22. *Amplitudes of P- and S-waves at Different Focal Distances.*

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It is well-ascertained fact that the amplitudes of P-waves are smaller than those of S-waves at an intermediate focal distance in general and this character becomes more and more distinct as the distance from the origin is increased.

Such a fact is by no means the consequence of the essential property of P- and S-waves, but due to the reason that the effective periods of original fluctuation of the amplitudes of P-waves are always shorter than those of S-waves. The shorter the periods of oscillations, the quicker is the diminution of the amplitudes. This comes out directly from the theory of propagation of waves in visco-elastic bodies.[1] The same theory also enables us to elucidate the nature that the amplitudes of S-waves rather approach those of P-waves at large focal distances where the oscillations of P-waves of small periods have almost died away and the periods of P-waves and those of S-waves become comparable. No theory without the consideration of visco-elastic waves can explain the small amplitudes of P-waves at an intermediate focal distance as well as the relatively large amplitudes of P-waves at a long distance. Reflection and refraction cannot, of course, give rise to such a result.

In the present paper we have dealt with the case where P- and S-waves of equal periods are generated from a point in a visco-elastic body and transmitted in all directions towards infinity. To get the solutions of waves of all probable cases we have considered the generation of waves of a certain distribution in different azimuth as well as different latitude. The case of some distribution has already been studied by one[2] of us a

1) K. SEZAWA, "On the Decay of Waves in Visco-Elastic Solid Bodies", *Bull. Earthq. Res. Inst.*, **3** (1927), 43-53; "Notes on Waves in Visco-Elastic Solid Bodies", *Bull. Earthq. Res. Inst.*, **10** (1932), 19-22.

2) K. SEZAWA, "Dilatational and Distortional Waves generated from a Cylindrical or a Spherical Origin", *Bull. Earthq. Res. Inst.*, **2** (1927), 13-20.

few years ago. The explanation of the smallness of the periods of the actually generated P-waves is very difficult, and we have left its study to future occasion.

For the sake of simplicity we have taken spherical coordinates. Let u, v, w be radial, colatitudinal and azimuthal components of displacement. The equations of motion of the body are expressed by

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2}&=\left\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\frac{\partial\Delta}{\partial r}\\ &\qquad-\left(2\mu+2\mu'\frac{\partial}{\partial t}\right)\left\{\frac{1}{r\sin\theta}\frac{\partial(\varpi_\phi\sin\theta)}{\partial\theta}-\frac{1}{r\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi}\right\},\\ \rho\frac{\partial^2 v}{\partial t^2}&=\left\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\frac{1}{r}\frac{\partial\Delta}{\partial\theta}\\ &\qquad-\left(2\mu+2\mu'\frac{\partial}{\partial t}\right)\left\{\frac{1}{r\sin\theta}\frac{\partial\varpi_r}{\partial\phi}-\frac{1}{r}\frac{\partial(r\varpi_\phi)}{\partial r}\right\},\\ \rho\frac{\partial^2 w}{\partial t^2}&=\left\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\frac{1}{r\sin\theta}\frac{\partial\Delta}{\partial\phi}\\ &\qquad-\left(2\mu+2\mu'\frac{\partial}{\partial t}\right)\left\{\frac{1}{r}\frac{\partial(r\varpi_\theta)}{\partial r}-\frac{1}{r}\frac{\partial\varpi_r}{\partial\theta}\right\}. \end{aligned}\right\}\quad\ldots.(1)$$

In these equations r, θ, ϕ are the radial distance, colatitude, and azimuth, the origin being taken at seismic focus; ρ the density: λ, μ Lamé's elastic constants; λ', μ' dilatational and shearing solid viscosities; while Δ, ϖ_r, ϖ_θ, ϖ_ϕ have the meanings as expressed in the following forms:

$$\left.\begin{aligned} \Delta&=\frac{1}{r^2\sin\theta}\left[\frac{\partial(ur^2\sin\theta)}{\partial r}+\frac{\partial(vr\sin\theta)}{\partial\theta}+\frac{\partial(wr)}{\partial\phi}\right],\\ 2\varpi_r&=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(wr\sin\theta)-\frac{\partial}{\partial\phi}(vr)\right],\\ 2\varpi_\theta&=\frac{1}{r\sin\theta}\left[\frac{\partial u}{\partial\phi}-\frac{\partial(wr\sin\theta)}{\partial r}\right],\\ 2\varpi_\phi&=\frac{1}{r}\left[\frac{\partial(vr)}{\partial r}-\frac{\partial u}{\partial\theta}\right]. \end{aligned}\right\}\quad\ldots(2)$$

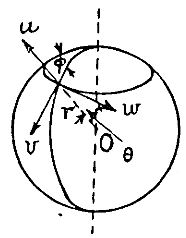

Fig. 1.

Eliminating u, v, w, in (1) by means of (2), we get

$$\begin{aligned} \rho\frac{\partial^2\Delta}{\partial t^2}&=\left\{(\lambda+2\mu)+(\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Delta}{\partial r}\right)\right.\\ &\qquad\left.+\frac{1}{r^2}\frac{\partial}{\sin\theta\partial\theta}\left(\sin\theta\frac{\partial\Delta}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Delta}{\partial\phi^2}\right], \end{aligned}$$

$$
\left.
\begin{aligned}
\rho\frac{\partial^2\varpi_r}{\partial t^2}&=\left\{\mu+\mu'\frac{\partial}{\partial t}\right\}\left[\frac{\partial^2\varpi_r}{\partial r^2}+\frac{4}{r}\frac{\partial\varpi_r}{\partial r}+\frac{2}{r^2}\varpi_r\right.\\
&\qquad\left.+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varpi_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi_r}{\partial\phi^2}\right],\\
\rho\frac{\partial^2\varpi_\theta}{\partial t^2}&=\left\{\mu+\mu'\frac{\partial}{\partial t}\right\}\left[\frac{1}{r}\frac{\partial^2(\varpi_\theta r)}{\partial r^2}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi_\theta}{\partial\phi^2}\right.\\
&\qquad\left.-\frac{1}{r^2\sin^2\theta}\frac{\partial^2(\varpi_\phi\sin\theta)}{\partial\phi\,\partial\theta}-\frac{1}{r}\frac{\partial^2\varpi_r}{\partial r\partial\theta}\right],\\
\rho\frac{\partial^2\varpi_\phi}{\partial t^2}&=\left\{\mu+\mu'\frac{\partial}{\partial t}\right\}\left[\frac{1}{r}\frac{\partial^2(\varpi_\phi r)}{\partial r^2}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial(\varpi_\phi\sin\theta)}{\partial\theta}\right.\\
&\qquad\left.-\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi}-\frac{1}{r\sin\theta}\frac{\partial^2\varpi_r}{\partial r\partial\phi}\right].
\end{aligned}
\right\}\quad\ldots\ldots(3)
$$

Put in (3)

$$\Delta=e^{ipt}\Delta',\quad \varpi_r=e^{ipt}\varpi_r',\quad \varpi_\theta=e^{ipt}\varpi_\theta',\quad \varpi_\phi=e^{ipt}\varpi_\phi',\ldots\ldots(4)$$

where $p/2\pi$ is the frequency of all waves; then we get

$$\frac{-\rho p^2}{(\lambda+2\mu)+ip(\lambda'+2\mu')}=\frac{1}{\Delta'}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Delta'}{\partial r}\right)+\frac{1}{r^2\sin\theta d\theta}\left(\sin\theta\frac{\partial\Delta'}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Delta'}{\partial\phi^2}\right]\equiv-h^2,\ \ldots\ldots(5_a)$$

$$\frac{-\rho p^2}{\mu+ip\mu'}=\frac{1}{\varpi'_r}\left[\frac{\partial^2\varpi'_r}{\partial r^2}+\frac{4}{r}\frac{\partial\varpi'_r}{\partial r}+\frac{2}{r^2}\varpi'_r+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varpi'_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi'_r}{\partial\phi_2}\right]\equiv-k^2,\ldots\ldots(5_b)$$

$$\frac{-\rho p^2}{\mu+ip\mu'}=\frac{1}{\varpi'_\theta}\left[\frac{1}{r}\frac{\partial^2(\varpi'_\theta r)}{\partial r^2}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi'_\theta}{\partial\phi^2}-\frac{1}{r^2\sin^2\theta}\frac{\partial^2(\varpi'_\phi\sin\theta)}{\partial\phi\,\partial\theta}-\frac{1}{r}\frac{\partial^2\varpi'_r}{\partial r\partial\theta}\right]\equiv-k^2,\ \ldots\ldots(5_c)$$

$$\frac{-\rho p^2}{\mu+ip\mu'}=\frac{1}{\varpi'_\phi}\left[\frac{1}{r}\frac{\partial^2(\varpi'_\phi r)}{\partial r^2}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial(\varpi'_\phi\sin\theta)}{\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial\varpi'_\theta}{\partial\phi}-\frac{1}{r\sin\theta}\frac{\partial^2\varpi'_r}{\partial r\partial\phi}\right]\equiv-k^2,\ \ldots\ldots(5_d)$$

where $2\pi/h$, $2\pi/k$ give the approximate wave lengths of P- and S-waves respectively, from which we find

$$\left(\frac{p}{h}\right)^2-\frac{ip}{h}\frac{(\lambda'+2\mu')}{\rho}h-\frac{\lambda+2\mu}{\rho}=0\quad\ldots\ldots\ldots\ldots(6_1)$$

for (5_a) and

$$\left(\frac{p}{k}\right)^2-\frac{ip}{k}\frac{\mu'}{\rho}k-\frac{\mu}{\rho}=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots(6_2)$$

for $(5_b), (5_c), (5_d)$; and we also obtain the equations of the types:

$$\left.\begin{aligned}
&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Delta'}{\partial r}\right)+\frac{1}{r^2}\frac{1}{\sin\theta\,\partial\theta}\left(\sin\theta\frac{\partial\Delta'}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Delta'}{\partial\phi^2}+h^2\Delta'=0,\\
&\frac{\partial\varpi'_r}{\partial r^2}+\frac{4}{r}\frac{\partial\varpi'_r}{\partial r}+\frac{2}{r^2}\varpi_r'+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varpi'_r}{\partial\theta}\right)\\
&\qquad\qquad+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi'_r}{\partial\phi^2}+k^2\varpi'_r=0,\\
&\frac{1}{r}\frac{\partial^2(\varpi'_\theta r)}{\partial r^2}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi'_\theta}{\partial\phi^2}-\frac{1}{r^2\sin^2\theta}\frac{\partial^2(\varpi_\phi'\sin\theta)}{\partial\phi\,\partial\theta}\\
&\qquad\qquad-\frac{1}{r}\frac{\partial^2\varpi_r'}{\partial r\,\partial\theta}+k^2\varpi_\theta'=0,\\
&\frac{1}{r}\frac{\partial^2(\varpi_\phi' r)}{\partial r^2}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial(\varpi'_\phi\sin\theta)}{\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial\varpi'_\theta}{\partial\phi}\\
&\qquad\qquad-\frac{1}{r\sin\theta}\frac{\partial^2\varpi'_r}{\partial r\,\partial\phi}+k^2\varpi'_\phi=0.
\end{aligned}\right\}\ldots(7)$$

The solutions of (6) are expressed by

$$\left.\begin{aligned}
&\frac{p}{h}=\frac{(\lambda'+2\mu')hi}{2\rho}\pm\sqrt{\frac{\lambda+2\mu}{\rho}-\left(\frac{\lambda'+2\mu'}{2\rho}\right)^2h^2}\\
&\qquad\qquad\fallingdotseq\frac{(\lambda'+2\mu')hi}{2\rho}\pm\sqrt{\frac{\lambda+2\mu}{\rho}},\\
&\frac{p}{k}=\frac{\mu'ki}{2\rho}\pm\sqrt{\frac{\mu}{\rho}-\left(\frac{\mu'}{2\rho}\right)^2k^2}\fallingdotseq\frac{\mu'ki}{2\rho}\pm\sqrt{\frac{\mu}{\rho}}.
\end{aligned}\right\}\ldots\ldots(8)$$

Solving (7) and using the result in (8), we obtain

$$\left.\begin{aligned}
&\Delta=A_{mn}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}}P_n^m(\cos\theta)\cos m\phi\, e^{-\frac{(\lambda'+2\mu')}{2\rho}h^2t}e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\
&2\varpi_r=B_{mn}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{3/2}}P_n^m(\cos\theta)\sin m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
&2\varpi_\theta=\left[C_{mn}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P_n^m(\cos\theta)}{\sin\theta}+\frac{B_{mn}}{n(n+1)}\frac{1}{r}\frac{d}{dr}\right.
\end{aligned}\right\}\ldots\ldots(9)$$

$$\times\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\}\frac{dP_n^m(\cos\theta)}{d\theta}\Big]\sin m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt},$$

$$2\varpi_\phi=\Big[C_{mn}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{m\sqrt{r}}\frac{dP_n^m(\cos\theta)}{d\theta}+\frac{B_{mn}m}{n(n+1)}\frac{1}{r}\frac{d}{dr}$$

$$\times\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\}\frac{P_n^m(\cos\theta)}{\sin\theta}\Big]\cos m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.$$

Displacement $(u_1, v_1, w_1,)$ answering to $\varDelta$ in (9) and satisfying $\varpi_r=\varpi_\theta=\varpi_\phi=0$ is expressed by

$$\left.\begin{aligned}
u_1&=-\frac{A_{mn}}{h^2}\frac{d}{dr}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}}P_n^m(\cos\theta)\cos m\phi\, e^{-\frac{(\lambda'+2\mu')h^2t}{2\rho}}\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\
v_1&=-\frac{A_{mn}}{h^2}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}\frac{dP_n^m(\cos\theta)}{d\theta}\cos m\phi\, e^{-\frac{(\lambda'+2\mu')h^2t}{2\rho}}\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\
w_1&=\frac{mA_{mn}}{h^2}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}\frac{P_n^m(\cos\theta)}{\sin\theta}\sin m\phi\, e^{-\frac{(\lambda'+2\mu')h^2t}{2\rho}}\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht}.
\end{aligned}\right\}\quad ..(10)$$

Displacement (u_2, v_2, w_2) answering to ϖ_r together with second terms in the expressions of ϖ_θ and ϖ_ϕ, under the condition that $\varDelta=0$, is expressed by

$$\left.\begin{aligned}
u_2&=0,\\
v_2&=\frac{mB_{mn}}{n(n+1)}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{P_n^m(\cos\theta)}{\sin\theta}\cos m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
w_2&=-\frac{B_{mn}}{n(n+1)}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\frac{dP_n^m(\cos\theta)}{d\theta}\sin m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.
\end{aligned}\right\}\quad ..(11)$$

Displacement (u_3, v_3, w_3) derived from the values of the first terms of ϖ_θ and ϖ_ϕ and fulfilling the conditions, $\varDelta=\varpi_r=0$, is written by

$$\left.\begin{aligned}
u_3&=-\frac{n(n+1)C_{mn}}{mk^2}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{3/2}}P_n^m(\cos\theta)\cos m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
v_3&=-\frac{C_{mn}}{mk^2}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\}\frac{dP_n^m(\cos\theta)}{d\theta}\cos m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
w_3&=\frac{C_{mn}}{k^2}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\}\frac{P_n^m(\cos\theta)}{\sin\theta}\sin m\phi\, e^{-\frac{\mu'}{2\rho}k^2t}\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.
\end{aligned}\right\}\quad (12)$$

The expressions[3] of displacements in (10), (11), (12) give the case of waves decaying with time even at the origin, and they are not suitable to the propagation of waves generated from the origin at a constant rate of energy transmission, so that we have introduced conventionally kinematical relations, such that $hr=pt$ in $exp.\left\{-\frac{(\lambda'+2\mu')}{2\rho}h^2t\right\}$ for (u_1, v_1, w_1) and $kr=pt$ in $exp.\left\{-\frac{\mu'}{2\rho}k^2t\right\}$ for (u_2, v_2, w_2) and (u_3, v_3, w_3). In spite of its incorrectness in strict mathematics we have employed such a manner, as it gives a dynamically equivalent form of expressions of waves transmitted in visco-elastic medium. The expressions thus obtained are as follows:

$$\left.\begin{aligned}
u_1&=-\frac{A_{mn}}{h^2}\frac{d}{dr}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}}\,e^{-\frac{(\lambda'+2\mu')h^2r}{2\sqrt{\rho(\lambda+2\mu)}}}P_n^m(\cos\theta)\cos m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\
v_1&=-\frac{A_{mn}}{h^2}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}\,e^{-\frac{(\lambda'+2\mu')h^2r}{2\sqrt{\rho(\lambda+2\mu)}}}\frac{dP_n^m(\cos\theta)}{d\theta}\cos m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\
w_1&=\frac{mA_{mn}}{h^2}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}\,e^{-\frac{(\lambda'+2\mu')h^2r}{2\sqrt{\rho(\lambda+2\mu)}}}\frac{P_n^m(\cos\theta)}{\sin\theta}\sin m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht}.
\end{aligned}\right\}\quad\ldots(13)$$

$$\left.\begin{aligned}
u_2&=0,\\
v_2&=\frac{mB_{mn}}{n(n+1)}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\,e^{-\frac{\mu'k^2r}{2\sqrt{\rho\mu}}}\frac{P_n^m(\cos\theta)}{\sin\theta}\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
w_2&=-\frac{B_{mn}}{n(n+1)}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{\sqrt{r}}\,e^{-\frac{\mu'k^2r}{2\sqrt{\rho\mu}}}\frac{dP_n^m(\cos\theta)}{d\theta}\sin m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.
\end{aligned}\right\}\quad\ldots(14)$$

$$\left.\begin{aligned}
u_3&=-\frac{n(n+1)C_{mn}}{mk^2}\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{3/2}}\,e^{-\frac{\mu'k^2r}{2\sqrt{\rho\mu}}}P_n^m(\cos\theta)\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
v_3&=-\frac{C_{mn}}{mk^2}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}H^{(2)}_{n+\frac{1}{2}}(kr)\}\,e^{-\frac{\mu'k^2r}{2\sqrt{\rho\mu}}}\frac{dP_n^m(\cos\theta)}{d\theta}\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
w_3&=\frac{C_{mn}}{k^2}\frac{1}{r}\frac{d}{dr}\{\sqrt{r}H^{(2)}_{n+\frac{1}{2}}(kr)\}\,e^{-\frac{\mu'k^2r}{2\sqrt{\rho\mu}}}\frac{P_n^m(\cos\theta)}{\sin\theta}\sin m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.
\end{aligned}\right\}\quad(15)$$

3) Similar expressions as (10), (11), (12) that are suitable to the propagation of waves on a spherical surface were obtained by one of us a few years ago. See *Bull. Earthq. Res. Inst.* **6** (1929), 14. Solutions (10), (11), (12) in the case $\lambda'=\mu'=0$ were published by one of us on July 7, 1931 and their special case where $m=0$ was published in *Bull. Earthq. Res. Inst.*, **2** (1927), 13-20.

For the purpose of getting numerical values of these displacements, we have used the relations:

$$\left.\begin{aligned} &H^{(2)}_{n+\frac{1}{2}}(hr)\approx\sqrt{\frac{2}{\pi hr}}\,e^{-i\left(hr-\frac{n+1}{2}\pi\right)},\\ &H^{(2)}_{n+\frac{1}{2}}(kr)\approx\sqrt{\frac{2}{\pi kr}}\,e^{-i\left(kr-\frac{n+1}{2}\pi\right)},\\ &P_n^m(\cos\theta)=(-1)^m\frac{d^mP_n(\cos\theta)}{d\theta^m}. \end{aligned}\right\}\quad\ldots\ldots\ldots(16)$$

By means of these expression we have

$$\left.\begin{aligned} u_1=&-\frac{A_{mn}}{h^2}\sqrt{\frac{2}{\pi h}}\left\{-\frac{ih}{r}e^{-i\left(hr-\frac{n+1}{2}\pi\right)}-\frac{1}{r^2}e^{-i\left(hr-\frac{n+1}{2}\pi\right)}\right\}\\ &\times e^{-\frac{\lambda'+2\mu'}{2\rho}\sqrt{\frac{\rho}{\lambda+2\mu}}h^2r}(-1)^m\frac{d^mP_n(\cos\theta)}{d\theta^m}\cos m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\ v_1=&-\frac{A_{mn}}{h^2}\frac{1}{r^2}\sqrt{\frac{2}{\pi h}}e^{-i\left(hr-\frac{n+1}{2}\pi\right)}\\ &\times e^{-\frac{\lambda'+2\mu'}{2\rho}\sqrt{\frac{\rho}{\lambda+2\mu}}h^2r}(-1)^m\frac{d^{m+1}P_n(\cos\theta)}{d\theta^{m+1}}\cos m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht},\\ w_1=&\frac{mA_{mn}}{h^2}\frac{1}{r^2}\sqrt{\frac{2}{\pi h}}e^{-i\left(hr-\frac{n+1}{2}\pi\right)}\\ &\times e^{-\frac{\lambda'+2\mu'}{2\rho}\sqrt{\frac{\rho}{\lambda+2\mu}}h^2r}\frac{1}{\sin\theta}(-1)^m\frac{d^mP_n(\cos\theta)}{d\theta^m}\sin m\phi\, e^{i\sqrt{\frac{\lambda+2\mu}{\rho}}ht}. \end{aligned}\right\}\quad\ldots(17)$$

$$\left.\begin{aligned} u_2=&0,\\ v_2=&\frac{mB_{mn}}{n(n+1)}\frac{1}{r}\sqrt{\frac{2}{\pi k}}e^{-i\left(kr-\frac{n+1}{2}\pi\right)}\\ &\times e^{-\frac{\mu'}{2\rho}\sqrt{\frac{\rho}{\mu}}k^2r}\frac{1}{\sin\theta}(-1)^m\frac{d^mp_n(\cos\theta)}{d\theta^m}\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\ w_2=&-\frac{B_{mn}}{n(n+1)}\frac{1}{r}\sqrt{\frac{2}{\pi k}}e^{-i\left(kr-\frac{n+1}{2}\pi\right)}\\ &\times e^{-\frac{\mu'}{2\rho}\sqrt{\frac{\rho}{\mu}}k^2r}(-1)^m\frac{d^{m+1}P_n(\cos\theta)}{d\theta^{m+1}}\sin m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt}. \end{aligned}\right\}\quad\ldots(18)$$

$$u_3=-\frac{n(n+1)}{mk^2}C_{mn}\frac{1}{r_2}\sqrt{\frac{2}{\pi k}}e^{-i\left(kr-\frac{n+1}{2}\pi\right)}$$

$$
\left.\begin{aligned}
&\times e^{-\frac{\mu'}{2\rho}\sqrt{\frac{\rho}{\mu}}k^2 r}(-1)^m\frac{d^m P_n(\cos\theta)}{d\theta^m}\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
v_3&=-\frac{C_{mn}}{mk^2}\frac{1}{r}\sqrt{\frac{2}{\pi k}}\,e^{-i\left(kr-\frac{n+1}{2}\pi\right)}(-ik)\\
&\times e^{-\frac{\mu'}{2\rho}\sqrt{\frac{\rho}{\mu}}k^2 r}(-1)^m\frac{d^{m+1} P_n(\cos\theta)}{d\theta^{m+1}}\cos m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt},\\
w_3&=\frac{C_{mn}}{k^2}\frac{1}{r}\sqrt{\frac{2}{\pi k}}\,e^{-i\left(kr-\frac{n+1}{2}\pi\right)}(-ik)\\
&\times e^{-\frac{\mu'}{2\rho}\sqrt{\frac{\rho}{\mu}}k^2 r}\frac{1}{\sin\theta}(-1)^m\frac{d^m P_n(\cos\theta)}{d\theta^m}\sin m\phi\, e^{i\sqrt{\frac{\mu}{\rho}}kt}.
\end{aligned}\right\}\quad\cdots(19)
$$

To make the problem simple we have taken $m=2$, $n=2$, and assumed the density of the solid, the velocities of propagation of P- and S-waves and damping constants for these respective waves as follows.

$$
\rho=2{\cdot}7,\quad \sqrt{\frac{\lambda+2\mu}{\rho}}=6{\cdot}0\ \text{km/sec},\quad \sqrt{\frac{\mu}{\rho}}=3{\cdot}7\ \text{km/sec},
$$

$$
\frac{\lambda'}{2\rho}=\frac{\mu'}{2\rho}=\frac{1}{5{\cdot}4\times10^3}\,\text{km}^2/\text{sec.}^{4)}\ \left(\begin{array}{l}\text{assuming that }\lambda',\mu'\text{ are}\\ \text{of equal qualification}\end{array}\right)
$$

The wave lengths for P-waves are assumed to be 6·0 km, 60 km, and 0·6 km, while those of the corresponding S-waves are taken to be 3.7 km, 37 km, and 0·37 km respectively. The amplitudes of all components of displacements in the directions i) $P_2(\cos\theta)=1$, $\cos2\phi=1$; ii) $P_2(\cos\theta)=1$, $\cos 2\phi=0$; iii) $P_2^2(\cos\theta)=0$, $\cos2\phi=1$; iv) $P_2^2(\cos\theta)=0$, $\cos2\phi=0$; v) $P_2(\cos\theta)=-\frac{1}{2}$, $\cos2\phi=1$; vi) $P_2(\cos\theta)=-\frac{1}{2}$, $\cos2\phi=0$; and at the radial distances, $r=30$ km, 60 km, 120 km, 240 km, 480 km, 960 km, 1920 km, 2400 km, are shown in the appended tables and Figs. 2–10.

The expressions of the displacements show that there are two kinds of S-waves, namely (u_2, v_2, w_2) and (u_3, v_3, w_3), while P-waves are only of one kind. We may call the waves of the type (u_3, v_3, w_3) to be S-waves of the first kind and those of the type (u_2, v_2, w_2) to be S-waves of the

4) These values were temporarily taken to be approximately equal to what Professor Suyehiro derived from the results of experiments due to Professor Tanakadate, Professor Omori and Dr. Taniguchi on brick columns. *Bull. Earthq. Res. Inst.*, 6 (1928), 1-8.

second kind. We find many features to distinguish the two kinds of S-waves.

When the effect of damping is neglected, the radial component ot the displacement of P-waves varies inversely as the radial distance from the origin, while the components transverse to the radial one vary as the inverse square of the radial distance.

The radial component of the displacement of S-waves of the first kind under the similar condition varies inversely as the square of the firsf radial disfance, while the components transverse to the radial one vary as the direct inverse of the radial distance.

The radial component of the displacement of S-waves of the second kind under the similar condition is zero for all radial distances, while the components transverse to the radial one vary again the direct inverse of the radial distance.

It may be concluded that in every kind of waves transmitted in a solid body without solid friction damping the main portion of the displacement varies as the inverse of the radial distance from the origin. Thus, with a mere comparison of the main amplitudes of waves we cannot discriminate P- and S-waves, and much less both kinds of S-waves.

When, however, we observe more closely the components of all kinds of waves, it is possible to get the identification of the waves. The transverse components of P-waves diminish very quickly as the radial distance is increased, and the component of S-waves of the first kind diminishes again in the same manner as in the above case, while that of S-waves of the second kind is zero for all radii.

The variation of the displacements in regard to the direction of propagation has a pecuriority corresponding to each kind of waves. In the direction of propagation where the transeverse component of P-waves is polarised in a certain sense, the same component of S-waves of the first kind is also polarised in the same sense, while the sense of polarisation of S-waves of the second kind is independent of that of P-waves as well as S-waves of the first kind. This may be attributed to the mechanism that P-waves should be acompanied by S-waves of the first kind owing to the conditions of displacements or stresses at the origin of seismic waves, but S-waves of the second kind is possible to occur independently.

In the case of coexistence of P-waves and S-waves of the first kind the distributions of the main amplitudes of both waves are different. In the region, where the amplitudes of P-waves are large, those of S-waves in question are small, and *vice versa*. The amplitudes of S-waves of the second kind, however, are independent of P-waves. These are also due to the mechanism of the seismic focus.

The above consideration is connected merely with the case where the effect of the solid friction of the material is neglected, although the results in the tables and diagrams contain this effect. When such an effect is taken into account and the periods of vibrations of all kinds of waves are the same, the amplitude of S-waves at a certain station is more damped than that of P-waves at the same station and this is due to the fact that the time of transmission of S-waves from the origin to this station is longer than that of P-waves. The shorter the period of both waves, the relative quickness of the rate of decrease of S-waves is more distinct. In order to demonstrate this nature, we have taken up u_1, v_2, v_3 of three cases i) $L_1=60$ km, $L_2=37$ km; ii) $L_1=6$ km; $L_2=3{\cdot}7$ km; iii) $L_1=0{\cdot}6$ km, $L_2=0{\cdot}37$ km; and kept constant the amplitudes of all waves at the focal distance 2400 km and illustrated the result in Figs. 11, 12 and 13. From the amplitudes of waves at other focal distances, we may understand how the damping of S-waves is quick in comparison with that of P-waves.

The foregoing conclusion cannot be much altered even in the cases where there are some boundaries which cause the reflection and refraction of waves and also where the medium of transmission is varying in a certain manner. This may be seen from the results of a number of our theoretical investigations already carried out to get concrete idea on several cases of boundaries etc.. At any rate, the abnormal smallness of the amplitudes of P-waves cannot be ascertained directly from the nature of the propagation of waves, but they may be cleared when the mechanism, that the period of P-waves is small compared with those of S-waves, will be completely determined.

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22. P 波 S 波の振幅が震源距離によつて變化する割合

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一般に震源距離が遠くなるに從て P 波の振幅は S 波の振幅に比較して益々小さくなる事はよく知られて居る事柄である。しかしながらこの事實は P 波 S 波の根本的性質の結果として現れるのではなく、P 波の振動週期が S 波のそれに比して概して短い爲に減衰作用を速に受けるといふ事

に他ならないのである。それで震源距離が一層大になる時は P 波の震幅は反て S 波の振幅に近づくといふ事の説明も出來るものである。減衰作用を用ひなければ、中間の震源距離で P 波が S 波よりも益々小になり、もつと遠方では P 波がS 波に比して再び增大するといふ説明を與へる事が出來ぬ。反射屈折其他の理論は餘り役に立たない事はこれまでの理論的研究で明かである。

計算の方法は震源から種々の方向性を有する P 波と S 波とが出るものと假定し、これまで種々の場合に行つたのと大體同じ様に考へ且つ原點の條件から P 波と S 波とを算出するのである。彈性體は勿論粘性を含むものとする。この様にして現れる S 波と P 波の形を見ると P 波の方は一種類しかないけれども S 波の方は第一種と第二種とから成立して居り、且つ P 波と第一種の S 波とは震源の條件の爲に必ず組合つて發生する事及び第二種の S 波は震源の適當なる條件によつて單獨に起り得る事がわかる。

次に固體に粘性のない時に P 波のその傳播方向の振幅は震源距離に反比例して小さくなるけれども同じ波のこれに直角の方向の振幅は距離の二乘に反比例して小さくなる事、第一種の S 波は其傳播方向の振幅が震源距離の二乘に反比例して小さくなるのにそれに直角の方向の振幅は距離それ自身に反比例して小さくなる事、又第二種の S 波は傳播方向に直角の方向の振幅は第一種の場合と同じ樣に變化するのに傳播方向のものは始めから零である事が知られるのである。

上の樣に P 波と各種の S 波とは固體粘性のない時振動方向や其他特別な事項は大分相違のある事がわかるけれども、絶對振幅のみの比較では餘り區別がつかない。しかしながら種々の觀測點の P 波や S 波の振幅分布を見ると著しい相違がある。即ち或場所で P 波の傳播の向に直角方向の變位が或向に polarise して居れば其場所で第一種の S 波の傳播の向に直角方向の變位も同じ向に polarise する事がわかる。又 P 波と第一種の S 波とが共存する時に P 波の振幅が大なる所では S 波の振幅は小さく、逆に P 波の振幅の小なる所では S 波の振幅が大なるものである。然るに第二種の S 波は P 波に無關係となる。此等は震源に於ける應力や變位の條件から定まるものである。

以上は固體の粘性を考へなくても出て來る事柄であるが、若し粘性を考へる時には、模樣が多少變更される。即ち P 波と S 波との粘性的減衰の作用を同じ割合に取る時は、或震源距離では S 波の方の振幅が同じ週期の P 波に比較して餘計に減衰して來るといふ事がわかるのである。この事は震源距離の非常に遠い所で P 波と S 波との振動週期が可なり近いものしか殘つて居らぬ樣な時に P 波の方が S 波に比較して振幅が餘り縮まないといふ事實とよく符合して居る樣に思はれる。何れにしても適當なる震源距離で P 波がS 波よりも振幅が一般に小であるといふ事柄の説明を一定週期の波についてなす事は出來ぬ。それにはどうしても震源又は原點附近から出る波について P 波の振動週期が S 波のものよりも小であるといふ事を確める事が先決問題である樣に思ふ。

Table I. The Case of $L_1=6{\cdot}0$ km, $L_2=3{\cdot}7$ km.

	r in km	30	60	120	240	480	960	1920	2400
$P_2(\cos\theta)=1$, $\cos 2\phi=0$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·09965	0·04970	0·02470	0·01220	0·00595	0·00284	0·00129	0·00099
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·0036143	0·0008997	0·0002230	0·0000548	0·0000132	0·0000031	0·0000007	0·0000004
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(to be continued.)

Table I. (*continued.*)

r in km		30	60	120	240	480	960	1920	2400
$P_2(\cos\theta)=1$, $\cos 2\phi=0$	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
$P_2^2(\cos\theta)=0$, $\cos 2\phi=1$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·0031709	−0·0007907	−0·0001965	−0·0000485	−0·0000118	−0·0000028	−0·0000006	−0·0000004
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·03070	−0·01528	−0·00758	−0·00372	−0·00180	−0·00084	−0·00036	−0·00027

(*to be continued.*)

Table I. (*continued*.)

r in km		30	60	120	240	480	960	1920	2400
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2^2(\cos\theta)=0$, $\cos 2\phi=0$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	−0·06140	−0·03057	−0·01515	−0·00745	−0·00360	−0·00168	−0·00073	−0·00055
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·09965	−0·04970	−0·02470	−0·01220	−0·00595	−0·00284	−0·00129	−0·00099
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued*.)

Table I. (*continued.*)

	r in km	30	60	120	240	480	960	1920	2400
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=1$	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0·12281	0·06114	0·03031	0·01489	0·00719	0·00336	0·00146	0·00109
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·0036143	−0·0008997	−0·0002230	−0·0000548	−0·0000132	−0·0000031	−0·0000007	−0·0000004
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·0063417	0·0015813	0·0003929	0·0000971	0·0000237	0·0000056	0·0000013	0·0000008
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=0$	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table I. (*contineud.*)

r in km	30	60	120	240	480	960	1920	2400
$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·06140	0·03057	0·01515	0·00745	0·00360	0·00168	0·00073	0·00055

Table II. The Case of $L_1=60$ km, $L_2=37$ km.

	r in km	300	600	1200	2400	4800	9600	19200	24000
$P_2(\cos\theta)=1$, $\cos 2\phi=1$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·09997	0·04997	0·02497	0·01247	0·00622	0·00309	0·00153	0·00122
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·0036285	0·0009066	0·0002265	0·0000565	0·0000141	0·0000035	0·00000086	0·00000055
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table II. (*continued.*)

r in km		300	600	1200	2400	4800	9600	19200	24000
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2(\cos\theta)=1$, $\cos 2\phi=0$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·0031810	−0·0007950	−0·0001986	−0·0000496	−0·0000124	−0·0000031	−0·00000075	−0·0000005
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table II. (*continued.*)

	r in km	300	600	1200	2400	4800	9600	19200	24000
$P_2^2(\cos\theta)=0$, $\cos 2\phi=1$	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·03082	−0·01540	−0·00770	−0·00384	−0·00191	−0·00095	−0·00047	−0·00037
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2^2(\cos\theta)=0$, $\cos 2\phi=0$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	−0·06164	−0·03080	−0·01540	−0·00768	−0·00382	−0·00190	−0·00094	−0·00074
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table II. (*continued.*)

	r in km	300	600	1200	2400	4800	9600	19200	24000
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=1$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·09997	−0·04997	−0·02497	−0·01247	−0·00622	−0·00309	−0·00153	−0 00122
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0·12329	−0·06161	0·03078	0·01536	0·00766	0 00380	0·00187	0·00149
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·0036285	−0·0009066	−0·0002265	−0·0000565	−0·0000141	−0·0000035	−0·00000086	−0 00000055
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table II. (*continued.*)

	r in km	300	600	1200	2400	4800	9600	19200	24000
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=0$	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·0063620	0·0015900	0·0003972	0·0000992	0·0000248	0·0000062	0·0000015	0·0000010
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·06164	0·03080	0·01540	0·00768	0·00382	0·00190	0·00094	0·00074

Table III. The Case of $L_1=0{\cdot}6$ km, $L_2=0{\cdot}37$ km.

	r in km	3	6	12	24	48	96	192	240
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·09700	0·04735	0·02213	0·00979	0·00384	0·00118	0·00022	0·00011
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table III. (*continued.*)

	r in km	3	6	12	24	48	96	192	240
$P_2(\cos\theta)=1$, $\cos 2\phi=1$	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·0034760	0·0008322	0·0001908	0·0000401	0·0000071	0·0000009	0·00000006	0·00000002
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2(\cos\theta)=1$, $\cos 2\phi=0$	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table III. (*continued.*)

r in km		3	6	12	24	48	96	192	240
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	—	—	—	—	—	—	—	—
$P_2^2(\cos\theta)=0$ $\cos 2\phi=1$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·0030864	−0·0007533	−0·0001760	−0·0000390	−0·0000076	−0·0000012	−0 0000001	−0·00000004
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·02953	−0·01414	−0·00648	−0·00273	−0 00096	−0·00024	−0·00003	−0·00001
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table III. (*continued.*)

	r in km	3	6	12	24	48	96	192	240
$P_2^2(\cos\theta)=0$, $\cos 2\phi=0$	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	−0·05905	−0.02828	0·01297	−0·00545	−0·00193	−0·00048	−0·00006	−0·00002
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=1$	$u_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	−0·09700	−0·04735	−0·02213	−0·00979	−0·00384	−0·00118	−0·00022	−0·00011
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0·11811	0·05655	0·02594	0·01091	0·00386	0·00096	0·00012	0·00005
	$w_2\Big/\frac{B_{22}}{2\cdot3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0

(*to be continued.*)

Table III. (*continued.*)

	r in km	3	6	12	24	48	96	192	240
$P_2(\cos\theta)=-\frac{1}{2}$, $\cos 2\phi=0$	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	−0·0034760	−0·0008322	−0·0001908	−0·0000401	−0·0000071	−0·0000009	−0·00000006	−0·00000002
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_1/A_{22}\sqrt{\frac{2}{\pi h}}\times\frac{3}{2\pi}$	0·0061728	0·0015065	0·0003521	0·0000779	0·0000153	0·0000023	0·0000002	0·0000001
	$u_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_2\Big/\frac{B_{22}}{2\cdot 3}\sqrt{\frac{2k}{\pi}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$u_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$v_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0	0	0	0	0	0	0	0
	$w_3/C_{22}\sqrt{\frac{2}{\pi k}}\times\frac{3}{2\pi}$	0·05905	0·02827	0·01297	0·00545	0·00193	0·00048	0·00006	0·00002

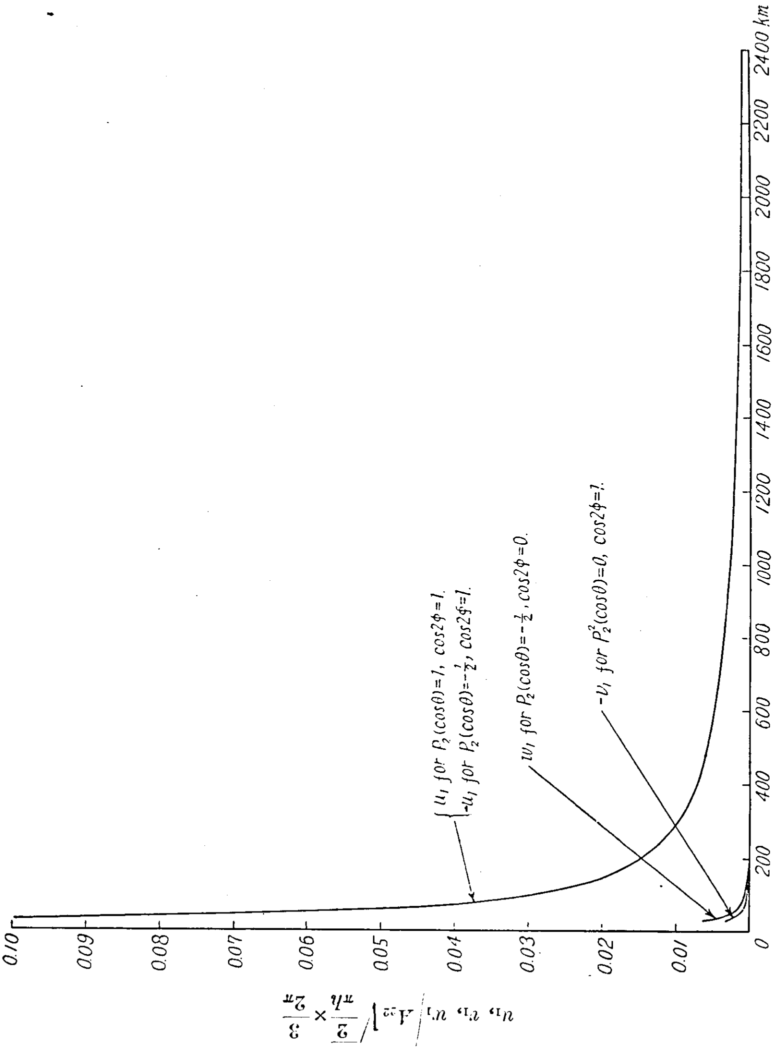

Fig. 2. $L_1 = 6{\cdot}0$ km, $L_2 = 3{\cdot}7$ km

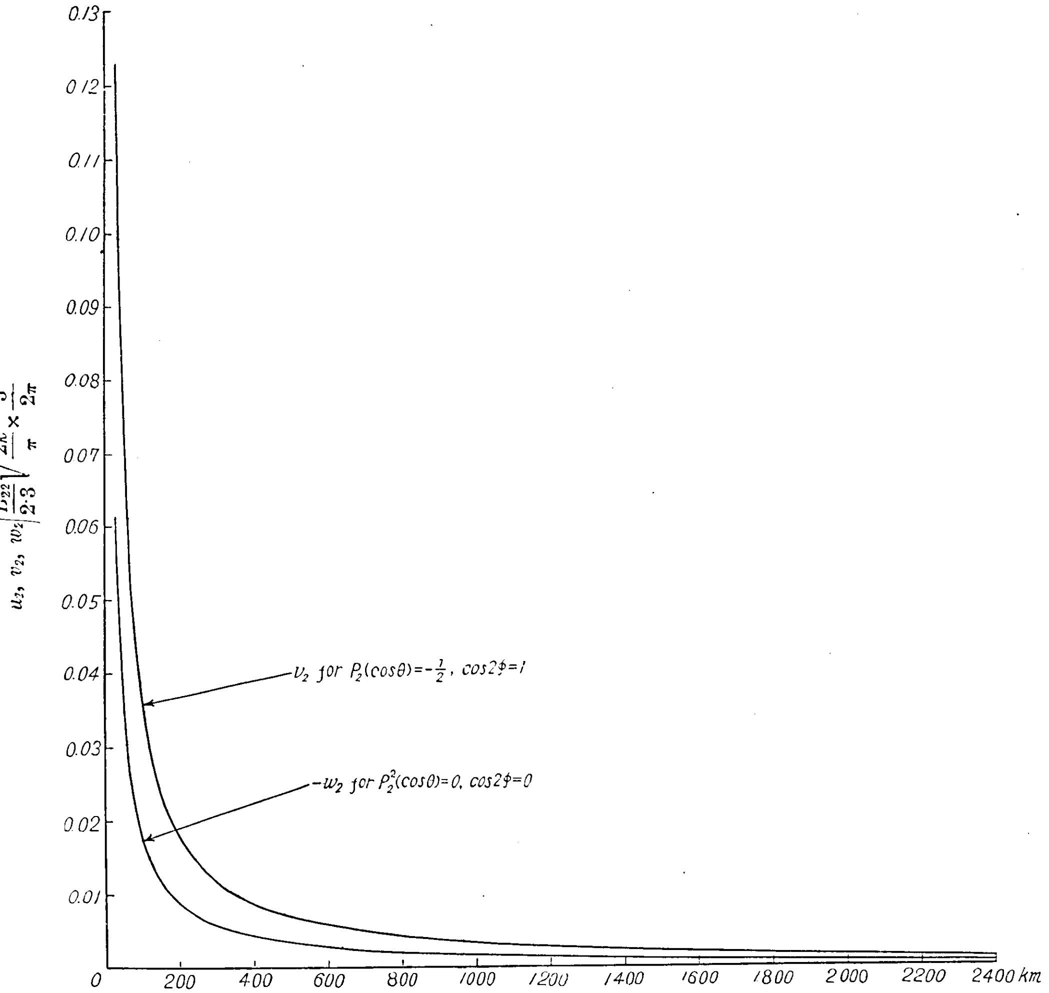

Fig. 3. $L_1=6{\cdot}0$ km, $L_2=3{\cdot}7$ km.

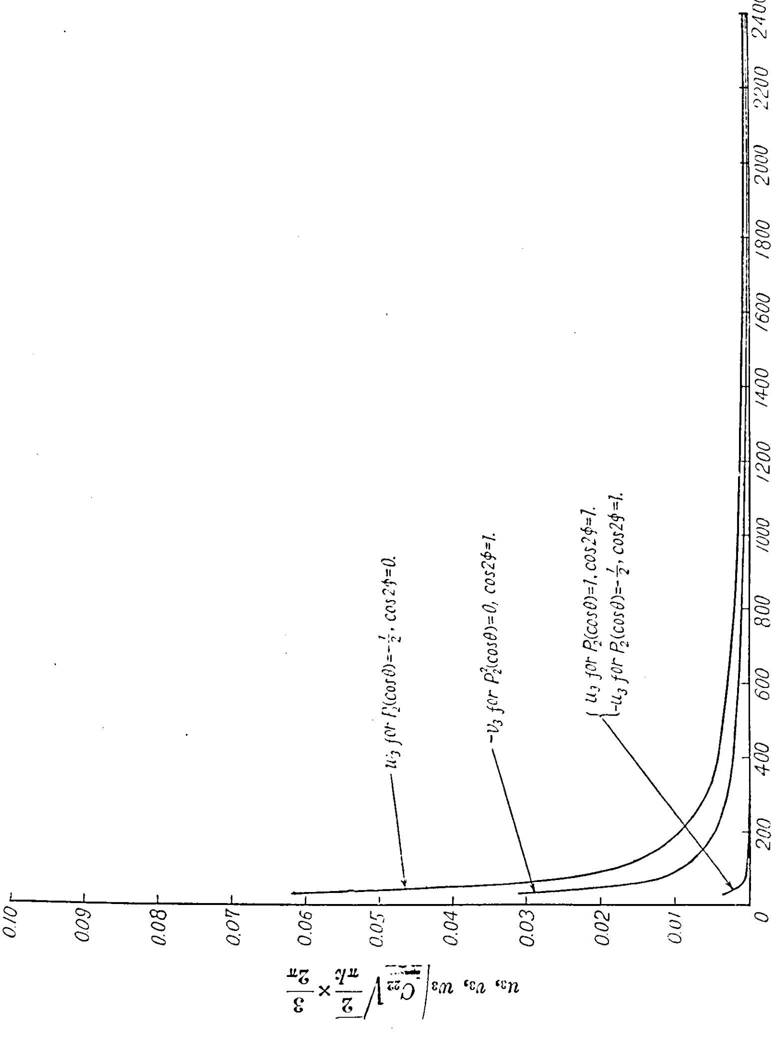

Fig. 4. $L_1 = 6{\cdot}0$ km, $L_2 = 3{\cdot}7$ km.

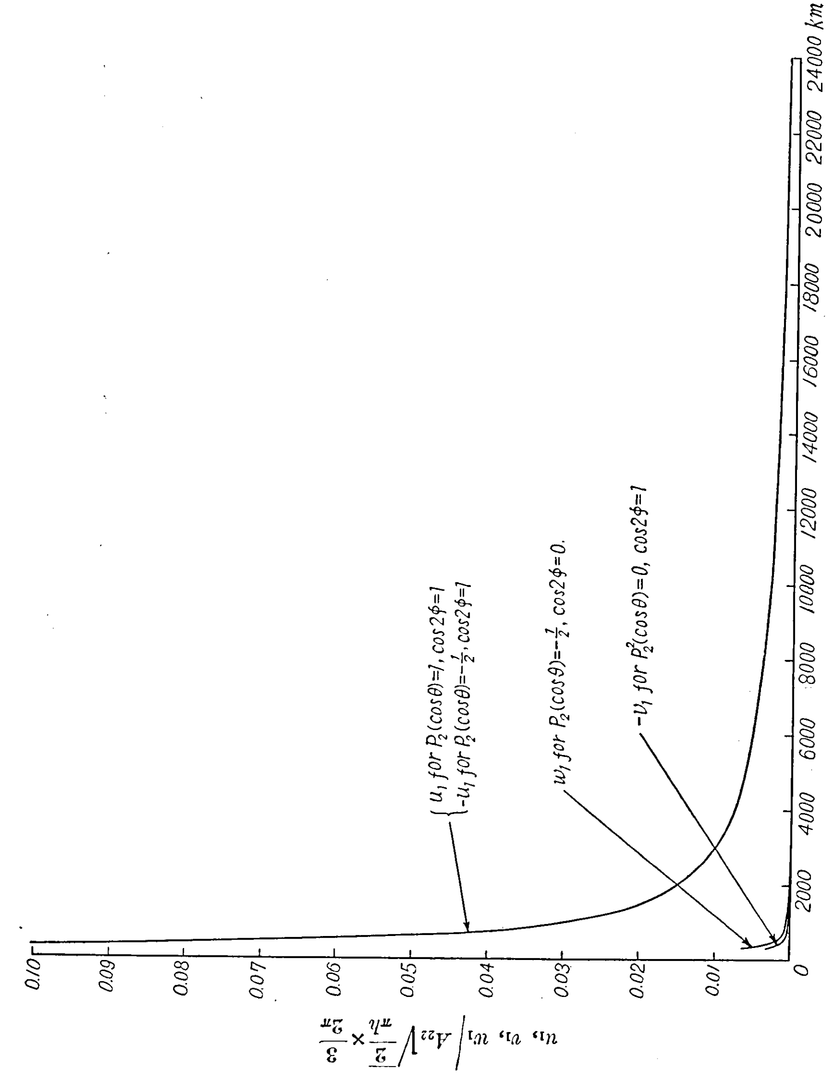

Fig. 5. $L_1=60$ km, $L_2=37$ km.

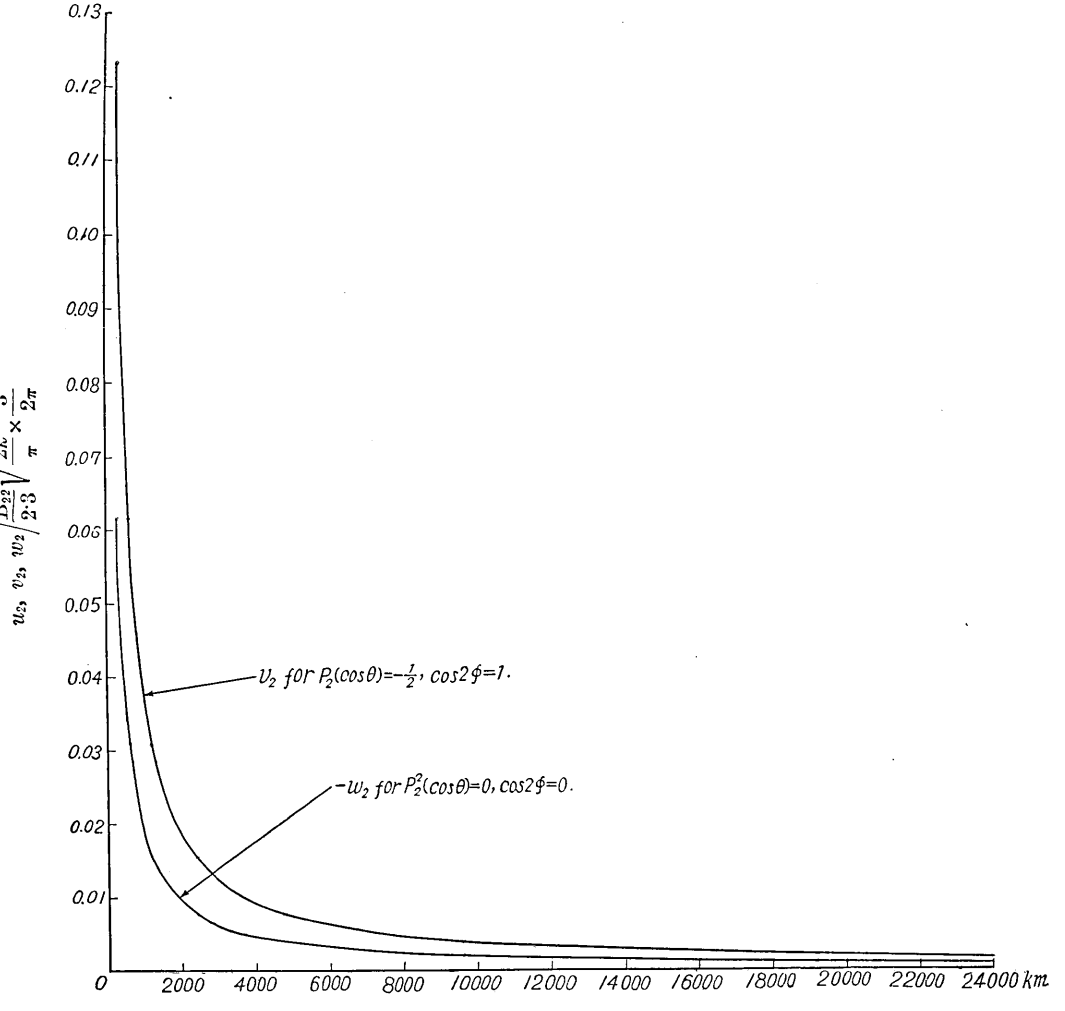

Fig. 6. $L_1=60$ km, $L_2=37$ km.

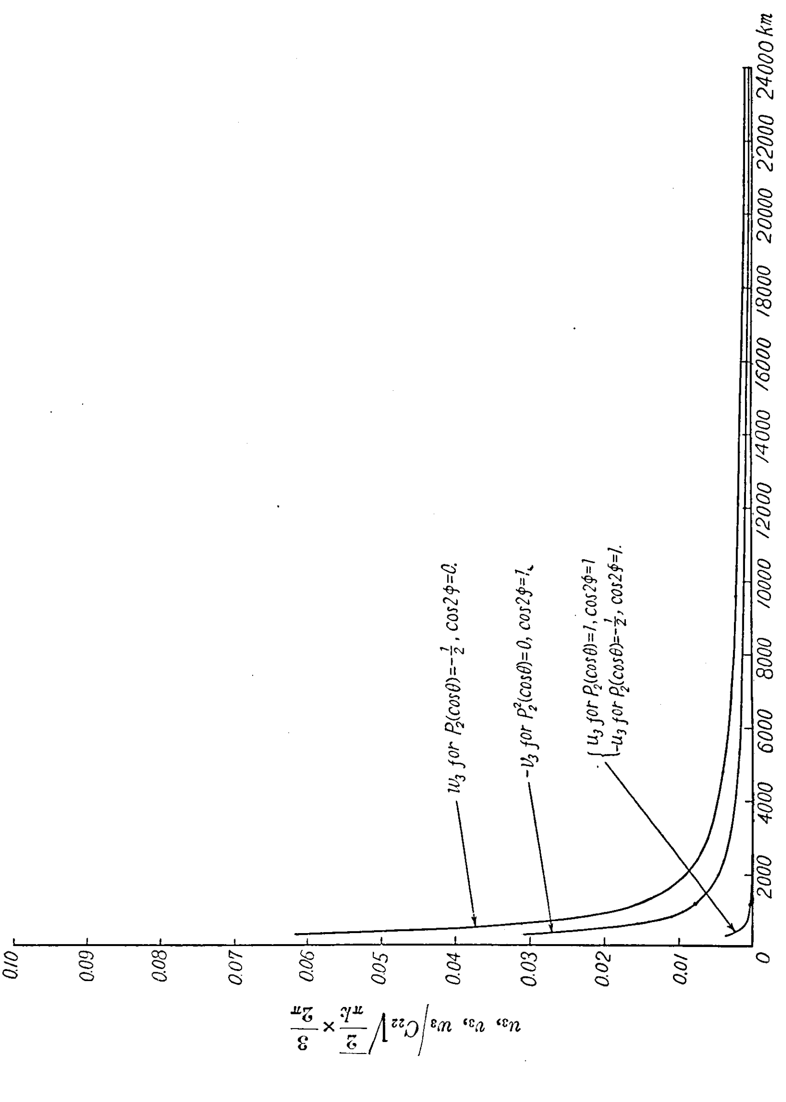

Fig. 7. $L_1=60$ km, $L_2=37$ km.

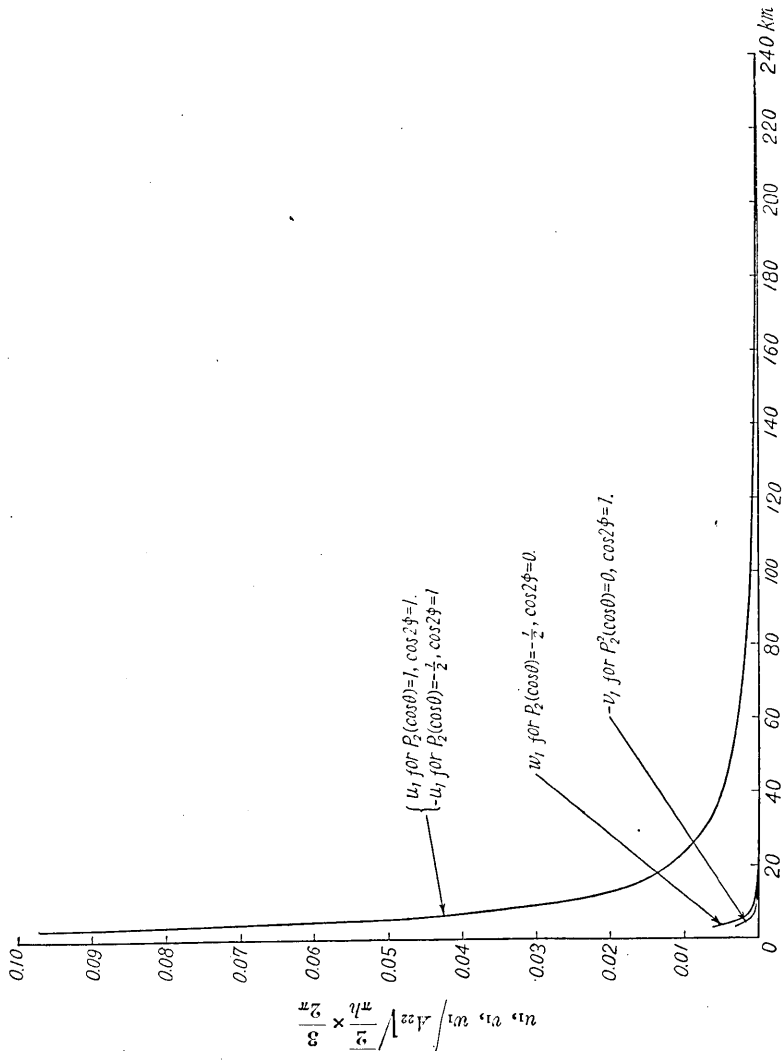

Fig. 8. $L_1 = 0{\cdot}6$ km, $L_2 = 0{\cdot}37$ km.

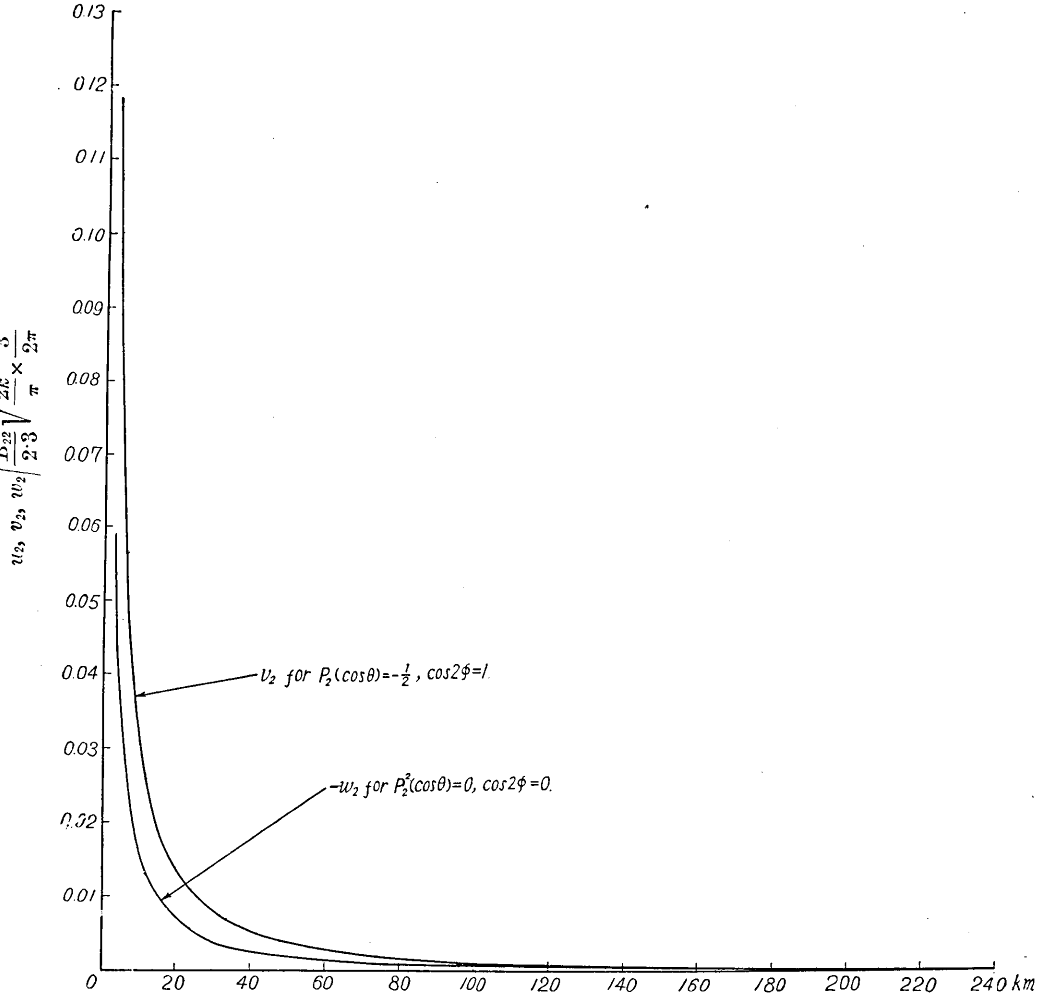

Fig. 9. $L_1=0{\cdot}6$ km, $L_2=0{\cdot}37$ km.

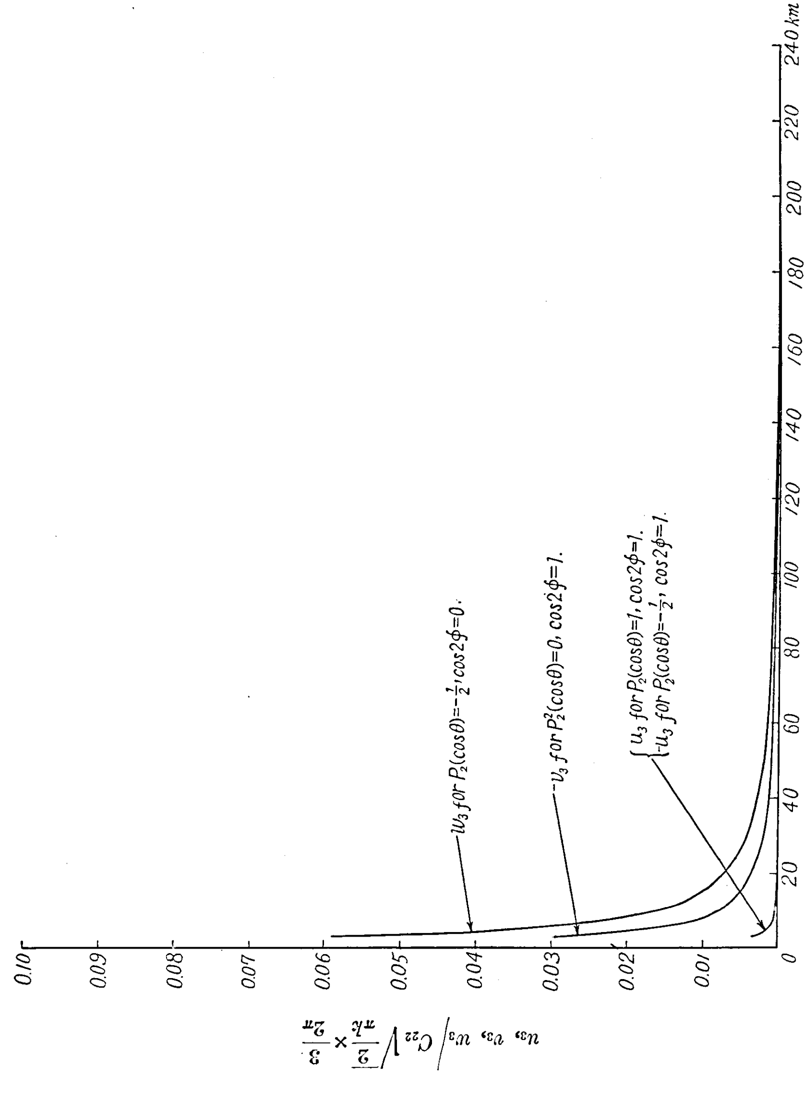

Fig. 10. $L_1=0{\cdot}6$ km, $L_2=0{\cdot}37$ km.

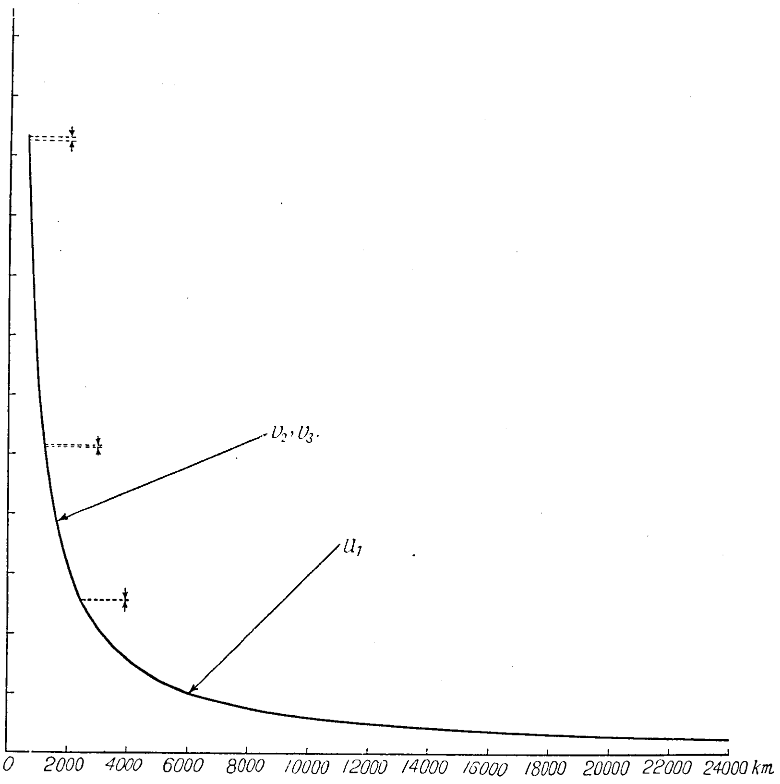

Fig. 11. $L_1=60$ km, $L_2=37$ km.

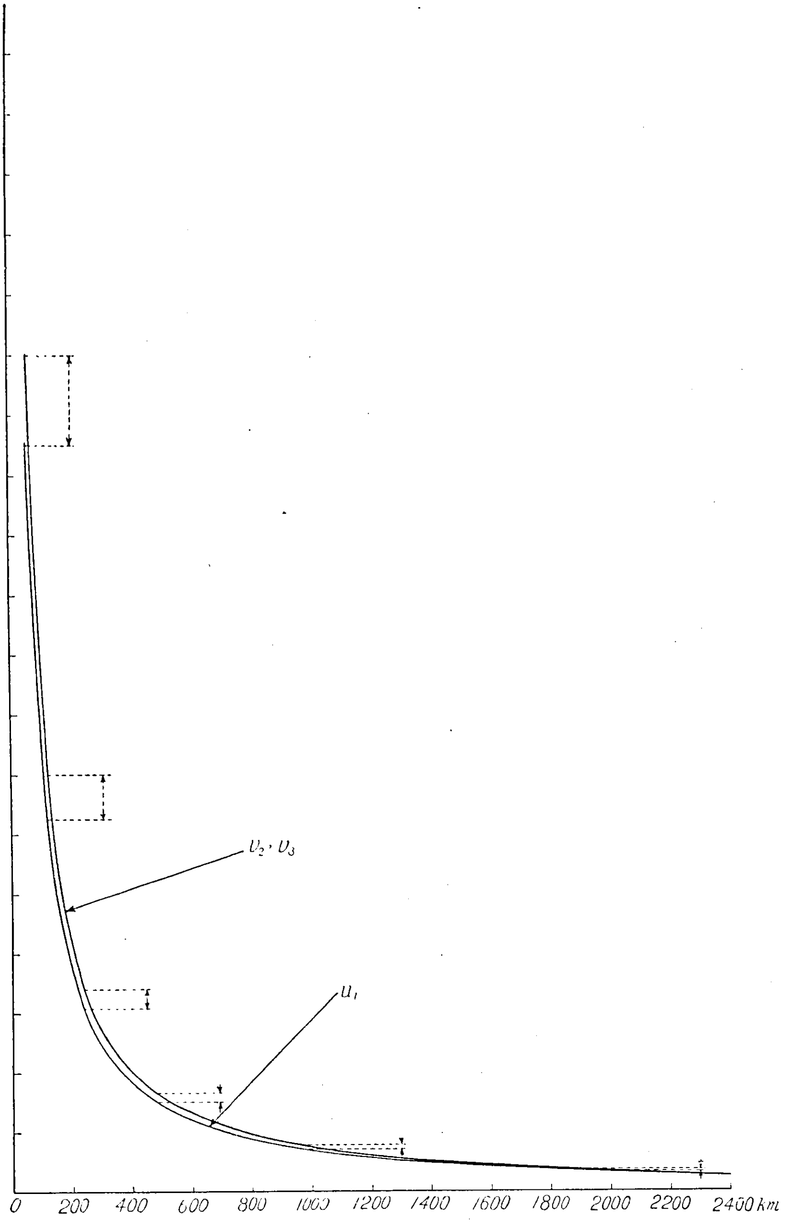

Fig. 12. $L_1=6{\cdot}0$ km, $L_2=3{\cdot}7$ km.

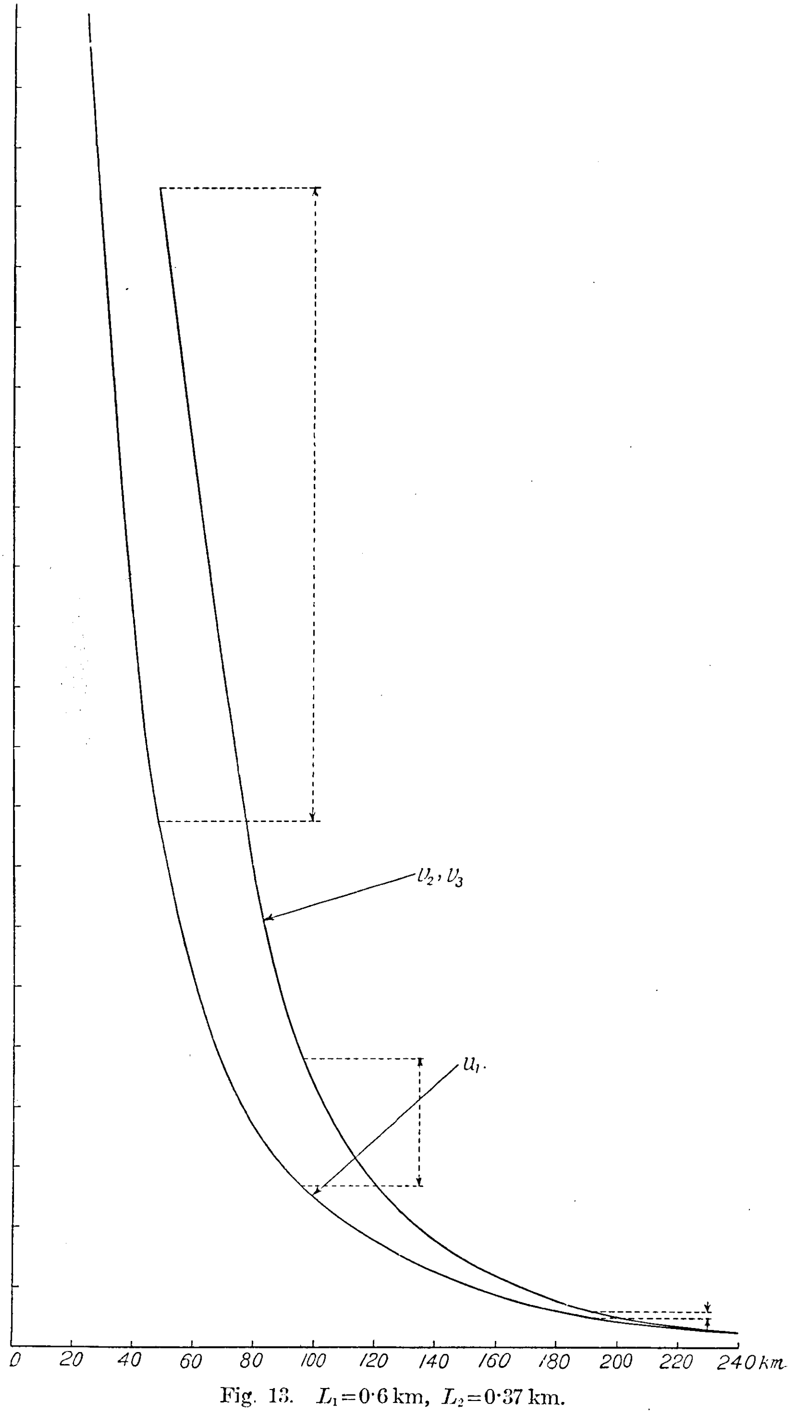

Fig. 13. $L_1 = 0{\cdot}6$ km, $L_2 = 0{\cdot}37$ km.