24. *On the Effect of a Spherical Cavity on the Equilibrium of the Gravitating Semi-infinite Elastic Solid.*

By Genrokuro NISHIMURA and Takeo TAKAYAMA,

Earthquake Research Institute.

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1. Due to Professor M. Ishimoto,[1] there are many pockets of magma in the interior of the crust of the earth and these cavities play an important *rôle* on the occurrence of an earthquake. Thus we think that it may be some importance to investigate the strength of these pockets or cavities under the gravitating body force of the crust.

Professor N. Yamaguti[2] studied the stress distribution in the vicinity of a horizontal circular hole in a gravitating elastic solid for the purpose of investigating the strength of a horizontal tunnel. Recently Dr. T. Sugihara[3] also studied the stress distribution in the neighbourhood of a shaft or of a inclined drift in a gravitating solid with some approximations. But the problem referring to a spherical cavity has not yet been studied.

We shall study, in the first part of this paper, the stress distribution in the neighbourhood of a spherical cavity in the interior of the gravitating semi-infinite elastic solid. In the second part the stress distributions in the neighbourhood of a spherical cavity in a infinite elastic solid under the uniform shearing force are investigated.

2. In the present study, the spherical coordinates (r, θ, ϕ) are used. The density and the gravity constant are taken to be ρ and g. Let the origin O of the coordinates be at the centre of a spherical cavity of the radius a, and the distance from O to the upper horizontal surface of solid be z_0. Fig. 1 indicates the relation between the spherical coordinates (r, θ, ϕ) and the rectangular coordinates (x, y, z). Let u, v, w be the components of displacement in the directions of radius r, co-latitude

1) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, 6 (1929), (in Japanese).

2) N. YAMAGUTI, "On the Stresses Around a Horizontal Circular Hole in Gravitating Elastic Solid," *Jour. Civil Eng.*, Tokyo, 15 (1929), 291.

3) T. SUGIHARA, *Jour. Mining Inst.*, Japan, 47 (1931), 560, (in Japanese).

θ, and azimuth ϕ, and $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{\phi\phi}$ the normal components of traction, $\widehat{r\theta}, \widehat{r\phi}, \widehat{\theta\phi}$ the shearing components of stress with regard to the spherical coordinates. Again, let u', v', w' be the components of displacement in the directions of x, y, and z, and $\widehat{xx}$, $\widehat{yy}, \widehat{zz}$ the normal components of stress and $\widehat{xy}, \widehat{yz}, \widehat{zx}$ the shearing components of stress respectively.

Now the top surface ($z=z_0$) of the solid is horizontal, and therefore there is no variation of the azimuthal components of displacement and stress.

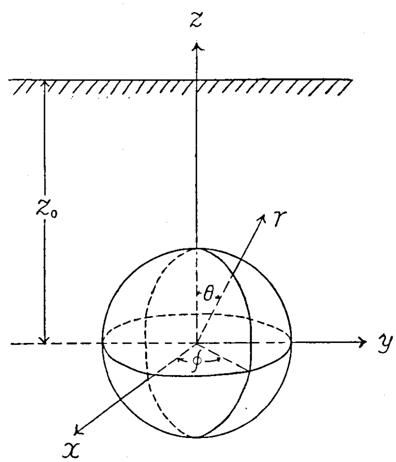

Fig. 1.

Then the stress equations of the equilibrium of the solid in the gravitating field are expressed by

$$\left.\begin{aligned} &\frac{\partial\widehat{rr}}{\partial r}+\frac{1}{r}\frac{\partial\widehat{r\theta}}{\partial\theta}+\frac{1}{r}\{2\widehat{rr}-\widehat{\theta\theta}-\widehat{\phi\phi}+\widehat{r\theta}\cot\theta\}=\rho g\cos\theta, \\ &\frac{\partial\widehat{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\widehat{\theta\theta}}{\partial\theta}+\frac{1}{r}\{(\widehat{\theta\theta}-\widehat{\phi\phi})\cot\theta+3\widehat{r\theta}\}=-\rho g\sin\theta. \end{aligned}\right\}\quad\ldots.(1)$$

The normal components of stress, $\widehat{rr}, \widehat{\theta\theta}, \widehat{\phi\phi}$ and the shearing component of stress $\widehat{r\theta}$ are expressed in the following forms:

$$\left.\begin{aligned} \widehat{rr}&=\frac{\lambda}{r^2\sin\theta}\left\{\frac{\partial}{\partial\theta}(r^2u\sin\theta)+\frac{\partial}{\partial r}(rv\sin\theta)\right\}+2\mu\frac{\partial u}{\partial r}, \\ \widehat{\theta\theta}&=\frac{\lambda}{r^2\sin\theta}\left\{\frac{\partial}{\partial r}(r^2u\sin\theta)+\frac{\partial}{\partial\theta}(rv\sin\theta)\right\}+2\mu\left\{\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right\}, \\ \widehat{\phi\phi}&=\frac{\lambda}{r^2\sin\theta}\left\{\frac{\partial}{\partial r}(r^2u\sin\theta)+\frac{\partial}{\partial\theta}(rv\sin\theta)\right\}, \\ \widehat{r\theta}&=\mu\left\{\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right\}, \end{aligned}\right\}(2)$$

where λ and μ are the Lamé's elastic constants of the solid.

In equation (1), we substitute for the components of stress the expression (2); and we thus obtain the following equations:

$$\left.\begin{aligned} &(\lambda+2\mu)\frac{\partial\Delta}{\partial r}-\frac{2\mu}{r}\frac{\partial\varpi}{\partial\theta}-\frac{2\mu}{r}\varpi\cot\theta=\rho g\cos\theta, \\ &(\lambda+2\mu)\frac{1}{r}\frac{\partial\Delta}{\partial\theta}+2\mu\frac{\partial\varpi}{\partial r}+2\mu\frac{\varpi}{r}=-\rho g\sin\theta, \end{aligned}\right\}\quad\ldots\ldots(3)$$

where

$$\left.\begin{aligned} \varDelta &= \frac{1}{r^2 \sin\theta}\left\{\frac{\partial}{\partial r}(r^2 u \sin\theta)+\frac{\partial}{\partial\theta}(rv\sin\theta)\right\}, \\ 2\varpi &= \frac{\partial v}{\partial r}+\frac{v}{r}-\frac{1}{r}\frac{\partial u}{\partial\theta}. \end{aligned}\right\} \quad \ldots\ldots(4)$$

Among the particular solutions satisfying (3), we take the following particluar solution which is favourable to the present study:

$$\left.\begin{aligned} \varDelta &= \frac{\rho g r}{(\lambda+2\mu)} P_1(\cos\theta), \\ 2\varpi &= 0, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

where $P_1(\cos\theta)$ is the Zonal Harmonics of the first order.

Next we must obtain the complementary solutions of (3) which are necessary to satisfy the boundary conditions of the solid. Now we obtain the following equation, using the equations (3), after some reductions:

$$\frac{\partial^2\varDelta}{\partial r^2}+\frac{2}{r}\frac{\partial\varDelta}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varDelta}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\varDelta}{\partial\theta}=0. \quad \ldots\ldots\ldots\ldots(6)$$

We obtain the following solutions of (6) easily:

$$\begin{aligned} \varDelta &= \left\{A_0+\frac{A_0'}{r}\right\}P_0(\cos\theta)+\left\{A_1 r+\frac{A_1'}{r^2}\right\}P_1(\cos\theta) \\ &\quad +\left\{A_2 r^2+\frac{A_2'}{r^3}\right\}P_2(\cos\theta)+\left\{A_3 r^3+\frac{A_3'}{r^4}\right\}P_3(\cos\theta), \end{aligned} \quad \ldots\ldots(7)$$

where A_0, A_0', A_1, A_1', A_2, A_2', A_3, A_3' are the arbitrary constants, and $P_0(\cos\theta)$, $P_2(\cos\theta)$, $P_3(\cos\theta)$ are the Zonal Harmonics of zero, second and third order.

Using (7), we obtain the following forms of 2ϖ which are the particular solutions of (3):

$$\begin{aligned} 2\varpi &= -\frac{(\lambda+2\mu)}{\mu}\left\{\frac{A_1 r}{2}-\frac{A_1'}{r^2}\right\}\frac{\partial P_1(\cos\theta)}{\partial\vartheta} \\ &\quad -\frac{(\lambda+2\mu)}{\mu}\left\{\frac{A_2 r^2}{3}-\frac{A_2'}{2r^3}\right\}\frac{\partial P_2(\cos\theta)}{\partial\theta} \\ &\quad -\frac{(\lambda+2\mu)}{\mu}\left\{\frac{A_3 r^3}{4}-\frac{A_3'}{3r^3}\right\}\frac{\partial P_3(\cos\theta)}{\partial\theta}. \end{aligned} \quad \ldots\ldots\ldots\ldots(8)$$

Now we find the following two differential equations in relation to $u, v, \varDelta$ and 2ϖ after some reductions from the relations expressed by (4):

$$\frac{\partial^2(r^2u)}{\partial r^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial(r^2u)}{\partial\theta}\right\}=\frac{\partial}{\partial r}(r^2\varDelta)-\frac{2r}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\varpi),\ldots(9)$$

$$\frac{\partial}{\partial r}\left\{r^2\frac{\partial(rv\sin\theta)}{\partial r}\right\}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial(rv\sin\theta)}{\partial\theta}\right\}+2\cos\theta\frac{\partial(r^2u)}{\partial r}$$

$$=\frac{r^2}{\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta\varDelta)+2\sin\theta\frac{\partial}{\partial r}(r^4\varpi). \quad \ldots.(10)$$

Substituting the relations (5) for the expressions of $\varDelta$ and 2ϖ in the equation (9), we obtain the following particular solutions of (9):

$$r^2u=\frac{3}{10}\frac{\rho g r^4}{(\lambda+2\mu)}P_1(\cos\theta)+\frac{1}{5}\frac{\rho g r^4}{(\lambda+2\mu)}P_3(\cos\theta). \quad \ldots.(11)$$

Next, substituting the expression (5) of $\varDelta$ and 2ϖ, and r^2u expressed by (11), for the expressions of $\varDelta$, 2ϖ, and r^2u in equation (10), we find the particular solution $rv\sin\theta$ of (10) in the following forms:

$$rv\sin\theta=\frac{1}{10}\frac{\rho g r^3}{(\lambda+2\mu)}\sin\theta\frac{\partial P_1(\cos\theta)}{\partial\theta}+\frac{1}{15}\frac{\rho g r^3}{(\lambda+2\mu)}\sin\theta\frac{\partial P_3(\cos\theta)}{\partial\theta}. \quad (12)$$

Substituting (7) and (8) for the expression of $\varDelta$ and 2ϖ in the right-hand term of the equation (9), we obtain the following particular solution of (9):

$$\begin{aligned}r^2u=&\left[\frac{1}{3}A_0r^3+\frac{(\lambda+3\mu)}{2\mu}A_0'r^2\right]P_0(\cos\theta)\\&+\left[\frac{(-\lambda+\mu)}{10\mu}r^4A_1-\frac{(\lambda+2\mu)}{\mu}A_1'r\right]P_1(\cos\theta)\\&-\left[\frac{\lambda}{7\mu}A_2r^5+\frac{(3\lambda+5\mu)}{6\mu}A_2'\right]P_2(\cos\theta)\\&-\left[\frac{(3\lambda+\mu)}{18\mu}A_3r^6+\frac{(2\lambda+3\mu)}{5\mu}\frac{A_3'}{r}\right]P_3(\cos\theta). \quad \ldots\ldots(13)\end{aligned}$$

Substituting (7) and (8) and (13) for the expressions of $\varDelta$, 2ϖ and r^2u in equation (10), we obtain the particular solution $rv\sin\theta$ of (10) in the following forms:

$$\begin{aligned}rv\sin\theta=&-\left[\frac{(2\lambda+3\mu)}{10\mu}A_1r^3+\frac{(\lambda+3\mu)}{2\mu}A_1'\right]\sin\theta\frac{\partial P_1(\cos\theta)}{\partial\theta}\\&-\left[\frac{(5\lambda+7\mu)}{42\mu}A_2r^4+\frac{A_2'}{6r}\right]\sin\theta\frac{\partial P_2(\cos\theta)}{\partial\theta}\\&-\left[\frac{(3\lambda+4\mu)}{36\mu}A_3r^5+\frac{(-\lambda+\mu)}{30\mu}\frac{A_3'}{r^2}\right]\sin\theta\frac{\partial P_3(\cos\theta)}{\partial\theta}. \quad (14)\end{aligned}$$

As the complementary solutions of the equation (9) we obtain the following expressions of r^2u which are favourable to the present study:

$$r^2u = -C_0'P_0(\cos\theta) + \left[C_1r^2 - \frac{2}{r}C_1'\right]P_1(\cos\theta)$$
$$+\left[2r^3C_2 - \frac{3}{r^2}C_2'\right]P_2(\cos\theta) + \left[3r^4C_3 - \frac{3C_4'}{r^3}\right]P_3(\cos\theta), \quad ..(15)$$

where C_0', C_1, C_1', C_2, C_2', C_3 and C_3' are the arbitrary constants which may be determined by the boundary conditions.

Corresponding to r^2u expressed by (15), we obtain the following particular solutions of $rv\sin\theta$ from the equation (10):

$$rv\sin\theta = \left[C_1r + \frac{C_1'}{r^2}\right]\sin\theta\frac{\partial P_1(\cos\theta)}{\partial\theta} + \left[C_2r^2 + \frac{C_2'}{r^3}\right]\sin\theta\frac{\partial P_2(\cos\theta)}{\partial\theta}$$
$$+\left[C_3r^3 + \frac{C_3'}{r^4}\right]\sin\theta\frac{\partial P_3(\cos\theta)}{\partial\theta}. \quad ....(16)$$

For obtaining the solutions of u and v expressed by (11), (12), (13), (14), (15) and (16), we used the following relations about the Zonal Harmonics:

$$\left.\begin{aligned}\frac{\partial\left[\sin\theta\frac{\partial P_n(\cos\theta)}{\partial\theta}\right]}{\partial\theta} &= -n(n+1)\sin\theta P_n(\cos\theta),\\ \frac{\partial^2}{\partial\theta^2}\left[\sin\theta\frac{\partial P_n(\cos\theta)}{\partial\theta}\right] &= -n(n+1)\sin\theta\frac{\partial P_n(\cos\theta)}{\partial\theta} - n(n+1)\cos\theta P_n(\cos\theta).\end{aligned}\right\} \quad (17)$$

The general solutions of u and v expressed by (11), (12), (13), (14), (15), (16) satisfy, of course,. the equation of equilibrium of the elastic solid (3). Using these general expressions of displacement, we find the general expressions of the components of stress $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{\phi\phi}$ and $\widehat{r\theta}$ as in the following forms:

$$\widehat{rr} = \left[\left(\lambda + \frac{2\mu}{3}\right)A_0 + \frac{\lambda A_0'}{r} + \frac{4\mu}{r^3}C_0'\right]P_0(\cos\theta)$$
$$+\left[\frac{(5\lambda+6\mu)}{5(\lambda+2\mu)}\rho gr + \frac{(3\lambda+2\mu)}{5}A_1r + \frac{(3\lambda+4\mu)}{r^2}A_1' + \frac{12\mu}{r^4}C_1'\right]P_1(\cos\theta)$$
$$+\left[\frac{\lambda}{7}A_2r^2 + \frac{(9\lambda+10\mu)}{3r^3}A_2' + 4\mu C_2 + \frac{24\mu}{r^5}C_2'\right]P_2(\cos\theta)$$
$$+\left[\frac{4\mu}{5(\lambda+2\mu)}\rho gr - \frac{(3\lambda+4\mu)}{9}A_3r^3 + \frac{(17\lambda+18\mu)}{5r^4}A_3'\right.$$
$$\left. + 12\mu C_3r + \frac{40\mu}{r^6}C_3'\right]P_3(\cos\theta), \quad ..(18)$$

$$\widehat{\theta\theta} = \left[\left(\lambda + \frac{2\mu}{3}\right)A_0 + \frac{(2\lambda+3\mu)}{r}A_0' - \frac{2\mu}{r^3}C_0'\right]P_0(\cos\theta)$$

$$+\left[\frac{(5\lambda+3\mu)}{5(\lambda+2\mu)}\rho gr-\frac{(\lambda+4\mu)}{r^2}A_1'+\frac{(4\lambda+\mu)}{5}A_1r+\frac{2\mu}{r}C_1-\frac{4\mu}{r^4}C_1'\right]P_1(\cos\theta)$$

$$+\left[\frac{5}{7}\lambda A_2r^2-\frac{5\mu}{3r^3}A_2'+4\mu C_2-\frac{6\mu}{r^5}C_2'\right]P_2(\cos\theta)$$

$$+\left[\frac{2\mu}{5(\lambda+2\mu)}\rho gr-\frac{(6\lambda+\mu)}{9}A_3r^3+\frac{(\lambda-6\mu)}{5r^4}A_3'+6\mu C_3r-\frac{8\mu}{r^6}C_3'\right]P_3(\cos\theta)$$

$$+\left[\frac{\mu}{5(\lambda+2\mu)}\rho gr-\frac{(2\lambda+3\mu)}{5}A_1r-\frac{(\lambda+3\mu)}{r^2}A_1'+\frac{2\mu}{r}C_1+\frac{2\mu}{r^4}C_1'\right]\frac{\partial^2P_1(\cos\theta)}{\partial\theta^2}$$

$$+\left[-\frac{(5\lambda+7\mu)}{21}A_2r^2-\frac{\mu}{3}\frac{A_2'}{r^2}+2\mu C_2+\frac{2\mu}{r^5}C_2'\right]\frac{\partial^2P_2(\cos\theta)}{\partial\theta^2}$$

$$+\left[\frac{2\mu}{15(\lambda+2\mu)}\rho gr-\frac{(3\lambda+4\mu)}{18}A_3r^3+\frac{(\lambda-\mu)}{15r^4}A_3'+2\mu C_3r+\frac{2\mu}{r^6}C_3'\right]\frac{\partial^2P_3(\cos\theta)}{\partial\theta^2}, \quad (19)$$

$$\widehat{\phi\phi}=\left[\left(\lambda+\frac{2\mu}{3}\right)A_0+\frac{(2\lambda+3\mu)}{r}A_0'-\frac{2\mu}{r^3}C_0'\right]P_0(\cos\theta)$$

$$+\left[\frac{(5\lambda+3\mu)}{5(\lambda+2\mu)}\rho gr+\frac{(4\lambda+\mu)}{5}A_1r-\frac{(\lambda+4\mu)}{r^2}A_1'+\frac{2\mu}{r}C_1-\frac{4\mu}{r^4}C_1'\right]P_1(\cos\theta)$$

$$+\left[\frac{5}{7}\lambda A_2r^2-\frac{5\mu}{3r^3}A_2'+4\mu C_2-\frac{6\mu}{r^5}C_2'\right]P_2(\cos\theta)$$

$$+\left[\frac{2}{5}\frac{\mu\rho gr}{(\lambda+2\mu)}+\frac{(6\lambda-\mu)}{9}A_3r^3+\frac{(\lambda-6\mu)}{5r^4}A_3'+6\mu C_3r-\frac{8\mu}{r^6}C_3'\right]P_3(\cos\theta)$$

$$+\left[\frac{\mu\rho gr}{5(\lambda+2\mu)}-\frac{(2\lambda+3\mu)}{5}A_1r-\frac{(\lambda+3\mu)}{r^2}A_1'+\frac{2\mu}{r}C_1+\frac{2\mu}{r^4}C_1'\right]\cot\theta\frac{\partial P_1(\cos\theta)}{\partial\theta}$$

$$+\left[-\frac{(5\lambda+7\mu)}{21}A_2r^2-\frac{\mu}{3r^3}A_2'+2\mu C_2+\frac{2\mu}{r^5}C_2'\right]\cot\theta\frac{\partial P_2(\cos\theta)}{\partial\theta}$$

$$+\left[\frac{2}{15}\frac{\mu\rho gr}{(\lambda+2\mu)}-\frac{(3\lambda+4\mu)}{18}A_3r^3+\frac{(\lambda-\mu)}{15r^4}A_3'+2\mu C_3r+\frac{2\mu}{r^6}C_3'\right]\cot\theta\frac{\partial P_3(\cos\theta)}{\partial\theta}, \ldots (20)$$

$$\widehat{r\theta}=\left[\frac{2}{5}\frac{\mu\rho gr}{(\lambda+2\mu)}-\frac{(3\lambda+2\mu)}{10}A_1r+\frac{\mu A_1'}{r^2}-\frac{6\mu}{r^4}C_1'\right]\frac{\partial P_1(\cos\theta)}{\partial\theta}$$

$$+\left[-\frac{(8\lambda+7\mu)}{21}A_2r^2-\frac{8\mu}{r^5}C_2'-\frac{(3\lambda+2\mu)}{6r^3}A_2'+2C_2\mu\right]\frac{\partial P_2(\cos\theta)}{\partial\theta}$$

$$+\left[\frac{4}{15}\frac{\mu\rho g r}{(\lambda+2\mu)}-\frac{(15\lambda+14\mu)}{36}A_3 r^3-\frac{(8\lambda+7\mu)}{15}\frac{A_3'}{r^4}\right.$$

$$\left.+4\mu C_3 r-\frac{10\mu}{r^6}C_3'\right]\frac{\partial P_3(\cos\theta)}{\partial\theta}. \quad \ldots\ldots(21)$$

The general expressions of the components of stresses thus obtained satisfy, of course, the equations of equilibrium of the elastic body under the gravitating field expressed by (1).

Using these general expressions of displacement and traction, we shall study the problem on the effect of an internal spherical cavity on the stresses of the gravitating semi-infinite elastic solid.

Next referring to Fig. 1, we have the following expressions of the components of displacement and stress which are the solutions of the gravitating semi-infinite elastic solid having no cavity in its interior:

$$u'=v'=0, \quad w'=\frac{\rho g}{2(\lambda+2\mu)}(z_0-z)^2, \quad \ldots\ldots\ldots\ldots(22)$$

$$\left.\begin{aligned}\widehat{xx}=\widehat{yy}&=-\frac{\lambda}{\lambda+2\mu}\rho g(z_0-z),\\ \widehat{zz}&=-\rho g(z_0-z),\\ \widehat{xy}=\widehat{yz}&=\widehat{zx}=0.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

These expressions satisfy, of course, the following conditions: the normal and the tangential components of stress $\widehat{zz}$, $\widehat{yz}$, and $\widehat{zx}$ vanish at the free surface $z=z_0$, and the stress at any point in the gravitating solid increases according to the increase of the position from the upper surface of the solid.

When we transform the components of displacement and stress expressed by (22), (23) into the components of displacement and stress in the polar coordinates $(r,\ \theta,\ \phi)$, we have the following forms of them:

$$\left.\begin{aligned}u=&-\frac{\rho g z_0}{2(\lambda+2\mu)}r+\left[\frac{\rho g z_0^2}{2(\lambda+2\mu)}+\frac{3\rho g r^2}{8(\lambda+2\mu)}\right]\cos\theta\\ &-\frac{\rho g z_0 r}{2(\lambda+2\mu)}\cos 2\theta+\frac{\rho g r^2}{8(\lambda+2\mu)}\cos 3\theta,\\ v=&-\left[\frac{\rho g z_0^2}{2(\lambda+2\mu)}+\frac{\rho g r^2}{8(\lambda+2\mu)}\right]\sin\theta+\frac{\rho g z_0 r}{2(\lambda+2\mu)}\sin 2\theta\\ &-\frac{\rho g r^2}{8(\lambda+2\mu)}\sin 3\theta,\end{aligned}\right\} \quad \ldots(24)$$

$$\left.\begin{aligned}
\widehat{rr} &= -\frac{(\lambda+\mu)}{(\lambda+2\mu)}\rho g z_0 + \frac{(2\lambda+3\mu)}{2(\lambda+2\mu)}\rho g r \cos\theta \\
&\quad - \frac{\mu}{(\lambda+2\mu)}\rho g z_0 \cos 2\theta + \frac{\mu}{2(\lambda+2\mu)}\rho g r \cos 3\theta, \\
\widehat{\theta\theta} &= -\frac{(\lambda+\mu)}{(\lambda+2\mu)}\rho g z_0 + \frac{(2\lambda+\mu)}{2(\lambda+2\mu)}\rho g r \cos\theta \\
&\quad + \frac{\mu}{(\lambda+2\mu)}\rho g z_0 \cos 2\theta - \frac{\mu}{2(\lambda+2\mu)}\rho g r \cos 3\theta, \\
\widehat{\phi\phi} &= -\frac{\lambda}{(\lambda+2\mu)}\rho g z_0 + \frac{\lambda}{(\lambda+2\mu)}\rho g r \cos\theta, \\
\widehat{r\theta} &= -\frac{\mu}{2(\lambda+2\mu)}\rho g r \sin\theta + \frac{\mu}{(\lambda+2\mu)}\rho g z_0 \sin 2\theta \\
&\quad - \frac{\mu}{2(\lambda+2\mu)}\rho g r \sin 3\theta.
\end{aligned}\right\} \quad \ldots\ldots (25)$$

The expressions (24), (25) are favourable to the study of the equilibrium of the gravitating semi-infinite solid containing no cavity as we have previously stated.

Now, the boundary conditions of the present problem are as follows :

1. The surface of the spherical cavity is free from traction.
2. The displacement and the stress at the whole space far from the spherical cavity in the solid are equal to the ones expressed by (24) and (25).

These are denoted by

$$r=a\,;\quad \widehat{rr}=0,\ \ \widehat{r\theta}=0. \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

$$\left.\begin{aligned}
r=\infty\,;\quad & u=u \text{ expressed by } (24), \\
& v=v \quad ,, \quad ,, \quad ,, \\
& \widehat{rr}=\widehat{rr} \quad ,, \quad ,, \quad (25), \\
& \widehat{\theta\theta}=\widehat{\theta\theta} \quad ,, \quad ,, \quad ,, \\
& \widehat{\phi\phi}=\widehat{\phi\phi} \quad ,, \quad ,, \quad ,, \\
& \widehat{r\theta}=\widehat{r\theta} \quad ,, \quad ,, \quad ,,
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (27)$$

Using the expressions of displacement (11), (12), (13), (14), (15), (16) and that of stress (18), (19), (20), (21) which are the general solutions of the equation of the equilibrium (1) or (3), and also the conditions (26) and (27), we find the following values of the arbitrary constants :

From (26) ;

$$\left.\begin{aligned} &A_0'=0, \quad A_1=A_2=A_3=C_3=0, \\ &A_0=-\frac{\rho g z_0}{(\lambda+2\mu)}, \quad C_1=\frac{\rho g z_0^2}{2(\lambda+2\mu)}, \quad C_2=-\frac{\rho g z_0}{2(\lambda+2\mu)}. \end{aligned}\right\} \dots (28)$$

From (26);

$$\left.\begin{aligned} &A_2'=\frac{20\mu\,\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)}, \quad C_0'=\frac{(3\lambda+2\mu)\rho g z_0 a^3}{12\mu(\lambda+2\mu)}, \\ &C_1'=\frac{\rho g a^5}{90(\lambda+2\mu)}, \quad A_1'=-\frac{\rho g a^3}{3(\lambda+2\mu)}, \\ &C_2'=-\frac{2(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)}, \quad C_3'=\frac{2\rho g(\lambda+\mu)a^7}{(\lambda+2\mu)(19\lambda+26\mu)}, \\ &A_3'=-\frac{28\mu\rho g a^5}{(\lambda+2\mu)(19\lambda+26\mu)}. \end{aligned}\right\} \dots (29)$$

Substituting these values (28), (29) for C_2', A_3', C_3', A_1', C_1', C_0', A_0', C_1, C_2, A_0, A_1, A_2, A_3, C_3, A_2', in the expressions (11), (12), (13), (14), (15), (16), (18), (19), (20), (21), we obtain the final results which are favourable to the present study as follows:

$$\begin{aligned} u=&-\left[\frac{\rho g z_0 r}{3(\lambda+2\mu)}+\frac{(3\lambda+2\mu)\rho g z_0 a^3}{12\mu(\lambda+2\mu)r^2}\right]P_0(\cos\theta) \\ &+\left[\frac{3}{10}\frac{\rho g r^2}{(\lambda+2\mu)}+\frac{\rho g a^3}{3\mu r}+\frac{\rho g z_0^2}{2(\lambda+2\mu)}-\frac{2\rho g a^5}{90(\lambda+2\mu)r^3}\right]P_1(\cos\theta) \\ &-\left[\frac{10(3\lambda+5\mu)\rho g z_0 a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^2}+\frac{2\rho g z_0 r}{3(\lambda+2\mu)}-\frac{6(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^4}\right]P_2(\cos\theta) \\ &+\left[\frac{1}{5}\frac{\rho g r^2}{(\lambda+2\mu)}+\frac{28\rho g(2\lambda+3\mu)a^5}{5(\lambda+2\mu)(19\lambda+26\mu)r^3}-\frac{8(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]P_3(\cos\theta), \end{aligned}$$

$$\dots\dots\dots\dots (30)$$

$$\begin{aligned} v=&\left[\frac{1}{10}\frac{\rho g r^2}{(\lambda+2\mu)}+\frac{(\lambda+3\mu)\rho g a^3}{6\mu(\lambda+2\mu)r}+\frac{\rho g z_0^2}{2(\lambda+2\mu)}+\frac{\rho g a^5}{90(\lambda+2\mu)r^3}\right]\frac{\partial P_1(\cos\theta)}{\partial\theta} \\ &-\left[\frac{10\mu\rho g z_0 a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^2}+\frac{\rho g z_0 r}{3(\lambda+2\mu)}+\frac{2(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^4}\right]\frac{\partial P_2(\cos\theta)}{\partial\theta} \\ &+\left[\frac{\rho g r^2}{15(\lambda+2\mu)}+\frac{14(-\lambda+\mu)\rho g a^5}{15(\lambda+2\mu)(19\lambda+26\mu)r^3}\right. \\ &\qquad\left.+\frac{2(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]\frac{\partial P_3(\cos\theta)}{\partial\theta}, \quad \dots (31) \end{aligned}$$

$$\widehat{rr}=\left[-\frac{(3\lambda+2\mu)}{3(\lambda+2\mu)}\rho g z_0+\frac{(3\lambda+2\mu)\rho g z_0 a^3}{3(\lambda+2\mu)r^3}\right]P_0(\cos\theta)$$

$$+\left[\frac{(5\lambda+6\mu)\rho gr}{5(\lambda+2\mu)}-\frac{(3\lambda+4\mu)\rho ga^3}{3(\lambda+2\mu)r^2}+\frac{2\mu\rho ga^5}{15(\lambda+2\mu)r^4}\right]P_1(\cos\theta)$$

$$+\left[-\frac{4\mu\rho gz_0}{3(\lambda+2\mu)}+\frac{20\mu(9\lambda+10\mu)\rho gz_0a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}-\frac{48\mu(\lambda+\mu)\rho gz_0a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]P_2(\cos\theta)$$

$$+\left[\frac{4\mu\rho gr}{5(\lambda+2\mu)}-\frac{28\mu(17\lambda+18\mu)\rho ga^5}{5(\lambda+2\mu)(19\lambda+26\mu)r^4}+\frac{80\mu(\lambda+\mu)\rho ga^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]P_3(\cos\theta), \quad \ldots\ldots(32)$$

$$\widehat{\theta\theta}=-\left[\frac{(3\lambda+2\mu)}{3(\lambda+2\mu)}\rho gz_0+\frac{(3\lambda+2\mu)\rho gz_0a^3}{6(\lambda+2\mu)r^3}\right]P_0(\cos\theta)$$

$$+\left[\frac{(5\lambda+3\mu)}{5(\lambda+2\mu)}\rho gr+\frac{\mu\rho gz_0^2}{(\lambda+2\mu)r}+\frac{(\lambda+4\mu)\rho ga^3}{3(\lambda+2\mu)r^2}-\frac{4\mu\rho ga^5}{90(\lambda+2\mu)r^4}\right]P_1(\cos\theta)$$

$$-\left[\frac{4\mu\rho gz_0}{3(\lambda+2\mu)}+\frac{100\mu^2\rho gz_0a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}-\frac{12\mu(\lambda+\mu)\rho gz_0a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]P_2(\cos\theta)$$

$$+\left[\frac{2\mu\rho gr}{5(\lambda+2\mu)}-\frac{28\mu(\lambda-6\mu)\rho ga^5}{5(\lambda+2\mu)(19\lambda+26\mu)r^4}-\frac{16\mu(\lambda+\mu)\rho ga^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]P_3(\cos\theta)$$

$$+\left[\frac{\mu\rho gr}{5(\lambda+2\mu)}+\frac{\mu\rho gz_0^2}{(\lambda+2\mu)r}+\frac{(\lambda+3\mu)\rho ga^3}{3(\lambda+2\mu)r^2}+\frac{2\mu\rho ga^5}{90(\lambda+2\mu)r^4}\right]\frac{\partial^2P_1(\cos\theta)}{\partial\theta^2}$$

$$-\left[\frac{2\mu\rho gz_0}{3(\lambda+2\mu)}+\frac{20\mu^2\rho gz_0a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}+\frac{4\mu(\lambda+\mu)\rho gz_0a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\frac{\partial^2P_2(\cos\theta)}{\partial\theta^2}$$

$$+\left[\frac{2\mu\rho gr}{15(\lambda+2\mu)}-\frac{28\mu(\lambda-\mu)\rho ga^5}{15(\lambda+2\mu)(19\lambda+26\mu)r^4}+\frac{4\mu(\lambda+\mu)\rho ga^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\frac{\partial^2P_3(\cos\theta)}{\partial\theta^2}, \quad \ldots\ldots(33)$$

$$\widehat{\phi\phi}=-\left[\frac{(3\lambda+2\mu)\rho gz_0}{3(\lambda+2\mu)}+\frac{(3\lambda+2\mu)\rho gz_0a^3}{6(\lambda+2\mu)r^3}\right]P_0(\cos\theta)$$

$$+\left[\frac{(5\lambda+3\mu)}{5(\lambda+2\mu)}\rho gr+\frac{\mu\rho gz_0^2}{(\lambda+2\mu)r}+\frac{(\lambda+4\mu)\rho ga^3}{3(\lambda+2\mu)r^2}\right]P_1(\cos\theta)$$

$$-\left[\frac{4\mu\rho gz_0}{3(\lambda+2\mu)}+\frac{100\mu^2\rho gz_0a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}-\frac{12\mu(\lambda+\mu)\rho gz_0a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]P_2(\cos\theta)$$

$$+\left[\frac{2\mu\rho gr}{5(\lambda+2\mu)}-\frac{28\mu(\lambda-6\mu)\rho ga^5}{5(\lambda+2\mu)(19\lambda+26\mu)r^4}-\frac{16\mu(\lambda+\mu)\rho ga^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]P_3(\cos\theta)$$

$$+\left[\frac{\mu\rho gr}{5(\lambda+2\mu)}+\frac{\mu\rho gz_0^2}{(\lambda+2\mu)r}+\frac{(\lambda+3\mu)\rho ga^3}{3(\lambda+2\mu)r^2}+\frac{2\mu\rho ga^5}{90(\lambda+2\mu)r^4}\right]\cot\theta\frac{\partial P_1(\cos\theta)}{\partial\theta}$$

$$-\left[\frac{2\mu\rho gz_0}{3(\lambda+2\mu)}+\frac{20\mu^2\rho gz_0a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}+\frac{4\mu(\lambda+\mu)\rho gz_0a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\cot\theta\frac{\partial P_2(\cos\theta)}{\partial\theta}$$

$$+\left[\frac{2\mu\rho g r}{15(\lambda+2\mu)}-\frac{28\mu(\lambda-\mu)\rho g a^5}{15(\lambda+2\mu)(19\lambda+26\mu)r^4}+\frac{4u(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cot\theta\frac{\partial P_3(\cos\theta)}{\partial\theta}, \quad ..(34)$$

$$\widehat{r\theta}=-\left[\frac{\mu\rho g a^3}{3(\lambda+2\mu)r^2}+\frac{6\mu\rho g a^5}{90(\lambda+2\mu)r^4}-\frac{2\mu\rho g r}{5(\lambda+2\mu)}\right]\frac{\partial P_1(\cos\theta)}{\partial\theta}$$
$$-\left[\frac{2\mu\rho g z_0}{3(\lambda+2\mu)}+\frac{10\mu(3\lambda+2\mu)\rho g z_0 a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}-\frac{16\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\frac{\partial P_2(\cos\theta)}{\partial\theta}$$
$$+\left[\frac{4\mu\rho g r}{15(\lambda+2\mu)}+\frac{28\mu(8\lambda+7\mu)\rho g a^5}{15(\lambda+2\mu)(19\lambda+26\mu)r^4}-\frac{20\mu(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\frac{\partial P_3(\cos\theta)}{\partial\theta}. \quad ..(35)$$

Now we now the relations such that

$$\left.\begin{aligned}&P_0(\cos\theta)=1, \quad P_1(\cos\theta)=\cos\theta, \quad P_2(\cos\theta)=\frac{3}{4}\cos2\theta+\frac{1}{4},\\&P_3(\cos\theta)=\left(\frac{5}{8}\cos3\theta+\frac{3}{8}\cos\theta\right).\end{aligned}\right\} \quad (36)$$

Using these formulae, we rewrite the final solutions (30), (31), (32), (33), (34), (35) in the following forms which are more or less convenient for the study of the natures of the equilibrium in the interior of the solid:

$$u=-\frac{\rho g z_0}{2(\lambda+2\mu)}r-\frac{(9\lambda^2+30\lambda\mu+26\mu^2)\rho g z_0 a^3}{4\mu(\lambda+2\mu)(9\lambda+14\mu)r^2}+\frac{3(\lambda+\mu)\rho g z_0 a^5}{2(\lambda+2\mu)(9\lambda+14\mu)r^4}$$
$$+\left[\frac{\rho g z_0^2}{2(\lambda+2\mu)}+\frac{3\rho g r^2}{8(\lambda+2\mu)}+\frac{\rho g a^3}{3\mu r}+\frac{(68\lambda+103\mu)\rho g a^5}{18(\lambda+2\mu)(19\lambda+26\mu)r^3}-\frac{3(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]\cos\theta$$
$$+\left[-\frac{\rho g z_0}{2(\lambda+2\mu)}r-\frac{5(3\lambda+5\mu)\rho g z_0 a^3}{2(\lambda+2\mu)(9\lambda+14\mu)r^2}+\frac{9(\lambda+\mu)\rho g z_0 a^5}{2(\lambda+2\mu)(9\lambda+14\mu)r^4}\right]\cos2\theta$$
$$+\left[\frac{\rho g r^2}{8(\lambda+2\mu)}+\frac{7(2\lambda+3\mu)\rho g a^5}{2(\lambda+2\mu)(19\lambda+26\mu)r^3}-\frac{5(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]\cos3\theta, \quad ....(37)$$

$$v=-\left[\frac{\rho g z_0^2}{2(\lambda+2\mu)}+\frac{\rho g r^2}{8(\lambda+2\mu)}+\frac{(\lambda+3\mu)\rho g a^3}{6\mu(\lambda+2\mu)r}+\frac{(-5\lambda+23\mu)\rho g a^5}{36(\lambda+2\mu)(19\lambda+26\mu)r^3}+\frac{3(\lambda+\mu)\rho g a^7}{4(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]\sin\theta$$

$$+\left[\frac{\rho g z_0 r}{2(\lambda+2\mu)}+\frac{5\mu\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)r^2}+\frac{3\rho g z_0 a^5}{(9\lambda+14\mu)r^4}\right]\sin 2\theta$$

$$-\left[\frac{\rho g r^2}{8(\lambda+2\mu)}+\frac{7(-\lambda+\mu)\rho g a^5}{4(\lambda+2\mu)(19\lambda+26\mu)r^3}+\frac{15(\lambda+\mu)\rho g a^7}{4(\lambda+2\mu)(19\lambda+26\mu)r^5}\right]\sin 3\theta, \quad \ldots\ldots\ldots\ldots(38)$$

$$\widehat{rr}=-\frac{(\lambda+\mu)}{(\lambda+2\mu)}\rho g z_0+\frac{(27\lambda^2+105\lambda\mu+78\mu^2)\rho g z_0 a^3}{3(\lambda+2\mu)(9\lambda+14\mu)r^3}-\frac{12\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}$$

$$+\left[\frac{(2\lambda+3\mu)\rho g r}{2(\lambda+2\mu)}-\frac{(3\lambda+4\mu)\rho g a^3}{3(\lambda+2\mu)r^2}-\frac{\mu(199\lambda+206\mu)\rho g a^5}{6(\lambda+2\mu)(19\lambda+26\mu)r^4}\right.$$

$$\left.+\frac{30\mu(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos\theta-\left[\frac{\mu\rho g z_0}{(\lambda+2\mu)}-\frac{5\mu(9\lambda+10\mu)\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)r^3}\right.$$

$$\left.+\frac{36\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\cos 2\theta+\left[\frac{\mu}{2(\lambda+2\mu)}\rho g r-\frac{7\mu(17\lambda+18\mu)\rho g a^5}{2(\lambda+2\mu)(19\lambda+26\mu)r^4}\right.$$

$$\left.+\frac{50\mu(\lambda+\mu)\rho g a^7}{(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos 3\theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots(39)$$

$$\widehat{\theta\theta}=-\frac{(\lambda+\mu)\rho g z_0}{(\lambda+2\mu)}-\frac{(9\lambda^2+20\lambda\mu+26\mu^2)\rho g z_0 a^3}{2(\lambda+2\mu)(9\lambda+14\mu)r^3}+\frac{3\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}$$

$$+\left[\frac{(2\lambda+\mu)}{2(\lambda+2\mu)}\rho g r+\frac{\mu\rho g a^3}{3(\lambda+2\mu)r^2}-\frac{\mu(16\lambda-61\mu)\rho g a^5}{6(\lambda+2\mu)(19\lambda+26\mu)r^4}\right.$$

$$\left.-\frac{15\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos\theta+\left[\frac{\mu}{(\lambda+2\mu)}\rho g z_0-\frac{5\mu^2\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)r^3}\right.$$

$$\left.+\frac{21\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\cos 2\theta-\left[\frac{\mu\rho g r}{2(\lambda+2\mu)}+\frac{7\mu(2\lambda+3\mu)\rho g a^5}{(19\lambda+26\mu)r2(\lambda+2\mu)^4}\right.$$

$$\left.+\frac{65\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos 3\theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots(40)$$

$$\widehat{\phi\phi}=-\frac{\lambda}{(\lambda+2\mu)}\rho g z_0-\frac{(9\lambda^2+20\lambda\mu+6\mu^2)\rho g z_0 a^3}{2(\lambda+2\mu)(9\lambda+14\mu)r^3}+\frac{9\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}$$

$$+\left[\frac{\lambda}{(\lambda+2\mu)}\rho g r+\frac{\mu\rho g a^3}{3(\lambda+2\mu)r^2}+\frac{\mu(26\lambda+19\mu)\rho g a^5}{6(\lambda+2\mu)(19\lambda+26\mu)r^4}\right.$$

$$\left.-\frac{45\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos\theta$$

$$+\left[-\frac{15\mu^2\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)r^3}+\frac{15\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\right]\cos 2\theta$$

$$+\left[\frac{35\mu^2\rho g a^5}{2(\lambda+2\mu)(19\lambda+26\mu)r^4}-\frac{35\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\right]\cos 3\theta, \quad \ldots(41)$$

$$\widehat{r\theta}=\left[-\frac{\mu}{2(\lambda+2\mu)}\rho g r+\frac{\mu\rho g a^3}{3(\lambda+2\mu)r^2}-\frac{\mu(26\lambda+19\mu)\rho g a^5}{6(\lambda+2\mu)(19\lambda+26\mu)r^4}\right.$$

$$+\frac{15\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\Big]\sin\theta+\Big[\frac{\mu}{(\lambda+2\mu)}\rho g z_0+\frac{5\mu(3\lambda+2\mu)\rho g z_0 a^3}{(\lambda+2\mu)(9\lambda+14\mu)r^3}$$
$$-\frac{24\mu(\lambda+\mu)\rho g z_0 a^5}{(\lambda+2\mu)(9\lambda+14\mu)r^5}\Big]\sin 2\theta+\Big[-\frac{\mu}{2(\lambda+2\mu)}\rho g r-\frac{7\mu(8\lambda+7\mu)\rho g a^5}{2(\lambda+2\mu)(19\lambda+26\mu)r^4}$$
$$+\frac{75\mu(\lambda+\mu)\rho g a^7}{2(\lambda+2\mu)(19\lambda+26\mu)r^6}\Big]\sin 3\theta. \quad\ldots\ldots\ldots\ldots(42)$$

These equations shew us the distributions of displacement and stress in the solid of two kinds: one of which is in a plane-symmetry with respect to the plane $\theta=\pi/2$, and the other is unsymmetrical about that plane. The former corresponds to the distributions of displacement and stress in a elastic solid having a spherical cavity due to the uniform compression $\rho g z_0$ at infinity, and the latter corresponds to the different distributions of displacement and stress.

For asertaining the distributions of displacement and stress in the solid, we take the following numerical example. In this example, we tentatively assume that $\lambda=\mu$, the poisson's ratio$=1/4$, and $a/z_0=1/5$. The units of displacement and stress are taken $\rho g z_0^2/5\mu$ and $\rho g z_0$ respectively. Under these assumptions, we have the six tables in which the displacement and stress corresponding to the values of r/a and θ are tabulated. (Table I, II, III, IV, V, VI.)

The values in these tables are also illustrated in six figures. (Fig. 2, 3, 4, 5, 6, 7.) The parameter of each curve in these figures is r/a, while the abscissa of the same figures is the angle of the colatitude θ and the ordinates of the curves represent the magnitude of components of displacement and stress of which the units are $\rho g z_0^2/5\mu$ and $\rho g z_0$ respectively.

From these figures it may be seen that the tractions in the solid are distributed symmetrically about the vertical axis ($\theta=0$) and unsymmetrically with respect to the plane $z=0$ passing the origin, and thus the tractions on the surface of the spherical cavity is largest on the circle of $r/a=1$, $\theta\fallingdotseq100°$. It may also be seen that the component of stress $\widehat{\theta\theta}$, which is the compression on the surface of the cavity in this case, is larger than the other components of the stress on this surface. In this numerical examples, all the components of stress have the negative sign (compression), but in the other cases where z_0/a is small and the poisson's ratio is less than that of this example, the stress in some portions (at the regions of $\theta=0°$, $\theta=180°$) of spherical surface of the cavity has the positive sign (tension).

As we have discussed, $\widehat{\theta\theta}$ on the surface of the cavity is the important component of stress on the failure of the cavity. We shall study the properties of the component $\theta\theta_{r=a}$ in detail.

We write $\widehat{\theta\theta}$, when $\lambda=\mu$, on the surface of the cavity in the following form:

$$\widehat{\theta\theta}_{r/a=1}=-0{\cdot}979\,\rho g z_0+0{\cdot}444\rho g a\cos\theta$$
$$+0{\cdot}870\rho g z_0\cos 2\theta-0{\cdot}778\rho g a\cos 3\theta. \qquad (43)$$

Using this equation, we get the result as in the following table (Table VII) which gives us the magnitudes of $\theta\theta_{r=a}$ in cases of $z_0/a=3, 5, 7, 9$, and 10, the unit of the magnitude of stress being taken $\rho g a$.

Table VII gives us the following two figures. (Fig. 8 and 9.) The unit of stress $\widehat{\theta\theta}_{r=a}$ in these figures is taken to be $\rho g a$.

Fig. 8 and 9 indicate us that the maximum magnitude of $\widehat{\theta\theta}_{r=a}$ occurs in the vicinity of $\theta=90°-100°$. Again, $\theta\theta_{r=a}$ increases linearly with the increase of depth of the centre of cavity from the upper surface of the elastic solid. The rate of increase of $\theta\theta_{r=a}$ with the depth, therefore, is maximum at $\theta=90°\sim100°$.

Table I.

(Magnitude of the component of displacement u. Unit $=\frac{\rho g z_0^2}{5\mu}$.)

θ \ r/a	1	2	3	4
0°	0·27	0·12	−0·13	−0·39
10°	0·27	0·12	−0·12	−0·37
20°	0·28	0·15	−0·09	−0·34
30°	0·29	0·16	−0·05	−0·29
45°	0·29	0·21	0·03	−0·17
60°	0·27	0·21	0·10	−0·04
70°	0·19	0·18	0·11	0·02
80°	0·09	0·12	0·09	0·04
90°	−0·03	0·01	0·01	0
100°	−0·20	−0·15	−0·14	−0·13
110°	−0·41	−0·36	−0·35	−0·35
120°	−0·64	−0·59	−0·62	−0·64
135°	−1·02	−1·04	−1·05	−1·20
150°	−1·33	−1·35	−1·54	−1·77
160°	−1·50	−1·53	−1·78	−2·07
170°	−1·60	−1·66	−1·94	−2·28
180°	−1·64	−1·70	−1·99	−2·35

Table II.

(Magnitude of the component of displacement v. Unit $=\frac{\rho g z_0^2}{5\mu}$.)

θ \ r/a	1	2	3	4
0°	0	0	0	0
10°	−0·04	−0·05	−0·02	0
20°	−0·08	−0·10	−0·05	−0·01
30°	−0·15	−0·17	−0·09	−0·03
45°	−0·27	−0·29	−0·19	−0·10
60°	−0·45	−0·46	−0·36	−0·26
70°	−0·59	−0·59	−0·50	−0·42
80°	−0·73	−0·73	−0·67	−0·62
90°	−0·87	−0·86	−0·85	−0·85
100°	−0·99	−0·98	−1·02	−1·08
110°	−1·07	−1·05	−1·16	−1·29
120°	−1·09	−1·08	−1·23	−1·43
135°	−1·01	−1·01	−1·21	−1·45
150°	−0·78	−0·79	−0·98	−1·20
160°	−0·56	−0·56	−0·71	−0·88
170°	−0·29	−0·29	−0·37	−0·46
180°	0	0	0	0

Table III.

(Magnitude of the component of stress $\widehat{rr}$. Unit $=\rho g z_0$.)

θ \ r/a	1	2	3	4
0°	0	−0·40	−0·33	−0·17
10°	0	−0·40	−0·34	−0·18
20°	0	−0·40	−0·35	−0·21
30°	0	−0·40	−0·36	−0·24
45°	0	−0·39	−0·36	−0·28
60°	0	−0·37	−0·34	−0·30
70°	0	−0·35	−0·33	−0·30
80°	0	−0·35	−0·33	−0·31
90°	0	−0·36	−0·35	−0·34
100°	0	−0·39	−0·40	−0·41
110°	0	−0·45	−0·49	−0·53
120°	0	−0·54	−0·63	−0·69
135°	0	−0·72	−0·90	−1·02
150°	0	−0·87	−1·19	−1·38
160°	0	−0·97	−1·35	−1·57
170°	0	−1·05	−1·46	−1·70
180°	0	−1·08	−1·50	−1·75

Table IV.

(Magnitude of the component of stress $\widehat{\theta\theta}$. Unit $=\rho g z_0$.)

θ \ r/a	1	2	3	4
0°	−0·17	−0·23	−0·15	−0·07
10°	−0·21	−0·25	−0·16	−0·08
20°	−0·30	−0·29	−0·19	−0·11
30°	−0·47	−0·37	−0·26	−0·16
45°	−0·81	−0·52	−0·40	−0·30
60°	−1·21	−0·72	−0·60	−0·51
70°	−1·48	−0·85	−0·75	−0·68
80°	−1·71	−0·97	−0·89	−0·85
90°	−1·85	−1·06	−1·01	−1·00
100°	−1·89	−1·11	−1·10	−1·12
110°	−1·81	−1·11	−1·13	−1·18
120°	−1·61	−1·06	−1·10	−1·17
135°	−1·15	−0·91	−0·97	−1·05
150°	−0·62	−0·72	−0·78	−0·85
160°	−0·32	−0·61	−0·66	−0·73
170°	−0·11	−0·53	−0·58	−0·64
180°	−0·04	−0·51	−0·55	−0·61

Table V.

(Magnitude of the component of stress $\widehat{\phi\phi}$. Unit $=\rho g z_0$.)

θ \ r/a	1	2	3	4
0°	−0·10	−0·23	−0·15	−0·08
10°	−0·11	−0·23	−0·15	−0·08
20°	−0·14	−0·24	−0·16	−0·09
30°	−0·19	−0·25	−0·17	−0·11
45°	−0·28	−0·26	−0·20	−0·15
60°	−0·39	−0·28	−0·24	−0·20
70°	−0·46	−0·30	−0.27	−0·24
80°	−0·51	−0·32	−0·30	−0·29
90°	−0·54	−0·34	−0·34	−0·33
100°	−0·55	−0·37	−0·37	−0·38
110°	−0·53	−0·39	−0·40	−0·43
120°	−0·48	−0·42	−0·44	−0·47
135°	−0·37	−0·45	−0·48	−0·52
150°	−0·24	−0·48	−0·52	−0·57
160°	−0·18	−0·50	−0·53	−0·59
170°	−0·13	−0·51	−0·54	−0·60
180°	−0·11	−0·51	−0·55	−0·61

Table VI.

(Magnitude of the component of stress $\widehat{r\theta}$. Unit $=\rho g z_0$.)

θ \ r/a	1	2	3	4
0°	0	0	0	0
10°	0	0·08	0·05	0·03
20°	0	0·15	0·10	0·06
30°	0	0·21	0·15	0·10
45°	0	0·27	0·20	0·15
60°	0	0·26	0·21	0·18
70°	0	0·21	0·18	0·16
80°	0	0·12	0·11	0·10
90°	0	0·01	0	0
100°	0	−0·12	−0·13	−0·13
110°	0	−0·25	−0·26	−0·28
120°	0	−0·36	−0·38	−0·41
135°	0	−0·45	−0·48	−0·52
150°	0	−0·41	−0·45	−0·49
160°	0	−0·31	−0·34	−0·38
170°	0	−0·17	−0·18	−0·21
180°	0	0	0	0

Table VII. (The magnitude of $\widehat{\theta\theta}_{r=a}$. Unit$=\rho g a$.)

θ \ z_0/a	3	5	7	9	10
0°	−0·66	−0·88	− 1·10	− 1·32	− 1·42
10°	−0·72	−1·04	− 1·32	− 1·69	− 1·85
20°	−0·91	−1·54	− 2·16	− 2·78	− 3·09
30°	−1·25	−2·34	− 3·42	− 4·52	− 5·06
45°	−2·07	−4·03	− 6·00	− 7·95	− 8·93
60°	−3·24	−6·07	− 8·90	−11·72	−13·14
70°	−4·11	−7·40	−10·70	−13·99	−15·63
80°	−4·92	−8·52	−12·16	−15·71	−17·50
90°	−5·55	−9·25	−12·94	−16·64	−18·49
100°	−5·85	−9·45	−13·09	−16·64	−18·43
110°	−5·76	−9·05	−12·25	−15·64	−17·29
120°	−5·24	−8·07	−10·90	−13·72	−15·14
135°	−3·80	−5·76	− 7·72	− 9·68	−10·65
150°	−2·02	−3·11	− 4·19	− 5·29	− 5·82
160°	−0·97	−1·59	− 2·21	− 2·84	− 3·15
170°	−0·25	−0·57	− 0·84	− 1·21	− 1·38
180°	−0·01	−0·21	− 0·43	− 0·65	− 0·76

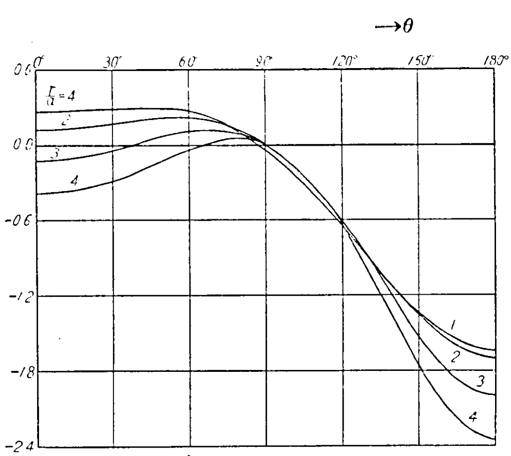

Fig. 2.

[Magnitude of u, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{5}$, unit$=\frac{\rho g z_0^2}{5\mu}$.]

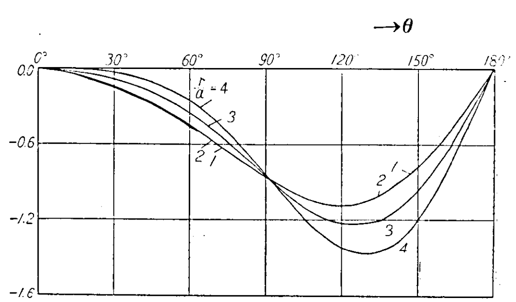

Fig. 3.

[Magnitude of v, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{5}$, unit$=\frac{\rho g z_0^2}{5\mu}$.]

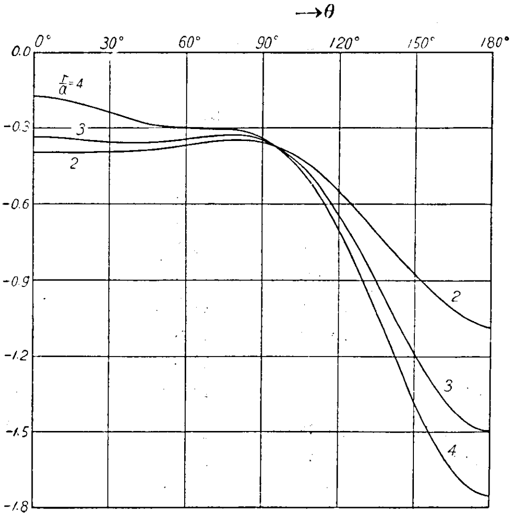

Fig. 4.

[Magnitude of $\widehat{rr}$, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{z_0}$, unit$=\rho g z_0$.]

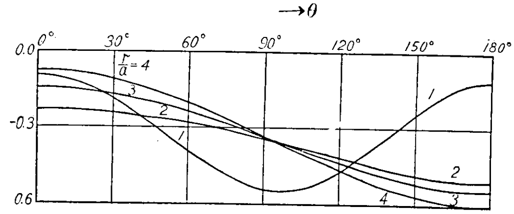

Fig. 6.

[Magnitude of $\widehat{\phi\phi}$, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{5}$, unit$=\rho g z_0$.]

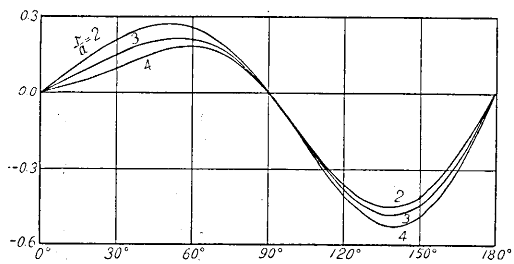

Fig. 7. →θ

[Magnitude of $\widehat{r\theta}$, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{5}$, unit$=\rho g z_0$.]

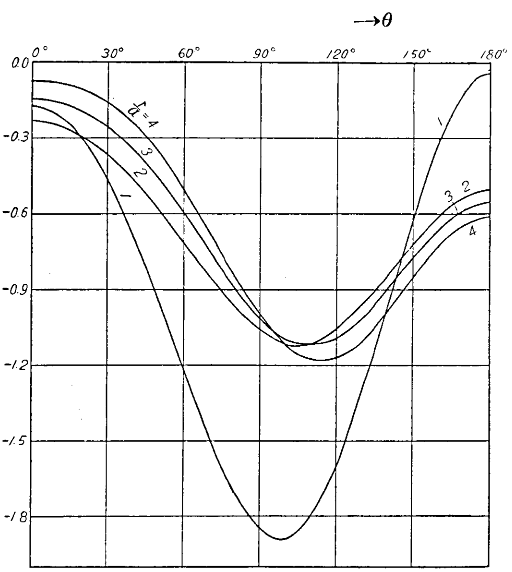

Fig. 5.

[Magnitude of $\widehat{\theta\theta}$, $\lambda=\mu$, $\frac{a}{z_0}=\frac{1}{5}$, unit$=\rho g z_0$.]

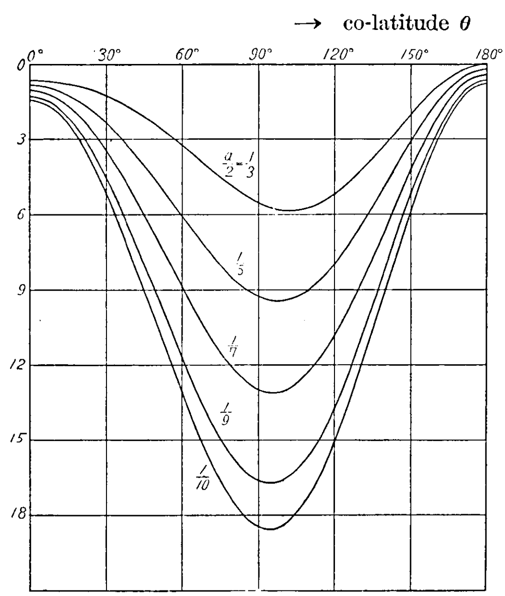

Fig. 8.

[Magnitude of $\widehat{\theta\theta}_{r=a}$, $\lambda=\mu$, unit$=\rho g a$.]

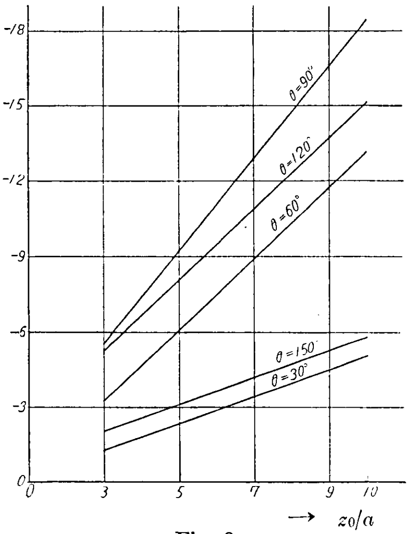

Fig. 9.

[Magnitude of $\widehat{\theta\theta}_{r=a}$, $\lambda=\mu$.
Unit$=\rho g a$.]

3. When the gravitating solid is incompressible, the displacement and the stress are formulated in the following forms:

$$\left.\begin{aligned} u &= -\frac{\rho g a z_0}{4\mu}\left(\frac{a}{r}\right)^2, \\ v &= -\frac{\rho g a^3}{6\mu}\frac{1}{r}\sin\theta, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (44)$$

and

$$\left.\begin{aligned} \widehat{rr} &= \rho g z_0\left[-1+\left(\frac{a}{r}\right)^3\right]+\rho g z_0\left[\frac{r}{z_0}-\frac{a^3}{z_0 r^2}\right]\cos\theta, \\ \widehat{\theta\theta} &= \rho g z_0\left[-1-\frac{1}{2}\left(\frac{a}{r}\right)^3\right]+\rho g r\cos\theta, \\ \widehat{\phi\phi} &= \rho g z_0\left[-1-\frac{1}{2}\left(\frac{a}{r}\right)^3\right]+\rho g r\cos\theta, \\ \widehat{r\theta} &= 0. \end{aligned}\right\} \quad \ldots (45)$$

It can be seen from the equations (45) that when the gravitating solid is incompressible, the interior of solid is maintained in hydrostatic pressure. The stress on the surface of cavity is compressive, and when the depth of the centre of cavity from the top surface of solid is very deep, and $a \ll z_0$, the stress components on the surface of cavity are approximately expressed by the following equations:

$$\left.\begin{aligned} \widehat{\theta\theta}_{r=a} &= -\frac{3}{2}\rho g z_0, \\ \widehat{\phi\phi}_{r=a} &= -\frac{3}{2}\rho g z_0. \end{aligned}\right\} \quad \ldots\ldots\ldots (46)$$

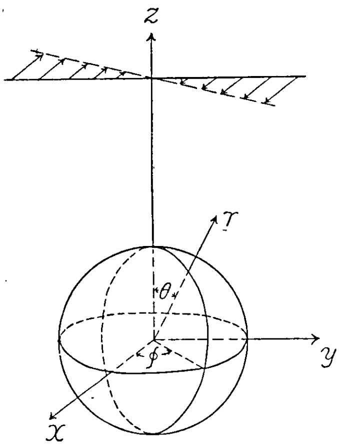

Fig. 10.

4. In the preceding sections, we have studied the stress distributions in a semi-infinite elastic gravitating solid having a spherical cavity in its interior. Now, we shall study the case where the semi-infinite gravitating solid containing a spherical cavity is subjected to a uniform simple shear. As the problem in this case becomes naturally free from gravitating body force, it is sufficient to study the case where the body is strained by the uniform shear. We can obtain the problem of the acutual case by superposing the results thus obtained on those of the preceding sections.

Referring to Fig. 10, we assume that the solid having a spherical cavity of radius a is subjected to a uniform simple shear:[4]

$$\widehat{xy} = S. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

The equations of equilibrium of elastic solid referring to spherical coordinates are as follows:

$$\left.\begin{aligned} &(\lambda+2\mu)\frac{\partial \Delta}{\partial r} - \frac{2\mu}{r\sin\theta}\frac{\partial(\sin\theta\,\varpi_\phi)}{\partial\theta} + \frac{2\mu}{r\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi} = 0, \\ &(\lambda+2\mu)\frac{1}{r}\frac{\partial \Delta}{\partial\theta} - \frac{2\mu}{r\sin\theta}\frac{\partial\varpi_r}{\partial\phi} + \frac{2\mu}{r}\frac{\partial(r\varpi_\phi)}{\partial r} = 0, \\ &(\lambda+2\mu)\frac{1}{r\sin\theta}\frac{\partial \Delta}{\partial\phi} - \frac{2\mu}{r}\frac{\partial(r\varpi_\theta)}{\partial r} + \frac{2\mu}{r}\frac{\partial\varpi_r}{\partial\theta} = 0, \end{aligned}\right\} \quad \ldots (48)$$

where

4) Professor Love studied the same problem, using the rectangular coordinates, yet I have fully studied the problem in spherical coordinates in order to get the solutions congruent with that of the preceding sections. LOVE, *The mathematical theory of Elasticity*, 4th ed., 252.

$$\left.\begin{aligned}\Delta&=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2u\sin\theta)+\frac{\partial}{\partial\theta}(rv\sin\theta)+\frac{\partial}{\partial\phi}(rw)\right],\\ 2\varpi_r&=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(rw\sin\theta)-\frac{\partial}{\partial\phi}(rv)\right],\\ 2\varpi_\theta&=\frac{1}{r\sin\theta}\left[\frac{\partial u}{\partial\phi}-\frac{\partial}{\partial r}(rw\sin\theta)\right],\\ 2\varpi_\phi&=\frac{1}{r}\left[\frac{\partial}{\partial r}(rv)-\frac{\partial u}{\partial\theta}\right].\end{aligned}\right\}\quad\ldots(49)$$

Using (48), (49), we have to obtain the general expressions of displacement and stress which are favourable to the present study.

Now we can obtain the following two equations concerning to Δ and $r\varpi_r$ from (48) after some reductions:

$$\frac{\partial^2\Delta}{\partial r^2}+\frac{2}{r}\frac{\partial\Delta}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Delta}{\partial\theta}\right)+\frac{1}{r^2\sin\theta}\frac{\partial^2\Delta}{\partial\phi^2}=0,\quad\ldots\ldots(50)$$

$$\frac{\partial^2(r\varpi_r)}{\partial r^2}+\frac{2}{r}\frac{\partial(r\varpi_r)}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial(r\varpi_r)}{\partial\theta}\right)+\frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2}(r\varpi_r)=0.\quad(51)$$

The solutions of (50), (51), which are necessary for the present study, are expresed by

$$\Delta=\left[Ar^2+\frac{A'}{r^3}\right]P_2^2(\cos\theta)\sin2\phi,\ldots\ldots\ldots\ldots\ldots(52)$$

$$2\varpi_r=-\left[Br+\frac{B'}{r^4}\right]P_2^2(\cos\theta)\cos2\phi,\ldots\ldots\ldots\ldots\ldots(53)$$

where $P_m^n(\cos\theta)$ is the associated function of m^{th} order and n^{th} degree, and A, A', B, B' are the arbitrary constants to be determined by the boundary conditions.

Substituting the expressions (52), (53) for Δ and $2\varpi_r$ in the equations (48), we can obtain the following forms of $2\varpi_\theta$ and $2\varpi_\phi$:

$$\begin{aligned}2\varpi_\theta=&\frac{2(\lambda+2\mu)}{\mu}\left\{\frac{A}{3}r^2-\frac{A'}{2r^3}\right\}\frac{P_2^2(\cos\theta)}{\sin\theta}\cos2\phi\\ &-\left\{\frac{B}{2}r-\frac{B'}{3r^4}\right\}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\cos2\phi,\quad\ldots(54)\end{aligned}$$

$$\begin{aligned}2\varpi_\phi=&-\frac{(\lambda+2\mu)}{\mu}\left\{\frac{Ar^2}{3}-\frac{A'}{2r^3}\right\}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin2\phi\\ &+\left\{Br-\frac{2}{3}\frac{B'}{r^4}\right\}\frac{P_2^2(\cos\theta)}{\sin\theta}\sin2\phi.\quad\ldots(55)\end{aligned}$$

Now, we can find the following three equations concerning $rw\sin\theta$,

$r^2u,\ rv\sin\theta,\ \Delta,\ 2\varpi_r,\ 2\varpi_\theta,\ 2\varpi_\phi$:

$$\frac{\partial}{\partial r}\left\{r^2\frac{\partial(rw\sin\theta)}{\partial r}\right\}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial}{\partial\theta}(rw\sin\theta)\right\}+\frac{1}{\sin^2\theta}\frac{\partial^2(rw\sin\theta)}{\partial\phi^2}$$
$$=r^2\frac{\partial\Delta}{\partial\phi}+\frac{2r^2}{\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta\,\varpi_r)-2\sin\theta\frac{\partial}{\partial r}(r^3\varpi_\theta),\quad\ldots\ldots(56)$$

$$\frac{\partial^2}{\partial r^2}(r^2u)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial(r^2u)}{\partial\theta}\right\}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2(r^2u)}{\partial\phi^2}$$
$$=\frac{\partial}{\partial r}(r^2\Delta)-\frac{2r}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,\varpi_\phi)+\frac{2r}{\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi},\quad\ldots\ldots\ldots(57)$$

$$\frac{\partial}{\partial r}\left\{r^2\frac{\partial(rv\sin\theta)}{\partial r}\right\}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial(rv\sin\theta)}{\partial\theta}\right\}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}(rv\sin\theta)$$
$$+2\cos\theta\frac{\partial(r^2u)}{\partial r}=\frac{r^2}{\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta\,\Delta)-2r^2\frac{\partial\varpi_r}{\partial\phi}+2\sin\theta\frac{\partial}{\partial r}(r^3\varpi_\phi).\quad(58)$$

Substituting (52), (53), (54), (55) for Δ, $2\varpi_r$, $2\varpi_\theta$, $2\varpi_\phi$ in the equations (56), (57), we obtain the particular solutions concerning $rw\sin\theta$ and r^2u :

$$rw\sin\theta=-\left\{\frac{(5\lambda+7\mu)}{21\mu}r^4A+\frac{1}{3}\frac{A'}{r}\right\}P_2^2(\cos\theta)\cos2\phi$$
$$+\frac{1}{6}\left\{Br^3+\frac{B'}{r^2}\right\}\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\cos2\phi,\quad\ldots\ldots\ldots\ldots(59)$$

$$r^2u=-\left\{\frac{\lambda}{7\mu}Ar^5+\frac{(3\lambda+5\mu)}{6\mu}A'\right\}P_2^2(\cos\theta)\sin2\phi.\quad\ldots\ldots(60)$$

For obtaining these solutions, use has been made of the following relations concerning the associated functions :

$$\left.\begin{aligned}&\frac{\partial}{\partial\theta}\left\{\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\right\}=\frac{4}{\sin\theta}P_2^2(\cos\theta)-6\sin\theta P_2^2(\cos\theta),\\&\frac{\partial^2}{\partial\theta^2}\left\{\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\right\}=\frac{4}{\sin\theta}\frac{\partial}{\partial\theta}P_2^2(\cos\theta)-6\left\{\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\right.\\&\qquad\left.+\cos\theta\,P_2^2(\cos\theta)\right\}-\frac{4\cos\theta}{\sin^2\theta}P_2^2(\cos\theta).\end{aligned}\right\}\ldots(61)$$

The complementary solutions of (56) and (57) are of the forms :

$$rw\sin\theta=2\left\{Cr^2+\frac{C'}{r^3}\right\}P_2^2(\cos\theta)\cos2\phi,\quad\ldots\ldots\ldots\ldots(62)$$

$$r^2u=\left\{2Cr^3-\frac{3C'}{r^2}\right\}P_2^2(\cos\theta)\sin2\phi,\quad\ldots\ldots\ldots\ldots(63)$$

were C, C' are also the arbitrary constants.

Substituting (52), (53), (55), (60), (63) for $\varDelta$, $2\varpi_r$, $2\varpi_\phi$ and r^2u in the equation of (58), we obtain the following particular solutions which are favourable to our present study:

$$rv\sin\theta = -\left\{\frac{(5\lambda+7\mu)}{42\mu}Ar^4 - \frac{A'}{6r}\right\}\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi$$
$$+\left\{\frac{B}{3}r^3 + \frac{B'}{3r^2}\right\}P_2^2(\cos\theta)\sin 2\phi$$
$$+\left\{Cr^2 + \frac{C'}{r^3}\right\}\sin\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi. \qquad (64)$$

Using the general expressions of the components of displacement (59), (60), (62), (63) and (64), we formulate the components of stress $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{\phi\phi}$, $\widehat{r\theta}$, $\widehat{r\phi}$, $\widehat{\theta\phi}$ in the following forms[5]:

$$\widehat{rr} = \left\{\frac{\lambda}{7}r^2A + \frac{(9\lambda+10\mu)}{3}\frac{A'}{r^3} + 4\mu C + \frac{24\mu}{r^5}C'\right\}P_2^2(\cos\theta)\sin 2\phi, \qquad (65)$$

$$\widehat{\theta\theta} = \left\{\frac{5\lambda}{7}Ar^2 - \frac{5\mu}{3}A'\frac{1}{r^3} + 4\mu C - \frac{6\mu C'}{r^5}\right\}P_2^2(\cos\theta)\sin 2\phi$$
$$-\left\{\frac{(5\lambda+7\mu)}{21}Ar^2 + \frac{\mu}{3}\frac{A'}{r^3} - 2\mu C - \frac{2\mu}{r^5}C'\right\}\frac{\partial^2 P_2^2(\cos\theta)}{\partial\theta^2}\sin 2\phi, \qquad (66)$$

$$\widehat{\phi\phi} = \left\{\frac{5\lambda}{7}Ar^2 - \frac{5\mu}{3}A'\frac{1}{r^3} + 4\mu C - \frac{6\mu C'}{r^5}\right\}P_2^2(\cos\theta)\sin 2\phi$$
$$+\left\{\frac{4(5\lambda+7\mu)}{21}Ar^2 + \frac{4\mu}{3}\frac{A'}{r^3} - 8\mu C - 8\mu\frac{C'}{r^5}\right\}\frac{P_2^2(\cos\theta)}{\sin^2\theta}\sin 2\phi$$
$$+\left\{-\frac{(5\lambda+7\mu)}{21}Ar^2 - \frac{\mu}{3}\frac{A'}{r^3} + 2\mu C + \frac{2\mu C'}{r^5}\right\}\cot\theta\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi, \qquad (67)$$

$$\widehat{r\theta} = -\left\{\frac{(8\lambda+7\mu)}{6}Ar^2 + \frac{(3\lambda+2\mu)}{6r^3}A' - 2\mu C + 8\mu\frac{C'}{r^5}\right\}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi, \qquad (68)$$

$$\widehat{r\phi} = -\left\{\frac{2(8\lambda+7\mu)}{21}Ar^2 + \frac{(3\lambda+2\mu)}{3r^3}A' - 4\mu C + \frac{16\mu C'}{r^5}\right\}\frac{P_2^2(\cos\theta)}{\sin\theta}\cos 2\phi, \qquad (69)$$

$$\widehat{\theta\phi} = \left\{-\frac{(5\lambda+7\mu)}{21}Ar^2 - \frac{\mu}{3}\frac{A'}{r^3} + 2\mu C + \frac{2\mu C'}{r^5}\right\}\frac{\partial}{\partial\theta}\left(\frac{P_2^2(\cos\theta)}{\sin\theta}\right)\cos 2\phi$$
$$+\left\{\frac{(5\lambda+7\mu)}{21}Ar^2 + \frac{\mu}{3}\frac{A'}{r^3} - 2\mu C - \frac{2\mu C'}{r^5}\right\}\frac{\cot\theta}{\sin\theta}P_2^2(\cos\theta)\cos 2\phi$$

5) In these expressions we omit the terms concerning to B from the nature of our problem.

$$-\left\{\frac{(5\lambda+7\mu)}{21}Ar^2+\frac{\mu}{3}\frac{A'}{r^3}-2\mu C-2\mu\frac{C'}{r^5}\right\}\frac{1}{\sin\theta}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\cos 2\phi. \qquad \ldots\ldots\ldots\ldots(70)$$

Now the boundary conditions which are necessray for the present case are as follows:

$$r=a\,;\quad \widehat{rr}=0,\ \widehat{r\theta}=0,\ \widehat{r\phi}=0, \qquad \ldots\ldots\ldots\ldots\ldots\ldots(71)$$

$$\left.\begin{aligned} r=\infty\,;\quad \widehat{rr}&=S\sin^2\theta\sin 2\phi,\\ \widehat{\theta\theta}&=S\cos^2\theta\sin 2\phi,\\ \widehat{\phi\phi}&=-S\sin 2\phi,\\ \widehat{r\theta}&=\frac{S}{2}\sin 2\theta\sin 2\phi,\\ \widehat{r\phi}&=S\sin\theta\cos 2\phi,\\ \widehat{\theta\phi}&=S\cos\theta\cos 2\phi. \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(72)$$

The conditions expressed by (71) mean that the surface of the spherical cavity is free from tractions, and the conditions (72) has the significance that the portions in the solid far from cavity are subjected to the uniform simple shear expressed by (47).

We find the following values of arbitrary constants in the expressions (65), (66), (67), (68), (69), (70) from the boundary conditions (71), (72):

$$\left.\begin{aligned} &B=B'=0,\quad A=0,\quad C=\frac{S}{12\mu},\\ &C'=\frac{(\lambda+\mu)Sa^5}{2\mu(9\lambda+14\mu)},\quad A'=-\frac{5a^3S}{(9\lambda+14\mu)}. \end{aligned}\right\} \qquad \ldots\ldots(73)$$

Substituting these values in the general expressions of displacement and stress (59), (60), (62), (63), (64), (65), (66), (67), (68), (69), (70), we obtain the final results as follows:

$$u=S\left\{\frac{r}{6\mu}+\frac{5}{6}\frac{(3\lambda+5\mu)}{\mu(9\lambda+14\mu)}\frac{a^3}{r^2}-\frac{3}{2}\frac{(\lambda+\mu)}{\mu(9\lambda+14\mu)}\frac{a^5}{r^4}\right\}P_2^2(\cos\theta)\sin 2\phi, \quad \ldots(74)$$

$$v=S\left\{\frac{r}{12\mu}+\frac{5}{6}\frac{1}{(9\lambda+14\mu)}\frac{a^3}{r^2}+\frac{1}{2}\frac{(\lambda+\mu)}{\mu(9\lambda+14\mu)}\frac{a^5}{r^4}\right\}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi, \quad \ldots(75)$$

$$w=S\left\{\frac{r}{6\mu}+\frac{5}{3}\frac{1}{(9\lambda+14\mu)}\frac{a^3}{r^2}+\frac{(\lambda+\mu)}{\mu(9\lambda+14\mu)}\frac{a^5}{r^4}\right\}\frac{P_2^2(\cos\theta)}{\sin\theta}\cos 2\phi, \quad \ldots\ldots(76)$$

$$\widehat{rr}=S\left\{\frac{1}{3}-\frac{5}{3}\frac{(9\lambda+10\mu)}{(9\lambda+14\mu)}\frac{a^3}{r^3}+\frac{12(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}P_2^2(\cos\theta)\sin 2\phi, \quad \ldots\ldots(77)$$

$$\widehat{\theta\theta}=S\left\{1+\frac{10\mu}{(9\lambda+14\mu)}\frac{a^3}{r^3}+\frac{6(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}\sin 2\phi$$

$$+S\left\{-\frac{1}{3}+\frac{5\mu}{3(9\lambda+14\mu)}\frac{a^3}{r^3}-\frac{7(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}P_2^2(\cos\theta)\sin 2\phi, \quad \ldots(78)$$

$$\widehat{\phi\phi}=S\left\{-1-\frac{10\mu}{(9\lambda+14\mu)}\frac{a^3}{r^3}-\frac{6(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}\sin 2\phi$$

$$+S\left\{\frac{5\mu}{(9\lambda+14\mu)}\frac{a^3}{r^3}-\frac{5(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}P_2^2(\cos\theta)\sin 2\phi, \quad \ldots\ldots\ldots(79)$$

$$\widehat{r\theta}=S\left\{\frac{1}{6}+\frac{5(3\lambda+2\mu)}{6(9\lambda+14\mu)}\frac{a^3}{r^3}-\frac{4(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\sin 2\phi, \quad \ldots\ldots\ldots(80)$$

$$\widehat{r\phi}=S\left\{\frac{1}{3}+\frac{5(3\lambda+2\mu)}{3(9\lambda+14\mu)}\frac{a^3}{r^3}-\frac{8(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}\frac{P_2^2(\cos\theta)}{\sin\theta}\cos 2\phi, \quad \ldots\ldots\ldots(81)$$

$$\widehat{\theta\phi}=S\left\{\frac{1}{6}+\frac{5\mu}{3(9\lambda+14\mu)}\frac{a^3}{r^3}+\frac{(\lambda+\mu)}{(9\lambda+14\mu)}\frac{a^5}{r^5}\right\}\frac{1}{\sin\theta}\frac{\partial P_2^2(\cos\theta)}{\partial\theta}\cos 2\phi. \quad \ldots(82)$$

We understand from these final expressions of displacement and stress that the displacement and the stress are expressed by Tesseral Harmonics of second order and second degree.

For the purpose of knowing the distributions of stress more fully, we take the following numerical example. Putting $\lambda=\mu$ in the expressions (77), (78), (79), (80), (81), (82), we obtain the results in the tables in which the displacement and the stress corresponding to different values of r/a and θ are tabulated. (Table VIII, IX, X, XI, XII, XIII, XIV, XV, XVI.)

These tables also indicate us the following figures. (Fig. 10, 11, 12, 13a, 13b, 14a, 14b, 15a, 15b, 16a, 16b, 17a, 17b, 18a, 18b).

From these tables and figures, we understand that the region effected by the presence of a spherical cavity is limited in the boundary region of that cavity and the effective radius is approximately $4a$ from the centre of the cavity, and stress component $\theta\theta_{r=a}$ is larger than the other components on the surface of cavity.

The values of $\widehat{\theta\theta}$ on the surface of cavity, when $\lambda=\mu$, is of the following form:

$$\theta\theta_{r=a}=\frac{15}{23}S\sin 2\phi+\frac{30}{23}S\cos 2\theta\sin 2\phi. \quad \ldots\ldots\ldots\ldots(83)$$

The expression (83) gives us the results shown in the following tables, in which the magnitudes of $\widehat{\theta\theta}_{r=a}$ are tabulated for different values of θ and ϕ. (Table XVII.)

At $\theta=0$, $\phi=45°$, or $\theta=0$, $\phi=225°$, the tension stress is 1.957 S, and at $\theta=0$, $\phi=135°$, or $\theta=0$, $\phi=315°$, the compression stress is 1·957 S.

Table VIII.

(Magnitude of u, unit$=\frac{Sa}{\mu}\sin 2\phi$, $\lambda=\mu$.)

θ \ r/a	1	2	3	4
0°	0·08	0·03	0·02	0·01
10°	0·10	0·07	0·06	0·07
20°	0·18	0·17	0·20	0·25
30°	0·29	0·32	0·41	0·52
45°	0·49	0·60	0·80	1·03
60°	0·70	0·88	1·19	1.54
70°	0·81	1·03	1·40	1·81
80°	0·88	1·13	1·53	1·99
90°	0·90	1·17	1·58	2·05

Table IX.

(Magnitude of v, unit$=\frac{Sa}{\mu}\sin 2\phi$, $\lambda=\mu$.)

θ \ r/a	1	2	3	4
0°	0	0	0	0
10°	0·17	0·18	0·26	0·34
20°	0·32	0·34	0·49	0·65
30°	0·42	0·46	0·66	0·87
45°	0·49	0·53	0·76	1·00
60°	0.42	0·46	0·66	0·87
70°	0·32	0·34	0·49	0·65
80°	0·17	0·18	0.26	0·34
90°	0	0	0	0

Table X.

(Magnitude of w, unit$=\frac{Sa}{\mu}\cos 2\phi$, $\lambda=\mu$.)

θ \ r/a	1	2	3	4
0°	0	0	0	0
10°	0·17	0·19	0·27	0·35
20°	0·33	0·37	0·52	0·69
30°	0·49	0·53	0·77	1·01
45°	0·69	0·76	1·08	1·42
60°	0·85	0·93	1·32	1·75
70°	0·92	1·01	1·44	1·89
80°	0·96	1·05	1·51	1·99
90°	0·98	1·07	1·53	2·02

Table XI.

(Magnitude of $\widehat{rr}$, unit$=S\sin 2\phi$, $\lambda=\mu$.)

θ \ r/a	1·0	1·05	1·1	1·2	1·25	1·5	1·75	2·0	2·5	2·75	3·0	3·5	3·75	4·0
0°	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10°	0							0·02			0·03			0·03
20°	0							0·07			0·70			0·11
30°	0							0·15			0·22			0·24
45°	0	−0·05	−0·08	- 0.07	−0·04	0·10	0·21	0·29	0·39	0·41	0·43	0·46	0·46	0·47
60°	0							0·44			0·65			0·70
70°	0							0·51			0·76			0·83
80°	0							0·56			0·83			0·91
90°	0	−0·10	−0·16	−0·13	−0·07	0·19	0·42	0·58	0·78	0·82	0·86	0·91	0·93	0·94

Table XII.

(Magnitude of $\widehat{\theta\theta}$, unit $= S \sin 2\phi$, $\lambda = \mu$.)

θ \ r/a	1·0	1·1	1·25	1·5	1·75	2·0	2·5	3·0	3·5	4·0
0°	1·96	1·65	1·39	1·19	1·11	1·07	1·03	1·02	1·01	1·01
10°	1·88					1·04		0·99		0·98
20°	1·65					0·95		0·90		0·89
30°	1·30					0·81		0·77		0·76
45°	0·65	0·66	0·65	0·60	0·58	0·56	0·53	0·52	0·51	0·51
60°	0					0·30		0·27		0·26
70°	−0·35					0·16		0·14		0·13
80°	−0·58					0·07		0·05		0·03
90°	−0·65	−0·33	−0·10	0·01	0·05	0·04	0·03	0·02	0·01	0·01

Table XIII.

(Magnitude of $\widehat{\phi\phi}$, unit $= S \cos 2\phi$, $\lambda = \mu$.)

θ \ r/a	1·0	1·1	1·2	1·25	1·5	1·75	2·0	2·5	3·0	3·5	4·0
0°	−1·96	−1·65	−1·46	−1·39	−1·20	−1·11	−1·07	−1·03	−1·02	−1·01	−1·01
10°	−1·98						−1·07		−1·02		−1·01
20°	−2·03						−1·07		−1·02		−1·01
30°	−2·12						−1·06		−1·01		−1·01
45°	−2·28	−1·81	−1·54	−1·44	−1·19	−1·09	−1·05	−1·02	−1·01	−1·01	−1·00
60°	−2·45						−1·04		−1·00		−1·00
70°	−2·53						−1·03		−1·00		−1·00
80°	−2·59						−1·03		−1·00		−1·00
90°	−2·61	−1·97	−1·61	−1·49	−1·18	−1·07	−1·03	−1·01	−1·00	−1·00	−1·00

Table XIV.

(Magnitude of $\widehat{r\theta}$, unit $= S \sin 2\phi$, $\lambda = \mu$.)

θ \ r/a	1·0	1·1	1·2	1·25	1·5	1·75	2·0	2·5	3·0	3·5	4·0
0°	0	0	0	0	0	0	0	0	0	0	0
10°	0						0·18		0·18		0·17
20°	0						0·35		0·33		0·33
30°	0						0·46		0·45		0·44
45°	0	0·26	0·40	0·44	0·52	0·54	0·54	0·53	0·52	0·51	0·51
60°	0						0·46		0·45		0·44
70°	0						0·35		0·33		0·33
80°	0						0·18		0·18		0·17
90°	0	0	0	0	0	0	0	0	0	0	0

Table XV.

(Magnitude of $\widehat{r\phi}$, unit $=S\cos 2\phi$, $\lambda=\mu$.)

θ \ r/a	1·0	1·1	1·2	1·5	1·75	2·0	2·5	3·0	3·5	4·0
0°	0	0	0	0	0	0	0	0	0	0
10°	0					0·19		0·18		0·18
20°	0					0·37		0·35		0·35.
30°	0					0·54		0·53		0·51
45°	0	0·32	0·56	0·74	0·76	0·76	0·74	0·73	0·72	0·72
60°	0					0·93		0·89		0·88
70°	0					1·01		0·97		0·96
80°	0					1·06		1·02		1·00
90°	0	0·52	0·79	1·05	1·08	1·07	1·05	1·03	1·02	1·02

Table XVI.

(Magnitude of $\widehat{\theta\phi}$, unit $=S\cos 2\phi$, $\lambda=\mu$.)

θ \ r/a	1·0	1·1	1·2	1·25	1·5	2·0	2·5	3·0	3·5	4·0
0°	1·33	1·03	0·84	0·77	0·57	0·45	0·41	0·39	0·39	0·38
10°	1·32					0·44		0·39		0·38
20°	1·25					0·42		0·37		0·36
30°	1·15					0·39		0·34		0·33
45°	0·94	0·73	0·59	0·54	0·40	0·31	0·29	0·28	0·27	0·27
60°	0·67					0·22		0·20		0·19
70°	0·46					0·15		0·14		0·13
80°	0·23					0·08		0·07		0·07
90°	0	0	0	0	0	0	0	0	0	0

Table XVII.

(Magnitude of $\widehat{\theta\theta}_{r=a}$)

ϕ \ θ	0°	60°	90°	120°
0°	0	0	0	0
45°	1·957S	0	−0·7S	0
90°	0	0	0	0
135°	−1·957S	0	0·7S	0
180°	0	0	0	0
225°	1·957S	0	−0·7S	0
270°	0	0	0	0
315°	−1·957S	0	0·7S	0

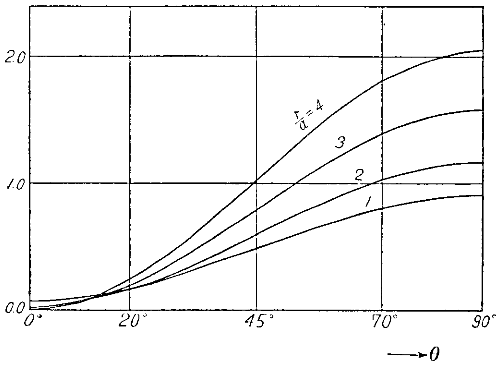

Fig. 10. (Magnitude of u, unit $=\dfrac{Sa}{\mu}\sin 2\phi$, $\lambda=\mu$.)

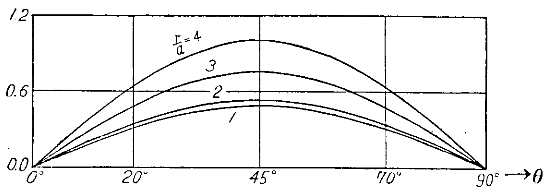

Fig. 11. (Magnitude of v, unit $=\dfrac{Sa}{\mu}\sin 2\phi$, $\lambda=\mu$.)

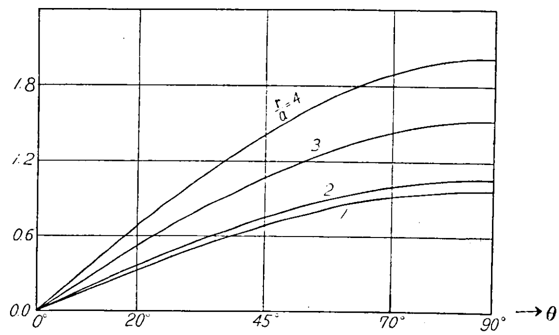

Fig. 12. (Magnitude of w, unit $=\dfrac{Sa}{\mu}\cos 2\phi$, $\lambda=\mu$.)

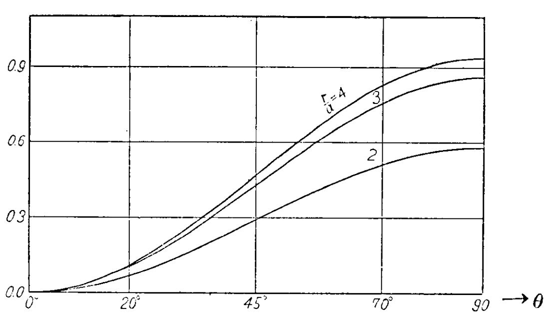

Fig. 13a. (Magnitude of $\widehat{rr}$, unit $=S\sin 2\phi$, $\lambda=\mu$.)

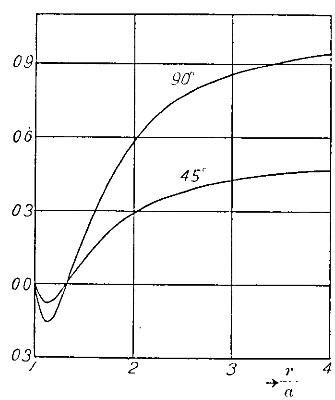

Fig. 13b. (Magnitude of $\widehat{rr}$, unit $=S\sin 2\phi$, $\lambda=\mu$.)

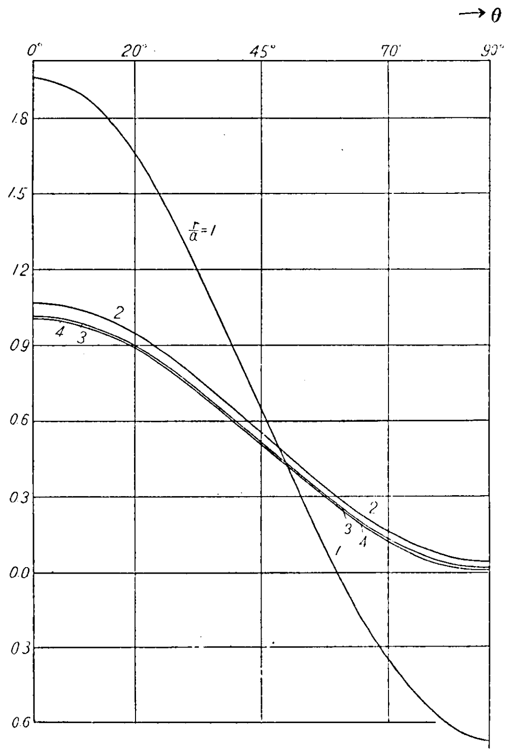

Fig. 14a.

(Magnitude of $\widehat{\theta\theta}$, unit $= S\sin 2\phi$, $\lambda=\mu$.)

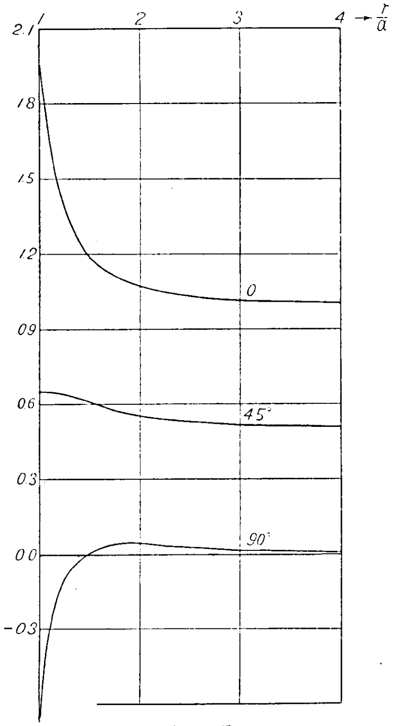

Fig. 14b.

(Magnitude of $\widehat{\theta\theta}$, unit $= S\sin 2\phi$, $\lambda=\mu$.)

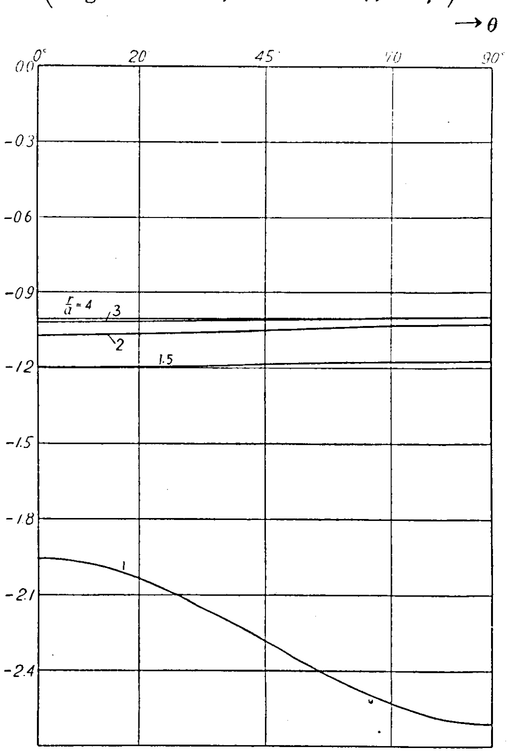

Fig. 15a.

(Magnitude of $\widehat{\phi\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

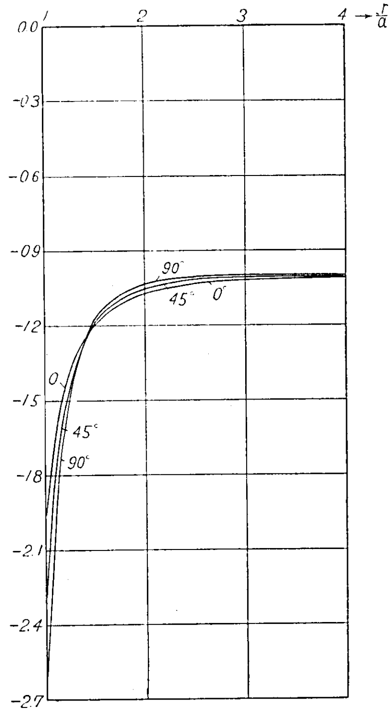

Fig. 15b.

(Magnitude of $\widehat{\phi\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

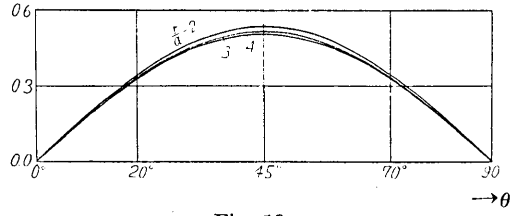

Fig. 16a.

(Magnitude of $\widehat{r\theta}$, unit $= S\sin 2\phi$, $\lambda=\mu$.)

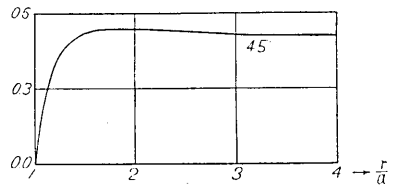

Fig. 16b.

(Magnitude of $\widehat{r\theta}$, unit $= S\sin 2\phi$, $\lambda=\mu$.)

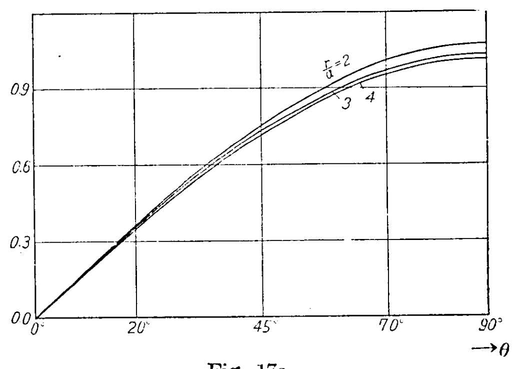

Fig. 17a.

(Magnitude of $\widehat{r\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

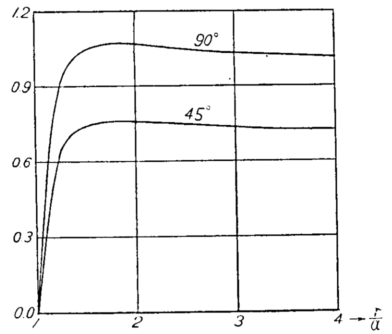

Fig. 17b.

(Magnitude of $\widehat{r\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

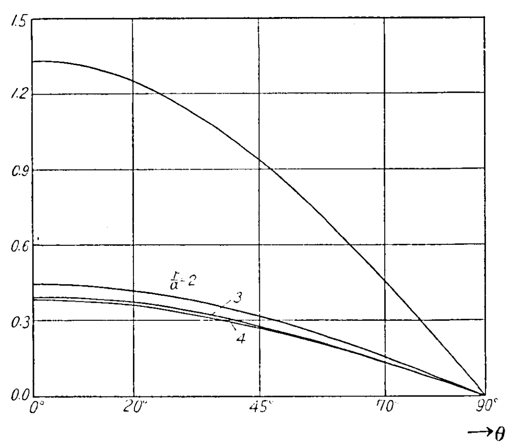

Fig. 18a.

(Magnitude of $\widehat{\theta\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

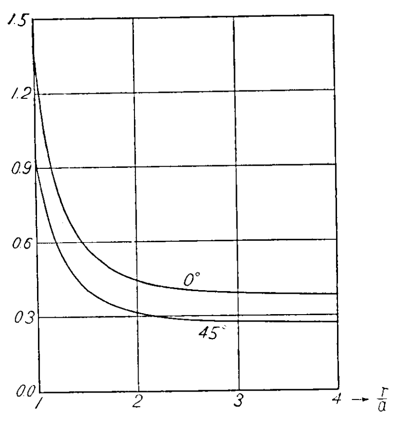

Fig. 18b.

(Magnitude of $\widehat{\theta\phi}$, unit $= S\cos 2\phi$, $\lambda=\mu$.)

6. We have studied the stress distribution in the neighbourhood of a spherical cavity in two cases; one of them is the equilibrium of the spherical cavity in a semi-infinite gravitating elastic solid, and the other is the equilibrium of the spherical cavity in an infinite elastic solid due to the uniform simple shear. The one may throw some lights on the mechanisms of the occurrence of an earthquake, and the other may give some clew to the investigations of the effect of a cavity in a semi-infinite gravitating elastic solid under dynamical forces.

We will mention some results obtained from the calculations:

1. The stress distribution in the neighbourhood of the inner surface of the cavity in a gravitating semi-infinite solid is different for different depth of the cavity from the top surface of the solid, and is affected by the dimensions of the cavity. The poisson's ratio is also effective upon the stress distribution in the solid.

2. The magnitude of stress at a point on the surface of cavity is proportional to the depth of the centre of the cavity from the upper surface of the solid.

3. The rate of increase of stress with depth of the cavity is different at different point of the surface of the cavity. The rate is maximum at the equator of the surface of the cavity.

4. At the equator of the surface of the cavity, the stress is compressive and is larger than the other components of tractions at the surface.

5. Some components of stress at the top and the bottom portions of the surface of cavity are tensile in the case of shallow cavity and small poisson's ratio, though their absolute magnitudes are very small.

6. When the gravitating semi-infinite elastic solid is incompressible, the interior of solid is maintained in hydrostatic pressure. The stresses on the surface of cavity are compression only. When the depth of the centre of the cavity from the upper surface of the solid is very deep, the stress components $(\theta\theta, \phi\phi)$ on the surface of cavity are approximately expressed by $-3/2\rho g z_0$.

7. When an infinite solid having a spherical cavity is subjected to a uniform simple shear, the traction is accumulated on the surface of cavity, and the region effected by the presence of cavity is limited in the boundary region of that cavity, and the effective radius from the centre of the cavity is approximately four times the radius of the cavity.

In conclusion, the present authors must express their sincere thanks to Professor K. Sezawa for his kind guidance.

24. 地殼內部に存在する空窩が地殼の平衡に及ぼす影響

地震研究所 {西村源六郎
高山威雄

先づ地殼を一つの gravitating な半無限彈性體と考へ、その內部に一つの球形な空窩が存在する時の平衡問題を研究し、次にこの彈性體が一樣な剪應力を受ける時どの樣な平衡上の變化を受けるかを研究する目的で一つの無限彈性體が一樣な剪應力を受ける時の平衡問題を取扱つてみた。色々面白いと思はれる結果を得たのであるが多少でも參考になる所があれば幸と思つてゐる。