20. *Movement of the Ground due to Atmospheric Disturbance in a Sea Region.*

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1. In a previous paper[1] one of us studied some problem of the transmission of seismic waves on the bottom surface of an ocean and it was found that the problem is of a great importance in connection with the dispersive nature of distant earthquakes. It occurred us very recently that the problem, if somewhat extended, may also be significant on the pulsatory motion of the ground occurring in disturbed weather in a sea region. Geophysical investigations of microseisms have already been made fully by Prof. Gutenberg,[2][3] Dr. Banerji[4] and others, chiefly by examining the data of the actual seismic records. It seems, however, that the accurate mathematical study concerning microseisms has not yet been made probably on account of the reason that the action of the water waves on the ground and on the steep coast is of a complex nature. Although it has already been concluded by Professor Gutenberg and others that the cause of the microseisms is the action of the surf against steep coasts but not the direct action of the wind pressure or of the water upon the bottom ground, yet we are now to study the problem of the movement of the ground due to the disturbed sea in rough weather for the purpose of ascertaining to what extent the change of the air pressure and also of the resulting water waves influence the microseisms in the neighbourhood of the sea.

From the result of calculation, however, it has been ascertained that

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1) K. SEZAWA, "On the Transmission of Seismic Waves on the Bottom Surface of an Ocean," *Bull. Earthq. Res. Inst.*, 9 (1931), 115–143.

2) B. GUTENBERG, "Bodenunruhe durch Brandung und durch Frost," *ZS. Geophys.*, 4 (1928), 246–250; and a number of his papers.

3) B. GUTENBERG "Die seismische Bodenunruhe," *Gerl. Beitr. Geophys.* 11., (1912), 414.

4) S. K. BANERJI, "Microseisms Associated with Disturbed Weather in the Indian Seas," *Phil Trans. Roy. Soc.*, 229 (1930), 287–328.

even the shallow water waves give no much effect directly on the generation of microseisms and that the principal cause of those microseisms, in fact, as Professor Gutenberg[5] suggested, seems to be of a kind such as the surf at the rocky cliff of a coast.

We shall assume that the horizontal surface of a sea, whose depth is h_0, is subjected to a pulsatory pressure:

$$p = f(r)e^{in_1t}, \qquad \cdots\cdots(1)$$

where r is the radial distance of a point from the pressure centre and $2\pi/n_1$ the period of the pulsation. The resulting movements of the sea water and of the surface of the solid will give us long water waves as well as microseisms of the ground caused by the atmospheric disturbance in a sea region.

2. Referring to the axes which are shown in the figure and using the same notations as in the preceding paper,[6] we may write the expressions of the displacement of the ground and the water, besides the potential of the water waves, in the following forms:

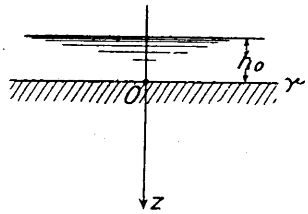

Fig. 1.

$$\left.\begin{aligned} u_1 &= -A_m \frac{1}{h^2} \frac{\partial H_m^{(2)}(kr)}{\partial r} e^{-\alpha z + int} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} m\theta, \\ v_1 &= A_m \frac{m}{h^2} \frac{H_m^{(2)}(kr)}{r} e^{-\alpha z + int} \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix} m\theta, \\ w_1 &= A_m \frac{\alpha}{h^2} H_m^{(2)}(kr) e^{-\alpha z + int} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} m\theta, \end{aligned}\right\} \qquad \cdots\cdots(2)$$

$$\left.\begin{aligned} u_3 &= C_m \frac{\beta}{mj^2} \frac{\partial H_m^{(2)}(kr)}{\partial r} e^{-\beta z + int} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} m\theta, \\ v_3 &= -C_m \frac{\beta}{j^2} \frac{H_m^{(2)}(kr)}{r} e^{-\beta z + int} \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix} m\theta, \\ w_3 &= -C_m \frac{k^2}{mj^2} H_m^{(2)}(kr) e^{-\beta z + int} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} m\theta, \end{aligned}\right\} \qquad \cdots\cdots(3)$$

5) B. GUTENBERG, "Microseisms in North America," *Bull. Seism. Soc. Amer.*, 21 (1931), 1–24.

6) K. SEZAWA, *loc. cit.*

$$\left.\begin{aligned}
\phi &= (D_m \operatorname{ch} k_1 z + E_m \operatorname{sh} k_1 z) J_m(kr) \left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} m\theta\, e^{int}, \\
u' &= \frac{i}{n}(D_m \operatorname{ch} k_1 z + E_m \operatorname{sh} k_1 z)\frac{\partial J_m(kr)}{\partial r} \left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} m\theta\, e^{int}, \\
v' &= -\frac{im}{n}(D_m \operatorname{ch} k_1 z + E_m \operatorname{sh} k_1 z)\frac{J_m(kr)}{r} \left.\begin{matrix}\sin\\ -\cos\end{matrix}\right\} m\theta\, e^{int}, \\
w' &= \frac{ik_1}{n}(D_m \operatorname{sh} k_1 z + E_m \operatorname{ch} k_1 z) J_m(kr) \left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} m\theta\, e^{int},
\end{aligned}\right\} \quad \cdots(4)$$

where

$$k_1^2 = k^2 - \frac{n^2}{c^2}. \quad \cdots\cdots(5)$$

The boundary conditions are such that

$$z = 0; \quad w = w', \quad \cdots\cdots(6)$$

$$z = 0; \quad \lambda\Delta + 2\mu\frac{\partial w}{\partial z} + g(\rho - \rho')w' - \rho'\frac{\partial \phi}{\partial t} = 0, \quad \cdots\cdots(7)$$

$$z = 0; \quad \left.\begin{aligned} &\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = 0, \\ &\frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z} = 0, \end{aligned}\right\} \quad \cdots\cdots(8)$$

$$z = -h_0; \quad p = e^{in_1 t} f(r) \left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} m\theta, \quad \cdots\cdots(9)$$

where p is the pulsatory pressure on the surface of water.

From (8), we get

$$\frac{C_m}{m} \bigg/ A_m = \frac{2\alpha j^2}{h^2(2k^2 - j^2)}. \quad \cdots\cdots(10)$$

From (6),

$$k_1 E_m = -in\left(\frac{\alpha}{h^2}A_m - \frac{k^2}{j^2}\frac{C_m}{m}\right). \quad \cdots\cdots(11)$$

From (7),

$$\left(\lambda - \frac{2\mu\alpha^2}{h^2}\right)A_m + \frac{2\mu k^2 \beta}{mj^2}C_m + \frac{ik_1}{n}g(\rho - \rho')E_m - in\rho' D_m = 0. \quad \cdots(12)$$

Comparing (10), (11), (12), we find

$$D_m = \frac{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2}{\rho' n^2\alpha j^2} \times k_1 E_m. \qquad \cdots\cdots\cdots\cdots(13)$$

By the use of the relation (13), we get the equation of the pressure distribution of the sea water as follows:

$$p = \rho'\frac{\partial\phi}{\partial t} + \rho' g w'$$

$$= iE_m \times \left[\left(nk_1 \operatorname{ch} k_1 z + \frac{k_1^2 g}{n} \operatorname{sh} k_1 z\right)\right.$$

$$\times \frac{(2k^2-j^2)(2\mu\alpha^2-\lambda h^2)-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2}{n^2\alpha j^2}$$

$$\left.+\left(n\rho' \operatorname{sh} k_1 z + \frac{\rho' k_1 g}{n} \operatorname{ch} k_1 z\right)\right] \times J_m(kr) \begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta\, e^{int}. \qquad \cdots\cdots\cdots(14)$$

When $z = -h_0$, this may be written in the form:

$$p = XJ_m(kr)\begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta\, e^{in_1 t}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

where n is equated to n_1 and

$$E_m = \frac{X}{i\mu k_1(n_1^2-gk_1^2h_0)}$$

$$\times \frac{n_1^3\alpha j^2}{\left[(2k^2-j^2)^2-4k^2\alpha\beta-\frac{g\alpha j^2}{\mu}+\frac{\rho'\alpha j^2}{\mu}\left(g-n_1^2\times\frac{g-n_1^2h_0}{n_1^2-gk_1^2h_0}\right)\right]}. \cdots(16)$$

Now, write

$$F(k) \equiv (2k^2-j^2)^2-4k^2\alpha\beta-\frac{g\rho\alpha j^2}{\mu}+\frac{\rho'\alpha j^2}{\mu}\times\frac{n_1^4h_0-g^2h_0k_1^2}{n_1^2-gh_0k_1^2}, \cdots\cdots\cdots(17)$$

or, neglecting $gh_0k_1^2/n_1^2$, write

$$F(k) \equiv (2k^2-j^2)^2-4\alpha\beta k^2-\frac{g\rho j^2}{\mu}\alpha+\rho'\frac{h_0\alpha j^4}{\rho}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(18)$$

then, by means of Fourier's integral expression of p on $z = -h_0$, such that

$$p = e^{in_1 t}\left\{\int_0^\infty J_m(kr)kdk\int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma\right\}\begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta, \cdots\cdots\cdots\cdots\cdots\cdots(19)$$

we obtain the expressions of the displacement of the water surface in the forms:

$$u'_{z=-h_0} = e^{in_1t} \begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{[\{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}k_1 \operatorname{ch} k_1h_0 - n_1^2\rho'\alpha j^2 \operatorname{sh} k_1h_0]}{\rho'\mu k_1(n_1^2-gk_1^2h_0)F(k)} \frac{\partial J_m(kr)}{\partial r} kdk$$

$$\times \int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(20)$$

$$v'_{z=-h} = -e^{in_1t}\, m \begin{matrix}\sin\\ -\cos\end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{[\{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}k_1 \operatorname{ch} k_1h_0 - n_1^2\rho'\alpha j^2 \operatorname{sh} k_1h_0]}{\rho'\mu k_1(n_1^2-gk_1^2h_0)F(k)} \frac{J_m(kr)}{r} kdk$$

$$\times \int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(21)$$

$$w'_{z=-h_0} = e^{in_1t} \begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{[\rho' n_1^2\alpha j^2 \operatorname{ch} k_1h_0 - \{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}k_1 \operatorname{sh} k_1h_0]}{\rho'\mu(n_1^2-gk_1^2h_0)F(k)} J_m(kr)kdk$$

$$\times \int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(22)$$

and similar expressions of the elastic body at the bottom surface of the sea in the forms:

$$u_{z=0} = e^{in_1t} \begin{matrix}\cos\\ \sin\end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{n_1^2\{(2k^2-j^2)^2-2\alpha\beta\}}{\mu(n_1^2-gk_1^2h_0)F(k)} \frac{\partial J_m(kr)}{\partial r} kdk \int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots(23)$$

$$v_{z=0} = -e^{in_1t}\, m \begin{matrix}\sin\\ -\cos\end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{n_1^2\{(2k^2-j^2)^2-2\alpha\beta\}}{\mu(n_1^2-gk_1^2h_0)F(k)} \frac{J_m(kr)}{r} kdk \int_0^\infty f(\sigma)J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots(24)$$

$$w_{z=0} = e^{in_1 t} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\theta$$

$$\times \int_0^\infty \frac{n_1^2 \alpha j^2}{\mu(n_1^2 - gk_1^2 h_0)F(k)} J_m(kr) k dk \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma. \quad \cdots\cdots\cdots\cdots(25)$$

When the depth h_0 is comparatively small, we find

$$w'_{z=-h_0} = \frac{e^{in_1 t}}{\rho'\mu} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\theta$$

$$\times \int_0^\infty \frac{[\mu(2k^2-j^2)^2 - 4\mu k^2 \alpha\beta + g(\rho-\rho')\alpha j^2 - \rho' n_1^2 \alpha j^2 h_0]}{(n_1^2 - gk_1^2 h_0)F(k)} \frac{\partial J_m(kr)}{\partial r} k dk$$

$$\times \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(26)$$

$$v'_{z=-h_0} = -\frac{e^{in_1 t}}{\rho'\mu} m \begin{Bmatrix}\sin\\ -\cos\end{Bmatrix} m\theta$$

$$\times \int_0^\infty \frac{[\mu(2k^2-j^2)^2 - 4\mu k^2 \alpha\beta + g(\rho-\rho')\alpha j^2 - \rho' n_1^2 \alpha j^2 h_0]}{(n_1^2 - gk_1^2 h_0)F(k)} \frac{J_m(kr)}{r} k dk$$

$$\times \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(27)$$

$$w'_{z=-h_0} = \frac{e^{in_1 t}}{\rho'\mu} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\theta$$

$$\times \int_0^\infty \frac{[\rho' n_1^2 \alpha j^2 - k_1^2 h_0\{\mu(2k^2-j^2)^2 - 4\mu k^2 \alpha\beta + g(\rho-\rho')\alpha j^2\}]}{(n_1^2 - gk_1^2 h_0)F(k)} J_m(kr) k dk$$

$$\times \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(28)$$

$$u_{z=0} = \frac{e^{in_1 t} n_1^2}{\mu} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\theta$$

$$\times \int_0^\infty \frac{(2k^2-j^2) - 2\alpha\beta}{(n_1^2 - gk_1^2 h_0)F(k)} \frac{\partial J_m(kr)}{\partial r} k dk \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots\cdots(29)$$

$$v_{z=0} = -\frac{e^{in_1t}n_1^2}{\mu} m \begin{matrix} \sin \\ -\cos \end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{(2k^2-j^2)-2\alpha\beta}{(n_1^2-gk_1^2h_0)F(k)} \frac{J_m(kr)}{r} kdk \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma, \quad \cdots\cdots\cdots(30)$$

$$w_{z=0} = \frac{e^{in_1t}n_1^2}{\mu} \begin{matrix} \cos \\ \sin \end{matrix}\Big\} m\theta$$

$$\times \int_0^\infty \frac{\alpha j^2}{(n_1^2-gk_1^2h_0)F(k)} J_m(kr)kdk \int_0^\infty f(\sigma) J_m(k\sigma)\sigma d\sigma. \quad \cdots\cdots\cdots(31)$$

When the disturbed pressure is accumulated in the neighbourhood of $r=0$ and symmetrical with respect to the vertical axis passing through the pressure centre, we may have

$$m = 0\,; \qquad \int_0^\infty f(\sigma) 2\pi\sigma d\sigma = 1, \quad \cdots\cdots\cdots(32)$$

so that

$$\left.\begin{aligned}
u'_{z=-h_0} &= \frac{e^{in_1t}}{2\pi\rho'\mu} \\
&\times \int_0^\infty \frac{[\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2-\rho' n_1^2\alpha j^2 h_0]}{(n_1^2-gk_1^2h_0)F(k)} \frac{\partial J_0(kr)}{\partial r} kdk, \\
v'_{z=-h_0} &= 0, \\
w'_{z=-h_0} &= \frac{e^{in_1t}}{2\pi\rho'\mu} \\
&\times \int_0^\infty \frac{[\rho' n_1^2\alpha j^2 - k_1^2h_0\{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}]}{(n_1^2-gk_1^2h_0)F(k)} J_0(kr)kdk,
\end{aligned}\right\} \quad (33)$$

$$\left.\begin{aligned}
u_{z=0} &= \frac{e^{in_1t}n_1^2}{2\pi\mu} \int_0^\infty \frac{(2k^2-j^2)-2\alpha\beta}{(n_1^2-gk_1^2h_0)F(k)} \frac{\partial J_0(kr)}{\partial r} kdk, \\
v_{z=0} &= 0, \\
w_{z=0} &= \frac{e^{in_1t}n_1^2}{2\pi\mu} \int_0^\infty \frac{\alpha j^2}{(n_1^2-gk_1^2h_0)F(k)} J_0(kr)kdk,
\end{aligned}\right\} \cdots\cdots\cdots(34)$$

Since we know the expression

$$J_m(kr) = -\frac{i}{\pi}\int_0^\infty \left(e^{i\left(kr\,\mathrm{ch}\,f-\frac{m\pi}{2}\right)} - e^{-i\left(kr\,\mathrm{ch}\,f-\frac{m\pi}{2}\right)}\right)\mathrm{ch}\,mfdf,^{7)} \quad \cdots\cdots\cdots(35)$$

we obtain

$$\left.\begin{aligned} u'_{z=-h_0} &= \frac{e^{in_1t}}{2\pi^2\rho'\mu}\int_0^\infty \mathrm{ch}\,fdf \\ &\times\int_{-\infty}^\infty \frac{[\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2-\rho' n_1^2\alpha j^2 h_0]}{(n_1^2-gk_1^2h_0)F(k)}\, e^{ikr\,\mathrm{ch}\,f}\,kdk, \\ w'_{z=-h_0} &= -\frac{ie^{in_1t}}{2\pi^2\rho'\mu}\int_0^\infty d \\ &\times\int_{-\infty}^\infty \frac{[\rho' n_1^2\alpha j^2-k_1^2h_0\{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}]}{(n_1^2-gk_1^2h_0)F(k)}\, e^{ikr\,\mathrm{ch}\,f}kdk, \end{aligned}\right\} \quad (36)$$

$$\left.\begin{aligned} u_{z=0} &= \frac{e^{in_0t}n_1^2}{2\pi^2\mu}\int_0^\infty \mathrm{ch}\,fdf\int_{-\infty}^\infty \frac{(2k^2-j^2)-2\alpha\beta}{(n_1^2-gk_1^2h_0)F(k)}\, e^{ikr\,\mathrm{ch}\,f}\,kdk, \\ w_{z=0} &= -\frac{ie^{in_1t}n_1^2}{2\pi^2\mu}\int_0^\infty df\int_{-\infty}^\infty \frac{\alpha j^2 e^{ikr\,\mathrm{ch}\,f}}{(n_1^2-gk_1^2h_0)F(k)}kdk. \end{aligned}\right\} \quad (37)$$

The evaluation of the integrals of the types:

$$\int_{-\infty}^\infty \frac{[\rho' n_1^2\alpha j^2-\{\mu(2k^2-j^2)^2-4\mu k^2\alpha\beta+g(\rho-\rho')\alpha j^2\}k_1^2h_0]}{(n_1^2-gk_1^2h_0)F(k)}\, e^{ikr\,\mathrm{ch}\,f}\,kdk, \cdots(38)$$

$$\int_{-\infty}^\infty \frac{\alpha j^2\, e^{ikr\,\mathrm{ch}\,f}\,kdk}{(n_1^2-gk_1^2h_0)F(k)}, \quad \cdots\cdots\cdots\cdots\cdots\cdots(39)$$

where $k_1^2=k^2-n_1^2/c^2$, can be performed in the following manner. Consider, for example, the integral

$$\int_c \frac{\sqrt{Z^2-h^2}\, e^{iZr\,\mathrm{ch}\,f}\,ZdZ}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}F(Z)} \quad \cdots\cdots\cdots\cdots\cdots\cdots(40)$$

7) WATSON, "Theory of Bessel Functions", (1922), 180.

taken round a contour shown in Fig. 2. In (40),

$$F(Z) = (2Z^2-j^2)^2-4Z^2\sqrt{(h^2-Z^2)(j^2-Z^2)}$$

$$-\frac{g\rho}{\mu}\sqrt{h^2-Z^2}\,j^2+\frac{\rho'\sqrt{h^2-Z^2}}{\mu}\,j^2\,\frac{n_1{}^4h_0-g^2h_0(Z^2-n_1{}^2/c^2)}{n_1{}^2-gh_0(Z^2-n_1{}^2/c^2)}, \quad \cdots\cdots(41)$$

and $h^2 = \rho n_1{}^2/(\lambda+2\mu)$, $j^2 = \rho n_1{}^2/\mu$. ....(42)

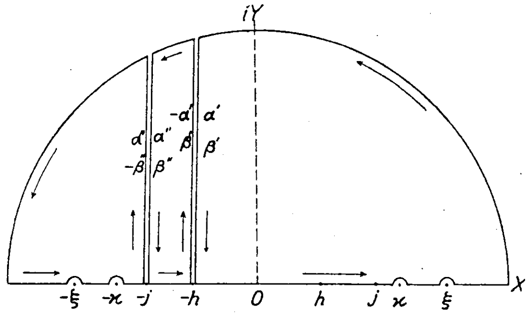

Fig. 2.

In Fig. 2, $h, j, -h, -j$ are branch points corresponding to the integrand of (40); $\pm\varkappa$ and $\pm\xi$ poles of the same integrand, namely $\pm\varkappa$ are a pair of real roots of the equation (41), while $\pm\xi$ are a pair of the roots of the equation $n_1{}^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2$. If we assume that there are certain terms of insensible frictional damping in the equations of the vibrations of the solid body, it will be clear that the branch points $Z=h$, $Z=j$ are shifted in the space of the lower quadrant of Fig. 2, so that these points do not influence the contour integration. The two grooves running between $(-h, 0)$, $(-h, i\infty)$ and between $(-j, 0)$, $(-j, i\infty)$ are branch lines. The positive and negative signs of $\alpha', \beta', \alpha'', \beta''$ on both sides of the lines $Z=-h$ and $Z=-j$ are taken suitably so as to satisfy the conditions of continuity. The resultant of these lines, when composed, may be equivalent to a branch line joining two points $(-h, 0)$, $(-j, 0)$. The result of the contour integral shows that

$$I = \int_{-\infty}^{\infty}\frac{\alpha j^2 e^{ikr\,\mathrm{ch}\,f}\,kdk}{\left\{n_1{}^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-k^2\right\}F(k)}$$

$$= \frac{i4\pi\alpha_1 j^2\xi}{\phi'(\xi)}\cos(\xi r\,\mathrm{ch}\,f)+\frac{i4\pi\alpha_2 j^2\varkappa}{\phi'(\varkappa)}\cos(\varkappa r\,\mathrm{ch}\,f)$$

$$+e^{-ijr\,\mathrm{ch}\,f}\int_0^\infty\left[\frac{j^2\alpha''}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left\{(2Z^2-j^2)^2-4Z^2\alpha''\beta''-\frac{g\rho}{\mu}\alpha''j^2+\frac{\rho' h_0}{\rho}\alpha''j^4\right\}}\right.$$

$$\left.-\frac{j^2\alpha''}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left\{(2Z^2-j^2)^2+4Z^2\alpha''\beta''-\frac{g\rho}{\mu}\alpha''j^2+\frac{\rho' h_0}{\rho}\alpha''j^4\right\}}\right]Ze^{-Yr\,\mathrm{ch}\,f}i\,dY$$

$$+e^{-ihr\,\mathrm{ch}\,f}\int_0^\infty\left[\frac{j^2\alpha'}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left\{(2Z^2-j^2)^2-4Z^2\alpha'\beta'-\frac{g\rho}{\mu}\alpha'j^2+\frac{\rho' h_0}{\rho}\alpha'j^4\right\}}\right.$$

$$\left.+\frac{j^2\alpha'}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left\{(2Z^2-j^2)^2+4Z^2\alpha'\beta'+\frac{g\rho}{\mu}\alpha'j^2-\frac{\rho' h_0}{\rho}\alpha'j^4\right\}}\right]Ze^{-Yr\,\mathrm{ch}\,f}i\,dY$$

$$=i\frac{4\pi\alpha_1 j^2\xi}{\phi'(\xi)}\cos(\xi r\,\mathrm{ch}\,f)+i\frac{4\pi\alpha_2 j^2\varkappa}{\phi'(\varkappa)}\cos(\varkappa r\,\mathrm{ch}\,f)$$

$$+8ie^{-ijr\,\mathrm{ch}\,f}\int_0^\infty\frac{j^2\alpha''e^{-Yr\,\mathrm{ch}\,f}\alpha''\beta''Z^3dY}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left[\left\{(2Z^2-j^2)^2-\frac{g\rho}{\mu}\alpha''j^2-\frac{\rho' h_0}{\rho}\alpha''j^4\right\}^2-16Z^4\alpha''^2\beta''^2\right]}$$

$$+2ie^{-ihr\,\mathrm{ch}\,f}\int_0^\infty\frac{j^2\alpha'e^{-Yr\,\mathrm{ch}\,f}Z(2Z^2-j^2)^2dY}{\left\{n_1^2\left(\frac{c^2+gh_0}{c^2gh_0}\right)-Z^2\right\}\left[(2Z^2-j^2)^4-\left(4Z^2\alpha'\beta'+\frac{g\rho}{\mu}\alpha'j^2-\frac{\rho' h_0}{\rho}\alpha'j^4\right)^2\right]},\qquad\cdots\cdots(43)$$

where

$$\alpha_1=\sqrt{\xi^2-h^2},\quad \alpha_2=\sqrt{\varkappa^2-h^2}.\qquad\cdots\cdots(44)$$

It is known that, for small values of Y,

$$\left.\begin{aligned}\alpha'&=-\sqrt{2hY}\,e^{-\frac{1}{4}i\pi},\quad &\beta'&=i\sqrt{(j^2-h^2)},\\ \alpha''&=\sqrt{(j^2-h^2)},&\beta''&=-\sqrt{2jY}\,e^{-\frac{1}{4}i\pi}\end{aligned}\right\}\qquad\cdots\cdots(45)^{8)}$$

may be taken with approximation. Hence we get

8) H. LAMB, *Phil. Trans. Roy. Soc.*, **203** (1904), 20.

$$I = i\frac{4\pi\alpha_1 j^2\xi}{\phi'(\xi)}\cos(\xi r\,\mathrm{ch}\,f) + i\frac{4\pi\alpha_2 j^2 \varkappa}{\phi'(\varkappa)}\cos(\varkappa r\,\mathrm{ch}\,f)$$

$$-\int_0^\infty \frac{j^2(j^2-h^2)\sqrt{2jY}(-j+iY)^3 e^{-\frac{1}{4}i\pi - Yr\,\mathrm{ch}\,f}df}{\left\{n_1^2\frac{(c^2+gh_0)}{c^2gh_0}-(-j+iY)^2\right\}\left[\left\{(2(-j+iY)^2-j^2)^2-\frac{\rho g}{\mu}j^2\sqrt{j^2-h^2}-\frac{\rho' h_0}{\rho}j^4\sqrt{j^2-h^2}\right\}^2-32(j^2-h^2)jYe^{-\frac{i\pi}{2}}(-j+iY)^4\right]}$$

$$-\int_0^\infty \frac{\sqrt{2h}\,j^2(-h+iY)\{2(-h+iY)^2-j^2\}^2 Y^{\frac{1}{2}} e^{-\frac{1}{4}i\pi - Yr\,\mathrm{ch}\,f}df}{\left\{n_1^2\frac{(c^2+gh_0)}{c^2gh_0}-(-h+iY)^2\right\}\left[\{2(-h+iY)^2-j^2\}^4-2hYe^{-\frac{i\pi}{2}}\left\{4i(-h+iY)^2\sqrt{j^2-h^2}+\frac{\rho g}{\mu}j^2-\frac{\rho' h_0}{\rho}j^4\right\}^2\right]}.$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\cdots\cdots\cdots\cdots(46)$$

We know again

$$\int_0^\infty Y^{1/2}\chi(Y)e^{-Yx}dY = \frac{\Pi(1/2)}{x^{3/2}}\chi(0) + \frac{\Pi(3/2)}{x^{5/2}}\frac{\chi'(0)}{1!} + \frac{\Pi(\ /2)}{x^{7/2}}\frac{\chi''(0)}{2!} + \cdots \quad (47)^{9)}$$

Hence

$$I = \frac{i4\pi\alpha_1 j^2\xi}{\phi'(\xi)}\cos(\xi r\,\mathrm{ch}\,f) + \frac{i4\pi\alpha_2 j^2\varkappa}{\phi'(\varkappa)}\cos(\varkappa r\,\mathrm{ch}\,f)$$

$$+\frac{\Pi(1/2)}{(r\,\mathrm{ch}\,f)^{3/2}}\left[\frac{\sqrt{2j}\,(j^2-h^2)e^{-\frac{1}{4}i\pi}j^5}{\left\{n_1^2\frac{(c^2+gh_0)}{c^2gh_0}-j^2\right\}\left\{j^2-\frac{\rho g}{\mu}j^2\sqrt{j^2-h^2}-\frac{\rho' h_0}{\rho}j^4\sqrt{j^2-h^2}\right\}^2}\right]+\text{etc.}$$

$$+\frac{\Pi(1/2)}{(r\,\mathrm{ch}\,f)^{3/2}}\left[\frac{hj^2\sqrt{2h}\,e^{-\frac{1}{4}i\pi}}{\left\{n_1^2\frac{(c^2+gh_0)}{c^2gh_0}-h^2\right\}(2h^2-j^2)^2}\right]+\text{etc.} \quad\cdots\cdots\cdots(48)$$

Substituting this in (37), we obtain

9) H. LAMB, *loc. cit.*

$$w_{z=0}=-\frac{ie^{in_1t}n_1^2}{2\pi^2\mu gh_0}\Big[\frac{i4\pi\alpha_1j^2\xi}{\phi'(\xi)}\int_0^\infty\cos(\xi r\,\mathrm{ch}\,f)df+\frac{i4\pi\alpha_2j^2\varkappa}{\phi'(\varkappa)}\int_0^\infty\cos(\varkappa r\,\mathrm{ch}\,f)df$$

$$+\frac{8i\Pi(1/2)(j^2-h^2)\sqrt{2j}\,j^5e^{-\frac{i\pi}{4}}}{r^{3/2}\Big\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-j^2\Big\}\Big\{j^4-\frac{\rho g}{\mu}j^2\sqrt{j^2-h^2}-\frac{\rho'h_0}{\rho}j^4\sqrt{j^2-h^2}\Big\}^2}\int_0^\infty\frac{e^{-ij\,\mathrm{ch}\,f}}{(\mathrm{ch}\,f)^{3/2}}df$$

$$+\frac{2i\Pi(1/2)\sqrt{2h}\,hj^2e^{-\frac{i\pi}{4}}}{r^{3/2}\Big\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-h^2\Big\}(2h^2-j^2)^2}\int_0^\infty\frac{e^{-ihr\,\mathrm{ch}\,f}}{(\mathrm{ch}\,f)^{3/2}}df\Big].\quad\cdots\cdots(49)$$

The integrals of the first and second terms of the right-hand side of this expression are easily obtained by means of the formula:

$$Y_0(\xi r)=-\frac{2}{\pi}\int_0^\infty\cos(\xi r\,\mathrm{ch}\,f)df.\cdots\cdots(50)$$

To evaluate the third and fourth integrals, we consider

$$K=\int_c\frac{e^{-\xi Z}dZ}{Z^{3/2}\sqrt{1+Z^2}}\quad\cdots\cdots(51)$$

taken round the contour shown in this page. Thus we know

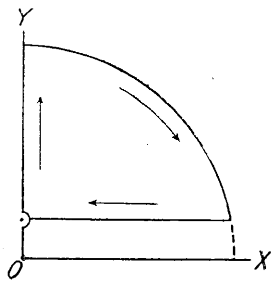

Fig. 3.

$$\int_0^\infty\frac{e^{i\xi\,\mathrm{ch}\,f}}{(\mathrm{ch}\,f)^{3/2}}df=e^{-i(\xi+\frac{\pi}{4})}\sqrt{\frac{\pi}{2\xi}}\Big(1+\frac{7}{4}i\frac{1}{2\xi}-\frac{15}{8}\frac{1\cdot3}{4\xi^2}+\cdots\cdots\Big).\quad(52)$$

Substituting from (50), (52) in (49), we get

$$w_{z=0}=\frac{ie^{in_1t}n_1^2}{2\pi^2\mu gh_0}\Big[\frac{i2\pi^2\alpha_1j^2\xi}{\phi'(\xi)}Y_0(\xi r)+\frac{i2\pi^2\alpha_2j^2\varkappa}{\phi'(\varkappa)}Y_0(\varkappa r)$$

$$-\frac{4i\pi(j^2-h^2)j\,e^{-i(jr+\frac{\pi}{2})}}{r^2\Big\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-j^2\Big\}\Big\{j^2-\frac{\rho g}{\mu}\sqrt{j^2-h^2}-\frac{\rho'h_0}{\rho}j^2\sqrt{j^2-h^2}\Big\}^2}$$

$$-\frac{\pi hj^2e^{-i(hr+\frac{\pi}{2})}}{r^2\Big\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-h^2\Big\}(2h^2-j^2)^2}+\cdots\cdots\Big].\quad\cdots\cdots(53)$$

This expression is suitable to the problem of transmission of waves of finite extent of disturbances. To get progressive waves of an infinite extent, we add to this expression other systems of standing vibrations corresponding to the water waves as well as the surface waves of the elastic solid body, the wave lengths and the amplitudes of both systems being respectively equal to the first and the second terms of the right-hand side of the equation (53), but the phases of the additive systems differ from the original ones about $\pi/2$. The final solution is thus expressed by

$$w_{z=0} \approx -\frac{ie^{in_1t}n_1^2}{2\mu g h_0}\Bigg[\frac{\alpha_1 j^2\xi}{\phi'(\xi)}H_0^{(2)}(\xi r)+\frac{\alpha_2 j^2\varkappa}{\phi'(\varkappa)}H_0^{(2)}(\varkappa r)$$

$$+\frac{4i(j^2-h^2)je^{-i\left(jr+\frac{\pi}{4}\right)}}{\pi r^2\left\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-j^2\right\}\left\{j^2-\frac{\rho g}{\mu}\sqrt{j^2-h^2}-\frac{\rho' h_0}{\rho}j^2\sqrt{j^2-h^2}\right\}^2}$$

$$+\frac{ihj^2e^{-i\left(hr+\frac{\pi}{2}\right)}}{\pi r^2\left\{n_1^2\frac{(c^2+gh_0)}{c^2gh_0}-h^2\right\}(2h^2-j^2)^2}\Bigg], \quad \cdots\cdots(54)$$

provided

$$\phi(k)=\left\{\frac{n_1^2(c^2+gh_0)}{c^2gh_0}-k^2\right\}\left\{(2k^2-j^2)^2-4\alpha\beta k^2-\frac{g\rho}{\mu}\alpha j^2+\frac{\rho' h_0}{\rho}\alpha j^4\right\}. \quad \cdots(55)$$

In the same manner of mathematical treatment we get the horizontal component of displacement of the surface of the solid, $u_{z=0}$, the similar component of the surface of water, $u'_{z=-h_0}$, and the vertical component of the surface displacement of the water, $w_{z=-h_0}$, as in the following forms:

$$u_{z=0}\approx\frac{ie^{in_1t}n_1^2}{2\mu g h_0}\left[\frac{(2\xi^2-j^2-2\alpha_1\beta_1)\xi^2}{\phi'(\xi)}H_1^{(2)}(\xi r)+\frac{(2\varkappa^2-j^2-2\alpha_2\beta_2)\varkappa^2}{\phi'(\varkappa)}H_1^{(2)}(\varkappa r)\right], \quad (56)$$

$$u'_{z=-h_0}\approx\frac{ie^{in_1t}}{2\rho'\mu g h_0}\Bigg[\frac{\{\mu(2\xi^2-j^2)^2-4\mu\xi^2\alpha_1\beta_1+g(\rho-\rho')\alpha_1j^2-\rho' n_1^2\alpha_1j^2h_0\}\xi^2}{\phi'(\xi)}H_1^{(2)}(\xi r)$$

$$+\frac{\{\mu(2\varkappa^2-j^2)^2-4\mu\varkappa^2\alpha_2\beta_2+g(\rho-\rho')\alpha_2j^2-\rho' n_1^2\alpha_2j^2h_0\}\varkappa^2}{\phi'(\varkappa)}H_1^{(2)}(\varkappa r)\Bigg], \quad \cdots\cdots(57)$$

$$w'_{z=-h_0} \approx -\frac{ie^{in_1t}}{2\rho'\mu g h_0}$$
$$\times\left[\frac{\left\{\rho' n_1^2\alpha_1 \mathfrak{f}^2-\left[\mu(2\xi^2-\mathfrak{f}^2)^2-4\mu\xi^2\alpha_1\beta_1+g(\rho-\rho')\alpha_1\mathfrak{f}^2\right]\left(\xi^2-\frac{n_1^2}{c^2}\right)h_0\right\}\xi}{\phi'(\xi)}H_0^{(2)}(\xi r)\right.$$
$$\left.+\frac{\left\{\rho' n_1^2\alpha_2 \mathfrak{f}^2-\left[\mu(2\varkappa^2-\mathfrak{f}^2)^2-4\mu\varkappa^2\alpha_2\beta_2+g(\rho-\rho')\alpha_2\mathfrak{f}^2\right]\left(\varkappa^2-\frac{n_1^2}{c^2}\right)h_0\right\}\varkappa}{\phi'(\varkappa)}H_0^{(2)}(\varkappa r)\right] \quad \cdots\cdots(58)$$

in which

$$\beta_1=\sqrt{\xi^2-\mathfrak{f}^2},\quad \beta_2=\sqrt{\varkappa^2-\mathfrak{f}^2}, \quad \cdots\cdots(59)$$

and the terms corresponding to the dilatational and the distortional waves are neglected.

3. It may be seen from (54) (or other expressions) that the movement of the solid body (or of water) is composed of four kinds of displacements, namely the displacement due to the transmission of shallow water waves, that due to Rayleigh-waves on the bottom surface of the water superposed upon a solid body, that due to distortional waves through the body and also that due to dilatational waves through the same body. The velocities of transmission of the waves of these displacements are, of course, equal to those peculiar to the respective kinds of waves. Thus the velocity of transmission of the waves of the displacement of the solid due to the shallow water waves is exceedingly slow in comparison with that of the Rayleigh-type deformation.

The amplitudes of the deformation of the solid body due to Rayleigh-waves and also to the shallow water waves change as inverse square root of the epicentral distance, while those due to the dilatational and distortional waves diminish as inverse square of the epicentral distance, so that it may be concluded that the principal movement at a certain distance from the centre of the pulsating pressure should be caused by the transmission of Rayleigh-waves as well as shallow water waves.

It is important to know the relative magnitudes of the deformations of the solid due to the transmission of shallow water waves and to that of Rayleigh-type waves, because it is yet uncertain, whether the microseisms occurring in a disturbed weather may be the effect of the seismic waves transmitted directly from the region of the original disturbance, or may be the vibratory motion of the ground due to the disturbed water waves approaching from the origin to the locality of the observing station, or even due to some other cause such as the surf against the steep coast.

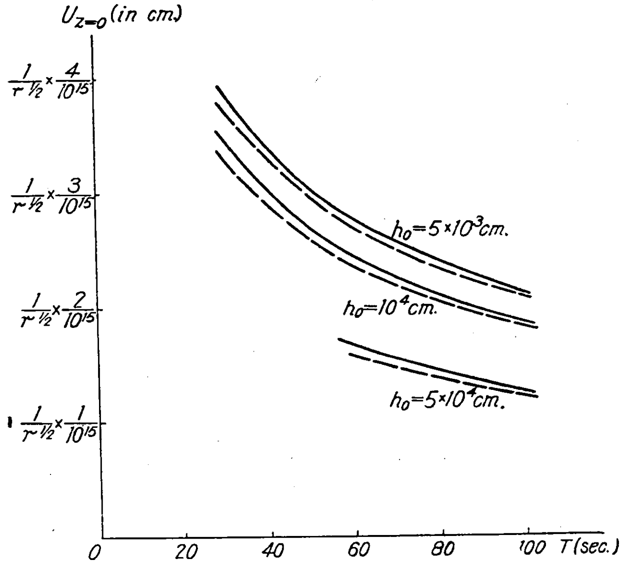

Fig. 4. (r in cm.)

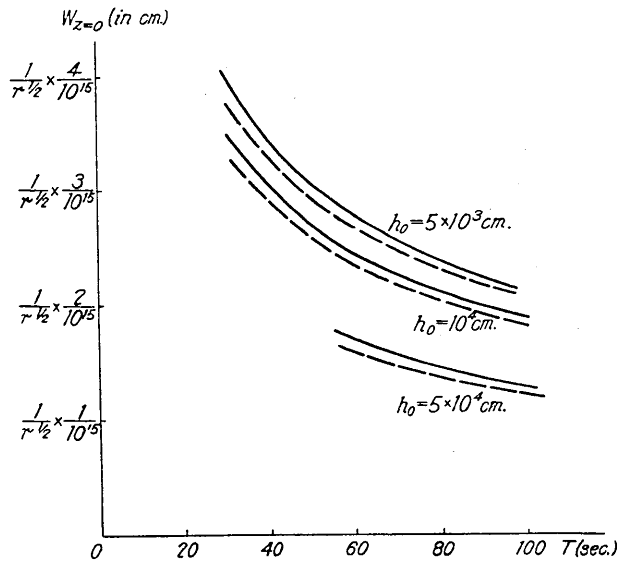

Fig. 5. (r in cm.)

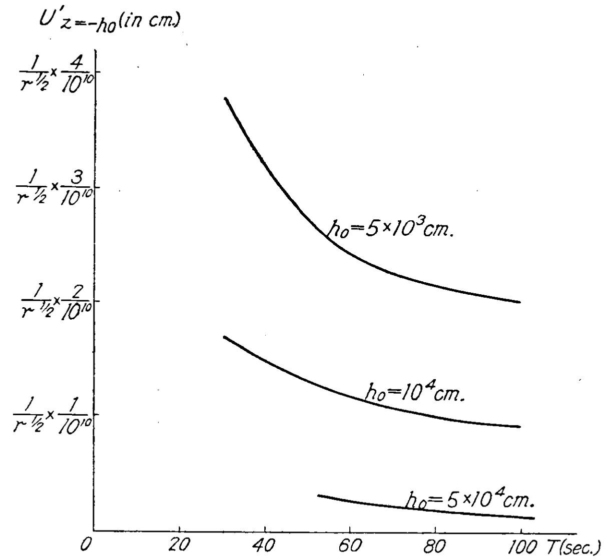

Fig. 6. (r in cm.)

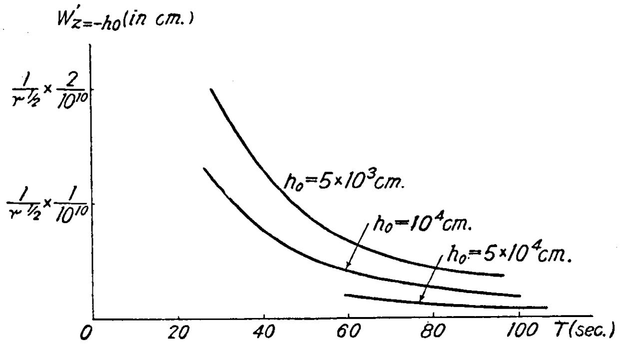

Fig. 7. (r in cm.)

Assuming $g = 980$, $c^2 = 2{\cdot}045\times10^{10}$, $\rho' = 1{\cdot}0$, $\rho = 2{\cdot}5$, $\mu/\rho = 3{\cdot}14^2\times10^{10}$ in C.G.S. unit, we have calculated the magnitudes of $u_{z=0}$, $w_{z=0}$, $u'_{z=-h_0}$, $w'_{z=-h_0}$ for different values of $T(=2\pi/n_1)$ and h_0. The results are plotted in Figs. 4, 5, 6, 7. The ordinates of the broken line from the base gives the effect due to the term involving $H_0^{(2)}(\xi r)$ and the vertical distance between the full line and this broken line indicates the effect due to the term containing $H_0^{(2)}(\varkappa r)$. In Figs. 6 and 7 the broken line coincides practically with the full line. The former cause, however, is principally of statical and special to shallow water, so that this is not yet sufficient for the full explanation of the microseisms. From these figures it will be clearly seen that the microseisms are rather due to long water waves near the observing station which are transmitted from the region of the disturbed weather, but not due to the seismic waves directly transmitted from such a region. Dr. Banerji[10] seemed to have the similar opinion. It is also to be noticed that the amplitude of the ground due to pulsatory original disturbance of long periods is smaller than that due to the disturbance of short periods, even though the amplitude of the disturbing pressure is kept constant. As Professor Gutenberg[11] suggested, this only fact is in good accordance with the phenomenon that the breakers play an important *rôle* on the microseisms of short periods. In fact, the pulsatory motion observed at a station seems partly to be the radiation, in the neighbourhood of a coast, of the deformation of the ground, which is transmitted with the velocity of long water waves. Besides the action of the surf against a steep coast, some portion of the shock of the breakers themselves must also be contributed from these radiated waves. The radiation of the deformation of the bottom ground and that of the shock due to breakers may be combined in certain cases owing to their approximately simultaneous radiation at the coast. Again, the comparison of Figs. 4, 5 and Figs 6, 7 shows that the displacement of the surface of the water is greater than that of the surface of the bottom ground about 10^4 times. Although it is obvious that the amplitude of water waves is exceedingly large in comparison with that of the ground, yet we may get some knowledge on the order of its magnitude from the result of the present calculation. It is, however, understood that the deformation of the bottom ground never corresponds with the statical head of the wave height. This reason will clearly be seen from the elementary theory of hydrodynamics with respect to the wave motion. The analytical treatment of the fact that the radiation of the deformation of the bottom ground due to the shallow water waves is still small compared with the shock of the surf against a steep coast, will be published in a future paper.

Summary.

We may now complete this paper with a brief summary.

1. The movement of the ground is composed of four kinds of displacements; namely the displacement due to the transmission of

10) S.K. Banerji, *loc. cit.*

11) B. Gutenberg, *loc. cit.*

shallow water waves, that due to Rayleigh-waves, that due to distortional waves and that due to dilatational waves.

2. The velocity of the transmission of the displacement of the body due to shallow water waves is equal to that of shallow water waves, while the transmission of the displacements of other kinds have their own velocities peculiar to the respective waves.

3. The amplitudes of the deformation of the solid body due to Rayleigh-waves and also to shallow water waves change as inverse square root of the epicentral distance, while those due to the dilatational and distortional waves diminish as inverse square of the epicentral distance.

4. Microseisms due to a disturbed weather occurring in a different region are chiefly due to long water waves, including breakers at the coast, advancing near the observing station, but not the seismic waves directly transmitted from the region of the disturbed weather. The action of the long water waves is, however, relatively small compared with that of breakers.

5. The amplitude of the ground due to pulsatory original disturbance of long periods is smaller than that due to short periods, even though the amplitude of the disturbing pressure is kept constant.

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20. 海面大氣壓の擾亂によつて起る地動に就て

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遠方の洋上に低氣壓が起つてゐる時、たとひ觀測點の附近は平穩であつても、地震計に脈動がよく現はれるといふ事が既に知られて居る。しかし其脈動は低氣壓の中心地方から直接地震波として來たものであるか、又は擾亂が一度海洋波として送られて來てから後其附近の海岸に於て地動に變化したものか未だ充分わかつて居らぬ樣に思はれる。

その機構を確める目的を以て筆者等は、半無限固體上に水の層を置き、其水面に適當な分布をなす氣壓の週期的變化を與へた場合に生ずる水の波と固體の變形とを計算し、脈動に對して如何なる原因をなすべきかを考へて見たのである。勿論この氣壓の週期的變化といふ事は、實際にはもつと複雜なものを簡單な形で代表させたまでである。

計算の結果によれば、固體の動き方は四種類の變位から成り、第一のものは淺海波の傳播によつて與へられる影響であつて其傳はる速さは恰度其淺海波の速度に等しいのである。第二のものはレーリー波の速度を以て傳はる固體其自身の波であり、他の二種は夫々固體の縱波と横波とである。

淺海波に伴つて起る波動とレーリー波に相當する波動とは擾亂氣壓の中心からの距離の平方根に反比例して振幅が減じて行く。然るに縱波と横波とは距離の二乘に反比例して振幅が少くなる。從て脈動が縱波や横波に基くものでない事は明である。

次に原點からのレーリー波による土地の振幅は、淺海波に伴つて起る振幅に比較する時は非常に小さい。從て脈動は原點から擾亂が一度淺海波として送られ、其に相當する土地の變形が傳はる事及び淺海波が海岸で碎けて生ずる振動とが非常に勢力のある原因をなす事がわかるのである。しかしこの論文では淺海波が碎けて斷崖などに直接衝動を與へる場合は態と避けた。これが主要な原因である事は明であるけれども餘りわかり切つた現象であるからである。實は淺海波による土地の變形が共儘傳播する事と淺海波が海岸で碎けて生ずる振動とは共生成の時間が大體同じである爲に之を分離する事は殆ど不可能といつても差支がない。しかし兩方とも同じ種類の原因であるから強ひて分る必要もないと思ふ。

尚大氣壓の擾亂振幅が同じでも、擾亂振動の週期が短いもの程土地の振幅卽ち脈動の振幅が大になる事は注意を要する事と思ふ。
