	107	500	577	118	643	946	232	7·0110	655	344
8	029	336	461	032	565	954	333	188	671	414
9	7·0928	165	305	6·9931	463	876	356	258	671	414
10	842	048	172	759	347	721	201	266	640	352
11	757	024	110	526	238	534	7·0889	219	585	352
12	663	063	032	409	175	410	476	196	585	383
13	577	141	040	471	199	339	6·9985	258	671	468
14	577	203	118	526	315	409	798	344	788	546
15	694	328	196	580	409	557	876	429	936	640
16	811	445	258	658	479	783	7·0063	554	7·1076	718
17	811	562	266	713	510	7·0032	539	694	162	811
18	811	577	258	721	518	235	546	889	201	905
19	772	538	211	713	479	437	577	7·1053	201	990
20	725	429	188	658	409	624	577	146	123	7·1021
21	663	359	126	588	331	772	492	123	021	7·0967
22	655	344	094	526	292	803	266	045	7·0873	819
23	655	344	017	456	276	686	6·9985	7·0811	671	671

			Nov.							
	30	31	1	2	3	4	5	6	7	8
h	″	″	″	″	″	″	″	″	″	″
0	7·0507	7·0188	6·9705	6·9682	6·9697	6·9565	6·9798	7·0585	7·0686	7·0500
1	359	6·9946	635	635	611	635	798	640	648	422
2	274	783	573	565	534	627	806	687	632	422
3	211	666	510	502	479	596	814	733	655	500
4	180	643	432	471	471	557	798	733	710	570
5	180	643	362	417	449	487	775	702	749	585
6	219	635	339	409	378	463	721	648	733	562
7	328	627	409	487	308	409	643	562	609	445
8	344	588	456	557	206	347	534	492	422	274
9	235	534	409	565	128	308	362	429	250	040
10	079	463	292	534	082	261	199	492	6·9985	6·9814
11	6·9915	401	175	440	027	253	175	655	798	635
12	791	331	097	308	019	308	253	928	798	573
13	721	284	035	222	012	471	393	7·1162	939	580
14	674	292	019	129	012	627	549	286	7·0141	713
15	697	362	027	097	004	643	721	349	344	876
16	767	432	051	113	004	635	931	356	531	7·0126
17	900	557	097	175	004	635	7·0196	356	733	328
18	7·0110	643	175	284	004	643	492	310	936	336
19	320	721	292	409	019	689	577	201	7·1037	266
20	453	775	440	510	090	752	577	123	7·0967	110
21	507	791	580	611	175	798	577	7·0990	811	6·9962
22	500	783	650	689	331	798	577	850	679	822
23	398	736	705	721	448	798	577	741	557	674

(to be continued.)

Table I. (*continued.*)

	9	10	11	12	13	14	15	16	17
h	″	″	″	″	″	″	″	″	″
0	6·9518	6·9386	7.0801	7·1894	7·2291	7.2089	7·2688	7·1886	7·1995
1	417	238	671	754	135	7·1980	541	730	941
2	409	175	585	629	7·1933	808	338	590	863
3	487	284	632	551	746	637	159	497	746
4	580	417	780	543	543	473	058	403	629
5	658	565	7·1006	645	434	427	026	349	536
6	721	635	201	754	512	434	042	356	497
7	721	643	349	824	590	528	112	434	442
8	611	565	434	824	590	676	198	442	442
9	347	370	434	738	504	824	198	419	473
10	167	214	341	520	356	948	120	356	543
11	097	105	162	310	271	7.2073	003	325	676
12	027	097	107	224	271	190	7.1886	279	948
13	019	206	123	279	349	283	769	279	7·2213
14	023	432	232	356	512	361	715	279	408
15	175	736	388	512	691	361	668	294	517
16	487	7·0063	575	746	832	276	621	380	525
17	705	492	816	980	972	213	629	551	525
18	822	928	995	7·2229	7·2073	291	668	777	564
19	876	7·1123	7·2151	470	159	463	730	964	603
20	884	138	213	650	229	603	808	7·2019	634
21	830	107	213	626	283	681	894	058	650
22	705	006	159	525	283	751	964	058	681
23	565	7.0905	058	400	213	759	956	050	774

1931 Pendulum II (Azimuth 165°)

	Oct. 20	21	22	23	24	25	26	27	28	29
h	″	″	″	″	″	″	″	″	″	″
0	5·4397	5·4264	5·4789	5·4468	5·4060	5·4193	5·5131	5·5758	5·6916	5·8242
1	366	182	652	264	5·3846	5·3984	5·4876	514	661	5·7946
2	366	162	570	121	652	836	692	274	406	554
3	381	295	519	060	519	800	524	080	182	243
4	407	447	560	065	504	907	463	5·4927	008	5·6987
5	407	555	621	137	591	5·4086	478	835	003	819
6	346	575	723	254	754	284	647	922	054	809
7	279	570	835	397	907	545	912	5·5187	202	916
8	223	636	973	585	5·4080	805	5·5243	590	457	5·7273
9	167	713	5·5019	723	172	5·5009	539	973	814	783
10	131	713	5·4978	672	213	090	641	5·6401	5·7191	5·8155
11	106	672	825	494	193	090	636	641	503	466
12	111	600	596	264	014	029	514	656	630	604
13	167	565	437	111	5·3831	5·4866	376	559	559	497
14	295	585	453	009	652	667	218	365	355	268
15	407	672	529	5·3999	601	519	131	263	263	038
16	514	876	621	5·4029	652	468	095	258	222	5·7936
17	641	5·5055	774	111	831	499	136	406	258	936
18	723	182	932	213	5·4009	621	274	610	375	5·8002
19	723	228	5·5080	366	193	830	437	834	554	109
20	682	187	131	519	341	5·5049	590	5·7018	783	232
21	606	131	080	550	494	228	753	120	5·8058	364
22	509	039	5·4876	427	570	335	876	171	222	476
23	381	5·4927	677	213	417	335	891	120	293	543

(*to be continued.*)

Table I. *(continued.)*

			Nov.							
	30	31	1	2	3	4	5	6	7	8
h	″	″	″	″	″	″	″	″	″	″
0	5·8497	5·8650	5·8273	5·7824	5·7936	5·7288	5·7278	5·6406	5·6151	5·5947
1	268	446	166	834	992	350	207	309	5·5998	718
2	5·7951	201	5·7987	752	997	375	161	304	937	559
3	630	5·7911	737	533	936	385	120	314	973	539
4	324	630	503	299	783	375	120	386	5.6126	590
5	084	334	273	038	579	283	115	457	253	743
6	018	181	120	5·6855	344	100	079	528	406	911
7	105	207	095	763	242	5·6931	018	610	508	5·6075
8	350	334	171	804	217	834	5·6947	681	610	202
9	732	528	370	901	222	778	840	743	661	330
10	987	737	503	5·7044	227	768	748	758	610	319
11	5·8186	936	610	222	273	789	630	661	508	049
12	278	5·8166	686	477	329	845	528	457	457	5·5743
13	308	293	732	681	380	931	498	330	483	488
14	293	222	747	834	421	5·7115	590	375	528	335
15	242	120	732	936	426	355	783	513	600	463
16	166	5·7962	656	982	416	579	967	712	681	769
17	125	834	528	911	375	732	5·7140	931	778	5·6029
18	140	763	395	814	324	783	258	5·7079	916	228
19	222	778	314	778	273	788	268	166	5·7018	406
20	344	849	350	758	222	732	151	069	5·6936	477
21	472	987	477	758	212	599	5·6931	5·6865	666	416
22	594	5·8094	574	783	222	477	732	636	432	166
23	665	222	701	860	248	375	523	381	151	5·5855

	9	10	11	12	13	14	15	16	17
h	″	″	″	″	″	″	″	″	″
0	5·5478	5·5641	5·5947	5·6100	5·5845	5·5029	5·3973	5 3009	5·3448
1	151	248	641	5·5845	498	5·4907	927	091	463
2	5·4851	5·4902	335	539	151	636	805	091	443
3	723	570	044	233	5·4810	402	576	5·2938	346
4	754	458	5·4907	5·4922	519	188	244	734	193
5	876	483	866	662	279	5·3973	5·2913	530	040
6	5·5080	590	937	565	091	902	637	367	5·2979
7	437	794	5·5131	621	014	897	448	224	964
8	794	5·5141	442	861	203	958	295	173	004
9	973	539	758	5·5182	596	5·4101	275	234	5·3127
10	993	682	978	549	927	315	352	377	295
11	774	641	5·6049	682	5·5100	570	505	556	550
12	452	457	5·5947	682	141	922	632	775	805
13	243	264	794	559	131	5·5090	724	938	5·4086
14	100	106	641	432	080	126	765	5·3071	366
15	141	049	488	310	5·4983	080	734	168	590
16	284	182	437	218	922	5·4856	663	259	779
17	508	417	503	182	825	570	581	341	896
18	702	677	614	243	723	172	530	341	937
19	896	896	830	417	657	5·3754	515	234	876
20	5·6090	5 6100	5·6008	614	647	596	550	213	749
21	182	228	182	820	738	611	632	275	621
22	136	268	299	896	891	744	754	336	570
23	5·5922	161	304	901	5·5004	907	887	397	652

Table II. Tidal constants of the ports.[12]

Places	Lat.		Long.		H_m	κ_m
Shanghai	31°	21′	121°	30′	95 cm	30°
Wei-hai-wei	37	29	122	13	61	297
Takou	38	59	117	42	94	95
Tang Luan Tsu	37	59	120	42	56	292
Nan Fan Chen Tao	38	21	120	54	60	309
Schichon Tao	39	25	121	15	43	41
Daikokuseki	38	58	121	18	56	359
Yang Tan Wa	38	47	121	08	61	331
Port Arthur	38	48	121	14	83	311
Hsiaoping Tao	38	49	121	31	89	304
Dairen Wan	38	56	121	39	100	297
Tseng Chia Tun	39	08	122	06	115	287
Tsao Tao	39	29	123	05	158	268
Taku Sham road	39	46	123	33	193	272
Autung	40	08	124	24	74	337
Neu-am Po	39	56	124	21	161	297
Tetasa To	39	48	124	25	215	263
Banzo Retto	39	41	124	27	205	253
Napu Somu	39	16	124	43	192	247
Unmu To	39	25	125	07	222	255
Dauchin I.	38	37	125	00	154	226
Pio Sem.	38	40	125	06	167	244
Ping Yang Inlet	38	39	125	35	204	262
Gets naï To	38	03	124	49	106	161
Taityoku To	37	50	124	43	99	157
Kotu Semi	37	44	125	33	198	139
Tyumun To	37	39	126	14	270	139
Chemulpho	37	30	126	34	293	134
Tokutyoku To	37	15	126	09	248	122
Tyonsu Wan	36	23	126	26	226	98

(*to be continued.*)

12) Taken from "*Tables of Harmonic Constants,*" Intern. Hydrogr. Bur. Monaco (1926), and "*Nippon Kinkai no Tyôseki* (*Tides in the Adjacent Seas of Japan.*)" Hydrogr. Bur. Japan (1905).

Table II. *(continued.)*

Place	Lat.		Long.		H_m	κ_m
Weyon To	36°	13′	126°	02′	175 cm	85°
Pikken To	36	07	125	59	179	81
Ȯtyon To	36	02	126	32	213	79
Kokunsan Ganto	35	49	126	25	226	85
Manma To	35	21	126	01	169	60
Koroku To	35	03	126	05	142	43
Hamupyon Wan	35	09	126	21	189	57
Mokuho (Yong Sau)	34	45	126	22	116	51
Pukkan Suido	34	52	126	06	142	34
Paruku Po	34	36	126	01	117	11
Hatai To	34	32	126	03	109	5
Sanse Somu	34	20	126	04	93	346
Tin To (Chin Do)	34	30	126	12	113	31
Tin To (Kapu To)	34	24	126	19	92	342
Gobaro To	34	27	126	25	98	308
Huwapukuni	33	31	126	35	66	301
Soki Po	33	14	126	33	77	271
Ietun Somu	34	10	127	18	92	266
Soan Ko	34	08	126	38	85	295
Tyotyokura	34	22	126	47	107	263
Mato Suido	34	25	126	51	113	285
Sontyuku Po	34	17	127	22	94	276
Kokkumu-Suido	34	30	127	09	110	275
Nano Retto	34	28	127	29	105	268
Kamakkupataan	34	44	127	45	102	252
Suni To	37	45	125	20	154	145

Place	Lat.		Long.		Spring rise	H. W. I. Full and Change	
Ki-kou	38°	35′	117°	32′	320 cm	4 h	30 m
Pei-tang Ho Kou	39	07	117	44	275	3	0
Tchi Ho	39	06	118	50	198	1	20
Luan Ho	39	16	119	70	152	1	30
Tai-shuchi-Ho	39	49	119	30	183	0	15

(to be continued.)

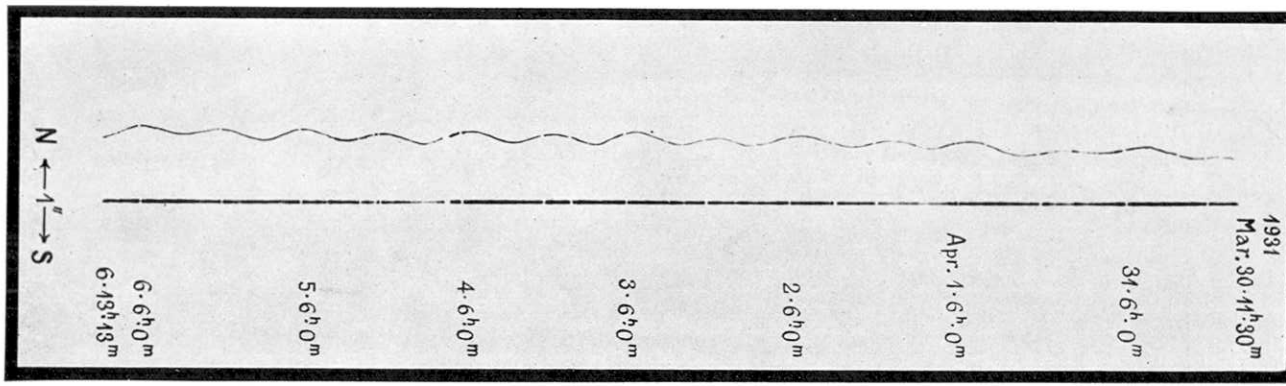

Fig. 3.

Record as obtained by the tiltmeter.

（震研彙報、第十號、圖版、高橋）

Table II. (*continued.*)

Place	Lat.		Long.		Spring rise	H. W. I. Full and Change	
	°	′	°	′	cm	h	m
Chan-Hai-kouan	39	59	119	49	183	0	00
St. Ubes Pt.	40	18	120	32	230	4	50
Kiu-tcheou	40	48	121	0	328	5	30
In-kou	40	43	122	14	394	5	0
Vansitarts Saddle	40	12	122	0	328	4	20
Collio-San. B.	39	30	121	17	262	2	30
Port-Adams	39	26	121	44	328	2	00
Kin-tcheou B.	39	06	121	42	262	0	15

Place	Lat.		Long.		Spring range	H. W. I. Full and Change	
	°	′	°	′	cm	h	m
Chan-toung Prom.	37	24	122	42	223	4	00
Sang Koon B.	37	08	122	27	226	0	45
Nanking	32	10	118	55	131	10	50
Hang-Tcheou B.	30	14	120	14	450	11	35
Chi-Pou	29	57	121	47	289	1	00
Tai-tcheou I.	28	24	121	52	463	8	50
Min-Kiang Kou	26	02	119	40	624	9	45
Min Kiang	26	03	119	24	633	0	30
Kiao-tcheou	36	00	120	20	374	4	50
Nioutchuang	40	35	122	00	357	4	30
Tieu-tsin	39	09	117	11	136	6	50
Hoang-Ho Kou	37	54	118	34	320	4	00