21. *Possibility of Free Oscillations of Strata excited by Seismic Waves. Part IV.*

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1. The present paper is the continuation of our research work on the determination of the possible range of free oscillations of strata due to seismic disturbances and deals with the case where primary seismic waves are incident upwards normally to doubly stratified layers residing on the surface of a semi-infinite body.

As to the thicknesses, densities and elastic constants of two layers and the bottom medium, we have employed the results of investigations due to a certain degree to Dr. Matuzawa[1] etc., and completely to Prof. Imamura[2] etc.. These are approximated as follows.

Layer	Thickness	Density	V for P-waves	V for S-waves
Upper layer (Granitic)	$H'-H=20$km	$\rho''=2\cdot7$	5·0 km/sec	3·15 km/sec
Second layer (Basaltic)	$H=30$km	$\rho'=3\cdot0$	6·1 km/sec	3·70 km/sec
Bottom medium (Ultrabasic)	∞	$\rho=3\cdot5$	7·5 km/sec	4·45 km/sec

2. Let ρ, λ, μ be the density and elastic constants of the bottom solid, ρ', λ', μ' those of the second layer of thickness H and ρ'', λ'', μ'' those of the first or upper layer of thickness $H'-H$. Again, let u, u', u'' be the respective displacements in the bottom medium, the second layer and the first layer; then the equations of motion of each medium are written as follows:

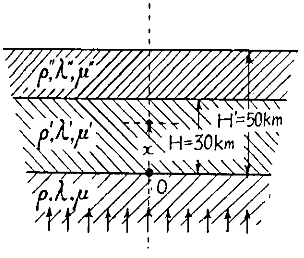

Fig. 1.

1) T. MATUZAWA, K. YAMADA and T. SUZUKI, "On the Forerunners of Earthquake-motions (The Second Paper)," *Bull. Earthq. Res. Inst.*, 7 (1929), 241-260.

2) A. IMAMURA, F. KISHINOUYE and T. KODAIRA, "The Effect of Superficial Sedimentary Layers upon the Transmission of Seismic Waves," *Bull. Earthq. Res. Inst.*, 7 (1929), 471-487, (in Japanese).

$$\left.\begin{aligned}\frac{\partial^2 u}{\partial t^2}&=V_1^2\frac{\partial^2 u}{\partial x^2},\\ \frac{\partial^2 u'}{\partial t^2}&=V_2^2\frac{\partial^2 u'}{\partial x^2},\\ \frac{\partial^2 u''}{\partial t^2}&=V_3^2\frac{\partial^2 u''}{\partial x^2},\end{aligned}\right\}\quad\cdots\cdots(1)$$

where $V_1^2=(\lambda+2\mu)/\rho$, $V_2^2=(\lambda'+2\mu')/\rho'$, $V_3^2=(\lambda''+2\mu'')/\rho''$ for dilatational waves and $V_1^2=\mu/\rho$, $V_2^2=\mu'/\rho'$, $V_3^2=\mu''/\rho''$ for distortional waves. The typical solutions of these equations are expressed by

$$\left.\begin{aligned}u&=e^{ih(V_1t-x)}+Ae^{ih(V_1t+x)},\\ u'&=Be^{ih'(V_2-x)}+Ce^{ih'(V_2t+x)},\\ u''&=De^{ih''(V_3t-x)}+Ee^{ih''(V_3t+x)},\end{aligned}\right\}\quad\cdots\cdots(2)$$

where A, B, C, D, E are arbitrary constants and h, h' and h'' have the meaning such that

$$h=p/V_1,\quad h'=p/V_2,\quad h''=p/V_3,\quad\cdots\cdots(3)$$

in which $p/2\pi$ is the frequency of waves.

The boundary conditions at the planes, $x=0$, $x=H$, $x=H'$ are as follows:

$$\left.\begin{aligned}&x=0;\quad u=u',\quad (\lambda+2\mu)\frac{\partial u}{\partial x}=(\lambda'+2\mu')\frac{\partial u'}{\partial x},\\ &x=H;\quad u'=u'',\quad (\lambda'+2\mu')\frac{\partial u'}{\partial x}=(\lambda''+2\mu'')\frac{\partial u''}{\partial x},\\ &x=H';\quad (\lambda''+2\mu'')\frac{\partial u''}{\partial x}=0,\end{aligned}\right\}\text{for dilatational waves,}\quad(4)$$

$$\left.\begin{aligned}&x=0;\quad u=u',\quad \mu\frac{\partial u}{\partial x}=\mu'\frac{\partial u'}{\partial x},\\ &x=H;\quad u'=u'',\quad \mu'\frac{\partial u'}{\partial x}=\mu''\frac{\partial u''}{\partial x},\\ &x=H';\quad \mu''\frac{\partial u''}{\partial x}=0.\end{aligned}\right\}\text{for distortional waves.}\quad(5)$$

Substituting (2) in (4) and (5) by means of (3), and using the following notations:

$$\left.\begin{aligned}\alpha&=\sqrt{\frac{\rho'(\lambda'+2\mu')}{\rho(\lambda+2\mu)}},\quad \alpha'=\sqrt{\frac{\rho''(\lambda''+2\mu'')}{\rho'(\lambda'+2\mu')}},\\ \beta&=\sqrt{\frac{\rho'(\lambda+2\mu)}{\rho(\lambda'+2\mu')}},\quad \beta'=\sqrt{\frac{\rho''(\lambda'+2\mu')}{\rho'(\lambda''+2\mu'')}}\end{aligned}\right\}\quad\cdots\cdots(6)$$

for dilatational waves; and

$$\left.\begin{aligned} \alpha&=\sqrt{\frac{\rho'\mu'}{\rho\mu}}, & \alpha'&=\sqrt{\frac{\rho''\mu''}{\rho'\mu'}}, \\ \beta&=\sqrt{\frac{\rho'\mu}{\rho\mu'}}, & \beta'&=\sqrt{\frac{\rho''\mu'}{\rho'\mu''}} \end{aligned}\right\} \quad\ldots\ldots\ldots\ldots\ldots(7)$$

for distortional waves, we find finally

$$A=\frac{(1-\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1+\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1-\alpha)(1-\alpha')e^{2i\beta hH}+(1+\alpha)(1+\alpha')}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}, \quad\ldots\ldots\ldots(8)$$

$$B=\frac{2\{(1+\alpha')e^{2i\beta\beta'hH'}+(1-\alpha')e^{2i\beta hH}\}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}, \quad\ldots\ldots\ldots(9)$$

$$C=\frac{2\{(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha')\}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}, \quad\ldots\ldots\ldots(10)$$

$$D=\frac{4e^{2i\beta\beta'hH'}e^{i\beta hH(1-\beta')}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}, \quad\ldots\ldots\ldots(11)$$

$$E=\frac{4e^{i\beta hH(1-\beta')}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}. \quad\ldots\ldots\ldots(12)$$

In the special case where $\alpha'=1$, $\beta'=1$ we find

$$A=\frac{(1-\alpha)+(1+\alpha)e^{-2i\beta hH'}}{(1+\alpha)+(1-\alpha)e^{-2i\beta hH'}}, \quad\ldots\ldots\ldots(8')$$

$$B=\frac{2}{(1+\alpha)+(1-\alpha)e^{-2i\beta hH'}}, \quad\ldots\ldots\ldots(9')$$

$$C=\frac{2e^{-2i\beta hH'}}{(1+\alpha)+(1-\alpha)e^{-2i\beta hH'}}, \quad\ldots\ldots\ldots(10')$$

$$D=\frac{2}{(1+\alpha)+(1-\alpha)e^{-2i\beta hH'}}, \quad\ldots\ldots\ldots(11')$$

$$E=\frac{2e^{-2i\beta hH'}}{(1+\alpha)+(1-\alpha)e^{-2i\beta hH'}} \quad\ldots\ldots\ldots(12')$$

These are coincident with the result which one[3] of us obtained a few years ago.

Decomposing u in (2) into primary waves u_1 and reflected waves u_2, and applying Fourier's integral we get

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3) K. SEZAWA, "Possibility of the Free-oscillations of the Surface-layer excited by the Seismic-waves," *Bull. Earthq. Res. Inst.*, 8 (1930), 1—11.

$$u_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t-x+\sigma)}d\sigma, \quad\dots\dots\dots\dots\dots(13)$$

$$u_2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{(1-\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1+\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1-\alpha)(1-\alpha')e^{2i\beta hH}+(1+\alpha)(1+\alpha')}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}\right]dh$$
$$\times\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t+x+\sigma)}d\sigma, \qquad (14)$$

$$u'=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\{(1+\alpha')e^{2i\beta\beta'hH'}+(1-\alpha')e^{2i\beta hH}\}dh}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$\times\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t-x+\frac{\sigma}{\beta}\right)}d\sigma$$
$$+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\{(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha')\}dh}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$\times\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t+x+\frac{\sigma}{\beta}\right)}d\sigma, \qquad (15)$$

$$u''=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{e^{2i\beta\beta'hH'}e^{i\beta hH(1-\beta')}dh}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$\times\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta\beta'h\left(V_3t-x+\frac{\sigma}{\beta\beta'}\right)}d\sigma$$
$$+\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{e^{i\beta hH(1-\beta')}dh}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$\times\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta\beta'h\left(V_3t+x+\frac{\sigma}{\beta\beta'}\right)}d\sigma, \qquad (16)$$

where the initial condition

$$t=0\,; \qquad u_0=f(x) \quad\dots\dots\dots\dots\dots\dots(17)$$

for the primary waves is applied.

Again, the integrand under the first integral sign of each first term of the right-hand side of (16) is transformed as follows:

$$\frac{e^{2i\beta\beta'hH'}e^{i\beta hH(1-\beta')}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$=\frac{e^{i\beta hH(\beta'-1)}}{(1+\alpha)(1+\alpha')+(1-\alpha)(1-\alpha')e^{-2i\beta hH}+(1+\alpha)(1-\alpha')e^{-2i\beta\beta'h(H'-H)}+(1-\alpha)(1+\alpha')e^{-2i\beta h(\beta'H'-\beta'H+H)}},$$
$$\dots\dots\dots\dots\dots(18)$$

$$\frac{e^{i\beta hH(1-\beta')}}{(1+\alpha)(1+\alpha')e^{2i\beta\beta'h(H'-H)}e^{2i\beta hH}+(1-\alpha)(1-\alpha')e^{2i\beta\beta'h(H'-H)}+(1+\alpha)(1-\alpha')e^{2i\beta hH}+(1-\alpha)(1+\alpha')}$$
$$=\frac{e^{i\beta h(\beta'H-2\beta'H'-H)}}{(1+\alpha)(1+\alpha')+(1-\alpha)(1-\alpha')e^{-2i\beta hH}+(1+\alpha)(1-\alpha')e^{-2i\beta\beta'h(H'-H)}+(1-\alpha)(1+\alpha')e^{-2i\beta h(\beta'H'-\beta'H+H)}}.$$
$$\dots\dots\dots\dots\dots(19)$$

The proper expansion of the factor corresponding to the denominator of the above expressions becomes

$$\{(1+\alpha)(1+\alpha')+(1-\alpha)(1-\alpha')e^{-2i\beta hH}+(1+\alpha)(1-\alpha')e^{-2i\beta\beta' h(H'-H)}$$
$$+(1-\alpha)(1+\alpha')e^{-2i\beta h(\beta' H'-\beta' H+H)}\}^{-1}$$

$$=(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}$$

$$\times\sum_{p=0}^{n}\frac{n!}{p!(n-p)!}\,\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}e^{2ih\beta\{(p-m)H+n\beta' H-n\beta' H'\}}. \quad (20)$$

When we put

$$M=(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}\sum_{p=0}^{n}\frac{n!}{p!(n-p)!}, \quad ..(21)$$

the expression (16) of the displacement of the upper layer becomes

$$u''=\frac{2}{\pi}\int_{-\infty}^{\infty}M\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}e^{ih\beta\{(2p-2m-1)H+(2n+1)\beta' H-2n\beta' H'\}}dh$$
$$\times\int_{-\infty}^{\infty}f(\sigma)e^{i\beta\beta' h\left(V_3 t-x+\frac{\sigma}{\beta\beta'}\right)}d\sigma$$
$$+\frac{2}{\pi}\int_{-\infty}^{\infty}M\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}e^{ih\beta\{(2p-2m-1)H+(2n+1)\beta' H-2(n+1)\beta' H'\}}dh$$
$$\times\int_{-\infty}^{\infty}f(\sigma)e^{i\beta\beta' h\left(V_3 t+x+\frac{\sigma}{\beta\beta'}\right)}d\sigma. \quad ..(22)$$

3. Let the form of the primary waves when $t=0$, that proceed upwards, be

$$\left.\begin{aligned} u_1&=\cos cx, && [-a<x<0] \\ &=0, && [0>x] \\ &=0; && [-a<x] \end{aligned}\right\} \quad ....(23)$$

then, integrating (13) and (22) with respect to σ, and again integrating with respect to h, we obtain the expressions of the displacement of the primary waves for any time and of the vibratory movement of the upper layer as follows:

$$\left.\begin{aligned} u_1&=\cos c(V_1 t-x), && [(V_1 t-a)<x<V_1 t] \\ &=0, && [V_1 t<x] \\ &=0; && [(V_1 t-a)>x] \end{aligned}\right\} \quad ....(24)$$

$$u''=4(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}$$
$$\times\sum_{p=0}^{n}\frac{n!}{p!(n-p)!}\,\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}\;f(m,n,p)$$

$$\times \cos c\beta\{\beta'(V_3t-x)+(2p-2m-1)H+(2n+1)\beta'H-2n\beta'H'\}$$

$$+4(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!\,(m-n)!}$$

$$\times\sum_{p=0}^{n}\frac{n!}{p!\,(n-p)!}\,\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}\;\varphi(m,n,p)$$

$$\times \cos c\beta\{\beta'(V_3t+x)+(2p-2m-1)H+(2n+1)\beta'H-2(n+1)\beta'H'\},\qquad\ldots\ldots\ldots\ldots(25)$$

where
$$f(m,n,p)=1,\quad\left[\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2n\beta'H'}{\beta'}\right\}>x>\left\{V_3t+\left\{\frac{(2p-2m-1)H+(2n+1)\beta'H-2n\beta'H'}{\beta'}-\frac{a}{\beta\beta'}\right\}\right]$$

$$=0,\quad\left[\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2n\beta'H'}{\beta'}\right\}<x\right]$$

$$=0,\quad\left[\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2n\beta'H'}{\beta'}-\frac{a}{\beta\beta'}\right\}>x\right]$$

$$\varphi(m,n,p)=1,\quad\left[-\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2(n+1)\beta'H'}{\beta'}\right\}<x<-\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2(n+1)\beta'H'}{\beta'}-\frac{a}{\beta\beta'}\right\}\right]$$

$$=0,\quad\left[-\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2(n+1)\beta'H'}{\beta'}\right\}>x\right]$$

$$=0.\quad\left[-\left\{V_3t+\frac{(2p-2m-1)H+(2n+1)\beta'H-2(n+1)\beta'H'}{\beta'}-\frac{a}{\beta\beta'}\right\}<x\right]$$

The values of u_1 at $x=0$ and u'' at $x=H'$ (free surface) are plotted in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. In these figures Figs. 2, 3, 4, 5, 6, are related to the transmission of dilatational waves so that the values $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec were used in these cases; while Figs. 7, 8, 9, 10, 11 are concerned with the propagation of distortional waves so that the values $V_1=4{\cdot}45$ km/sec, $V_2=3{\cdot}70$ km/sec, $V_3=3{\cdot}15$ km/sec were employed. Fig. 2 gives the case of wave length $L=\frac{2\pi}{c}=10$ km, and similarly Fig. 3 $L=40$ km; Fig. 4 $L=60$ km; Fig. 5 $L=100$ km; Fig. 6 $L=150$ km. The remaining figures correspond with the propagation of distortional waves and Fig. 7 gives the case of wave length $L=\frac{2\pi}{c}=6$ km; Fig. 8 $L=24$ km; Fig. 9 $L=36$ km; Fig. 10 $L=60$ km; Fig. 11 $L=90$ km. In all cases we have assumed that $a=2L$.

4. Let the form of the primary waves when $t=0$, that proceed upwards, be

$$u_1 = e^{-\frac{x^2}{c^2}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

then, integrating (12) and (21) with respect to σ, and again integrating with respect to h, we get the expressions of the displacement of the primary waves for any time and of the movement of the upper layer as follows:

$$u_1 = e^{-\frac{(V_1 t - x)^2}{c^2}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and

$$u'' = 4(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}$$

$$\times\sum_{p=0}^{n}\frac{n!}{p!(n-p)!}\,\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}$$

$$\times e^{\frac{-\beta^2\{\beta'(V_3 t-x)+(2p-2m-1)H+(2n+1)\beta' H-2n\beta' H'\}^2}{c^2}}$$

$$+4(1+\alpha)^{-1}(1+\alpha')^{-1}\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}$$

$$\times\sum_{p=0}^{n}\frac{n!}{p!(n-p)!}\,\frac{(1-\alpha)^{m-p}(1-\alpha')^{m+p-n}}{(1+\alpha)^{m-p}(1+\alpha')^{m+p-n}}$$

$$\times e^{\frac{-\beta^2\{\beta'(V_3 t+x)+(2p-2m-1)H+(2n+1)\beta' H-2(n+1)\beta' H'\}^2}{c^2}}. \quad (28)$$

The fact, that the expressions of displacements in (25), (28) are of series forms, is not only due to mathematical convenience, but also due to the nature that the series forms indicate the successive formation of the groups of waves by the multiple reflection at the discontinuities.

The values of u_1 at $x=0$ and u'' at $x=H'$ (free surface) are plotted in Figs. 12, 13, 14, 15, 16, 17, 18, 19, 20, 21. In these figures Figs. 12, 13, 14, 15, 16 give the cases of dilatational waves and correspond respectively to the cases of $c=5$ km, 20 km, 30 km, 50 km, 75 km; while Figs. 17, 18, 19, 20, 21 those of distortional waves and correspond respectively to the cases $c=3$ km, 12 km, 18 km, 30 km, 45 km.

5. It will be seen from these figures that, even when the primary waves are of a regular harmonic type or of a type of a single shock, the movements on the surface of the ground are oscillatory in complicated forms as in the cases of preceding papers. These complicated oscillations, however, may be separated in successive groups, each of which corresponds to the effect of reflection of waves at each discontinuous surface of the layer. Thus, the periodicity of occurrence of the successive groups depends on the thicknesses of the strata and on the proper velocities in these strata. In the present case, too, the successive groups become smaller and smaller. The intermittence or overlapping of the groups of oscillations according with the ratio of the thicknesses of strata to the length of disturbed portion abides also by the same law as shown in preceding papers.

When the primary disturbance is of a regular harmonic type of a finite extent, each group of the oscillations at the surface may be generally identified by the small steps superposed on the principal oscillatory movement, while in the case of the primary disturbance of a type of a single shock, all fluctuating amplitudes at the surface may be taken as the respective groups. From this consideration it may be concluded that, though in the case of the primary disturbance of the type of a single shock the vibratory movements on the surface indicate the sign of free oscillations of strata, the oscillations on the surface due to primary waves of some oscillatory type are by no means the consequence of self-oscillations of the strata, but each interval between small steps superposed on the principal vibrations corresponds, indeed, with the period of free oscillations of a stratum or of strata.

In actual seismic waves the primary disturbance, either of a pure harmonic type or of a single shock, is of rare occurrence and some intermediate type of disturbance between the above two cases are expected to take place, so that the category belonging to each individual case would not be of a perfect use. As, however, the two cases give critical types of the free-oscillations of strata, it seems possible that the moving type of the ground in general may also be intermediate between the above two.

6. This article is not directly connected with the main part of the present paper. The problem of the transmission of dilatational disturbance of a harmonic type of a finite extent in a body having a single surface layer was dealt with as a supplement to one[4] of our preceding papers.

Let the axis of x be drawn upwards from the lower boundary of a surface layer, and let u be the displacement of the bottom medium and ρ, λ, μ the density and Lamé's elastic constants of this medium. The similar displacement, density and elastic constants of the surface layer of thickness H are to be expressed by u', ρ', λ', μ'. In the same manner

4) K. SEZAWA, *loc. cit.*, in foot note 3).

of mathematical treatment as in the preceding paper[5] we arrive at

$$u_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t-x+\sigma)}d\sigma, \quad \ldots\ldots\ldots\ldots(29)$$

$$u_2=\frac{1}{2\pi}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m+1}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t+x+\sigma)}d\sigma$$
$$+\frac{1}{2\pi}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m}\int_{-\infty}^{\infty}e^{-2i(m+1)\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t+x+\sigma)}d\sigma, \ [\alpha<1]\ldots(30)$$

$$u'=\frac{1}{\pi}\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{i\beta h\left(V_2t-x+\frac{\sigma}{\beta}\right)}d\sigma$$
$$+\frac{1}{\pi}\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\int_{-\infty}^{\infty}e^{-2i(m+1)\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{i\beta h\left(V_2t+x+\frac{\sigma}{\beta}\right)}d\sigma, \ [\alpha<1]$$
$$\ldots\ldots\ldots(31)$$

$$u_2=-\frac{1}{2\pi}\sum_{m=0}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^{m+1}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t+x+\sigma)}d\sigma$$
$$+\frac{1}{2\pi}\sum_{m=0}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^{m}\int_{-\infty}^{\infty}e^{-2i(m+1)\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{ih(V_1t+x+\sigma)}d\sigma, \ [\alpha>1]\ (32)$$

$$u'=\frac{1}{\pi}\sum_{m=0}^{\infty}\frac{(\alpha-1)^m}{(\alpha+1)^{m+1}}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{i\beta h\left(V_2t-x+\frac{\sigma}{\beta}\right)}d\sigma$$
$$+\frac{1}{\pi}\sum_{m=0}^{\infty}\frac{(\alpha-1)^m}{(\alpha+1)^{m+1}}\int_{-\infty}^{\infty}e^{-2i(m+1)\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)e^{i\beta h\left(V_2t+x+\frac{\sigma}{\beta}\right)}d\sigma, \ [\alpha>1]\quad(33)$$

where

$$\left.\begin{aligned}&V_1=\sqrt{\frac{\lambda+2\mu}{\rho}}, \quad V_2=\sqrt{\frac{\lambda'+2\mu'}{\rho'}}, \quad h=p/V_1,\\ &\alpha=\sqrt{\frac{\rho'(\lambda'+2\mu')}{\rho(\lambda+2\mu)}}, \quad \beta=\sqrt{\frac{\rho'(\lambda+2\mu)}{\rho(\lambda'+2\mu')}},\end{aligned}\right\}\ldots\ldots(34)$$

u_1 is incident waves, u_2 reflected waves in the bottom medium and $f(x)$ is the type of the original waves. Now we take the initial disturbance of the type,

$$t=0; \quad f(x)=\cos cx, \quad [-a<x<0] \quad \ldots\ldots\ldots\ldots(35)$$

then, substituting this in (29), (30), (31), (32), (33), we find

$$\left.\begin{aligned}u_1&=\cos c(V_1t-x), &&[(V_1t-a)<x<V_1t]\\ &=0, &&[V_1t<x]\\ &=0, &&[(V_1t-a)>x]\end{aligned}\right\}\ldots\ldots\ldots(36)$$

5) K. SEZAWA, *loc. cit.*, in foot note 3).

$$u_2=\sum_{m=0}^{\infty} f(m)(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m+1}\cos c(V_1t+x-2m\beta H)$$
$$+\sum_{m=0}^{\infty}\varphi(m)(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\cos c\{V_1t+x-2(m+1)\beta H\},\ldots\ldots(37)$$

where

$$f(m)=1,\quad [-(V_1t-2m\beta H)<x<-(V_1t-2m\beta H-a)]$$
$$=0,\quad [-(V_1t-2m\beta H)>x]$$
$$=0,\quad [-(V_1t-2m\beta H-a)<x]$$
$$\varphi(m)=1,\quad [-\{V_1t-2(m+1)\beta H\}<x<-\{V_1t-2(m+1)\beta H-a\}]$$
$$=0,\quad [-\{V_1t-2(m+1)\beta H\}>x]$$
$$=0,\quad [-\{V_1t-2(m+1)\beta H-a\}<x]$$

$$u'=2\sum_{m=0}^{\infty} f(m)(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\cos c\beta(V_2t-x-2mH)$$
$$+2\sum_{m=0}^{\infty}\varphi(m)(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\cos c\beta\{V_2t+x-2(m+1)H\},\ldots.(38)$$

where

$$f(m)=1,\quad \left[(V_2t-2mH)>x>\left(V_2t-2mH-\frac{a}{\beta}\right)\right]$$
$$=0,\quad [V_2t-2mH)<x]$$
$$=0,\quad \left[\left(V_2t-2mH-\frac{a}{\beta}>x\right]\right.$$
$$\varphi(m)=1,\quad \left[-\{V_2t-2(m+1)H\}<x<-\left\{V_2t-2(m+1)H-\frac{a}{\beta}\right\}\right]$$
$$=0,\quad [-\{V_2t-2(m+1)H\}>x]$$
$$=0.\quad \left[-\left\{V_2t-2(m+1)H-\frac{a}{\beta}\right\}<x\right]$$

A few numerical examples are shown in Figs. 22, 23, 24, 25, 26, 27. In Figs. 22, 23, 24 the cases of $H=\frac{L}{2}$, $H=L$, $H=2L$, besides the condition that $\alpha=\frac{1}{2}$, $\beta=1$, $a=2L$ in common, are plotted respectively, while in Figs. 25, 26, 27 the cases of $H=\frac{L}{2}$, $H=L$, $H=2L$, besides the conditions that $\alpha=2$, $\beta=1$, $a=2L$ in common, are illustrated.

In conclusion we are indebted to Dr. Nasu who has shown us the proper values of the constants due to Prof. Imamura and Prof. Matuzawa concerning the strata of the ground.

21. 地震波によつて土地の固有振動が誘起される可能度 第四報

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この論文は著者等が以前からやつて居る研究の續きであつて、ここでは地表に二つの層がある時に下から垂直に縱波及横波が來る時の地表の振動のしかたを論じたものである。表面の二つの層の性質に關しては種々說があるかも知れぬけれども、今村、松澤兩博士其他の人々の結果が大體に於て正しいものと見て其を適當に調節して下の如く假定した。

層	厚さ	密度	縱波の速度	横波の速度
表面層（花崗岩層）	20 km	2·7	5·0 km/sec	3·15 km/sec
第二層（玄武岩層）	30 km	3·0	6·1 km/sec	3·70 km/sec
下　層（超鹽基性）	∞	3·5	7·5 km/sec	4·45 km/sec

この樣な狀態にある層狀體に下方から有限長の調和波と衝擊型單波とを送る場合を考へた。但し調和波と云つても層の厚さに比較して種々の波長を有する場合を計算し、又衝擊型波動についても其の有效長の種々の場合をあたつて見たのである。

有限長調和波を送る時に生ずる表面の振動は非常に複雜ではあるけれども、分析して見ると以前の研究の時と同じく、順々の波動群になり、この波動群が漸次小振幅となるものである。而して此等波動群は計算の結果を圖示せるものに現れて居る樣な所々の階段狀のものがそれであつて、大なる繰替しは始めからの振動である。從て層の固有振動に當るものも階段狀の所から次又は其次の階段狀の所までが其週期であつて、大なる繰替しの週期は始めからの振動週期である。

次に送られる波動が單一の衝擊型の時には、次々の振動繰替しが夫々の波動群に相當するものであり、從て此等の振動繰替しの週期が層の大體の固有振動週期であるといふ事が出來る。この場合にも次々の振動が漸次小振幅となる事は他の場合と大差がないのである。

實際の地震波では上述の二つの極限の間にある種々の狀態の波動の事が多いに違ひない。それ故表面の振動型や地層の固有振動の現れ方も大體前記二種の場合の間にある事が多い事と思はれる。從つて固有振動を推定する爲の分析は可なり困難を伴ふかも知れない。それにしても上の樣に兩極端がわかつて居る以上は大凡の範圍限界は想像がつく事と思ふ。

尙この論文の末尾には以前の論文の補ひとして、表面層が一つしかなく原波動が有限長の調和波である場合を附加へて置いた。その性質についての委しい說明は省略する事にした。

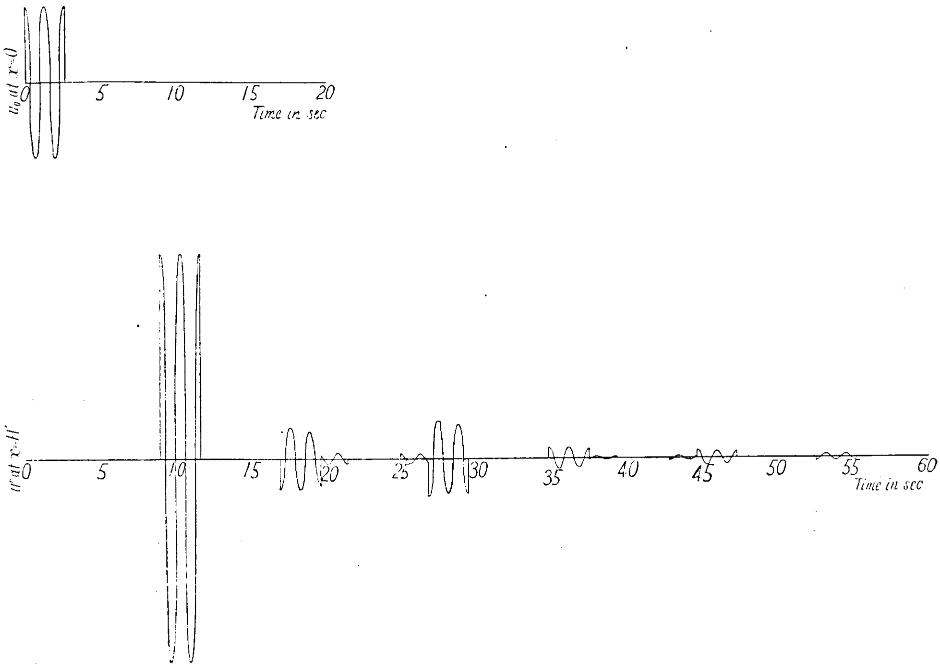

Fig. 2. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $L=10$ km.

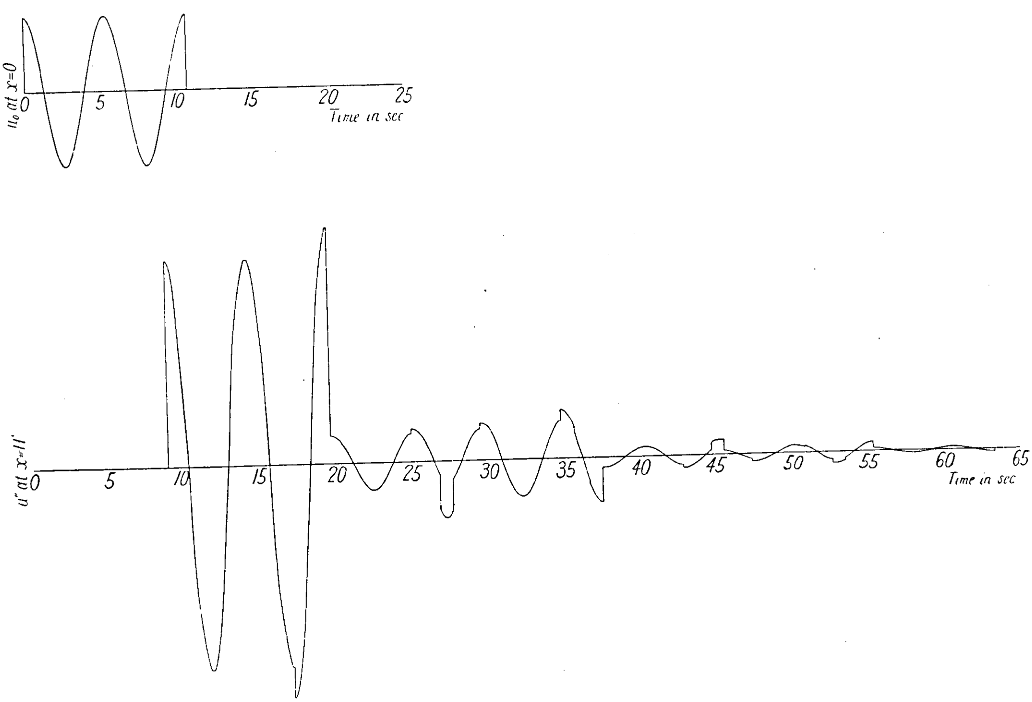

Fig. 3. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $L=40$ km.

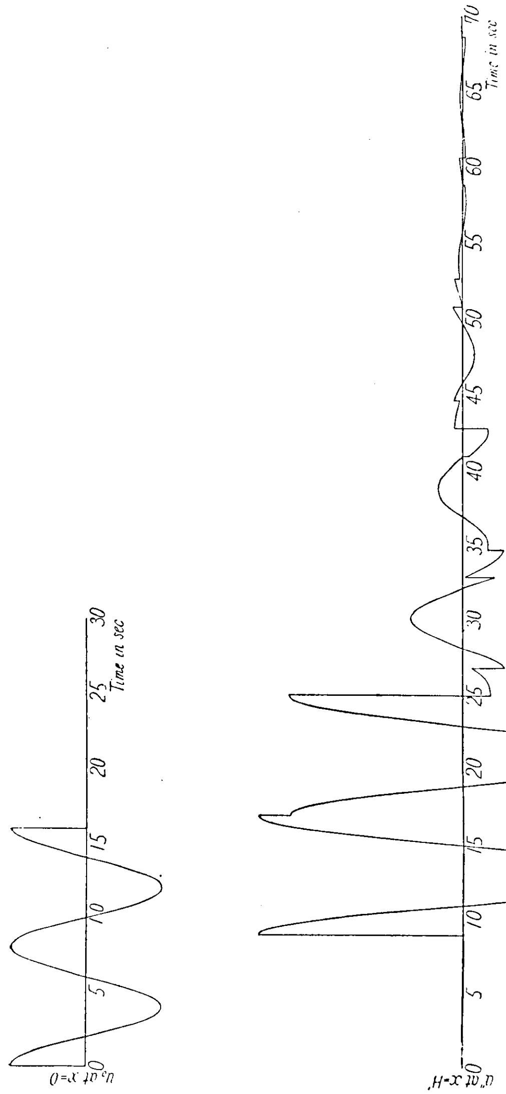

Fig. 4. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $L=60$ km.

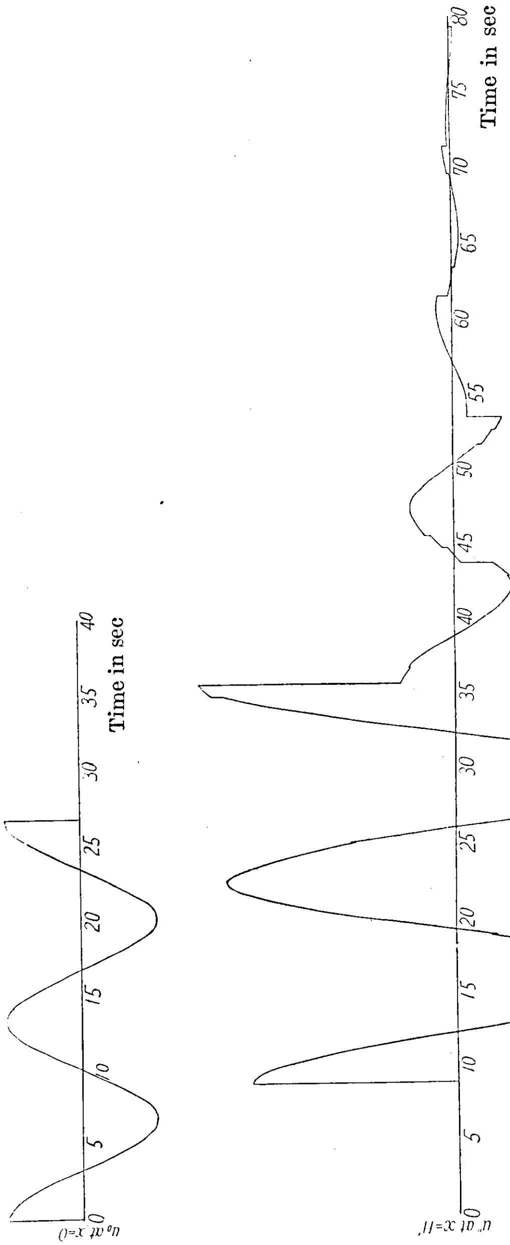

Fig. 5. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $L=100$ km.

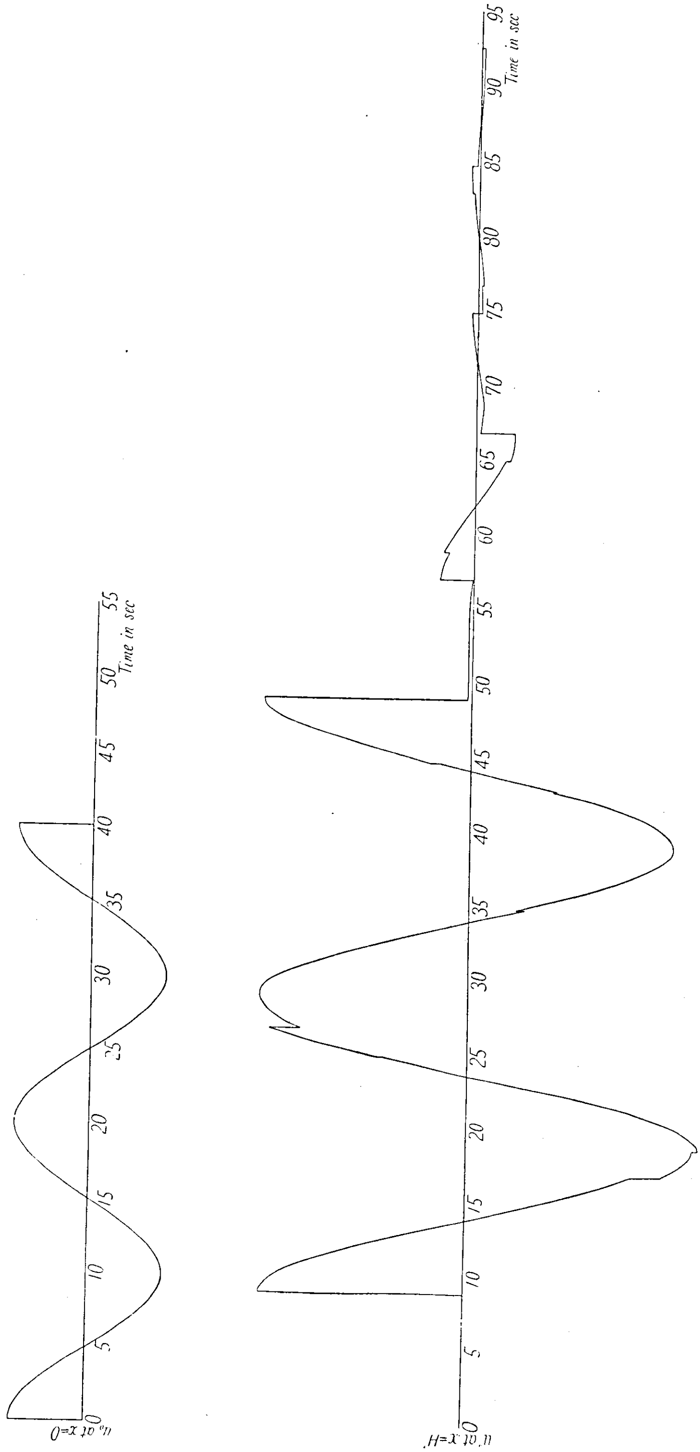

Fig. 6. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $L=150$ km.

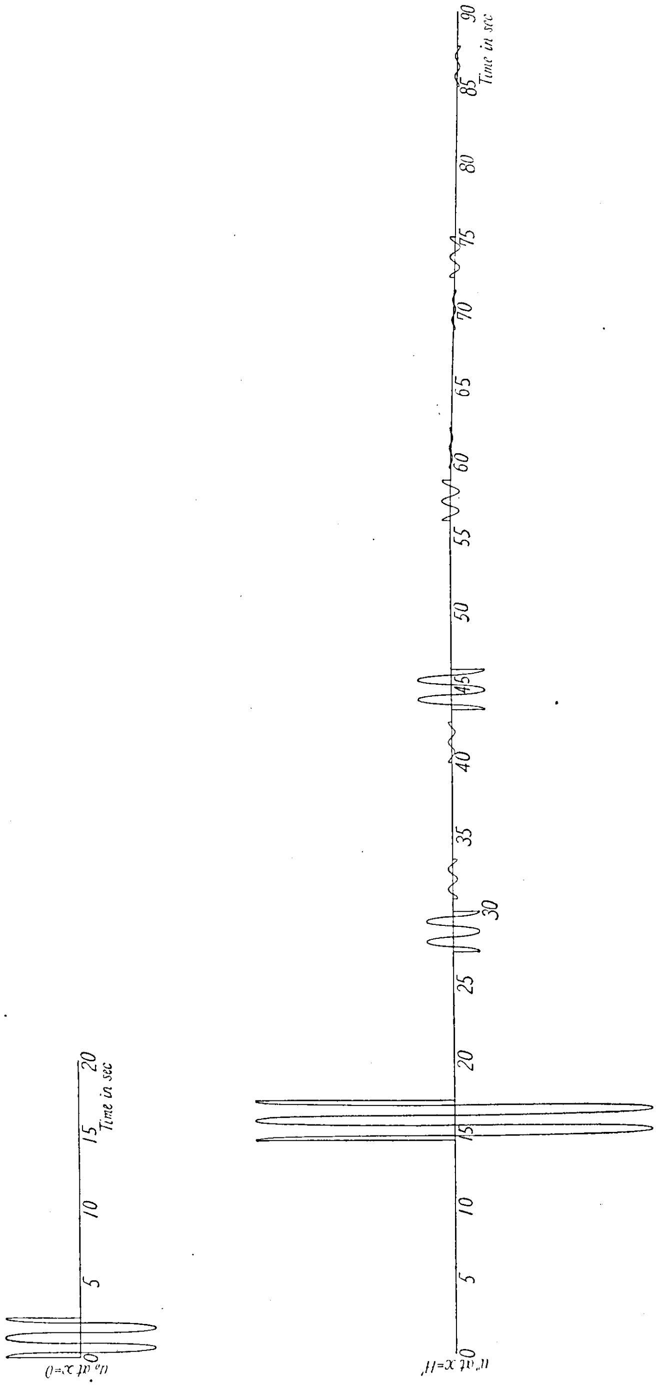

Fig. 7. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $L=6$ km.

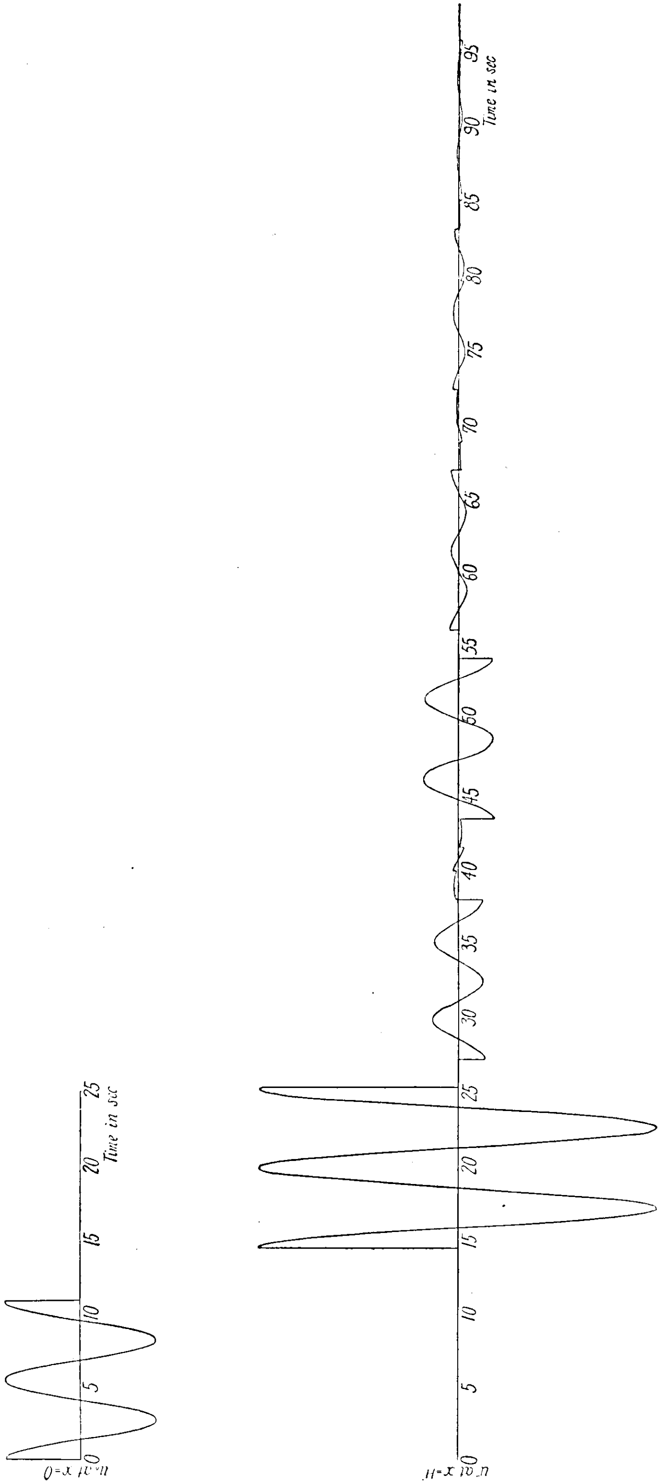

Fig. 8. $V'_1=4{\cdot}45$ km/sec, $V'_2=3{\cdot}70$ km/sec, $V'_3=3{\cdot}17$ km/sec, $L=24$ km.

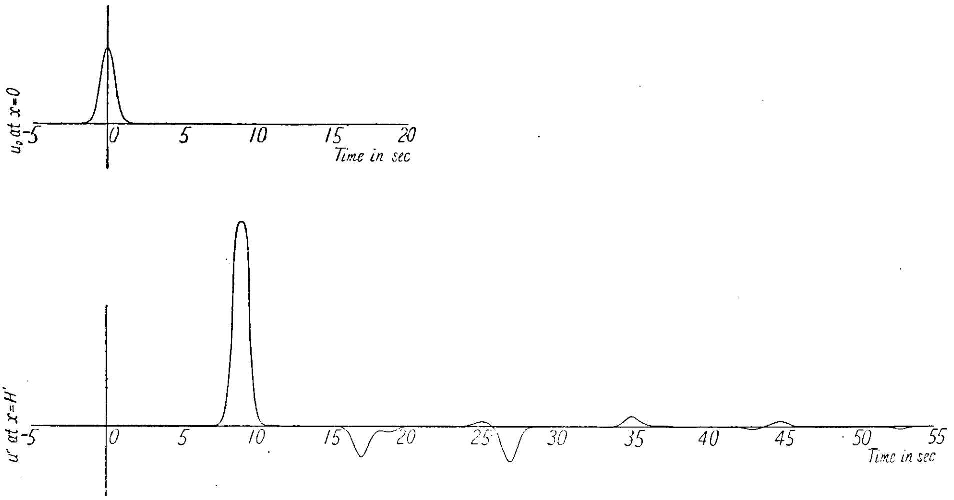

Fig. 12. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$km/sec, $c=5$ km.

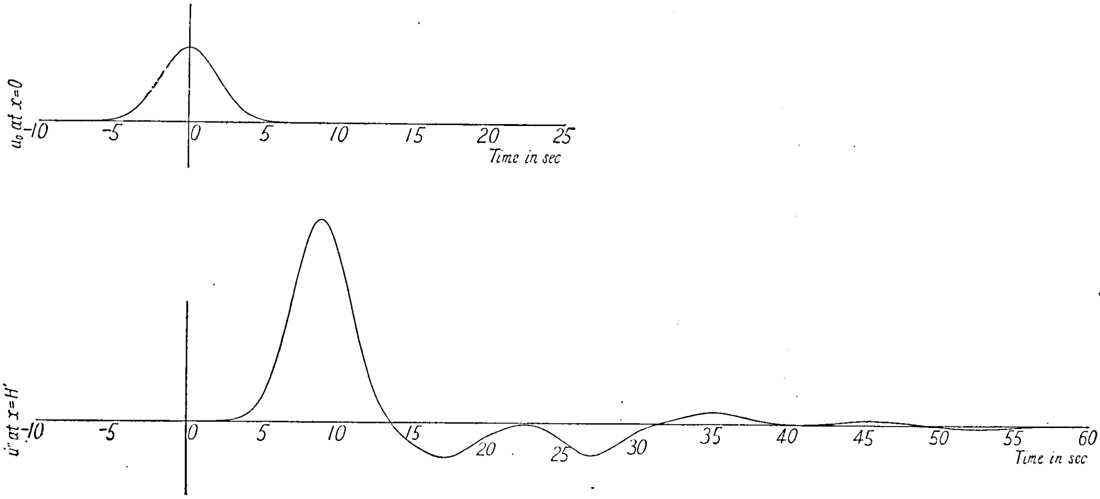

Fig. 13. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $c=20$ km.

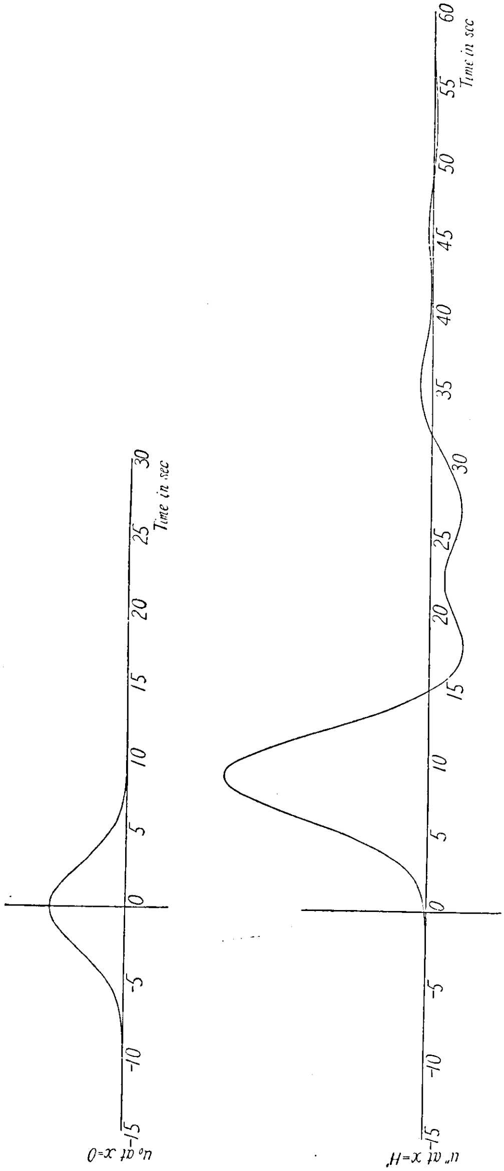

Fig. 14. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $c=30$ km.

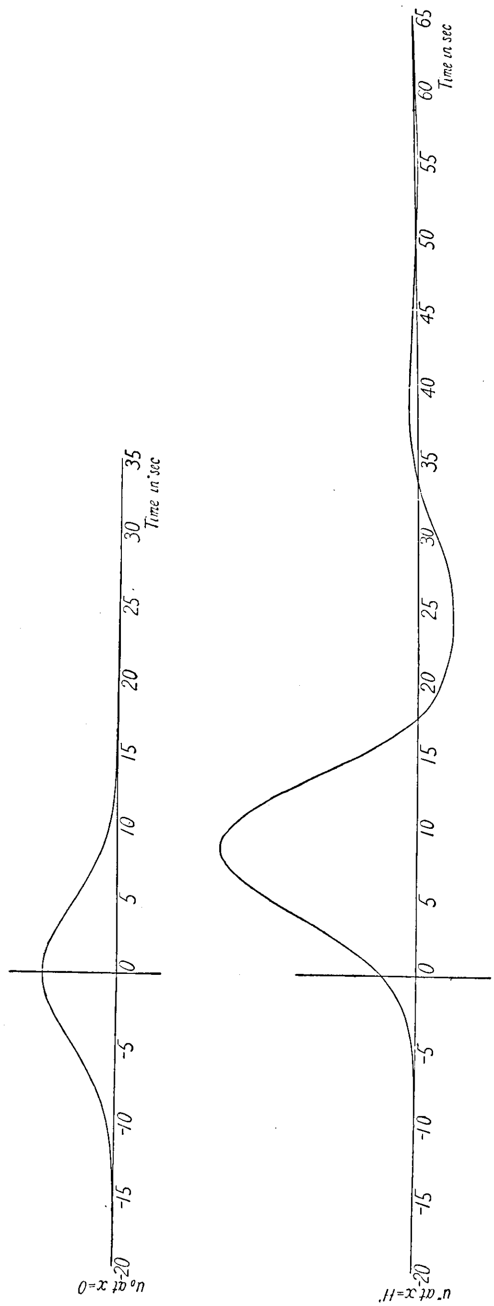

Fig. 15. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $c=50$ km.

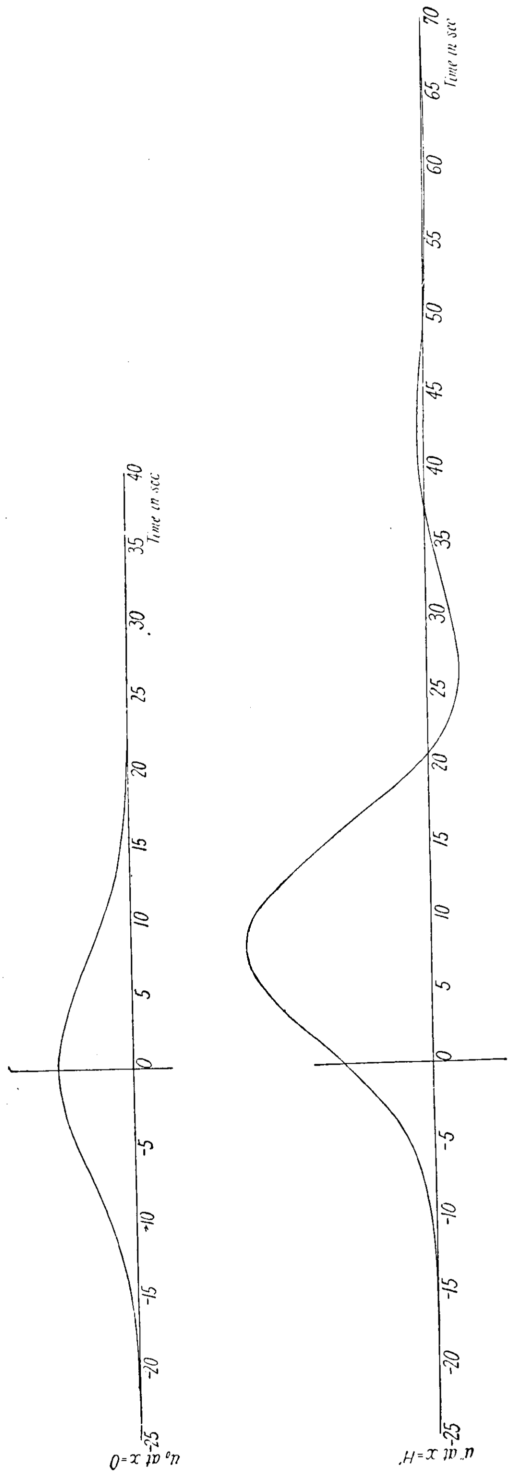

Fig. 16. $V_1=7{\cdot}5$ km/sec, $V_2=6{\cdot}1$ km/sec, $V_3=5{\cdot}0$ km/sec, $c=75$ km.

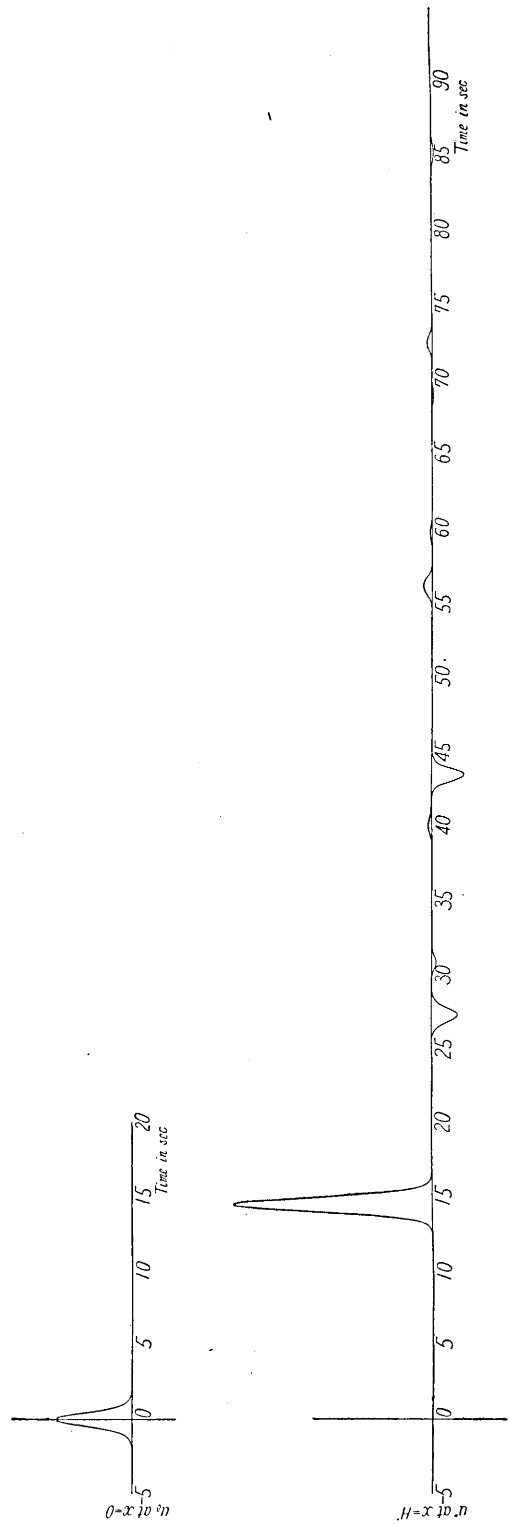

Fig. 17. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $c=3$ km

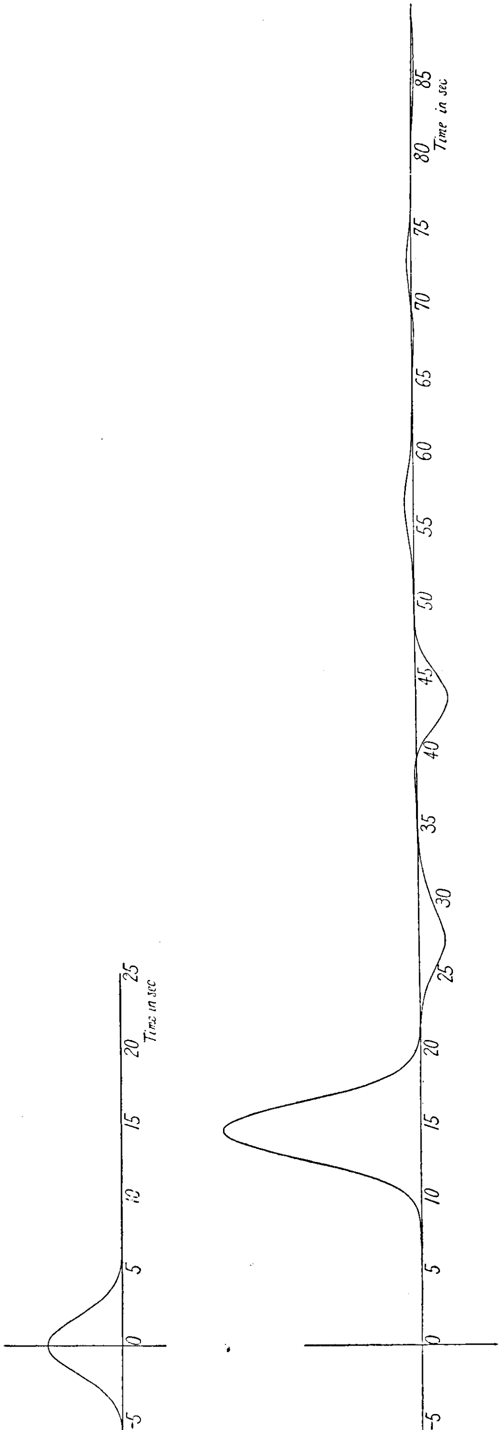

Fig. 18. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $c=12$ km.

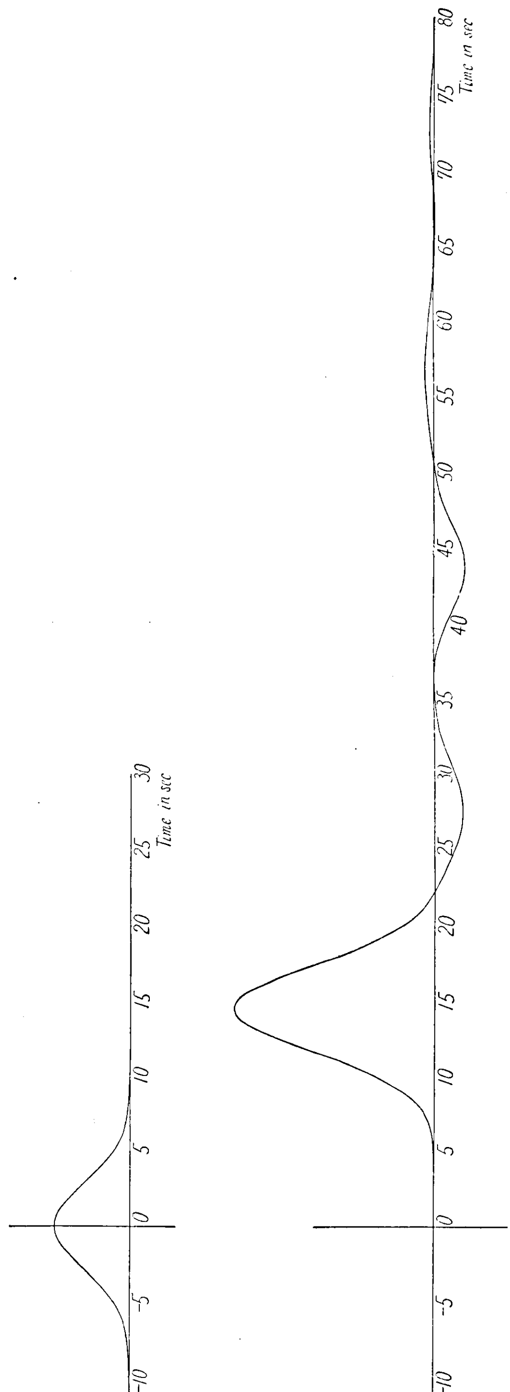

Fig. 19. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $c=18$ km.

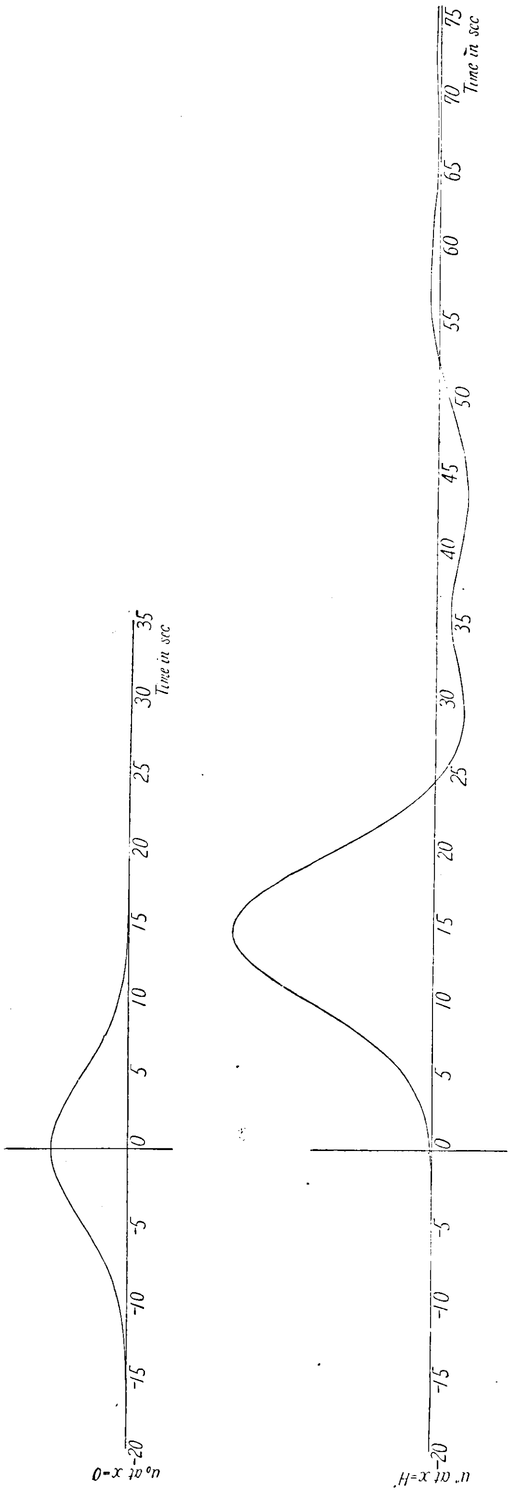

Fig. 20. $V_1'=4\cdot45$ km/sec, $V_2'=3\cdot70$ km/sec, $V_3'=3\cdot15$ km/sec, $c=30$ km.

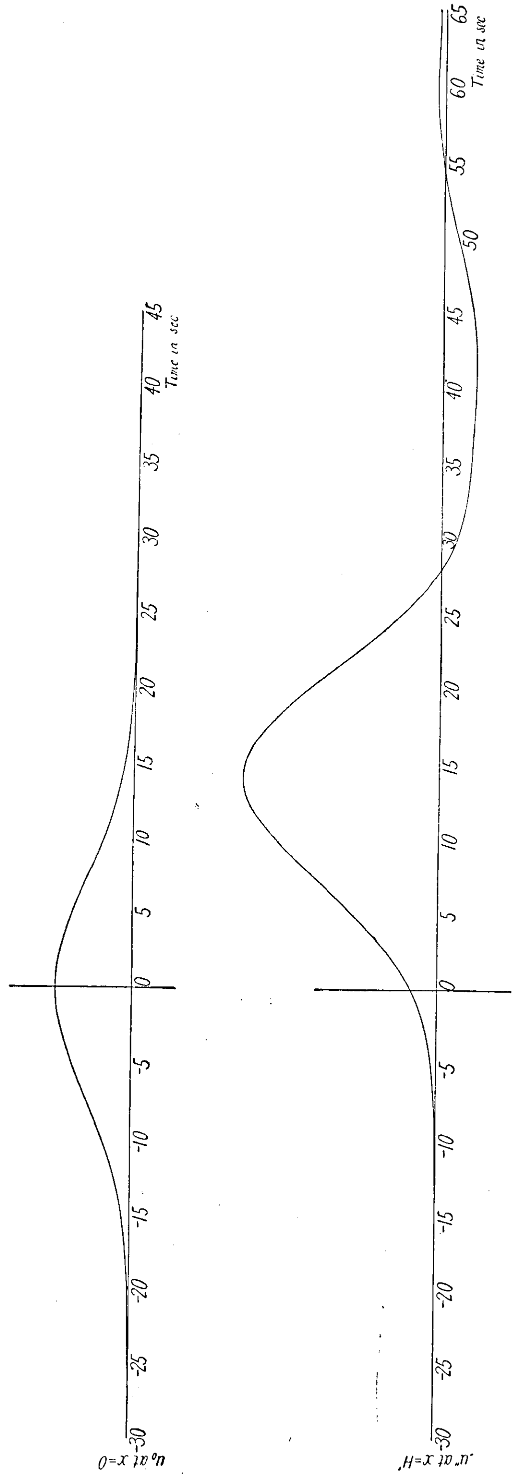

Fig. 21. $V_1'=4\cdot45$ km/sec, $V_2'=3\cdot70$ km/sec, $V_3'=3\cdot15$ km/sec, $L=45$ km.

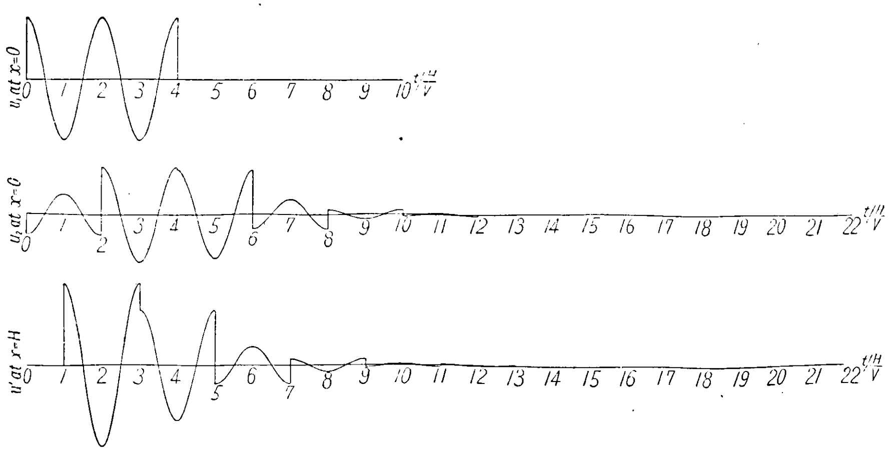

Fig. 22. $\alpha=1/2$, $\beta=1$, $H=L/2$, $a=2L$, $c=2\pi/L$.

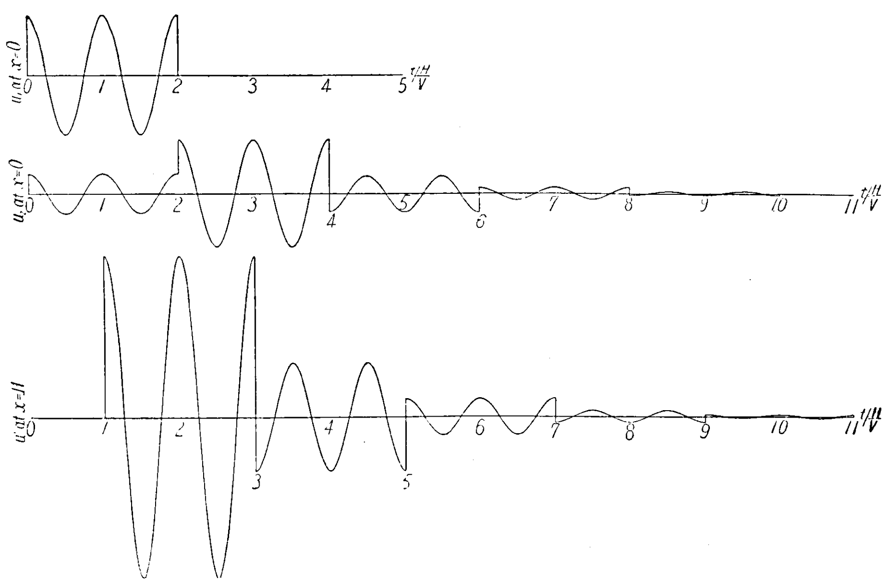

Fig. 23. $\alpha=1/2$, $\beta=1$, $H=L$, $a=2L$, $c=2\pi/L$.

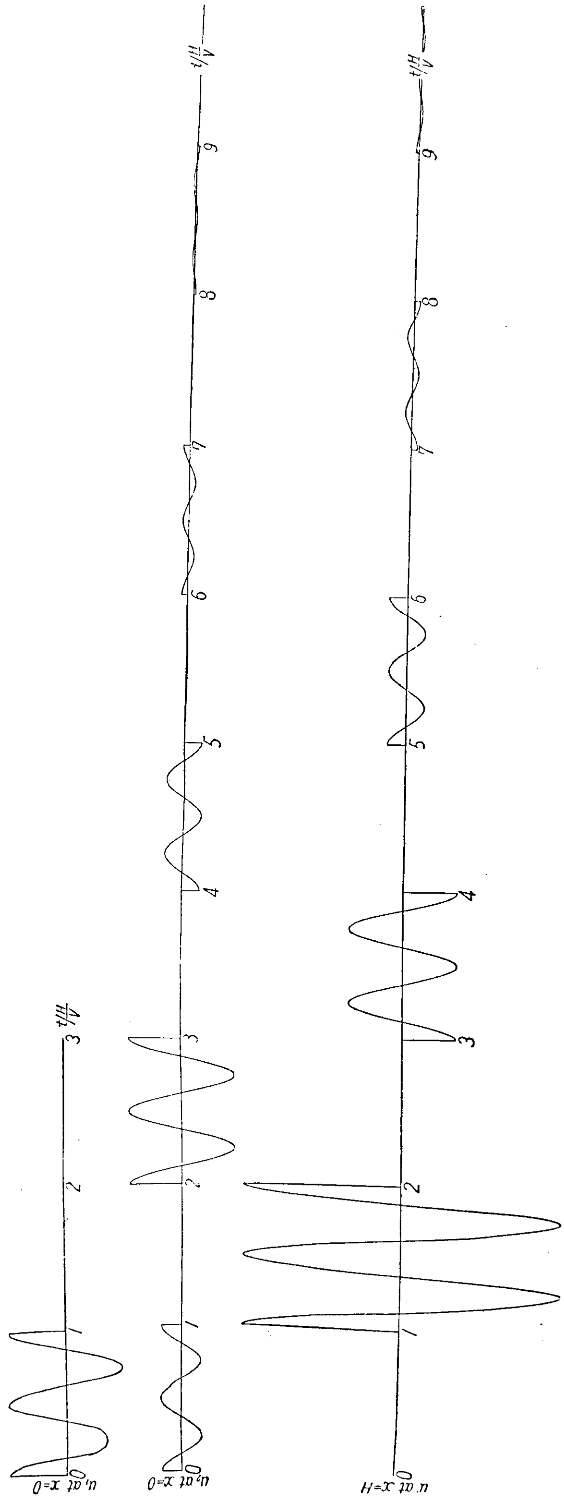

Fig. 24. $\alpha=1/2$, $\beta=1$, $H=2L$, $a=2L$, $c=2\pi/L$.

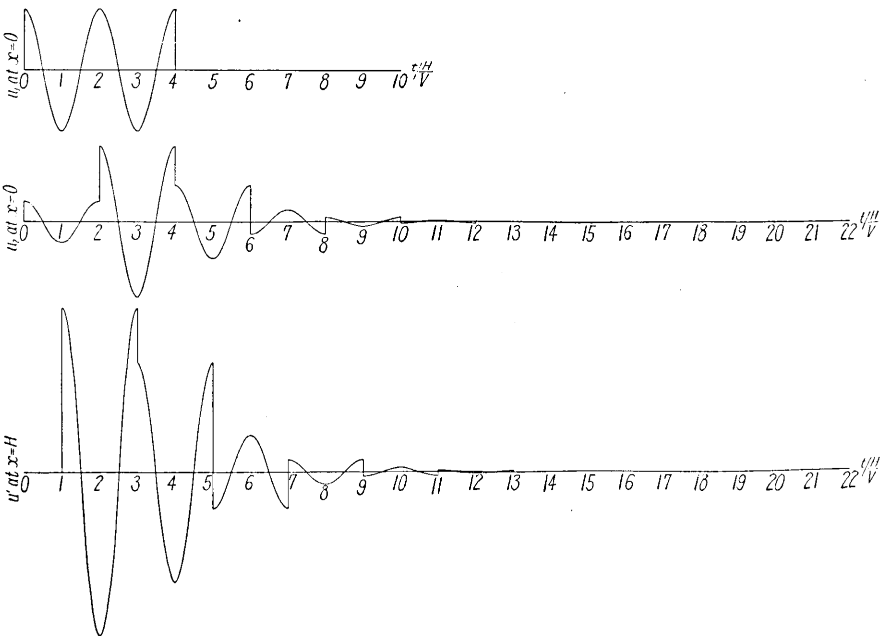

Fig. 25. $\alpha=2$, $\beta=1$, $H=L/2$, $a=2L$, $c=2\pi/L$.

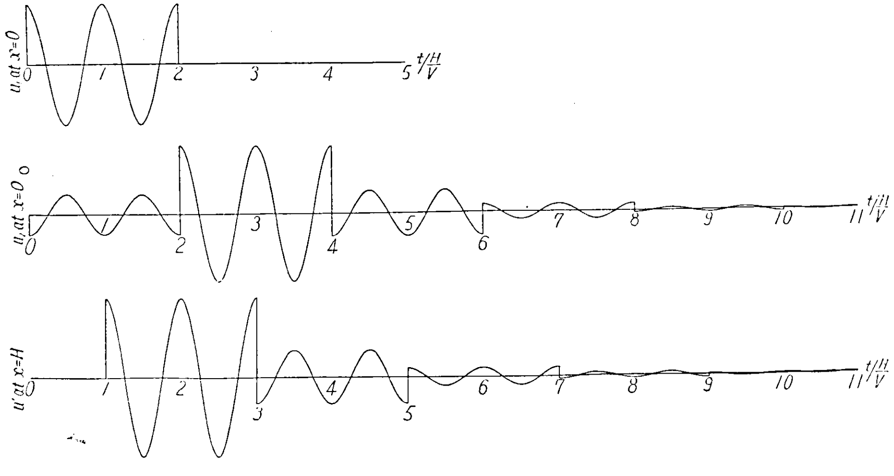

Fig. 26. $\alpha=2$, $\beta=1$, $H=L$, $a=2L$, $c=2\pi/L$.

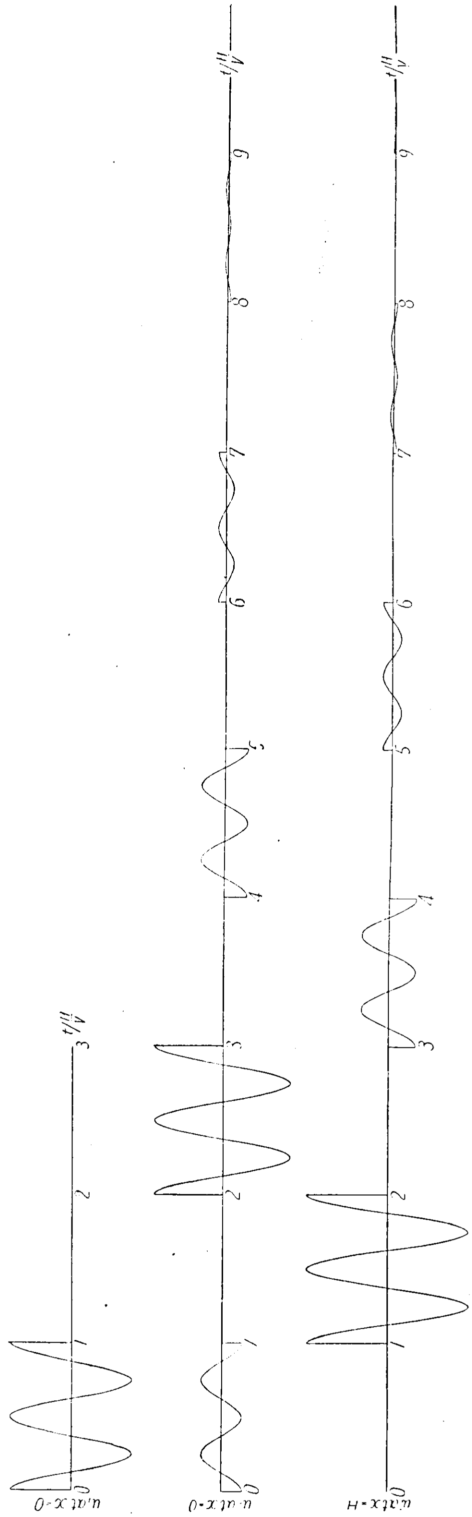

Fig. 27. $\alpha=2$, $\beta=1$, $H=2L$, $a=2L$, $c=2\pi/L$.

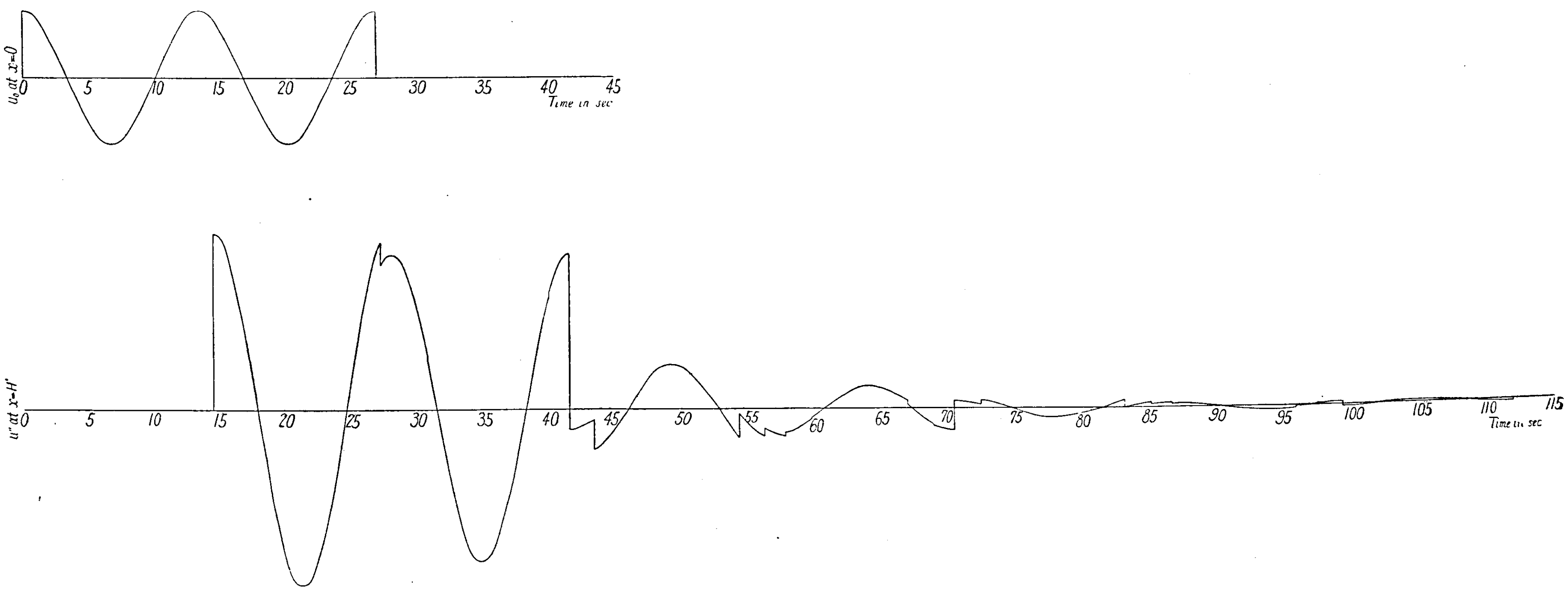

Fig. 10. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $L=60$ km.

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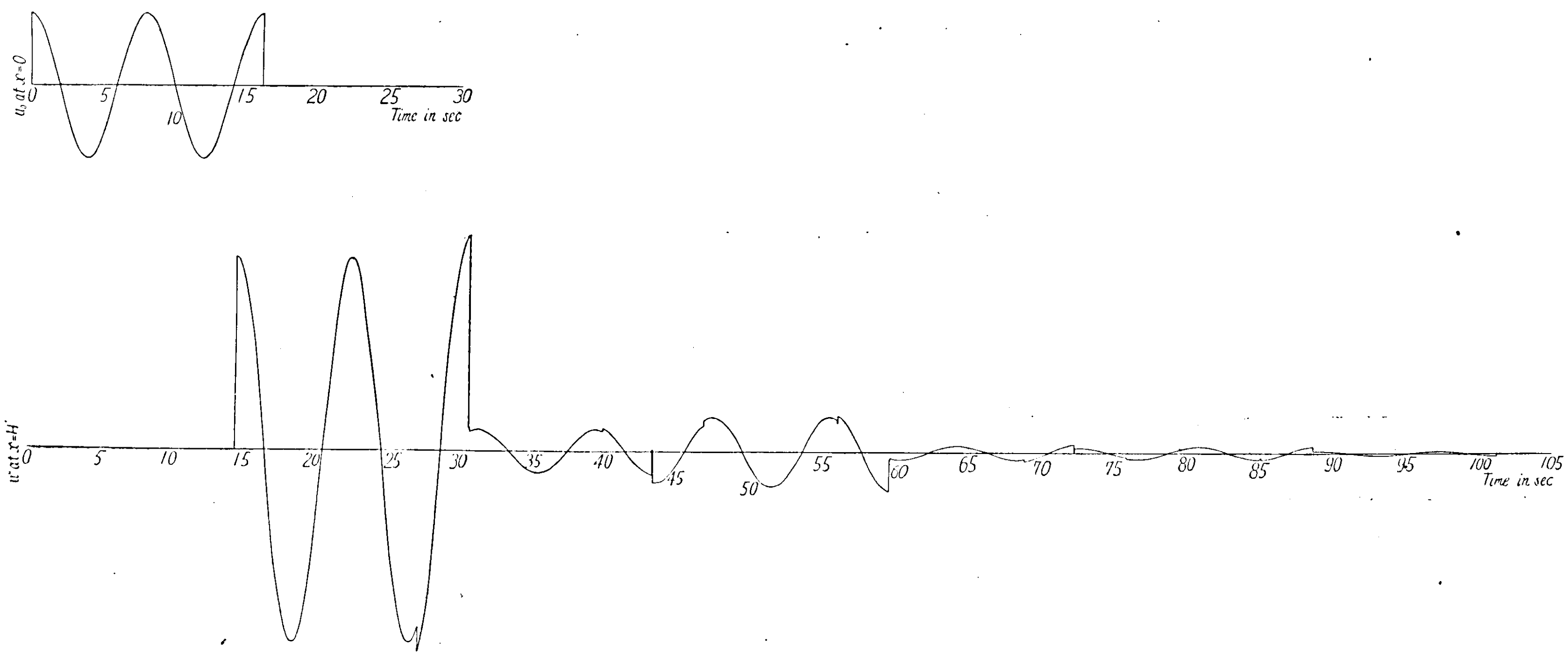

Fig. 9. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $L=36$ km.

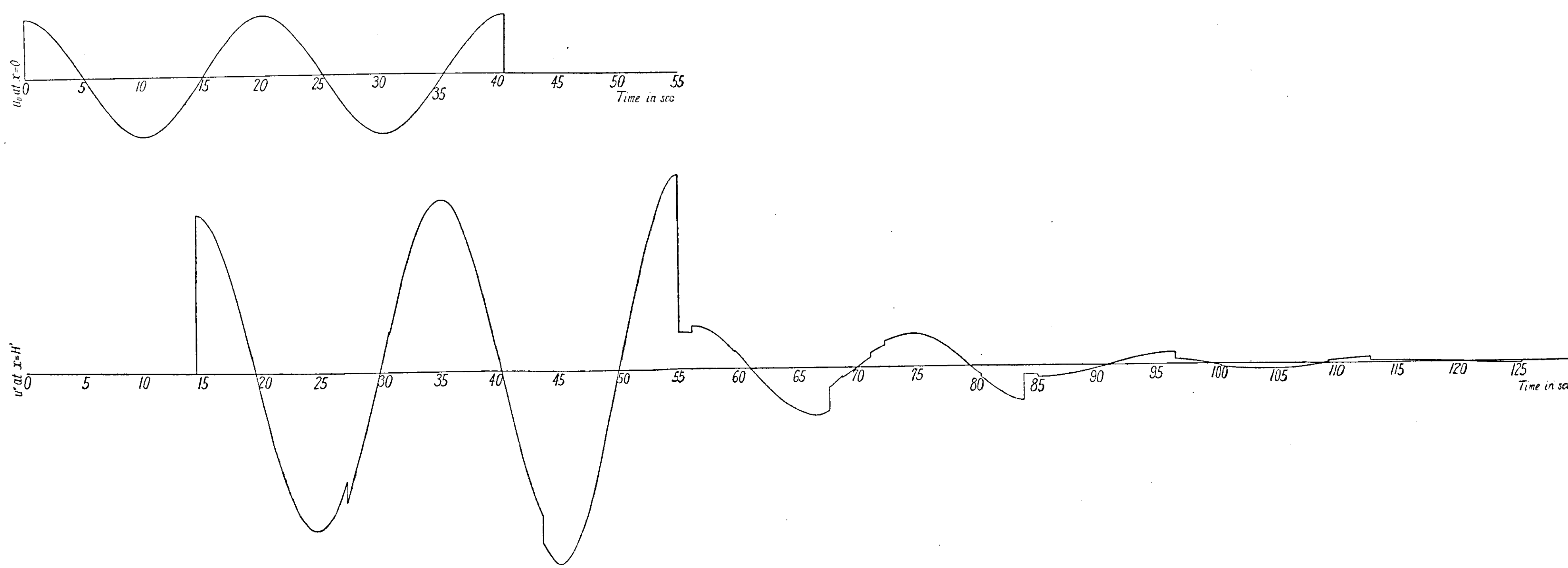

Fig. 11. $V_1'=4{\cdot}45$ km/sec, $V_2'=3{\cdot}70$ km/sec, $V_3'=3{\cdot}15$ km/sec, $L=90$ km.