# 12. *Dispersion of a Shock in Echoing- and Dispersive-Elastic Bodies.**

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**1.** It is well-ascertained fact that the earthquake movement of the ground at a distant station from the seismic origin is oscillatory even when the motion in the neighbourhood of the source is of the type of a single shock. The law that, the farther the epicentral distance, the more increase the repetition of the oscillations and also the duration of the disturbance, appears now to be quite established. Although a certain part of this phenomenon may be caused by the vibratory motion induced in the seismometrical instrument, yet the main motion of the ground itself seems doubtless to obey the above law. Such a law will, of course, be explained from several points of view. The most probable explanation will be that the chief feature of the oscillations is partly due to the ordinary dispersion of elastic waves in the stratified layers of the ground and partly due to the echoing nature of the solid body which has some discontinuities in the distribution of density and elasticity. Professor Terada and Professor Fujiwhara[1] appeared to have long ago somewhat similar views on the above nature of dispersion.

As to the dispersive nature of waves, the problems of Love-waves[2] and Rayleigh-waves[3] propagated along a stratified body are perhaps striking instances. The cases which were considered by the present

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* The principal calculation contained in this paper has been carried out by G. NISHIMURA. (K. SEZAWA.)

1) For example, Prof. TERADA's & Prof. FUJIWHARA's criticisms on SEZAWA's conception concerning the seismic radiation from multiple sources of land block type, Colloq. Meeting at Seism. Inst., Oct. 1924.

2) A. E. H. LOVE, *Some Problems of Geodynamics* (Cambridge, 1911).

3) K. SEZAWA, " Dispersion of Elastic Waves propagated on the Surface of Stratified Bodies and On Curved Surfaces ", *Bull. Earthq. Res. Inst.*, 3 (1927), 1–18.

authors[4] and others,[5] may belong to the similar problems. These problems are, however, connected with the propagation of an infinite train of harmonic waves and the criterion of dispersion in these enquiries has its basis merely upon the velocity of harmonic waves, which is proper to the wave length, but not concerned with the character of the deformation of arbitrary waves progressing towards distant points. A few years ago one of the present authors studied two cases of the deformation of the arbitrary waves in dispersive elastic bodies. One[6] of these deals with the transmission of harmonic disturbances of a limited extent and of the displacement of Rayleigh-type waves when the velocity of propagation in the medium is assumed to vary as wave length in addition to a certain constant, while the other[7] treats of the excitation of periodic Rayleigh-waves due to an arbitrary disturbance in a body in which the velocity of elementary harmonic waves varies as the power series of wave length taken to the second degree. In the present paper the criterion of dispersion has been taken to be based upon the deformation of a single shock with the condition that the elementary harmonic waves satisfy the velocity equations which can be formulated from the curves compiled already by one of the present authors and indicating the exact relation between the velocity and length of Rayleigh-waves on a stratified body.

The phenomenon of apparently dispersive transmission of seismic waves appears to depend also upon the echoing character of the elastic waves when the construction of the crust is heterogeneous in regard to the distribution of density and elasticity. The simplest example of this problem is to take some discontinuous layers which are perpendicular to the direction of propagation of elastic waves. The postulation of a train of harmonic waves of an infinite extent with a constant period cannot indicate any sign of the deformation of the waves, but, when a certain arbitrary shock is applied at an assigned portion of the body, the shock

4) K. SEZAWA & G. NISHIMURA, "Rayleigh-type Waves propagated along an Inner Stratum of a Body", *Bull. Earthq. Res. Inst.*, 5 (1928), 85–96.

5) R. STONELEY, "Elastic Waves at Surface of Separation of Two Bodies", *Proc. Roy. Soc.*, 106 (1924), 416–428.

T. MATUZAWA, "Propagation of Love Waves along a doubly Stratified Layer", *Proc. Phys.-Math. Soc.* Japan, [3], 10 (1928), 25–33.

6) K. SEZAWA, "On the Propagation of the Leading and Trailing Parts of a Train of Elastic Waves", *Bull. Earthq. Res. Inst.*, 4 (1928), 111–116.

7) K. SEZAWA, "Periodic Rayleigh-waves caused by an Arbitrary Disturbance", *Bull. Earthq. Res. Inst.*, 7 (1929), 193–206.

may be deformed into a complex form of waves in transmission through the body and gives us an apparently dispersive nature of elastic waves.

It may be doubted that a clear distinction between the true dispersion of elastic waves and the apparent dispersion of echoed waves can be made from the seismic records. As the result of the present investigation which will appear later enables us to know many facts identifying two cases, the obscurity which attaches to the analysis of seismic records can be avoided to a certain extent. We think that, when a few other cases of examples of the same nature as that in this paper were solved, the determination of the significance of all the seismic records would not necessarily be impossible.

In the first section of this paper the problem of the apparent dispersion of a shock in echoing elastic body is solved, in the second section the true dispersion of a shock in dispersive elastic body is dealt with, while in the remaining section the comparison of the results of both cases is contained.

## Dispertion of a Shock in Echoing Elastic Bodies.

**2.** Let us first consider a simplest case where there is one discontinuous layer perpendicular to the direction of the propagation of waves in an infinitely-extended uniform medium and let the waves be of pure plane-type. Let the axis of $x$ be drawn parallel to the direction of the propagation, the origin being taken at a boundary surface of the layer. The displacements, the density and elastic constants of the outer medium are to be $u_0+u_0'$, $u_2$, $\rho$, $\lambda$, $\mu$. The similar displacement, density, elastic constants of the layer of thickness $H$ are to be expressed by $u_1$, $\rho'$, $\lambda'$, $\mu'$.

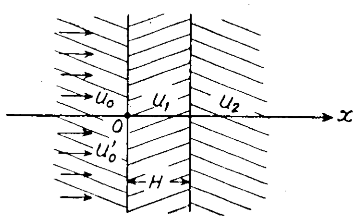

Fig. 1.

The equations of motion of both medium can be written by

$$\rho\,\frac{\partial^2(u_0+u_0')}{\partial t^2}=(\lambda+2\mu)\,\frac{\partial^2(u_0+u_0')}{\partial x^2},\ \text{(and similarly for } u_2,) \qquad \cdots\cdots(1)$$

$$\rho'\,\frac{\partial^2 u_1}{\partial t^2}=(\lambda'+2\mu')\,\frac{\partial^2 u_1}{\partial x^2}. \qquad \cdots\cdots\cdots\cdots(2)$$

The solutions of these equations can be denoted by

$$u_0+u_0' = e^{ih(Vt-x)}+C_0\, e^{ih(Vt+x)}, \quad \cdots\cdots(3)$$

$$u_1 = B_1\, e^{i\beta h(V't-x)}+C_1\, e^{i\beta h(V't+x)}, \quad \cdots\cdots(4)$$

$$u_2 = B_2\, e^{ih(Vt-x)}, \quad \cdots\cdots(5)$$

where $C_0$, $B_1$, $C_1$, $B_2$ are arbitrary constants to be determined from the boundary conditions and

$$V=\sqrt{\frac{\lambda+2\mu}{\rho}}, \quad V'=\sqrt{\frac{\lambda'+2\mu'}{\rho'}}, \quad \beta=\sqrt{\frac{\rho'(\lambda+2\mu)}{\rho(\lambda'+2\mu')}}. \quad \cdots\cdots(6)$$

At the boundaries $x=0$ (and $H$), we have

$$u_0+u_0' = u_1, \qquad (u_1=u_2,) \quad \cdots\cdots(7)$$

$$(\lambda+2\mu)\frac{\partial(u_0+u_0')}{\partial x} = (\lambda'+2\mu')\frac{\partial u_1}{\partial x}. \quad \left[(\lambda'+2\mu')\frac{\partial u_1}{\partial x} = (\lambda+2\mu)\frac{\partial u_2}{\partial x}.\right] \quad (8)$$

From (7) and (8), we can determine $C_0$, $B_1$, $C_1$, $B_2$ in the forms:

$$\left.\begin{aligned} C_0 &= \frac{(1-\alpha^2)(1-e^{-2i\beta hH})}{(1+\alpha)^2-(1-\alpha)^2\, e^{-2i\beta hH}}, \quad B_1 = \frac{2(1+\alpha)}{(1+\alpha)^2-(1-\alpha)^2\, e^{-2i\beta hH}}, \\ C_1 &= -\frac{2(1-\alpha)\, e^{-2i\beta hH}}{(1+\alpha)^2-(1-\alpha)^2\, e^{-2i\beta hH}}, \quad B_2 = \frac{4\alpha e^{-i(\beta-1)hH}}{(1+\alpha)^2-(1-\alpha)^2\, e^{-2i\beta hH}}. \end{aligned}\right\} \cdots(9)$$

Decomposing $(u_0+u_0')$ in (3) into the primary waves $u_0$ and the secondary waves $u_0'$ and applying Fourier's formula such that

$$\varphi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dh \int_{-\infty}^{\infty} \varphi(\sigma)\, e^{ih(x-\sigma)} d\sigma, \quad \cdots\cdots(10)$$

we obtain

$$u_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} dh \int_{-\infty}^{\infty} \varphi(\sigma)\, e^{ih(Vt-x+\sigma)} d\sigma, \quad \cdots\cdots(11)$$

$$u_0' = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(1-\alpha^2)(1-e^{-2i\beta hH})\,dh}{(1+\alpha)^2-(1-\alpha)^2\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{ih(Vt+x+\sigma)}d\sigma, \quad\cdots\cdots(12)$$

$$u_1 = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{(1+\alpha)\,dh}{(1+\alpha)^2-(1-\alpha)^2\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{i\beta h(V't-x+\frac{\sigma}{\beta})}d\sigma$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{(1-\alpha)\,e^{-2i\beta hH}dh}{(1+\alpha)^2-(1-\alpha)^2\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{i\beta h(V't+x+\frac{\sigma}{\beta})}d\sigma, \quad\cdots(13)$$

$$u_2 = \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\alpha\,e^{-i(\beta-1)hH}dh}{(1+\alpha)^2-(1-\alpha)^2\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{ih(Vt-x+\sigma)}d\sigma. \quad\cdots\cdots\cdots(14)$$

These are the solutions of displacements for a given primary waves of the type

$$t = 0\,; \qquad u = \varphi(x). \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

Again, we know the expressions

$$\frac{1}{(1+\alpha)^2-(1-\alpha)^2\,e^{-2i\beta hH}} = \sum_{m=0}^{\infty}\frac{(1-\alpha)^{2m}}{(1+\alpha)^{2m+2}}\,e^{-2im\beta hH}. \quad\cdots\cdots\cdots(16)$$

Hence (12), (13), (14) reduce to

$$u_0' = \frac{1}{2\pi}\sum_{m=0}^{\infty}\left(\frac{1-\alpha}{1+\alpha}\right)^{2m+1}\int_{-\infty}^{\infty}e^{-2im\beta hH}\,dh\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{ih(Vt+x+\sigma)}d\sigma$$

$$-\frac{1}{2\pi}\sum_{m=0}^{\infty}\left(\frac{1-\alpha}{1+\alpha}\right)^{2m+1}\int_{-\infty}^{\infty}e^{-2i\overline{m+1}\beta hH}dh\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{ih(Vt+x+\sigma)}d\sigma, \quad\cdots\cdots\cdots(12')$$

$$u_1 = \frac{1}{\pi}\sum_{m=0}^{\infty}\frac{(1-\alpha)^{2m}}{(1+\alpha)^{2m+1}}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{i\beta h(V't-x+\frac{\sigma}{\beta})}d\sigma$$

$$-\frac{1}{\pi}\sum_{m=0}^{\infty}\frac{(1-\alpha)^{2m+1}}{(1+\alpha)^{2m+2}}\int_{-\infty}^{\infty}e^{-2i\overline{m+1}\beta hH}dh\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{i\beta h(V't+x+\frac{\sigma}{\beta})}d\sigma, \quad\cdots\cdots(13')$$

$$u_2 = \frac{2}{\pi}\sum_{m=0}^{\infty}\frac{\alpha(1-\alpha)^{2m}}{(1+\alpha)^{2m+2}}\int_{-\infty}^{\infty} e^{i(\overline{2m+1}\beta+1)hH}dh\int_{-\infty}^{\infty}\varphi(\sigma)\,e^{ih(Vt-x+\sigma)}d\sigma. \quad \cdots\cdots(14')$$

**3.** We may now take a simple case such that

$$t=0, \quad u_0=\varphi(x)=e^{-\frac{x^2}{c^2}}, \quad \cdots\cdots(17)$$

then we find from (11), (12′), (13′), (14′),

$$u_0 = e^{-\frac{(Vt-x)^2}{c^2}}, \quad \cdots\cdots(18)$$

$$u_0' = \sum_{m=0}^{\infty}\left(\frac{1-\alpha}{1+\alpha}\right)^{2m+1} e^{-\frac{(Vt+x-2m\beta H)^2}{c^2}}$$

$$-\sum_{m=0}^{\infty}\left(\frac{1-\alpha}{1+\alpha}\right)^{2m+1} e^{-\frac{(Vt+x-\overline{2m+1}\beta H)^2}{c^2}}, \quad \cdots\cdots(19)$$

$$u_1 = 2\sum_{m=0}^{\infty}\frac{(1-\alpha)^{2m}}{(1+\alpha)^{2m+1}} e^{-\frac{\beta^2(V't-x-2m\beta H)^2}{c^2}}$$

$$-2\sum_{m=0}^{\infty}\left(\frac{1-\alpha}{1+\alpha}\right)^{2m+1} e^{-\frac{\beta^2(V't+x-\overline{2m+1}H)^2}{c^2}}, \quad \cdots\cdots(20)$$

$$u_2 = 4\sum_{m=0}^{\infty}\frac{\alpha(1-\alpha)^{2m}}{(1+\alpha)^{2m+2}} e^{-\frac{[Vt-x-(\overline{2m+1}\beta+1)H]^2}{c^2}}. \quad \cdots\cdots(21)$$

The values of $u_0$, $u_0'$, $u_1$, $u_2$ in a special case $\alpha=\frac{1}{2}$, $\beta=1$, $2c=H$ are plotted in Fig. 2. The similar values in the case $\alpha=2$, $\beta=1$, $2c=H$ are given in Fig. 3.

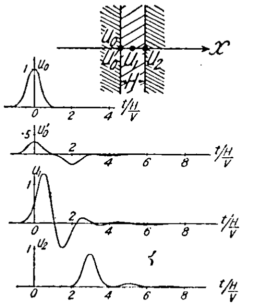

Fig. 2. ($\alpha=\frac{1}{2}$, $\beta=1$, $2c=H$.)

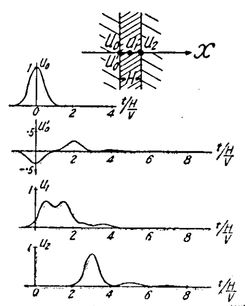

Fig. 3. ($\alpha=2$, $\beta=1$, $2c=H$.)

**4.** In the same manner of mathematical treatment we can find the solutions of the problem where a number of layers are assumed to reside perpendicular to the direction of propagation. We must, however, notice that, each term of the series of $u_2$ in (21) is essentially of the like form as that of $u_0$ in (18), so that it will be rather convenient to suppose that each term of the series of $u_2$ belonging to the first layer is the primary waves of the second layer and hence we can apply the results in (18), (19), (20), (21) to the problem of this second layer with a small modification in the coefficients of the amplitude equations and the retardation of phases. In this case the reflected waves of the type (19) belonging to the second layer become again primary waves of the first layer, though the direction of their propagation is opposite to that of the first primary waves. In this way the multiple reflection and transmission of elastic waves through and between two layers take place. The same process can be available to obtain the solutions of the problem having any number of layers. Fig. 4 and Fig. 5 give us the case of two layers and Fig. 6 and Fig. 7 indicate that of three layers.

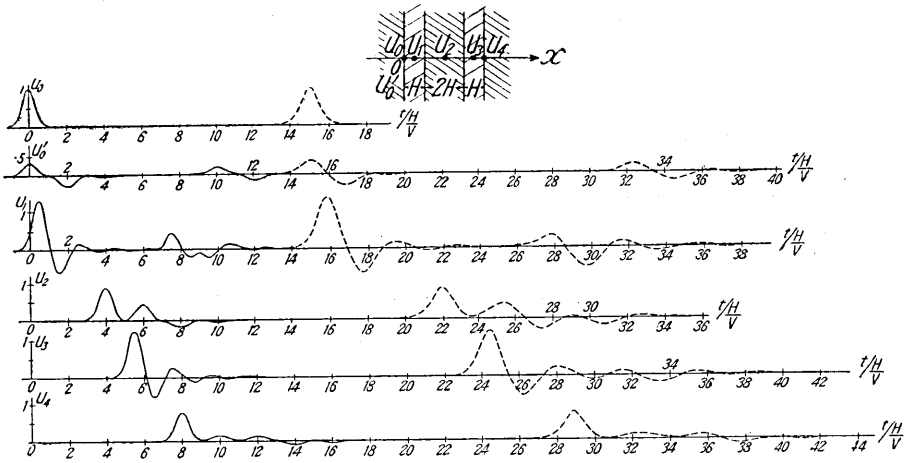

Fig. 4. ($\alpha=\frac{1}{2}$, $\beta=1$, $2c=H$.)

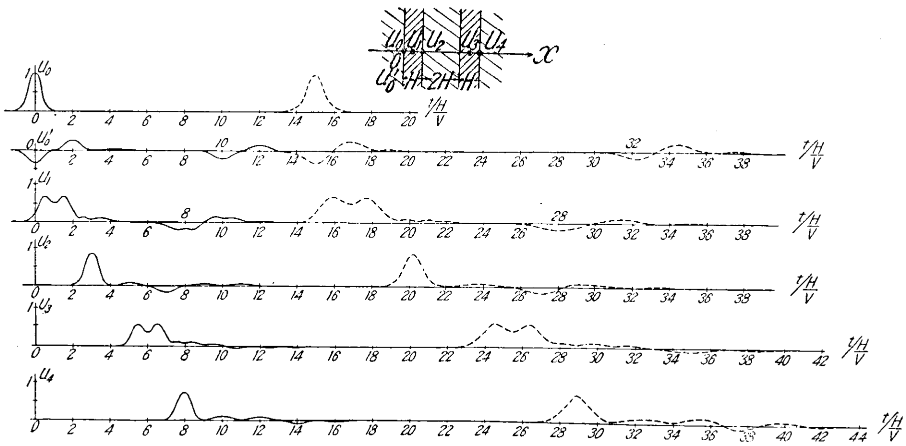

Fig. 5. ($\alpha=2$, $\beta=1$, $2c=H$.)

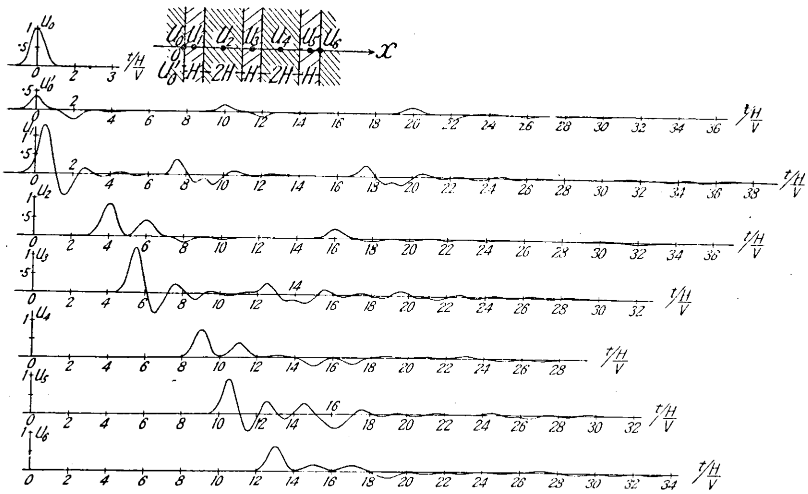

Fig. 6. ($\alpha=\frac{1}{2}$, $\beta=1$, $2c=H$.)

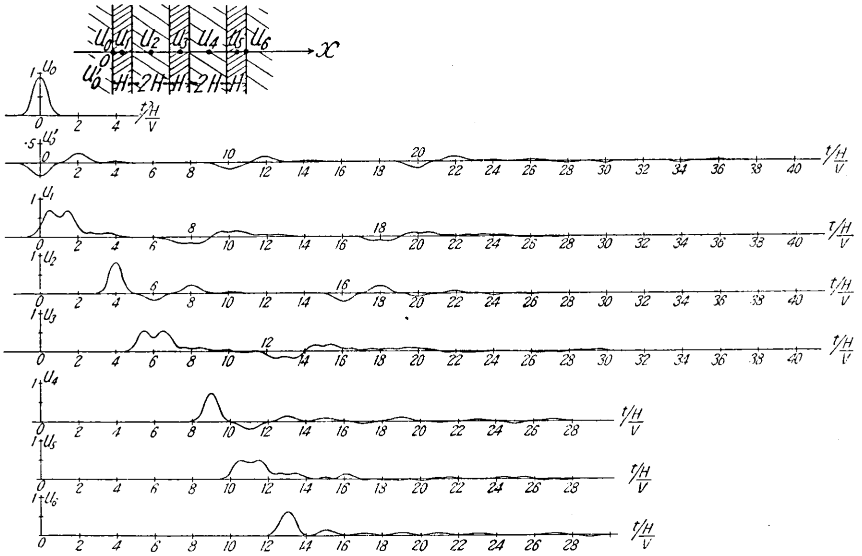


Fig. 7. ($\alpha=2$, $\beta=1$, $2c=H$.)

**5.** It may be seen from these diagrams that, even when the primary waves are of a type of a single shock, the vibrations in echoing medium or the waves through the echoing layers are oscillatory in general. But when the range of the primary disturbance is wide compared with the thickness of the layers, no periodic oscillation can be produced. The periodicity of the usual pulsatory motion depends on the size of the heterogeneous blocks as well as on their successive arrangement. Again, in the case of one layer the repeated oscillations are purely periodic and have the nature of decaying character, while in the case of more than one layer the vibrations are composed of several groups, each of which is a damped and periodic type. This feature of the problem sometimes obliges us to imagine the seismic waves as if they were radiated from multiple sources.[8)]

The leading part of the train of oscillations is transmitted through the heterogeneous medium with a definite velocity depending upon the effective density and elasticity of the medium. This is perhaps the most favourable fact, unless the medium is dispersive or viscous, for the application of the conception of the time-distance relation to the analysis of seismic records. The phase of leading part of the pulses is always maintained, so that the law of pull-push is justified to hold even in the case of the heterogeneous distribution of the medium in the direction of the wave transmission. The amplitude of the leading part of the train of

8) A. IMAMURARA, *Proc. Imp. Acad.*, 5 (1929), 330.

oscillations is large compared with those of following waves. This seems to conform with the usual character of all the earthquake movements. It is also important fact that the initial motion in the soft medium caused by the primary waves from the rigid medium and that of the opposite case can be clearly distinguished in their type as seen in the figures. The latter case is sometimes confused with the so-called waves of tilting movements. It is to be added that the amplitudes of the reflected waves are relatively small. This may become some guidance to the recognization of the side of the original disturbance.

When the shocks is diffused in a large extent, the amplitudes of all the vibrations are exceedingly diminished, the principle of energy being thus naturally satisfied. It is also to be noticed that the vibrations in the neighbourhood of the source are oscillatory to the same degree as that at large distances when there are many layers. This phenomenon which is proper to the echoing waves is much different from that of the ordinary dispersed elastic waves.

## Dispersion of a Shock in Dispersive Elastic Bodies.

**6.** We may now proceed to the question of the dispersion of a single shock in a dispersive elastic body. We shall introduce the relation[3] between the wave length of Rayleigh-waves and their velocity, which was obtained by one of the present authors a few years ago.

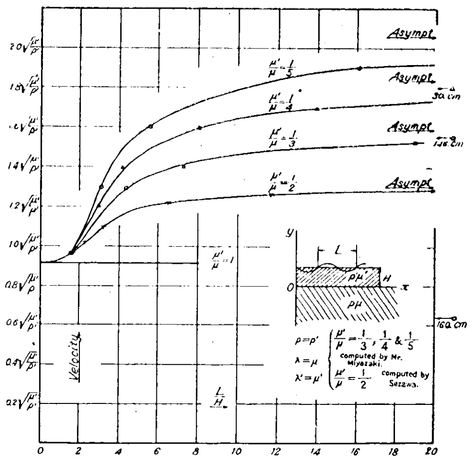


Fig. 8. Dispersion of Reyleigh-waves on Stratified Surface.
[From Bull. Earthq. Res. Inst., 3 (1927), p. 5.]

3) K. SEZAWA, *loc. cit.*, p. 321

Let us formulate the above relation for each $\mu'/\mu$ in the following form:

$$V = V_1 + \frac{(V_2 - V_1)a^2}{a^2 + f^2}, \qquad \text{(22)}$$

where $V_1$, $V_2$ are the least and the greatest values of $V$, $a$ is a constant to be adjusted and $f = 2\pi/\mathrm{L}$.

The vertical and the horizontal components of Rayleigh-waves on the surface can be expressed by

$$v = \frac{1}{\pi}\int_0^\infty \cos fVt\, df \int_{-\infty}^\infty \varphi(\sigma) \cos f(x-\sigma)\, d\sigma, \qquad \text{(23)}$$

$$u = \frac{q}{\pi}\int_0^\infty \cos fVt\, df \int_{-\infty}^\infty \varphi(\sigma) \sin f(x-\sigma)\, d\sigma, \qquad \text{(24)}$$

where $V$ is given by (22), $\varphi(\sigma)$ is the initial vertical disturbance and $q$ is the ratio of horizontal and vertical components of the elementary harmonic waves.

**7.** Let us take the initial disturbance of the form similar to that in the echoing medium, i.e.

$$\varphi(x) = e^{-\frac{x^2}{c^2}}, \qquad \text{(25)}$$

then, in virtue of this and (22), $v$ and $u$ become

$$v = \frac{c}{2\sqrt{\pi}}\int_0^\infty e^{-\frac{c^2f^2}{4}}\left[\cos f\left\{x + \left(V_1 + \frac{V_2 - V_1\, a^2}{a^2+f^2}\right)t\right\} + \cos f\left\{x - \left(V_1 + \frac{V_2 - V_1\, a^2}{a^2+f^2}\right)t\right\}\right] df, \qquad \text{(26)}$$

$$u = \frac{qc}{2\sqrt{\pi}}\int_0^\infty e^{-\frac{c^2f^2}{4}}\left[\sin f\left\{x + \left(V_1 + \frac{V_2 - V_1\, a^2}{a^2+f^2}\right)t\right\} + \sin f\left\{x - \left(V_1 + \frac{V_2 - V_1\, a^2}{a^2+f^2}\right)t\right\}\right] df. \qquad \text{(27)}$$

To integrate these expressions, we may introduce the idea of stationary point of an oscillating function due to P. Debye[9] and Lord Kelvin[10].

9) P. Debye, *Math. Ann.*, 67, 535.
10) W. Thomson, *Proc. Roy. Soc.*, 42 (1887), 80.

Consider

$$\Delta = \int_0^\infty e^{-\frac{c^2 f^2}{4}} e^{if\left\{x \pm \left(V_1 + \frac{(V_2 - V_1)a^2}{a^2+f^2}\right)\right\}} df, \quad \cdots\cdots\cdots\cdots\cdots(28)$$

and put

$$\frac{d}{df}\left[fx \pm fV_1 t \pm \frac{fbt}{a^2+f^2}\right] \equiv 0, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(29)$$

where $b = (V_2 - V_1)a^2$. Then the real possible root $f_1$ of the equation (29) in $f$ is expressed by

$$f_1 = \sqrt{-\left(a^2 \mp \frac{bt}{2(x \pm V_1 t)}\right) + \sqrt{\left\{a^2 \mp \frac{bt}{2(x \pm V_1 t)}\right\}^2 - \left\{a^4 \pm \frac{a^2 bt}{(x \pm V_1 t)}\right\}}}. \quad \cdots\cdots\cdots(30)$$

Again, if we write

$$\frac{d^2}{df^2}\left[fx \pm fV_1 t \pm \frac{fbt}{a^2+f^2}\right]_{f=f_1} = \frac{\mp 2bt\{3a^2 - f_1^2\} f_1}{(a^2+f_1^2)^3} \left\{\equiv \frac{d^2\phi(f_1)}{d\,{}_1^2}\right\}, \quad (31)$$

we have

$$\Delta = \frac{\sqrt{2\pi}\, e^{-\frac{c^2 f_1^2}{4}}}{\sqrt{\left|\frac{d^2\phi(f_1)}{df_1^2}\right|}} e^{i\left[f_1\left\{x \pm \left(V_1 + \frac{b}{a^2+f_1^2}\right) t \pm \frac{\pi}{4}\right]}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(32)$$

provided

$$\frac{d^3\phi({}_1)}{df_1^3} \bigg/ \left[\left|\frac{d^2\phi(f_1)}{df_1^2}\right|\right]^{3/2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(33)$$

is small. In (32), the upper and the lower sign of $\pm\frac{\pi}{4}$ should be taken according as $\frac{d^2\phi(f_1)}{df_1^2}$ is positive or negative.

Now, substituting the result of (32) in (26) and (27), we get

$$v = \frac{c}{\sqrt{2}} \frac{e^{-\frac{c^2 f_1^2}{4}}}{\sqrt{\left|\frac{d^2\phi(f_1)}{df_1^2}\right|}} \left\{ \cos\left[ f_1 \left\{ x + \left( V_1 + \frac{V_2 - V_1 a^2}{a^2 + f_1^2} \right) t \right\} \pm \frac{\pi}{4} \right] \right.$$

$$\left. + \cos\left[ f_1 \left\{ x - \left( V_1 + \frac{V_2 - V_1 a^2}{a^2 + f_1^2} \right) t \right\} \pm \frac{\pi}{4} \right] \right\}, \quad \cdots\cdots(34)$$

$$u = \frac{qc}{\sqrt{2}} \frac{e^{-\frac{c^2 f_1^2}{4}}}{\sqrt{\left|\frac{d^2\phi(f_1)}{df_1^2}\right|}} \left\{ \sin\left[ f_1 \left\{ x + \left( V_1 + \frac{V_2 - V_1 a^2}{a^2 + f_1^2} \right) t \right\} \pm \frac{\pi}{4} \right] \right.$$

$$\left. + \sin\left[ f_1 \left\{ x - \left( V_1 + \frac{V_2 - V_1 a^2}{a^2 + f_1^2} \right) t \right\} \pm \frac{\pi}{4} \right] \right\}, \quad \cdots(35)$$

together with the value of $f_1$ in (30).

To find compiled results, we shall take three cases, i.e. i) $\mu'/\mu = 1/2$, ii) $\mu'/\mu = 1/3$, iii) $\mu'/\mu = 1/5$. The original disturbance is assumed to be

$$e^{-\frac{x^2}{H^2}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(36)$$

in each case. From the diagrams in Fig. 8, we find with sufficient approximation the following relations:

TABLE

| $\mu'/\mu$ | $1/2$ | $1/3$ | $1/5$ |
|---|---|---|---|
| $V_2/V_1$ | 1·51 | 1·68 | 2·10 |
| $aH$ | 1·30 | 1·71 | 1·48 |

Applying these relations in (34) and analysing the resulting equation with respect to $x/H$, $V_1t/H$. we obtain the vertical component of wave forms of three cases i) $\mu'/\mu=1/2$, ii) $\mu'/\mu=1/3$, iii) $\mu'/\mu=1/5$ as shown in the drawings in Fig. 9, 10 and 11. The value of $V_1t/H$ has been taken to be 50, 75, 100 in all cases. In these figures the mathematical point representing the leading part of the waves should be excluded from consideration owing to the dissatisfaction of the condition (33).

Though the true displacement at this point can be evaluated by a certain exact manner, the process has been omitted here on account of the reason that it is not so important in the present case. Such a calculation will be carried out in some future occasion.

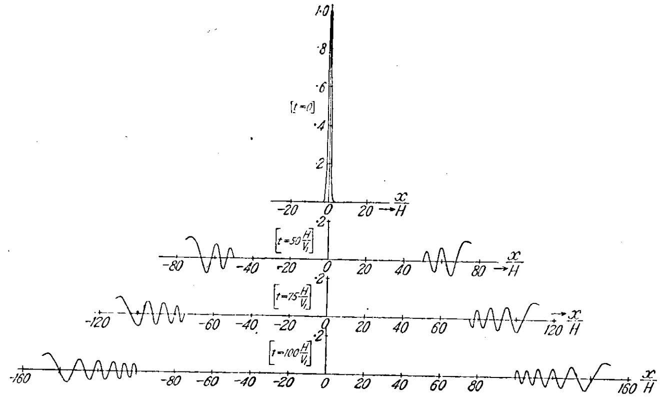

Fig. 9. $[\mu'/\mu = 1/2.]$

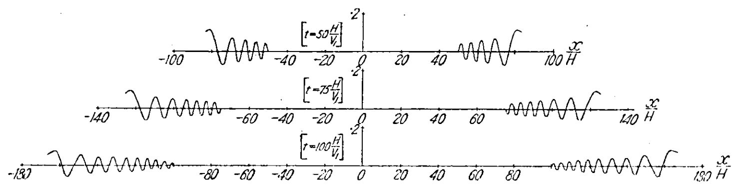

Fig. 10. $[\mu'/\mu = 1/3.]$

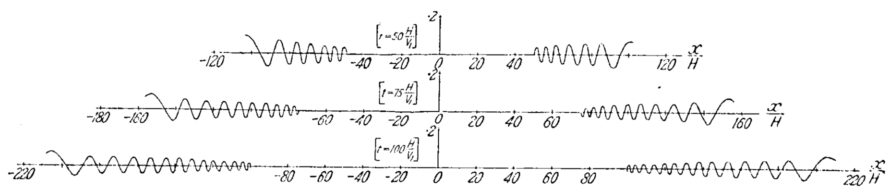

Fig. 11. $[\mu'/\mu = 1/5.]$

**8**. From these diagrams, it can be seen that, even when the initial disturbance is of the type of a single shock, the transmitted waves at a distance are of oscillatory and continually varying wave length. According as the epicentral distance is farther or nearer, the repetition of oscillations is greater or smaller and the extent of disturbed portion becomes wide or more narrow. The leading and the trailing parts of the disturbed portion take their positions at distances $V_2t$ and $V_1t$ respectively from the origin. The spaces outside the disturbed portion become quiescent on account of the imaginary nature of $f_1$ in (30) and of some other functions.

It is to be noticed, moreover, that the portion of the longer wave length resides nearer to the leading part, while that of shorter length in the vicinity of the trailing part. This can be easily understood from the physical sense that the longer waves have large velocities in the stratified elastic bodies. The most dominent length of waves in the oscillatory part is controlled by the dispersion-relation of the stratum as shown in the preceding table, but not by the type of the original motion of the source. This may be a very important fact on the problem of the transmission of the seismic waves.

Although the absolute amplitudes of the vibrations depend upon the amplitude of the initial shock and the value of $c$ which defines the sharpness of the shock, yet they are much affected by the dispersion formula already cited. In regard to the sharpness $c$, there is its certain value which makes the amplitude of waves maximum. The most common feature to be remarked in connection with the dispersed waves is that, the longer the range of the disturbed portion, the more decreases the general amplitudes of the vibratory motion. This phenomenon well conforms with the law of the conservation of the energy of waves.

It appears also that the greater ratio of $\mu/\mu'$ gives us the wide-spread disturbed portion of transmitted waves and the frequent oscillations of wave forms.

## Comparsion of Both Cases of Dispersion.

1. The disturbed portion of the transmitted waves which are dispersed by the echoing nature of the medium is not of a regular form, while in the ordinary dispersed waves the oscillatory part is quite regular and has gradually varying wave length.

2. In the echoed dispersion the leading part of the disturbed portion is distinct and has a large amplitude, while in the ordinary dispersed waves the leading as well as the trailing parts have their own velocities of propagation, the amplitude of the leading part in this case being also large.

3. In the case of echoed dispersion the disturbed portion may be partially of the same wave length and of decaying amplitudes, while in the ordinary dispersed waves even the partial equality of the wave length is impossible.

4. In both cases of dispersion the general amplitudes of the oscillations are diminished as the disturbed portion is diffused in a larger extent.

5. Partial portion of the vibrations in the echoing dispersion is of a gradually decaying type, while in the ordinary dispersion such a nature cannot take place in general.

6. When there are a number of layers, the vibrations due to echoed dispersion are repeated in a great interval of time even in the neighbourhood of the source, though the amplitudes of the succeeding shocks are relatively small compared with the initial shock. In the vibrations due to the ordinary dispersion, the disturbances in the neighbourhood of the source die away in a short time, but the vibratory disturbances at a large epicentral distance do not cease for a long time.

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## 12. 彈性波の反響性及び分散性分散に就て

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震源の衝撃が極めて單純な場合でも遠方での震動は繰替し的のものであり、而も其の距離が遠ければ遠い程繰替しが長く續くと云ふ事は既に一般に知られて居る事柄である。斯る性質の説明として誰にも想像出來る事は彈性波の反響性と分散性とである。しかしその何れが最も可能的であるかといふ事は勿論の事、其の各の性質さへも未だ餘り研究されて居らぬ様に思はれる。

彈性波の波長によつて其傳播速度が異なるといふ分散の基本的の性質に就ては著者の一人が三四年前に研究した事があるけれども勝手な衝撃が傳播につれて變形して行く性質は充分に手がつけてなかつたのである。但し波動速度を極く簡單な公式に直して取扱ふ事は一二の場合に就てはしらべた事もある。反響性の分散問題は彈性波の場合としては問題が非常に簡單であるにも拘らず未だ手が著けられて居らぬ様に見える。しかし彈性波を分散させるといふ事にはやはり重要な役割をなして居るものである。

兩方の分散性は共に簡單な數學、卽ちフーリエの積分や振動函數の定常値を求める方法などで解決が出來さうであつたから其等を適當に應用したのではあるけれども、實際に數値を入れて其結果を出す事は必ずしも容易でなかつた。

反響性の分散で直ちに氣の付く事は、初めの衝撃が單純な場合でも傳播する波動は複雜な波型となつて進み、擾亂部分の所々が週期的の振動となる事である。而して其等の所々が各減衰的の傾向を持つものである。斯くの如く所々が減衰性を持つ事は震源が恰も多數あつて順次に震動勢力を發散したかの如く思はせる。又擾亂部分の先頭は不均一彈性體の効果的速度で傳播される。卽ち彈性體が粘性や速度的分散性がなければ地震の初動によつて震源距離等を確めるが如き事に對して好都合な譯である。又反響性分散をやる場合に初動部は振幅が大きく且つ其位相が保持される。この事も亦驗震學的に重要であらう。特に注意すべき事柄は彈性波が軟い媒體から硬い媒體へ進む時に其初動部が普通と違つた波形を作る事であらう。但しこの場合は媒體が塊狀をなす時であつて一般の反射屈折の時には起らぬ現象である。又、震源の近所で層が變る爲めに第一に反射される波は振幅が非常に小である。之は震源附近の振動型に特別の型を與へるものである。尙反響性分散の爲に擾亂部分が次第に長くなるに從つて一般の振幅が小さくなる事は勢力保持の考にもよく符合して居る事柄である。

分散性の分散で第一に注目すべき事は、初動が單純でも傳播擾亂部分が矢張振動性を帶び、且つ震源からの距離が遠くなればなる程其擾亂部分の長さは長くなり、振動性は益々增大する事である。この部分が長くなればなる程一般の振幅が少くなる事や、其先頭部分が一定の速度を持つ事は反響性分散の場合に似て居る。しかしこの場合には尾部も一定の傳播速度を持つて居る。次に擾亂部分は次第に變化する波長を有し且つその始めの方は長波が占め、終りの方は短波が占めて居るけれども、一般的の波長は或る度合のものになる。この度合は震源の形には殆ど關係がなく、媒體の性質によつて決定されるものである。尙又注意すべきは $\mu/\mu'$ が大きければ大きい程擾亂部分の長さが非常に速に延び且つ振動の繰替しも割合に多くなる事である。

最後に反響性分散と分散性分散との比較を示して見ると下の如くになる。

1. 反響性媒體中の振動は不規則なのが通例であるが、分散性媒體中の分散波は割合によく規則的になり且つ次第に變化する波長を持つて居る。

2. 反響性分散波は其先頭部だけが明瞭な速度を持つけれども、分散性分散波は先頭部及び尾部共に各一定の速度を持つ。且つ又先頭部は兩方の場合共に大なる振幅になる。

3. 反響性分散では部分的に一定の波長及び減衰性振幅を持つけれども分散性分散では斯る事柄がなく一般に變化する波長を持つのである。

4. 兩方の場合共に擾亂部分の長さが長くなる程一般の振幅が減少して行き、其傾向が波動勢力保持の法則と一致して居る。

5. 反響性分散では層の數が、增大すればする程、振動の繰替しが增加する。しかし其は震源から遠い所のみでなく震源の近所でも多少同じ性質がある。然るに普通の分散性分散では振動源から遠方程擾亂部分の長さと振動の繰替しが增すのであつて、振動源に近い所では擾亂の長さが極めて短く振動の繰替しが非常に少い。