18. *A Note on the Analytical Treatments of the Horizontal Deformation of the Earth's Crust.*

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The horizontal deformation of the earth's crust which is going on in present days, whether chronic or acute, often has sufficiently large amount to be detected by means of precise triangulations. We have a number of such examples in this country, most of which were produced connected with earthquakes.[1] These unique data have been much discussed in this country from several points of view. It has been attempted by T. Terada, S. Fujiwhara, N. Miyabe and the writer to treat these data by means of analytical method.[2] From the observed horizontal displacements of the triangulation points in the district in question, they calculated the following quantities, viz. divergence, rotation and shear of the horizontal displacements and the directions and magnitudes of the axes of strain ellipses. They discussed the distributions of these quantities, compared them with those of other geophysical quantities, or interpreted them from geological point of view, and so on. The most fundamental assumption, implicitly taken as granted, underlying these investigations was that these calculated quantities are really exsisting ones but are not produced as a result of mere geometrical effect due to the accidental displacements to which the triangulation points were subjected. For the purpose of making these circumstances clear, we will reconsider for an example the process of calculation of the divergence

1) A. IMAMURA, *Pub. Earthq. Inv. Comm.*, 25 (1930).

2) S. FUJIWHARA and T. TAKAYAMA, *Bull. Earthq. Res. Inst.*, 6 (1928), 149.
T. TERADA and N. MIYABE, *ibid.*, 7 (1929), 223.
T. TERADA and N. MIYABE, *Proc. Imp. Acad.*, 5 (1929), 322.
N. MIYABE, *Bull. Earthq. Res. Inst.*, 8 (1930), 45.
T. TERADA and N. MIYABE, *Proc. Imp. Acad.*, 6 (1930), 49.
C. TSUBOI, *Bull. Earthq. Res. Inst.*, 8 (1930), 153.

of the horizontal displacement within a triangle with triangulation points at its vertices. The method of calculation initiated by T. Terada[3] was as follows. Let $o\ o$, x_2y_2, and x_3y_3 be the rectangular co-ordinates of the three triangulation points and u_1v_1, u_2v_2, and u_3v_3 be the respective components of the displacements σ_1, σ_2, σ_3 of the triangulation points in x and y directions. It is assumed that u and v of any point within the triangle is a linear function of x and y such that

$$u = u_1 + a_1x + b_1y,$$

$$v = v_1 + a_2x + b_2y.$$

Thus we have

$$\frac{\partial u}{\partial x} = a_1, \quad \frac{\partial v}{\partial y} = b_2\,;$$

$$\operatorname{div} \sigma = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = a_1 + b_2,$$

where

$$a_1 = \frac{u_2y_3 - u_3y_2 - u_1(y_3 - y_2)}{x_2y_3 - x_3y_2},$$

$$b_2 = -\frac{v_2x_3 - v_3x_2 - v_1(x_3 - x_2)}{x_2y_3 - x_3y_2}.$$

In this calculation, div σ naturally comes out to be constant throughout the triangle.

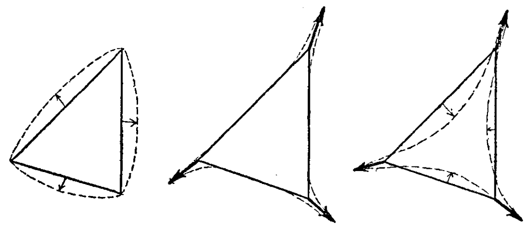
The method adopted by the writer[4] consists of, first, drawing contour lines of equal u and v over the region in question, and secondly calculating $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ for every point in the region and lastly taking the sum of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ which is the divergence of horizontal displacement of that point.

It is clear that both methods involve the assumption that u and v within the triangle and whence the divergence are produced as a result of general deformation of the earth's crust, but are not produced by a merely accidental disturbances of the triangulation points. If the earth's crust underwent other kinds of deformation than is assumed in the

3) T. Terada and N. Miyabe, *Bull. Earthq. Res. Inst.*, 7 (1927), 223.

4) C. Tsuboi, *Bull. Earthq. Res. Inst.*, 8 (1930), 153.

above calculation, there will be no physical significances in them. Some of the remarkable examples of such cases will be shown in the following figure. In the first example, the real divergence of the horizontal displacement in the triangle is a large positive value



(1) (2) (3)

Fig. 1.

though the calculated divergence will be zero. In the second example the real divergence in the traiangle is practically zero though the calculated divergence will be of a large positive value. Still in the third example, the real divergence is a large negative value though the calculated divergence will be of a large positive value. In such cases, the calculated quantity has no physical significance being merely produced as a geometrical effect.

Though the assumptions made in the methods of the above mentioned authors seem to be plausible, yet it is of importance to make some check on the validity of these assumptions for developing any further discussion in this line.

Now we have an excellent case in which this crucial test is to be made. After the great Kwanto earthquake of 1923, the Military Land Survey Department[5] made revisions of triangulations of the first, second and third orders over the southern part of the Kwanto district where the damages caused by the earthquake were large. Still we have a standard rhombus base lines remeasured in the compound of the Tokyo Astronomical Observatory at Mitaka. They are a special set of base lines forming a rhombus which consists of two equilateral triangles. The successive changes of the lengths of the sides of the rhombus have been repeatedly measured. Then, if we can compare the divergence of Mitake region by four different sets of data, i.e. the results of the rhombus measurements and those of the triangulation of the first, second and third orders, we will get some informations regarding the validity of the assumptions made in the calculation of the divergence.

5) *Bull. Earthq. Inv. Comm.*, 11 (1930), 1.

The first order triangle, which contains in it Mitaka rhombus is composed of three points, Renkôzi, Kaminumabe and Tokumaru. The co-ordinates and the displacements of these points are given in table 1.

Table I.

	x	y	u	v
	m	m	m	m
Renkozi	0	0	−0·187	−1·438
Kaminumabe	18400	−4000	−0·053	−1·332
Tokumaru	17600	15600	−0·206	−0·793

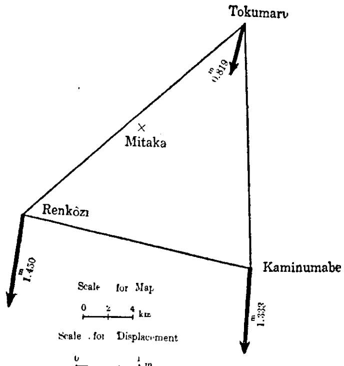


Fig. 2.

The divergence of horizontal displacements in this triangle was calculated after Terada's method. The value obtained was $3{\cdot}4 \times 10^{-5}$.

The second order triangle which contains in it Mitaka rhombus is composed of three points, Hitomi, Mure and Hosoyama. The co-ordinates and the displacements of these points are given in table 2.

Table II.

	x	y	u	v
	m	m	m	m
Hitomi	0	0	+0·088	−0·851
Mure	7300	950	−0·128	−0·841
Hosoyama	1500	−6750	+0·035	−1·322

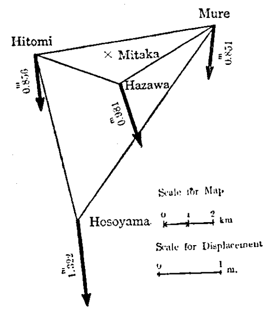


Fig. 3.

The divergence of horizontal displacements in this triangle was calculated similarly as in the case of the first order triangle. The value obtained was $3{\cdot}8 \times 10^{-5}$.

The smallest triangle which contains in it Mitaka rhombus is composed of Hitomi, Mure which are secondary points and Hazawa which is a tertiary triangulation point. The co-ordinates and the displacements of these points are given in table 3.

Table III.

	x	y	u	v
	m	m	m	m
Hitomi	0	0	+0·088	−0·851
Mure	7300	950	−0·128	−0·841
Hazawa	3400	−1300	+0·328	−0·925

The divergence of horizontal displacements in this triangle was calculated similarly, and the value obtained was $4{\cdot}5 \times 10^{-5}$.

The standard rhombus in the compound of the Tokyo Astronomical Observatory at Mitaka is composed of four base lines, each with the length of 100 m. The diagonals of this rhombus have the direction of EW and NS. The lengths of these sides have been measured fifteen times by the experts of the Military Land Survey Department under the supervision of the Imperial Japanese Geodetic Commission. The results obtained are shown in the following table.

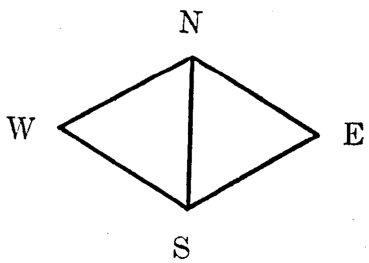


Fig. 4.

Table IV.

Date		NE	ΔNE	ES	ΔES	SW	ΔSW	WN	ΔWN	NS	ΔNS
		m 100+		m 100		m 100+		m 100+		m 100+	
		mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
1916	June	+0·92		+0·26		+0·72		+0·78		+0·59	
			0		−0·01		−0·03		0		−0·05
1917	Feb.	+0·92		+0·25		+0·69		+0·78		+0·54	
			+0·23		+0·30		+0·22		+0·23		+0·30
1917	Oct.	+0·15		+0·55		+0·91		+0·01		+0·84	
			−0·18		−0·29		−0·25		−0·82		−0·37
1918	Feb.	+0·97		+0·26		+0·66		+0·09		+0·47	
			−0·66		+0·01		+0·03		+0·67		+0·04
1918	Oct.	+0·31		+0·27		+0·69		+0·76		+0·51	
			+0·72		+0·25		+0·25		+0·10		+0·12
1919	Nov.	+1·03		+0·52		+0·94		+0·86		+0·63	
			+0·23		+0·03		+0·07		+0·16		+0·22
1920	Nov.	+1·26		+0·55		+1·07		+1·02		+0·85	
			−0·01		+0·01		+0·04		+0·09		+0·08
1921	Nov.	+1·25		+0·56		+1·11		+1·11		+0·93	
			−0·14		−0·14		−0·09		−0·05		−0·11
1922	Nov.	+1·11		+0·42		+1·02		+1·06		+0·82	
			+0·62		+0·11		+0·32		+0·06		+3·54
1923	Sept.	+1·73		+0·53		+1·34		+1·12		+4·40	
			−0·09		−0·20		+0·13		−0·17		−0·40
1923	Oct.	+1·64		+0·33		+1·47		+0·95		+4·00	
			+0·27		−0·26		−0·08		−0·23		−0·24
1924	Jan.	+1·91		+0·07		+1·39		+0·72		+3·76	
			−0·33		−0·12		+0·11		−0·01		−0·34
1924	Aug.	+1·58		−0·05		+1·50		+0·71		+3·42	
			+0·26		+0·30		−0·06		+0·11		+0·27
1925	Dec.	+1·84		+0·25		+1·44		+0·82		+3·69	
			+0·54		+0·55		+0·72		+0·56		+0·29
1927	Dec.	+2·38		+0·80		+2·16		+1·38		+3·91	

By these data, we can calculate the increase of areas of triangles NWS and NSE both of which are equilateral. If we denote by S the area of a triangle whose sides are a, b and c, and if $s = \frac{1}{2}(a+b+c)$, we have

$$S = \sqrt{s(s-a)(s-b)(s-c)},$$

hence
$$\frac{dS}{S} = \frac{1}{2}\left\{\frac{ds}{s} + \frac{ds-da}{s-a} + \frac{ds-db}{s-b} + \frac{ds-dc}{s-c}\right\}.$$

If $a = b = c$ as in the present case,

$$da + db + dc = 2ds,$$

so that
$$\frac{dS}{S} = \frac{1}{2}\left\{\frac{2s-a}{s(s-a)}\right\}ds.$$

In the present case, $s = 150$ m and $a = 100$ m,

therefore
$$\frac{dS}{S} = \frac{ds}{75000},$$ ds being measured in mm.

In this way, we can calculate easily the increase of areas of the triangles NWS and NSE. The results are shown in the following table and figure.

Table V.

Date	Δ(NE+ES+NS)	div. (NES)		Δ(SW+WN+NS)	div. (SWN)	
			Integrated			Integrated
	mm			mm		
1916 June	−0·06	$-0{\cdot}04\times10^{-5}$	$-0{\cdot}04\times10^{-5}$	−0·08	$-0{\cdot}05\times10^{-5}$	$-0{\cdot}05\times10^{-5}$
1917 Feb.	+0·83	+0·55	+0·51	+0·75	+0·50	+0·45
1917 Oct.	−0·84	−0·56	−0·05	−1·54	−1·03	−0·58
1918 Feb.	−0·61	−0·41	−0·46	+0·73	+0·49	−0·09
1918 Oct.	+1·09	+0·73	+0·27	+0·47	+0·31	+0·22
1919 Nov.	+0·48	+0·32	+0·59	+0·51	+0·34	+0·56
1920 Nov.	+0·08	+0·05	+0·64	+0·21	+0·14	+0·70
1921 Nov.	−0·39	−0·26	+0·38	−0·25	−0·17	+0·53
1922 Nov.	+4·31	+2·87	+3·25	+3·96	+2·64	+3·17
1923 Sept.	−0·69	−0·46	+2·79	−0·44	−0·29	+2·88
1923 Oct.	−0·23	−0·15	+2·65	−0·55	−0·37	+2·51
1924 Jan.	−0·79	−0·53	+2·12	−0·24	−0·16	+2·35
1924 Aug.	+0·83	+0·55	+2·67	+0·32	+0·21	+2·56
1925 Dec.	+1·31	+0·87	+3·55	+1·50	+1·00	+3·56
1927 Dec.						

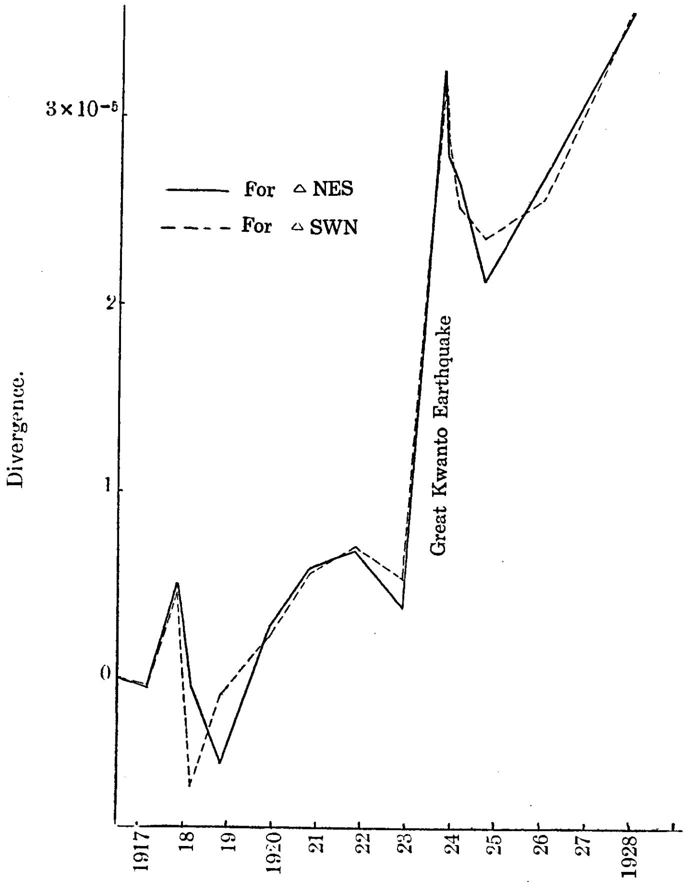


Fig. 5.

Two curves in Fig. 5 which represent the divergence of triangles NES and SWN respectively are alomost exactly similar in their forms. This seems to be a sufficient proof that the measurements of the variations in the lengths of the rhombus sides were precise enough and the deformation of the triangles were produced as the effect of the general deformation of the neighbouring earth's crust but not as the effect of the accidental displacements of one or some of the base stones. At the time of the great Kwanto earthquake of 1923, both triangles expanded in area and the divergences of the horizontal displacement in both triangles were,

$$2{\cdot}9\times10^{-5} \text{ for NES.}$$

$$2{\cdot}6\times10^{-5} \text{ for SWN.}$$

Now we have calculated the divergence of horizontal displacements of the neighbouring region of Mitake in four difference ways. The results are :—

(1) $3{\cdot}4\times 10^{-5}$ ···for 1st order triangle.
(2) $3{\cdot}8\times$,, ···for 2nd order triangle.
(3) $4{\cdot}5\times$,, ···for 3rd order triangle.
(4) $2{\cdot}9\times$,, ···for NES.
$2{\cdot}6\times$,, ···for SWN.

As Prof. T. Terada kindly remarked the writer, the results (1), (2) and (3) must not be regarded as entirely independent, the displacements of the second order triangulation points having been calculated using the data of the displacements of the first order triangulation points. The results for rhombus are on the other hand entriely independent from those obtained from the results of triangulations. The values of divergence as calculated from these different date are seen to be of the same sign and their magnitudes are approximatey equal. This is a proof that the assumptions made in analytical treatments on the horizontal deformation of the earth's crust are to be justified.

One important point to be remarked in the above is that the divergence calculated from the data of the triangulations is that in the interval between the former and the later triangulations, which is some thirty years long. On the other hand, the divergence calculated from the data of the rhombus is that in the interval of about one year. As we can see from Fig. 5, the earth's crust in the neighbourhood of Mitaka has been generally expanding even when there was no remarkable earthquake. Thus we have no reason to expect that the divergence as calculated in different periods must come out to be exactly equal. It is rather plausible that the divergence calculated from the data of the triangulations comes out larger than that calculated from the data of the rhombus at the time of the earthquake of 1923.

In Fig. 5, the areas of the triangles of the rhombus are seen to have been subjected to gradual expansion. This expansion of the areas seems to have been disturbed in 1922 and suddenly came the earthquake of 1923 and after the earthquake the triangles decreased in their areas for one year and after that they again took their general course of expansion. In this respect the great earthquake of 1923 seems to have been an accidental event in the course of the expansion of the general area. It is of special interest to note that a similar curve is obtained with other quantities such as the relative height of the mean sea level at Aburatubo which is the nearest mareographic station to the epicentre of the great Kwanto earthquake. This curve is shown in Fig. 6. The similarity in the forms of the curves in Fig. 5 and 6 suggests that there was some connections in the mechanisms of producing the divergence in Mitaka on the one hand and the vertical displacements at Aburatubo on the other hand.

In conclusion, the writer wishes to express his deep thanks to Professor T. Terada for his interests and advices in this work.

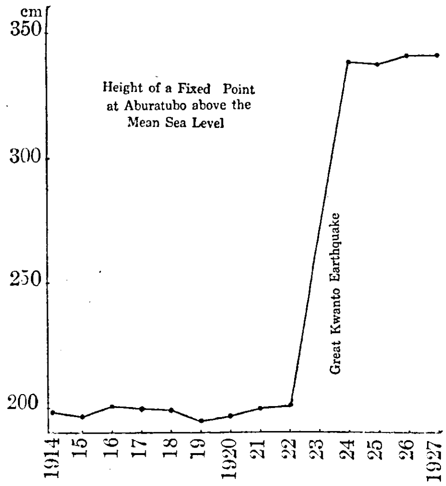


Fig. 6.

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18. 地殼の水平變動に就いて

地震研究所　坪　井　忠　二

地震の際の測量によつて明かにされた三角點の移動から該地域の地殼の變形を論ずる際には、此の三角點の移動が偶發的のものでなく近傍の一般の移動を代表して居ると看做されて居る。之は當然の假定であるが念の爲め事實果してさうであるかを調べた。1923年の關東地震後一、二、三等三角測量が改測せられたのみならず三鷹村東京天文臺構内に於ける變形基線の改測も行はれてゐるので、之等四つの材料から三鷹村近傍の divergence を求めた。その結果

$3{\cdot}4\times10^{-5}$	一等三角から
$3{\cdot}8\times10^{-5}$	二等三角から
$4{\cdot}5\times10^{-5}$	三等三角から
$2{\cdot}9\times10^{-5}$, $2{\cdot}6\times10^{-5}$	變形から

なる値を得た。先づ大體に於いて一致してゐると見られるから、三角點の移動から地殼の一般の變動を求めても正鵠に近いものが得られるであらう。

變形から求めた値が小さいのは、前後の測量の間の時間が三角測量では數十年、變形では約一年であるからで、三鷹地方が緩慢に展張しつてあつた事實ともよく符合する。

變形の約十年間に於ける面積の增減の有樣が油壺に於ける平均水面の上下と形が似てゐるのは注意すべき事である。