東京帝國大學
地震研究所彙報
第八號

1. *Possibility of the Free-oscillations of the Surface-layer excited by the Seismic-waves.*

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A few months ago Professor Imamura[1] showed that in some of the recent seismic records at Kii District there is a superposition of small vibrations of a very quick type on the main oscillations of the earthquake movement, and from this fact and other geological data he concluded that these small vibrations are chiefly the effect of self-oscillations of the surface layer as excited by the seismic waves from a very deep origin. Such an explanation will, of course, meet with the objections from the theoretical investigators in various ways. The instrumental defects may in some cases be mentioned as the cause of the oscillations and I think that the effects of the other behaviours of waves such as the dispersion and the scattering of the seismic rays will be equally important on the explanation of the present problem. These facts, however, have no much bearings on the possibility of the excitation of the free oscillations of the surface layer. Thus it is most urgent to know in the first place to what extent the free viblations of the layer can be excited in the idealised case, leaving out the question to the causes of the small oscillations of the ground.

In the present paper I have attempted to examine a simplest case in which a dilatational pulse of a purely plane type propagated vertically upwards in an elastic solid medium is partly transmitted through the

1) A. IMAMURA, "On the Earth-vibrations Induced in some Localities at the Arrival of Seismic Waves," *Bull. Earthq. Res. Inst.*, 7 (1929), 489-494.

bottom boundary of the superficial layer and partly reflected at this bottom as well as the surface bodunaries of the same layer.

1. Let the axis of x be drawn vertically upwards from the lower boundary of the layer, and let u be the displacement of the bottom medium and ρ, λ, μ the density and Lamé's elastic constants of this medium. The similar displacement, the density and the elastic constants of the surface layer of the thickness H are to be expressed by u', ρ', λ', μ'.

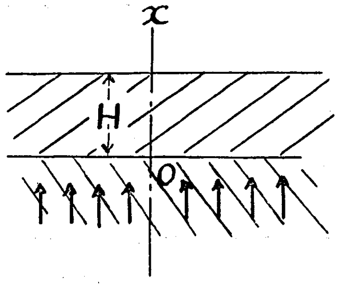

Fig. 1.

The equations of motion of the both medium can be written by

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2},\qquad\text{(1)}$$

$$\rho'\frac{\partial^2 u'}{\partial t^2}=(\lambda'+2\mu')\frac{\partial^2 u'}{\partial x^2}.\qquad\text{(2)}$$

The solutions of these equations can be easily denoted in the forms:

$$u=e^{ih(V_1t-x)}+Ae^{ih(V_1t+x)},\qquad\text{(3)}$$

$$\Delta=-ih\{e^{ih(V_1t-x)}-Ae^{ih(V_1t+x)}\},\qquad\text{(4)}$$

$$u'=Be^{ih'(V_2t-x)}+Ce^{ih'(V_2t+x)},\qquad\text{(5)}$$

$$\Delta'=-ih'\{Be^{ih'(V_2t-x)}-Ce^{ih'(V_2t+x)}\},\qquad\text{(6)}$$

where A, B, C are arbitrary constants to be determined from the boundary conditions and

$$V_1=\sqrt{\frac{\lambda+2\mu}{\rho}},\qquad V_2=\sqrt{\frac{\lambda'+2\mu'}{\rho'}},\qquad\text{(7)}$$

$$h=p/V_1,\qquad h'=p/V_2,\qquad\text{(8)}$$

$$-\frac{1}{h^2}\frac{\partial\Delta}{\partial x}=u,\qquad -\frac{1}{h^2}\frac{\partial\Delta'}{\partial x}=u'.\qquad\text{(9)}$$

At the lower boundary, $x=0$, we have

$$\left.\begin{aligned}u&=u',\\ \lambda\Delta+2\mu\frac{\partial u}{\partial x}&=\lambda'\Delta'+2\mu'\frac{\partial u'}{\partial x},\end{aligned}\right\}\qquad\text{(10)}$$

and at the upper boundary, $x=H$, we must have

$$\lambda'\Delta'+2\mu'\frac{\partial u'}{\partial x}=0. \qquad\qquad (11)$$

From (10) and (11), we get

$$\left.\begin{aligned} A &= \frac{(1-\alpha)+(1+\alpha)\,e^{-2i\beta hH}}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}},\\ B &= \frac{2}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}},\\ C &= \frac{2\,e^{-2i\beta hH}}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}, \end{aligned}\right\} \qquad\qquad (12)$$

where

$$\alpha=\sqrt{\frac{\rho'(\lambda'+2\mu')}{\rho(\lambda+2\mu)}};\quad \beta=\frac{h'}{h}=\sqrt{\frac{\rho'(\lambda+2\mu)}{\rho(\lambda'+2\mu')}}. \qquad\qquad (13)$$

The displacement u in (3) is composed of two parts, one of them being of the type of the primary waves and the other that of reflected waves. It is therefore convenient to denote u in the form:

$$u=u_1+u_2, \qquad\qquad (14)$$

where

$$u_1=e^{ih(V_1t-x)}, \qquad\qquad (15)$$

$$u_2=Ae^{ih(V_1t+x)}. \qquad\qquad (16)$$

To generalise the expressions of displacements, we introduce Fourier's double integral such that

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{ih(x-\sigma)}d\sigma. \qquad\qquad (17)$$

By applying this theorem, we obtain

$$u_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{ih(V_1t-x+\sigma)}d\sigma, \qquad\qquad (18)$$

$$u_2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(1-\alpha)+(1+\alpha)\,e^{-2i\beta hH}}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}\,dh\int_{-\infty}^{\infty}f(\sigma)\,e^{ih(V_1t+x+\sigma)}d\sigma, \qquad\qquad (19)$$

$$\begin{aligned} u' &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\,dh}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t-x+\frac{\sigma}{\beta}\right)}\,d\sigma\\ &+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\,e^{-2i\beta hH}dh}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t+x+\frac{\sigma}{\beta}\right)}\,d\sigma. \end{aligned} \qquad\qquad (20)$$

These are the solutions of displacements at all time and positions for given primary waves of the type

$$t=0\,;\quad u=f(x). \qquad\cdots\cdots(21)$$

Now we know the expressions:

$$\left.\begin{aligned}\frac{1}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}&=\frac{1}{1+\alpha}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m e^{-2im\beta hH}, &[\alpha<1]\\ \frac{1}{(1+\alpha)+(1-\alpha)\,e^{-2i\beta hH}}&=\frac{1}{\alpha+1}\sum_{m=0}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^m e^{-2im\beta hH}, &[\alpha>1]\end{aligned}\right\}\qquad(22)$$

so that (19) and (20) reduce to

$$\begin{aligned}u_2=&\frac{1}{2\pi}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m+1}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{ih(V_1t+x+\sigma)}\,d\sigma\\ &+\frac{1}{2\pi}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m}\int_{-\infty}^{\infty}e^{-2i\overline{m+1}\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{ih(V_1t+x+\sigma)}\,d\sigma,\\ &\qquad[\alpha<1]\qquad\cdots\cdots(19')\end{aligned}$$

$$\begin{aligned}u'=&\frac{1}{\pi}\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\int_{-\infty}^{\infty}e^{-2im\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t-x+\frac{\sigma}{\beta}\right)}\,d\sigma\\ &+\frac{1}{\pi}\sum_{m=0}^{\infty}(-1)^m\frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\int_{-\infty}^{\infty}e^{-2i\overline{m+1}\beta hH}dh\int_{-\infty}^{\infty}f(\sigma)\,e^{i\beta h\left(V_2t+x+\frac{\sigma}{\beta}\right)}\,d\sigma,\\ &\qquad[\alpha<1]\qquad\cdots\cdots(20')\end{aligned}$$

and to similar expressions of some modified forms, when $\alpha>1$.

2. We may now take a simplest case such that

$$t=0,\quad f(x)=e^{-\frac{x^2}{c^2}}, \qquad\cdots\cdots(23)$$

then we find in virtue of (18), (19′), (20′),

$$\begin{aligned}u_1&=e^{-\frac{(V_1t-x)^2}{c^2}},\\ u_2&=\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m+1}e^{-\frac{(V_1t+x-2m\beta H)^2}{c^2}}\\ &\quad+\sum_{m=0}^{\infty}(-)^m\left(\frac{1-\alpha}{1+\alpha}\right)^{m}e^{-\frac{(V_1t+x-2\overline{m+1}\beta H)^2}{c^2}},\end{aligned}$$

$$u' = 2\sum_{m=0}^{\infty}(-1)^m \frac{(1-\alpha)^m}{(1+\alpha)^{m+1}} e^{-\frac{\beta^2(V_2t-x-2mH)^2}{c^2}}$$

$$+2\sum_{m=0}^{\infty}(-1)^m \frac{(1-\alpha)^m}{(1+\alpha)^{m+1}} e^{-\frac{\beta^2(V_2t+x-2\overline{m+1}H)^2}{c^2}},$$

$$[\alpha<1] \quad \cdots\cdots\cdots(24)$$

and similarly

$$u_1 = e^{-\frac{(V_1t-x)^2}{c^2}},$$

$$u_2 = -\sum_{m=0}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^{m+1} e^{-\frac{(V_1t+x-2m\beta H)^2}{c^2}}$$

$$+\sum_{m=0}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^{m} e^{-\frac{(V_1t+x-2\overline{m+1}\beta H)^2}{c^2}},$$

$$u' = 2\sum_{m=0}^{\infty}\frac{(\alpha-1)^m}{(\alpha+1)^{m+1}} e^{-\frac{\beta^2(V_2t-x-2mH)^2}{c_2}}$$

$$+2\sum_{m=0}^{\infty}\frac{(\alpha-1)^m}{(\alpha+1)^{m+1}} e^{-\frac{\beta^2(V_2t+x-2\overline{m+1}H)^2}{c^2}}.$$

$$[\alpha>1] \quad \cdots\cdots\cdots(25)$$

The fact that the series forms of the expressions of the displacements have entered the present analysis is very reasonable. Each term of the series gives us every component of the succession of the disturbances produced by the multiple reflection and the transmission of the waves at the stratified layer, so that the series forms of the expressions have by no means been taken for the easiness of the procedure of the calculation as ordinarilly believed in other cases.

Let us here consider the viblations of the surface of the body due to the whole disturbance besides those of the most shallow portion of the bottom medium due to the receding waves vertically downwards. Putting $x=0$ in the expressions of u_2 in (24) and (25), and $x=H$ in those of u', we get

$$\left.\begin{aligned} u_2 &= \frac{1-\alpha}{1+\alpha}\, e^{-\frac{V_1^2t^2}{c^2}} - 4\sum_{m=1}^{\infty}(-1)^m \frac{\alpha}{1-\alpha^2}\left(\frac{1-\alpha}{1+\alpha}\right)^m e^{-\frac{(V_1t-2m\beta H)^2}{c^2}}, \\ u' &= 4\sum_{m=0}^{\infty}(-1)^m \frac{(1-\alpha)^m}{(1+\alpha)^{m+1}}\, e^{-\frac{\beta^2(V_2t-\overline{2m+1}H)^2}{c^2}}, \end{aligned}\right\}$$

$$[\alpha<1] \quad \cdots\cdots\cdots(26)$$

$$\left.\begin{aligned} u_2 &= -\frac{\alpha-1}{\alpha+1}\, e^{-\frac{V_1^2t^2}{c^2}} + 4\sum_{m=1}^{\infty} \frac{\alpha}{\alpha^2-1}\left(\frac{\alpha-1}{\alpha+1}\right)^m e^{-\frac{(V_1t-2m\beta H)^2}{c^2}}; \\ u' &= 4\sum_{m=0}^{\infty} \frac{(\alpha-1)^m}{(\alpha+1)^{m+1}}\, e^{-\frac{\beta^2(V_2t-\overline{2m+1}H)^2}{c^2}}, \end{aligned}\right\}$$

$$[\alpha>1] \quad \cdots\cdots\cdots(27)$$

where α and β are given by (13). Some special cases are plotted in the following drawings. In Figs. 2, 3, 4 the cases of $c=H$, $c=5H$, $5c=H$, besides the conditions $\alpha=\frac{1}{2}$, $\beta=1$ in common, are plotted respectively.

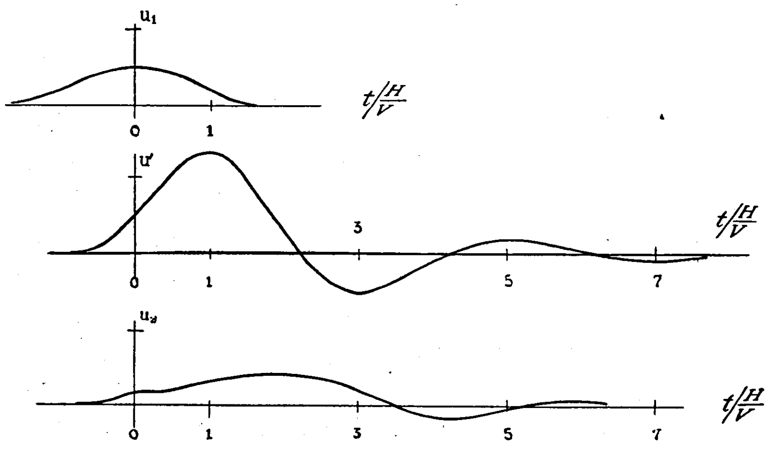

Fig. 2.

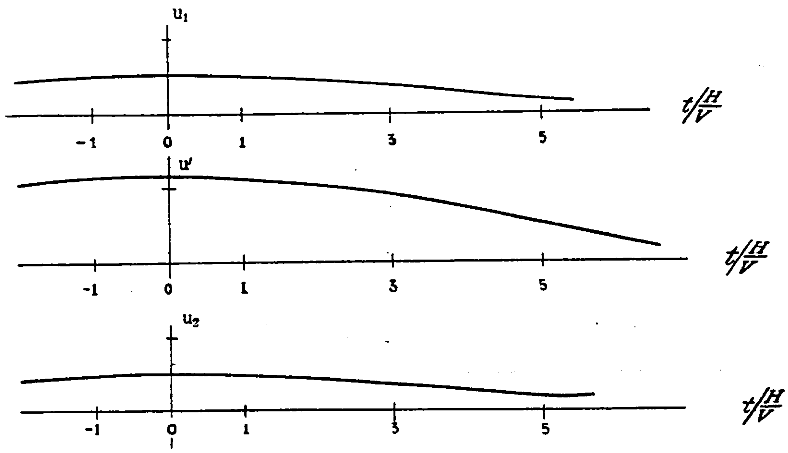

Fig. 3

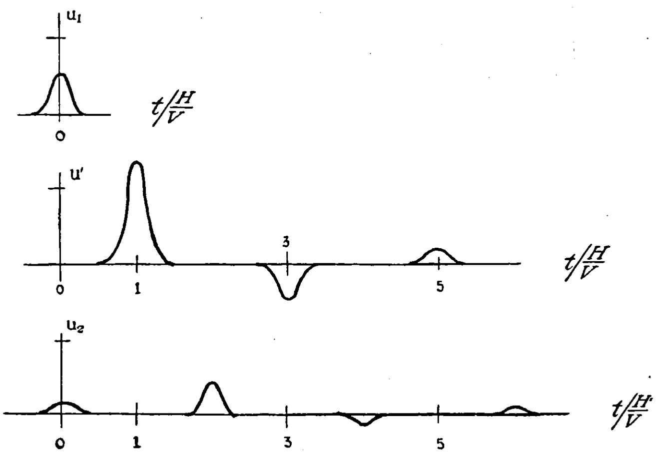

Fig. 4.

Figs. 5, 6, 7 give the similar cases under the conditions $\alpha=2$, $\beta=1$, while Fig. 8 gives us the case $\alpha=1$, $\beta=1$, i.e. the case of the semi-infinite isotropic body.

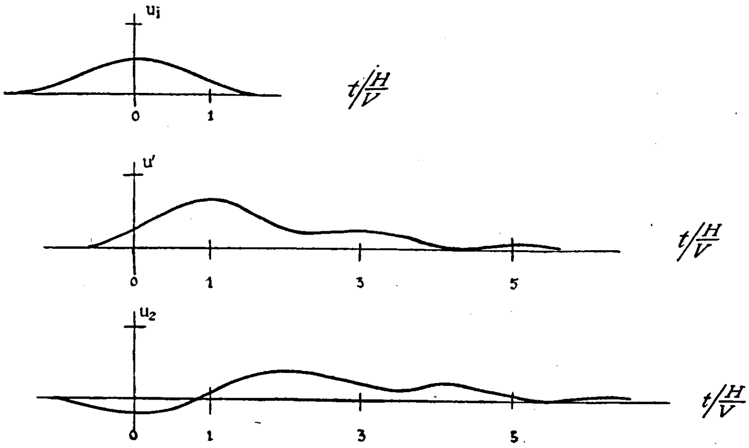

Fig. 5.

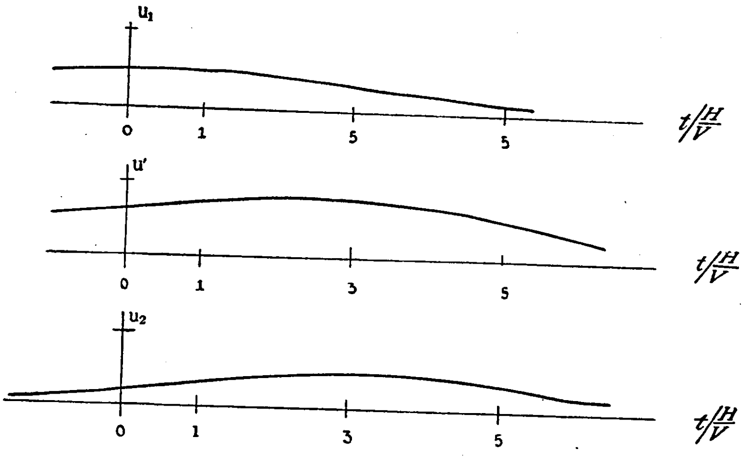

Fig. 6.

3. We may complete the present investigation by a brief notice on the results of the mathematical calculation.

It has been shewn in the preceding diagrams that, when the length of the disturbed portion of the primary shocks is small compared with

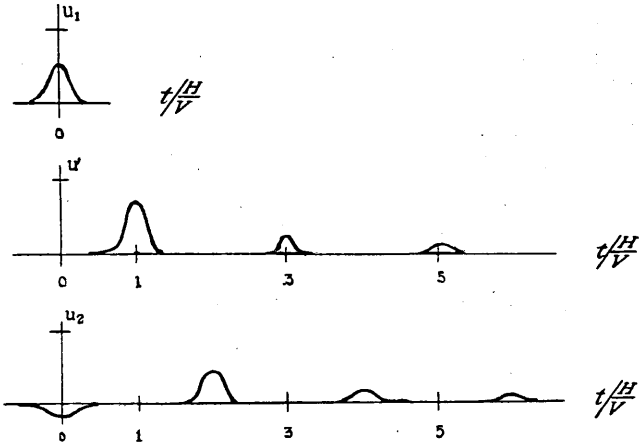

Fig. 7.

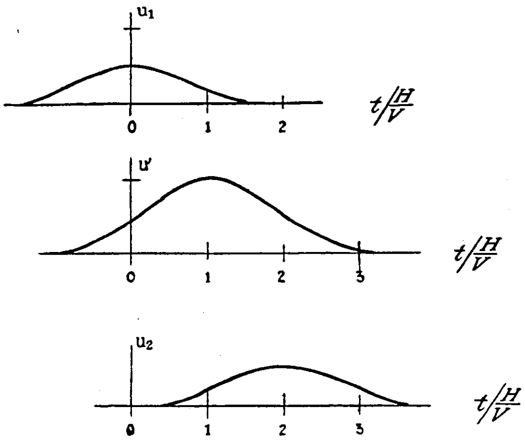

Fig. 8.

the depth of the surface stratum, the surface displacement and downward-receding waves are of periodic pulsatory types, and when the extent of the primary disturbance is large compared with the depth of the layer, no periodic pulsation can be obtained both in the surface movements and in the reflected waves. Thus, in the extreme case of the primary disturbance of very short duration, the surface displacement and the reflected waves are bestowed with the nature of the intermittent pulsation, while the disturbance of the intermediate length gives us the resulting surface motion and the reflected waves, each of nearly harmonic type.

We must, however, notice that these repeated oscillations can never be of the unceasing stationary type, but they reveal the nature of the decaying character. Neither the solid viscosity nor the other dispersive nature has been assumed from the start. The decaying character of the oscillations in the present problem depends entirely upon the fact that the partial emission of the waves into the lower medium during the multiple reflection of the pulses at the surface stratum gradually diminishes the energy of the waves. The above nature has the direct connection with the problem[2] of the decay of the restitutive origins emitting elastic waves.

When the surface layer is more rigid than the bottom medium, the oscillatory character and the decaying nature of the waves due to the intermediate primary pulsation shew a somewhat different tendency. In this case the harmonic character is not distinct as will be seen in Fig. 5.

It is equally important fact that the intermittent oscillations of the body having the soft surface stratum are of alternatingly different signs of phases, while the oscillations of the body having the rigid layer are of the phases of the same sign excepting that of the first reflected waves.

As I have not fully studied Professor Imamura's seismic records, I cannot examine that the criterions here given are all fulfilled in the actual case. We may, nevertheless, conclude that, for the purpose of the excitation of free-oscillations of the surface layer by the seismic waves, sharp tremors of very quick type should exist at the lower medium from the start and that, even when such oscillations are excited, they should obey all the conditions given in the above discussion.

My many thanks are due to Dr. G. Nishimura who has assisted me in preparing this paper.

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2) K. SEZAWA, "Scattering of Elastic Waves and Some Allied Problems," *Bull. Earthq. Res. Inst.*, 3 (1927), 35–39 (§§ 6, 7, 8).

1. 地震波によつて土地の固有振動が誘起される可能度に就いて

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數ケ月前に今村博士は紀伊地方の或地震では土地の固有振動が誘起され得る事を地震記象の說明によつて提唱された。斯る事柄は土地の固有振動を考へなくても地震波の分散や散逸を以てすれば亦容易に說明する事が出來る。しかしこゝでは土地の固有振動を考へる事が最も適當であると假定して如何なる振動現象が可能的であるかを硏究して見たのである。

問題を簡單にする爲に一つの表面層が下の彈性體の上に置かれ，深層地震の如く平らな前進面を有する任意の第一次波が與へられる場合，地表上の點が如何なる振動をするか又反射波が如何に變形されるかもしらべた。

計算の重要な結果を摘錄すれば，

1. 衝擊波の擾動部分の長さが表面層の厚さに比較して短い時には表面上の變位と地中の反射波とは週期性を保持する事になる。しかし擾動部分の長さが表面層の厚さに比較して大きい時は斯る週期性は決して存在せぬ。
2. 又、衝擊波の擾亂部分の長さが極めて短い時は表面の振動や反射波の誘起は間歇的となる。
3. 週期的の振動が誘起されてもその次々の振動は必ず減衰的傾向を持たねばならぬ。
4. 表面層が下の媒體よりも剛い時は衝擊波が多少短くても週期性はあまり明瞭ではない。
5. 間歇的振動が誘起される樣な場合表面層が下の媒體よりも軟い時には地表面に於ける次々の振動及び地中の次々の波動形は其位相が互に逆となる。しかし表面層の剛い時は各振動は同位相となる。