2. *On the Possibility of the Block Movements of the Earth Crust.*[1]

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1. Several years ago one[2] of the present authors studied the problem of some earthquake motions, in which the energy of the seismic waves is radiated in succession from multiple sources consisting of a series of land blocks. Taking the size of each block to be of a few kilometers in the linear scale and assuming that the volume of the earth crust as large as the faulted region at the occurrence of an earthquake is filled with the above blocks, the energy of the surface waves in a visco-elastic solid generated from these block origins was approximately equal to that estimated from the seismic records at various stations. The actual example was taken to the Great Kwanto Earthquake of 1923 and in that case, besides the total energy, even the order of the number of the oscillations was very like to that, in which the successive blocks radiate in series the respective energy of waves. Of course, the radiated energy was assumed to be equal to the strain energy of the earth crust accumulated in the earth blocks to the critical limit of the break-down of the material of the crust.

Since the time of the above investigation, many geophysicists have studied the problem of the block movement: S. Ono,[3] M. Ishimoto,[4] N. Nasu,[5] Ch. Tsuboi,[6] A. Imamura,[7] K. Muto—K. Atumi[8] are most notable among them. The works of the geological side have been excluded from the present consideration owing to our incomplete

1) The principal calculation involved in this paper has been carried out by G. NISHIMURA. (Added K. SEZAWA).

2) K. SEZAWA, *Colloq. Meeting of the Fac. Eng., Tokyo Imp. Univ.* (Sep. 1924).

3) S. ONO, *General Meeting of Math.-Phys. Soc., Japan,* (April 1925).

4) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, 4 (1928), 203; 6 (1929), 127.

5) N. NASU, *Bull. Earthq. Res. Inst.*, 6 (1929), 245; 7 (1929), 133.

6) Ch. TSUBOI, *Bull. Earthq. Res. Inst.*, 6 (1929), 71; 7 (1929), 103.

7) A. IMAMURA, *Proc. Imp. Acad.*, 5 (1929), 330.

8) K. MUTO-K. ATUMI, *Bull. Earthq. Res, Inst.*, 7 (1929), 503.

knowledge on such a line.[9]

These investigators seem to have explained the structures of the earth crust and the phenomenon of the earthquake principally by means of the hypothesis of the land blocks; and their excessive belief on this hypothesis appears now to be beyond the necessity of explaining the natural phenomenon to such an extent that they frequently violate both the actual facts and the well known principles of theoretical mechanics at the interpretation of the observed results.

By the encouragement of Professor Terada, one of the present authors has exerted his orthodox efforts to obtain the solutions of the stability problem[10] of determining the rough dimensions of the land blocks; but he could not get a full result with which he would satisfy. This fact meant that the nature of the configulation of the blocks at the land crust is not so distinct as to be analysed in any theoretical manner, rather than that his method of attack was incomplete. Thus his interest and persistence on this problem have been lost in the course of time.

The present authors do not think that the idea itself of the block movement is not to be excluded. Such an idea would rather become a good guidance to the discovery of the unknown nature of the geophysical evidence. Very recent investigation of some authors, however, on the earth crust in the manner of block movement gives the conclusion that the land blocks are heaped like a mosaic work and each block can be assumed to be very rigid, without any sensible action of the contact forces among respective blocks. The present authors cannot imagine that such blocks as larger than a few kilometers scale should behave like a toy mosaic in the actual gravitating and in the ordinary elastic medium. Even though the medium may be plastic, such a special nature can never be expected. These can be easily seen from very simple theories of the statics of the deformable body and of the dimensional analysis. Again, the confirmation of the actual results of the observation of the Land Survey Department made on the disturbed districts gives us the fact that any portion of the land surface has scarcely moved in the manner of rigid blocks like a mosaic which may be assumed to be too rigid for its size and weight.

9) Late Prof. N. YAMASAKI is said to have read a paper on block movements almost at the same time as that of one of the present authors. Dr. F. TADA, who is an excellent successor of Professor YAMASAKI, has published a few papers on the similar line of the research.

10) K. SEZAWA, *Colloquium at the Seism. Inst., Tokyo Imp. Univ.* (Feb. 18, 1925.)

According to the theoretical discussion of the present authors, the block movements cannot occur in the form of the assemblage of rigid blocks having no sensible connecting forces. The discontinuity of the displacement, of course, may be permitted to localise at some particular portions of the ground surface owing to the heterogeneity of the elasticity and the pre-exitence of the faulted boundaries. The general deformation, nevertheless, of the earth crust should be curved along its surface. The particular portions may give rise to the movement discontinuous in the field of the crust in combination with the general continuous movement. But such ideal displacements as an uniform overthrust or a tilting of the rigid block can never take place. It may be added that, in consequence of the combined movements of the continuous and discontinuous characters even in the most probable cases, the determination of the actual blocks and the discovery of the actual faults from the observed data would be much complicated.

Apart from the theoretical consideration, it may easily be seen that the original curvature of a tilted block among other blocks cannot be maintained owing to the geometrical configulation of the medium and the condition of the surface which is free from the stresses.

It may be asserted by some authors that in the case of the pure plastic earth crust the bodily movement of a block will be very large and the deformation relating to the curvature of the surface cannot be possible. The recent study on the plastic deformation chiefly involving the effect of the time tells us that the deformation of a plastic body is very similar to the case of the elastic body under the condition that the deformation of a body at any time is made with the state of the body at that instance as the ordinary conditions of stresses. From this fact we find that the displacement peculiar to the deformable body will become more conspicuous than the bodily displacement.

In the following calculations the pure elastic body has been assumed to exist from the start and the behaviour of the block under certain unbalanced forces has been examined. In the first section the case of the action of the uniform vertical force has been studied, and in the second place the case of the action of the varying vertical force has been discussed; while in the remaining sections the problem of the action of the horizontal forces at the vertical walls of the block is involved.

2. We shall take a two-dimensional problem in which the axis of x is directed downwards and the axis of y resides on the free surface of a body. Let us introduce Airy's function χ which satisfies

$$\nabla^4\chi = 0, \qquad (1)$$

where ∇^4 stands for

$$\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}. \qquad (2)$$

Again, take a complementary function ψ[11] which fulfils the conditions:

$$\frac{\partial^2\psi}{\partial x\partial y} = \nabla^2\chi, \qquad (3)$$

$$\nabla^2\psi = 0. \qquad (4)$$

Then the stresses and the displacements in a gravitating body are expressed in the forms:

$$X_x = \frac{\partial^2\chi}{\partial y^2} - \rho g x, \qquad (5)^{12)}$$

$$Y_y = \frac{\partial^2\chi}{\partial x^2}, \qquad (6)$$

$$X_y = -\frac{\partial^2\chi}{\partial x\partial y}, \qquad (7)$$

$$2\mu u = -\frac{\partial\chi}{\partial x} + \frac{\lambda+2\mu}{2(\lambda+\mu)}\frac{\partial\psi}{\partial y} + \frac{\mu\rho g}{\lambda+2\mu}(a^2-x^2), \qquad (8)$$

$$2\mu v = -\frac{\partial\chi}{\partial y} + \frac{\lambda+2\mu}{2(\lambda+\mu)}\frac{\partial\psi}{\partial x}. \qquad (9)$$

When the boundary conditions of stresses and displacements are specified, the problem can be uniquely determined.

3. Let us first suppose that a gravitating rectangular block of the elastic material is wedged between other blocks and that the bottom of this block is pressed or tracted by some unbalanced forces besides the isostatic force. Then, at the contacting surfaces of the vertical walls of the block the frictional resistance of the contacting solid body will neccessarily be induced

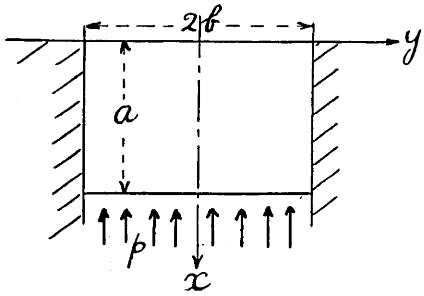

Fig. 1.

11) R. C. J. HOWLAND, *Proc. Roy. Soc., London,* 125 (1929), and other books on the theory of elasticity.

12) N. YAMAGUTI, *Jour. of Civil Eng. Soc., Tokyo,* 15 (1929), 291–304; Love, *Mathematical Theory of Elasticity, 4th ed.* (1927), 212.

owing to the lateral pressure of the gravitating mass of the block. This lateral pressure must be expected in the plastico-elastic medium, excepting the special case such as a sharp cliff.

Now we write the general solutions of the equations (1), (2), (3), (4) in the following forms:

$$\chi = \sum_m \left\{B_m y \,\mathrm{sh}\, my + C_m \,\mathrm{ch}\, my\right\} \cos mx + \sum_{m'} \left\{A'_{m'} x \,\mathrm{ch}\, m'x + B'_{m'} x \,\mathrm{sh}\, m'x + C'_{m'} \,\mathrm{ch}\, m'x + D'_{m'} \,\mathrm{sh}\, m'x\right\} \cos m'y, \quad \cdots\cdots\cdots(10)$$

$$\psi = \sum_m \frac{2}{m} B_m \,\mathrm{sh}\, my \cos mx + \sum_{m'} \frac{2}{m'} \left\{A'_{m'} \,\mathrm{ch}\, m'x + B'_{m'} \,\mathrm{sh}\, m'x\right\} \sin m'y. \quad \cdots\cdots\cdots(11)$$

Then the components of displacement and the stresses can be written by means of (5), (6), (7), (8), (9) in the forms:

$$2\mu u = \sum_m \left\{B_m \left(my \,\mathrm{sh}\, my + \frac{\lambda+2\mu}{\lambda+\mu} \,\mathrm{ch}\, my\right) + C_m m \,\mathrm{ch}\, my\right\} \sin mx - \sum_{m'} \left\{A'_{m'} \left(m'x \,\mathrm{sh}\, m'x - \frac{\mu}{\lambda+\mu} \,\mathrm{ch}\, m'x\right) + B'_{m'} \left(m'x \,\mathrm{ch}\, m'x - \frac{\mu}{\lambda+\mu} \,\mathrm{sh}\, m'x\right) + C'_{m'} m' \,\mathrm{sh}\, m'x + D'_{m'} m' \,\mathrm{ch}\, m'x\right\} \cos m'y + \frac{\mu\rho g}{\lambda+2\mu} (a^2 - x^2), \quad \cdots\cdots\cdots(12)$$

$$2\mu v = \sum_m \left\{B_m \left(\frac{\mu}{\lambda+\mu} \,\mathrm{sh}\, my - my \,\mathrm{ch}\, my\right) - C_m \,\mathrm{sh}\, my\right\} \cos mx + \sum_{m'} \left\{A'_{m'} \left(m'x \,\mathrm{ch}\, m'x + \frac{\lambda+2\mu}{\lambda+\mu} \,\mathrm{sh}\, m'x\right) + B'_{m'} \left(m'x \,\mathrm{sh}\, m'x + \frac{\lambda+2\mu}{\lambda+\mu} \,\mathrm{ch}\, m'x\right) + C'_{m'} m' \,\mathrm{ch}\, m'x + D'_{m'} \,\mathrm{sh}\, m'x\right\} \sin m'y, \quad \cdots\cdots\cdots(13)$$

$$X_x = \sum_m m\left\{B_m(my\,\mathrm{sh}\,my+2\,\mathrm{ch}\,my)+C_m\,m\,\mathrm{ch}\,my\right\}\cos mx$$
$$-\sum_{m'} m'^2\left\{A'_{m'}\,x\,\mathrm{ch}\,m'x+B'_{m'}\,x\,\mathrm{sh}\,m'x\right.$$
$$\left.+C'_{m'}\,\mathrm{ch}\,m'x+D'_{m'}\,\mathrm{sh}\,m'x\right\}\cos m'y-\rho gx, \quad \cdots\cdots\cdots(14)$$

$$Y_y = -\sum_m m^2\left\{B_m y\,\mathrm{sh}\,my+C_m\,\mathrm{ch}\,my\right\}\cos mx$$
$$+\sum_{m'} m'\left\{A'_{m'}(m'x\,\mathrm{ch}\,m'x+2\,\mathrm{sh}\,m'x)+B'_{m'}(m'x\,\mathrm{sh}\,m'x+2\,\mathrm{ch}\,m'x)\right.$$
$$\left.+C'_{m'}\,m'\,\mathrm{ch}\,m'x+D'_{m'}\,m'\,\mathrm{sh}\,m'x\right\}\cos m'y, \quad \cdots\cdots\cdots(15)$$

$$X_y = \sum_m m\left\{B_m(my\,\mathrm{ch}\,my+\mathrm{sh}\,my)+C_m\,m\,\mathrm{sh}\,my\right\}\sin mx$$
$$+\sum_{m'} m'\left\{A'_{m'}(m'x\,\mathrm{sh}\,m'x+\mathrm{ch}\,m'x)+B'_{m'}(m'x\,\mathrm{ch}\,m'x+\mathrm{sh}\,m'x)\right.$$
$$\left.+C'_{m'}\,m'\,\mathrm{sh}\,m'x+D'_{m'}\,m'\,\mathrm{ch}\,m'x\right\}\sin m'y. \quad \cdots\cdots\cdots(16)$$

Now we have the boundary conditions such that

$$y=\pm b,\ \begin{cases} u = \dfrac{\rho g}{2(\lambda+2\mu)}(a^2-x^2), & \cdots\cdots(17)\\ v = 0, & \cdots\cdots(18)\\ X_y = \pm f\dfrac{\lambda}{\lambda+2\mu}\rho gx. & \cdots\cdots(19)\end{cases}$$

$$x=0,\quad X_x = X_y = 0, \quad \cdots\cdots(20),(21)$$

$$x=a,\quad X_x = -f\frac{\lambda}{\lambda+2\mu}\frac{\rho g a^2}{2b}\quad [=-\Delta p]$$

except the isostatic pressure, $\cdots\cdots\cdots(22)$

in which Δp is the isostatically unbalanced pressure and the neighbouring blocks are supposed to be very rigid, except the deformation due to its own gravitating force. The case of the ordinary deformable neighbouring blocks can easily be calculated in the manner of the addition of the correctional

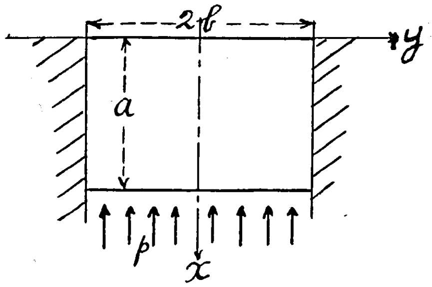

Fig. 2.

terms. In the equations (19) and (22), f is the coefficient of the friction due to the contact of the solid bodies corresponding the prescribed state. When f is smaller than a critical value, say ½, the relative movement at the contact surface cannot take place.

From (17), we find

$$m' = \frac{(2n'+1)\pi}{2b}, \qquad [n' = 0,\ 1,\ 2,\cdots] \cdots\cdots\cdots(23)$$

$$B_m\left(mb \operatorname{sh} mb + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} mb\right) + C_m\, m \operatorname{ch} mb = 0. \cdots\cdots\cdots(24)$$

From (18), we get

$$\begin{aligned}\sum_m \left\{B_m\left(\frac{\mu}{\lambda+\mu} \operatorname{sh} mb - mb \operatorname{ch} mb\right) - C_m\, m \operatorname{sh} mb\right\} \cos mx \\ + \sum_{n'} (-1)^{n'} \left\{A'_{m'}\left(m'x \operatorname{ch} m'x + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{sh} m'x\right)\right. \\ + B'_{m'}\left(m'x \operatorname{sh} m'x + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} m'x\right) \\ \left. + C'_{m'}\, m' \operatorname{ch} m'x + D'_{m'}\, m' \operatorname{sh} m'x\right\} = 0. \cdots\cdots\cdots(25)\end{aligned}$$

From (19),

$$\begin{aligned}\sum_m m\left\{B_m(mb \operatorname{ch} mb + \operatorname{sh} mb) + C_m\, m \operatorname{sh} mb\right\} \sin mx \\ + \sum_{n'} (-1)^{n'}\, m' \left\{A'_{m'}(m'x \operatorname{sh} m'x + \operatorname{ch} m'x)\right. \\ + B'_{m'}(m'x \operatorname{ch} m'x + \operatorname{sh} m'x) + C'_{m'} \operatorname{sh} m'x \\ \left. + D'_{m'}\, m' \operatorname{ch} m'x\right\} = f\frac{\lambda}{\lambda+2\mu}\rho g x. \cdots\cdots\cdots(26)\end{aligned}$$

From (20),

$$\begin{aligned}\sum_m m\left\{B_m(my \operatorname{sh} my + 2 \operatorname{ch} my) + C_m\, m \operatorname{ch} my\right\} \\ - \sum_{m'} m'^2\, C'_{m'} \cos m'y = 0. \cdots\cdots\cdots(27)\end{aligned}$$

From (21),

$$A'_{m'} + m'\, D'_{m'} = 0. \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(28)$$

From (22),

$$m = \frac{(2n+1)\pi}{2a}, \qquad [n = 0, 1, 2,\cdots] \cdots\cdots\cdots\cdots\cdots\cdots(29)$$

and

$$\sum_{m'} m'^2 \Big\{ A'_{m'} a \,\mathrm{ch}\, m'a + B'_{m'} a \,\mathrm{sh}\, m'a + C'_{m'} \,\mathrm{ch}\, m'a + D'_{m'} \,\mathrm{sh}\, m'a \Big\} \cos m'y = f \frac{\lambda}{\lambda+2\mu} \frac{\rho g a^2}{2b}. \quad \cdots\cdots\cdots(30)$$

By applying the method of Fourier's analysis on (25), (26), (27), (30) and making use of (23), (24), (28), (29) we obtain the relations:

$$\frac{\mathrm{sh}\, mb \,\mathrm{ch}\, mb - mb}{\mathrm{ch}\, mb} B_m + (-1)^{n'} \frac{2}{a} \times \Big\{ B'_{m'} \frac{(-1)^n m'}{m^2+m'^2} \Big(ma \,\mathrm{sh}\, m'a + \frac{m(m^2-m'^2)}{m'(m^2+m'^2)} \,\mathrm{ch}\, m'a \Big) + C'_{m'} \frac{(-1)^n mm'}{m^2+m'^2} \,\mathrm{ch}\, m'a - D'_{m'} \frac{m'^2}{m^2+m'^2} \Big((-1)^n ma \,\mathrm{ch}\, m'a - (-1)^n \frac{2mm'}{m^2+m'^2} \,\mathrm{sh}\, m'a - \frac{m^2-m'^2}{m^2+m'^2} \Big) \Big\} = 0, \quad \cdots\cdots\cdots(31)$$

$$\frac{m^2 b}{\mathrm{ch}\, mb} B_m + (-1)^{n'} \frac{2m'}{a} \times \Big\{ B'_{m'} \frac{(-1)^n m'}{m^2+m'^2} \Big(m'a \,\mathrm{sh}\, m'a + \frac{2m^2}{m^2+m'^2} \,\mathrm{ch}\, m'a \Big) + C'_{m'} \frac{(-1)^n m'^2}{m^2+m'^2} \,\mathrm{ch}\, m'a - D'_{m'} \frac{m'^2}{m^2+m'^2} \Big((-1)^n m'a \,\mathrm{ch}\, m'a - (-1)^n \frac{m^2-m'^2}{m^2+m'^2} \,\mathrm{sh}\, m'a - \frac{2mm'}{m^2+m'^2} \Big) \Big\} = f\rho g \frac{2}{a} \frac{(-1)^n}{m^2}, \quad \cdots\cdots\cdots(32)$$

$$(-1)^{n'} \frac{2m}{b} \frac{m^2-m'^2}{(m^2+m'^2)^2} \,\mathrm{ch}\, mb \, B_m + m' C'_{m'} = 0, \quad \cdots\cdots\cdots(33)$$

$$a \,\mathrm{sh}\, m'a \, B'_{m'} + \mathrm{ch}\, m'a \, C'_{m'} + (\mathrm{sh}\, m'a - m'a \,\mathrm{ch}\, m'a) D'_{m'} = f \frac{\rho g a^2}{b^2} \frac{(-1)^{n'}}{m'^3}, \quad \cdots\cdots\cdots(34)$$

where $m = \dfrac{(2n+1)\pi}{2a}$ and $m' = \dfrac{(2n'+1)\pi}{2b}$.

From (31), (32), (33), (34), we can construct the following determinants:

$$\vartheta = \begin{vmatrix} \dfrac{\operatorname{sh} mb \operatorname{ch} mb - mb}{\operatorname{ch} mb}, & (-1)^{n+n'} \dfrac{2}{a} \dfrac{m(m^2-m'^2)}{(m^2+m'^2)^2} \operatorname{ch} m'a, & 0, & (-1)^{n'} \dfrac{2}{a} \dfrac{m'(m^2-m'^2)}{a(m^2+m'^2)^2}\left(m' - (-1)^n m \operatorname{sh} m'a\right) \\ \dfrac{mb}{\operatorname{ch} mb}, & (-1)^{n+n'} \dfrac{4}{a} \dfrac{mm'^2}{(m^2+m'^2)^2} \operatorname{ch} m'a, & 0, & (-1)^{n'} \dfrac{4}{a} \dfrac{m'^3}{(m^2+m'^2)^2}\left(m' - (-1)^n m \operatorname{sh} m'a\right) \\ (-1)^{n'} \dfrac{2m}{b} \dfrac{m^2-m'^2}{(m^2+m'^2)^2} \operatorname{ch} mb, & 0, & m', & 0 \\ 0, & a \operatorname{sh} m'a, & \operatorname{ch} m'a, & \operatorname{sh} m'a - m'a \operatorname{ch} m'a \end{vmatrix}, \quad \cdots\cdots\cdots(35)$$

and

$$\vartheta' = \begin{vmatrix} \dfrac{\operatorname{sh} mb \operatorname{ch} mb - mb}{\operatorname{ch} mb}, & (-1)^{n+n'} \dfrac{2}{a} \dfrac{m'}{m^2+m'^2}\left(ma \operatorname{sh} m'a + \dfrac{m(m^2-m'^2)}{m'(m^2+m'^2)} \operatorname{ch} m'a\right) & (-1)^{n+n'} \dfrac{2}{a} \dfrac{mm'}{m^2+m'^2} \operatorname{ch} m'a, & 0 \\ \dfrac{m^2 b}{\operatorname{ch} mb}, & (-1)^{n+n'} \dfrac{2}{a} \dfrac{m'^2}{m^2+m'^2}\left(m'a \operatorname{sh} m'a + \dfrac{2m^2}{m^2+m'^2} \operatorname{ch} m'a\right), & (-1)^{n+n'} \dfrac{2}{a} \dfrac{m'^3}{m^2+m'^2} \operatorname{ch} m'a, & \dfrac{2}{a} \dfrac{(-1)^n}{m^2} \\ (-1)^{n'} \dfrac{2m}{b} \dfrac{m^2-m'^2}{(m^2+m'^2)^2} \operatorname{ch} mb, & 0, & m', & 0 \\ 0, & a \operatorname{sh} m'a, & \operatorname{ch} m'a, & \dfrac{a^2}{b^2} \dfrac{(-1)^{n'}}{m'^3} \end{vmatrix}. \quad \cdots\cdots\cdots(36)$$

Then the vertical displacement on the surface $x = 0$, can be easily expressed, when $\lambda = \infty$, in the form:

$$u = -\frac{f\rho g}{2\mu} \sum_m \sum_{m'} \frac{m'}{m} \frac{\vartheta'}{\vartheta} \cos m'y. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(37)$$

The computed result of this equation is illustrated in Fig. 3. The displacement thus obtained is the vertical movement of the surface due to the unbalanced bottom pressure Δp, where

$$\Delta p = f\frac{\rho g a^2}{2b} \qquad (38)$$

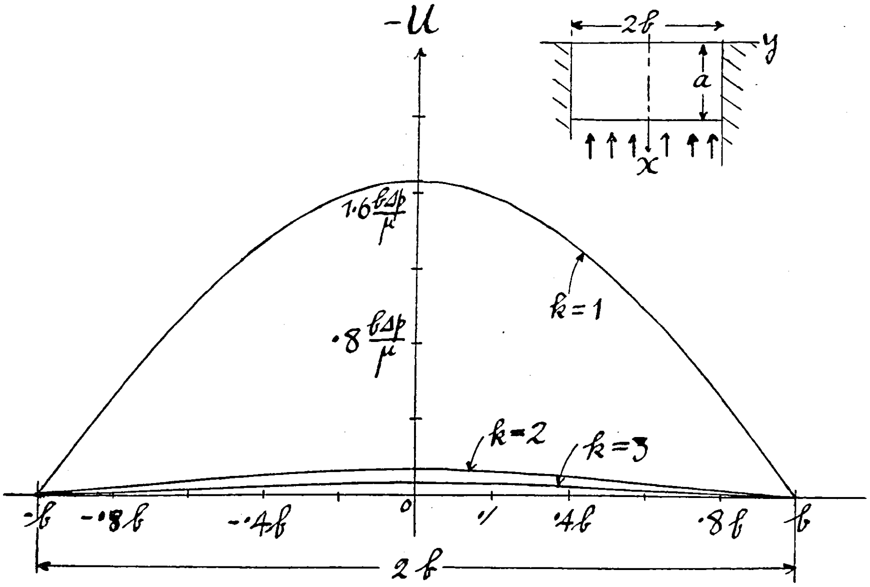

Fig. 3.

Some examples which are most probable in the actual case are tabulated below:

$k\ \left(=\frac{a}{b}\right)$	$2b$	Δp	f	max. $u_{x=0}$
½	10km.	15atm.	·0992	750cm.
1	〃	〃	·0247	42cm.
2	〃	〃	·0062	3·5cm.
3	〃	〃	·0027	1·75cm.

As the limit of f which gives the relative displacement at the contact surfaces $y = \pm b$ should take such a great value as $\frac{1}{4} \sim \frac{1}{2}$ in the ordinary rocks, the above results tell us the fact that even in the case of very thin blocks the relative displacement at the surface of contact can hardly take place. The pressure change of 15 atmosphere is not small. Even in the case of the high mountain, the pressure change at the base of the isostatic support cannot be so large, because the unbalanced pressure at the surface in the neighbourhood of the mountain is diffused in the interior of the earth crust in a vast horizontal area of the crust. Some question on the nature of the solid friction at the surface of contact may be raised. Though the coefficient of rolling friction of a solid body is very small, we cannot imagine that the friction of the present case has the nature of such a rolling quality. We think that the friction at the cracks of the earth crust will be not only of the sliding type but also of normally pressing type, so that the nature of the friction will be of an eddying quality (so to call), in which the coefficient of friction is perhaps greater than that of the ordinary solid friction. Thus the sliding fault which we observe at the ground may reside in the very vicinity of the surface. This can be clearly known from the fact that the solid friction at the shallow depth, where the hydrostatic pressure of the ground is very small, cannot give its full effect on the resistance of the sliding.

It is also seen from the diagram that the surface displacement cannot take place in a straight line form. The distribution of the displacements is curved along its surface in general. Even though the sliding takes place locally at very shallow portion of the ground, the general displacement is not of a mosaic type, but of the flexure type. It may also be added that, in the present case of the pressure distribution, even the local sliding cannot be expected. Because the region of the action of the unbalanced force is very far from the surface of the body.

4. Let us next consider a similar block which is subjected to the bottom pressure varying along the edge of this bottom.

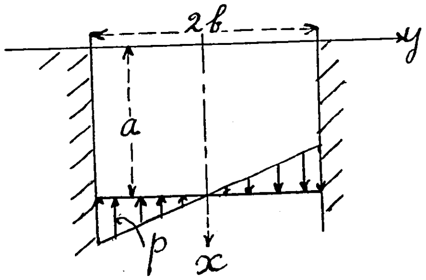

Fig. 4.

The general solutions of the equations (1), (2), (3), (4) are given in the following forms:

$$\chi = \sum_m \left\{A_m y \operatorname{ch} my + D_m \operatorname{sh} my\right\} \cos mx$$
$$+ \sum_{m'} \left\{A'_{m'} x \operatorname{ch} m'x + B'_{m'} x \operatorname{sh} m'x\right.$$
$$\left. + C'_{m'} \operatorname{ch} m'x + D'_{m'} \operatorname{sh} m'x\right\} \sin m'y, \qquad \cdots\cdots\cdots(39)$$

$$\psi = \sum_m \frac{2}{m} A_m \operatorname{ch} my \sin mx$$
$$- \sum_{m'} \frac{2}{m'} \left\{A'_{m'} m' \operatorname{ch} x + B'_{m'} \operatorname{sh} m'x\right\} \cos m'y. \qquad \cdots\cdots\cdots(40)$$

The components of displacement and the stresses can be written in the forms:

$$2\mu u = \sum_m \left\{A_m \left(my \operatorname{ch} my + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} my\right) + D_m m \operatorname{sh} my\right\} \sin mx$$
$$+ \sum_{m'} \left\{A'_{m'} \left(-m'x \operatorname{sh} m'x + \frac{\mu}{\lambda+\mu} \operatorname{ch} m'x\right)\right.$$
$$+ B'_{m'} \left(-m'x \operatorname{ch} m'x + \frac{\mu}{\lambda+\mu} \operatorname{sh} m'x\right)$$
$$\left. - C'_{m'} m' \operatorname{sh} m'x - D'_{m'} m' \operatorname{ch} m'x\right\} \sin m'y + \frac{\mu\rho g}{\lambda+2\mu} (a^2 - x^2), \qquad \cdots\cdots\cdots(41)$$

$$2\mu v = \sum_m \left\{A_m \left(-my \operatorname{sh} my + \frac{\mu}{\lambda+\mu} \operatorname{ch} my\right) - D_m m \operatorname{ch} my\right\} \cos mx$$
$$- \sum_{m'} \left\{A'_{m'} \left(m'x \operatorname{ch} m'x + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{sh} m'x\right)\right.$$
$$+ B'_{m'} \left(m'x \operatorname{sh} m'x + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} m'x\right)$$
$$\left. + C'_{m'} m' \operatorname{ch} m'x + D'_{m'} m' \operatorname{sh} m'x\right\} \cos m'y, \qquad \cdots\cdots\cdots(42)$$

$$X_x = \sum_m m \left\{A_m (my \operatorname{ch} my + 2 \operatorname{sh} my) + D_m m \operatorname{sh} my\right\} \cos mx$$
$$- \sum_{m'} m'^2 \left\{A'_{m'} x \operatorname{ch} m'x + B'_{m'} x \operatorname{sh} m'x\right.$$
$$\left. + C'_{m'} \operatorname{ch} m'x + D'_{m'} \operatorname{sh} m'x\right\} \sin m'y - \rho g x, \qquad \cdots\cdots\cdots(43)$$

$$Y_y = -\sum_m m^2\Big\{A_m\,y\,\mathrm{ch}\,my + D_m\,\mathrm{sh}\,my\Big\}\cos mx$$
$$+\sum_{m'} m'\Big\{A'_{m'}(m'x\,\mathrm{ch}\,m'x+2\,\mathrm{sh}\,m'x)+B'_{m'}(m'x\,\mathrm{sh}\,m'x+2\,\mathrm{ch}\,m'x)$$
$$+C'_{m'}\,m'\,\mathrm{ch}\,m'x+D'_{m'}\,m'\,\mathrm{sh}\,m'x\Big\}\sin m'y, \qquad \cdots\cdots\cdots(44)$$

$$X_y = -\sum_m m\Big\{A_m\,(my\,\mathrm{sh}\,my+\mathrm{ch}\,my)+D_m\,m\,\mathrm{ch}\,my\Big\}\sin mx$$
$$+\sum_{m'} m'\Big\{A'_{m'}(m'x\,\mathrm{sh}\,m'x+\mathrm{ch}\,m'x)+B'_{m'}(m'x\,\mathrm{ch}\,m'x+\mathrm{sh}\,m'x)$$
$$+C'_{m'}\,m'\,\mathrm{sh}\,m'x+D'_{m'}\,m'\,\mathrm{ch}\,m'x\Big\}\cos m'y. \qquad \cdots\cdots\cdots(45)$$

Now we have boundary conditions:

$$y=\pm b,\quad \begin{cases} u = \dfrac{\rho g}{2(\lambda+2\mu)}(a^2-x^2), & \cdots\cdots(46)\\ v=0, & \cdots\cdots(47)\\ X_y = f\dfrac{\lambda}{\lambda+2\mu}\rho g x, & \cdots\cdots(48)\end{cases}$$

$$x=0,\quad \begin{cases} v=0, & \cdots\cdots(49)\\ X_y=0, & \cdots\cdots(50)\end{cases}$$

$$x=a,\quad X_x = -\Delta p = -f\frac{\lambda}{\lambda+2\mu}\frac{3\rho g a^2}{2b^2}y$$

except the isostatic pressure. $\cdots\cdots\cdots(51)$

In the equation (51), the moment equilibrium of the block due to the bottom varying pressure and the solid friction of the surface of the contact caused by the hydrostatic pressure are considered.

From (46),

$$m' = \frac{n'\pi}{b}, \qquad [n'=1,2,3,\cdots]\cdots\cdots\cdots(52)$$

$$A_m\Big(my\,\mathrm{ch}\,my+\frac{\lambda+2\mu}{\lambda+\mu}\,\mathrm{sh}\,my\Big)+D_m\,m\,\mathrm{sh}\,my = 0. \qquad \cdots\cdots\cdots(53)$$

From (47),

$$\sum_m \left\{A_m\left(-mb \operatorname{sh} mb+\frac{\mu}{\lambda+\mu} \operatorname{ch} mb\right)-D_m\, m \operatorname{ch} mb\right\}\cdot\cos mx$$
$$-\sum_{n'}(-1)^{n'}\left\{A'_{m'}\left(m'x \operatorname{ch} m'x+\frac{\lambda+2\mu}{\lambda+\mu} \operatorname{sh} m'x\right)\right.$$
$$+B'_{m'}\left(m'x \operatorname{sh} m'x+\frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} m'x\right)$$
$$\left.+C'_{m'}\, m' \operatorname{ch} m'x+D'_{m'}\, m' \operatorname{sh} m'x\right\}=0. \quad \cdots\cdots(54)$$

From (48),

$$-\sum_m m\left\{A_m(mb \operatorname{sh} mb+\operatorname{ch} mb)+D_m\, m \operatorname{ch} mb\right\} \sin mx$$
$$+\sum_{m'}(-1)^{n'} m'\left\{A'_{m'}(m'x \operatorname{sh} m'x+\operatorname{ch} m'x)\right.$$
$$+B'_{m'}(m'x \operatorname{ch} m'x+\operatorname{sh} m'x)+C'_{m'}\, m' \operatorname{sh} m'x$$
$$\left.+D'_{m'} \operatorname{ch} m'x\right\}=f\frac{\lambda}{\lambda+2\mu}\rho g x. \quad \cdots\cdots\cdots\cdots(55)$$

From (49) and (50),

$$\sum_m m\left\{A_m(my \operatorname{ch} my+2 \operatorname{sh} my)+D_m\, m \operatorname{sh} my\right\}$$
$$-\sum_{m'} m'^2 C'_{m'} \sin m'y=0, \quad \cdots\cdots\cdots(56)$$
$$A'_{m'}+m' D'_{m'}=0. \quad \cdots\cdots(57)$$

From (51),

$$m=\frac{(2n+1)\pi}{2a}, \qquad [n=0,1,2,\cdots] \cdots\cdots\cdots(58)$$

$$-\sum_{m'} m'^2\left\{A'_{m'}\, a \operatorname{ch} m'a+B'_{m'}\, a \operatorname{sh} m'a+C'_{m'} \operatorname{ch} m'a\right.$$
$$\left.+D'_{m'} \operatorname{sh} m'a\right\} \sin m'y=f\frac{\lambda}{\lambda+2\mu}\frac{3\rho g a^2}{2b^2} y. \quad \cdots\cdots\cdots(59)$$

By applying the Fourier's analysis on (54), (54), (56), (59), making use of (52), (53), (57), (58) in the same manner as in the preceding section and simplifying the result, we get

$$(-1)^n \frac{m}{m^2+m'^2}\left\{m'a \operatorname{sh} m'a+\frac{(m^2-m'^2)}{(m^2+m'^2)} \operatorname{ch} m'a\right\} B'_{m'}$$
$$+\left\{(-1)^n \frac{mm'}{m^2+m'^2} \operatorname{ch} m'a-\frac{abm'(m^2+m'^2)^2(mb+\operatorname{sh} mb \operatorname{ch} mb)}{8\, m^3 \operatorname{sh}^2 mb}\right\} C'_{m'}$$
$$+\frac{m'^2}{m^2+m'^2}\left\{(-1)^n \frac{2mm'}{m^2+m'^2} \operatorname{sh} m'a-(-1)^n ma \operatorname{ch} m'a+\frac{m^2-m'^2}{m^2+m'^2}\right\} D'_{m'}=0.$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(60)$$

$$(-1)^n \frac{m'}{m^2+m'^2}\left\{m'a \operatorname{sh} m'a+\frac{2m^2}{m^2+m'^2}\operatorname{ch} m'a\right\} B'_{m'}$$
$$+\left\{(-1)^n \frac{m'^2}{m^2+m'^2}\operatorname{ch} m'a+\frac{ab^2(m^2+m'^2)^2}{8m \operatorname{sh}^2 mb}\right\} C'_{m'}$$
$$+\frac{m^2}{m^2+m'^2}\left\{\frac{2mm'}{m^2+m'^2}-(-1)^n m'a \operatorname{ch} m'a\right.$$
$$\left.-(-1)^n \frac{m^2-m'^2}{m^2+m'^2}\operatorname{sh} m'a\right\} D'_{m'}=(-1)^{n+n'}\frac{f\rho g}{m^2m'}, \qquad \cdots\cdots\cdots(61)$$

$$a \operatorname{sh} m'a\, B'_{m'}+\operatorname{ch} m'a\, C'_{m'}+(\operatorname{sh} m'a-m'a \operatorname{ch} m'a)\, D'_{m'}$$
$$=(-1)^{n'}\frac{3f\rho g a^2}{b^2m'^3}. \qquad \cdots\cdots\cdots(62)$$

From (60), (61), (62), we can construct two determinants:

$$\vartheta=\begin{vmatrix} (-1)^n \dfrac{m(m^2-m'^2)}{(m^2+m'^2)^2}\operatorname{ch} m'a, & -\dfrac{abm'(m^2+m'^2)^2(mb+\operatorname{sh} mb \operatorname{ch} mb)}{8m^3 \operatorname{sh}^2 mb}, & \dfrac{m'^2}{(m^2+m'^2)^2}\left\{(-1)^{n'} mm' \operatorname{sh} m'a+m^2-m'^2\right\} \\ (-1)^n \dfrac{2m^2m'}{(m^2+m'^2)^2}\operatorname{ch} m'a, & \dfrac{ab^2(m^2+m'^2)^2}{8m \operatorname{sh}^2 mb}, & 0 \\ a \operatorname{sh} m'a, & \operatorname{ch} m'a, & \operatorname{sh} m'a-m'a \operatorname{ch} m'a \end{vmatrix}, \qquad \cdots\cdots\cdots(63)$$

and

$$\vartheta'=\begin{vmatrix} (-1)^n \dfrac{m'}{m^2+m'^2}\left\{m'a \operatorname{sh} m'a+\dfrac{m^2-m'^2}{m^2+m'^2}\operatorname{ch} m'a\right\}, & \left\{(-1)^n \dfrac{mm'}{m^2+m'^2}\operatorname{ch} m'a-\dfrac{abm'(m^2+m'^2)^2}{8m^3}\dfrac{(mb+\operatorname{sh} mb \operatorname{ch} mb)}{\operatorname{sh}^2 mb}\right\}, & 0 \\ (-1)^n \dfrac{m'}{m^2+m'^2}\left\{m'a \operatorname{sh} m'a+\dfrac{2m^2}{m^2+m'^2}\operatorname{ch} m'a\right\}, & \left\{(-1)^n \dfrac{m'^2}{m^2+m'^2}\operatorname{ch} m'a+\dfrac{ab^2(m^2+m'^2)^2}{8m \operatorname{sh}^2 mb}\right\}, & (-1)^{n+n'}\dfrac{1}{m^2m'} \\ a \operatorname{sh} m'a, & \operatorname{ch} m'a, & (-1)^{n'}\dfrac{3a^2}{b^2m'^2} \end{vmatrix}. \qquad \cdots\cdots\cdots(64)$$

The vertical displacement on the surface $x = 0$, is expressed, when $\lambda = \infty$, in the form :

$$u = -\frac{f\rho g}{2\mu} \sum_m \sum_{m'} m' \frac{\vartheta'}{\vartheta} \sin m'y. \quad \cdots\cdots\cdots\cdots(65)$$

The computed result of this solution is shown in Fig. 5, in which the unbalanced pressure Δp is of the form :

$$\Delta p = -f \frac{3\rho g a^2}{2b^2} y. \quad \cdots\cdots\cdots\cdots(66)$$

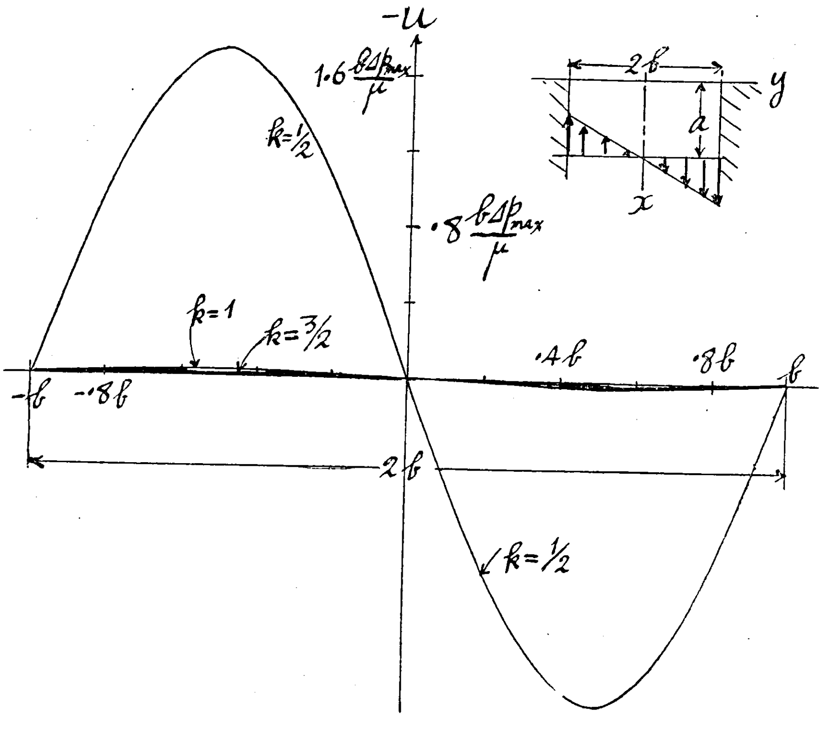

Fig. 5.

A few examples of numerical calculation are shown in the following table :

$k\left(=\frac{a}{b}\right)$	$2b$	$\Delta p_{max.}$	f	max. $u_{x=0}$
½	$10^{km.}$	$15^{atm.}$	·0327	$43{\cdot}25^{cm}$
¾	〃	〃	·0147	$\cdot 35^{cm}$
1	〃	〃	·0082	$\cdot 06^{cm}$
2	〃	〃	·0020	$\cdot 005^{cm}$

It will be worth noticing that, as the induced coefficient of friction at the contact surface is very small, the slipping of the block in an ordinary state is almost impossible. The distribution of the displacements at the surface is of a flexural type. The curve of the flexure is much different from that of the preceding case. The maximum amplitude of the displacement diminishes more quickly than that of the preceding case as the ratio of the depth to the breadth of the block is increased. This is of a similar nature as that[13] in a semi-infinite body subject to internal nuclei of strain of a multiplet type.

5. In this section, we shall suppose that the block is subjected to an uniform compression or tension acting horizontally. Let $\frac{ca}{2}$ denotes the intensity of the horizontal force, then the boundary conditions can be written in the forms:

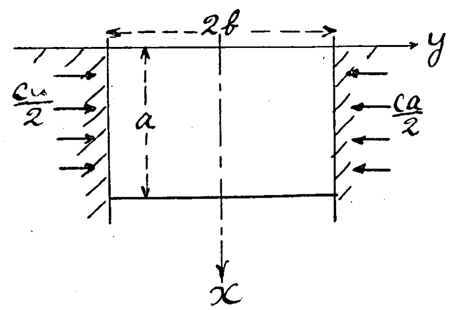

Fig. 6.

$$y = \pm b, \begin{cases} u = 0, & \cdots\cdots(67) \\ Y_y = \frac{Ca}{2}, & \cdots\cdots(68) \\ X_y = f\frac{\lambda}{\lambda+2\mu}\rho g x, & \cdots\cdots(69) \end{cases}$$

$$x = 0, \qquad X_x = X_y = 0, \qquad \cdots\cdots(70), (71)$$

$$x = a, \qquad X_x = 0. \qquad \cdots\cdots(72)$$

13) K. SEZAWA, "The Tilting of the Surface of a Semi-infinite Solid due to Internal Nuclei of Strain", *Bull. Earthq. Res. Inst.*, *7* (1929), 1–14.

The Airy's function χ, complementary function ψ, the components of displacement (u, v) and the stresses X_x, Y_y, X_y are quite of the same forms as those in (10), (11), (12), (13), (14), (15), (16).

From the nature of the problem, we can write

$$X_y \fallingdotseq 0, \qquad \cdots\cdots(69')$$

instead of (69) in the present case.

Then we have the following system of the relations.

From (67),

$$m' = \frac{(2n'+1)\pi}{2b}, \qquad [n' = 0, 1, 2, \cdots] \cdots\cdots(73)$$

$$B_m\left(mb \operatorname{sh} mb + \frac{\lambda+2\mu}{\lambda+\mu} \operatorname{ch} mb\right) + C_m\, m \operatorname{ch} mb = 0. \qquad \cdots\cdots(74)$$

From (68),

$$-\sum_m m^2 \left\{B_m\, b \operatorname{sh} mb + C_m \operatorname{ch} mb\right\} \cos mx = \frac{Ca}{2}, \qquad \cdots\cdots(75)$$

From (69'),

$$\sum_m m \left\{B_m (mb \operatorname{ch} mb + \operatorname{sh} mb) + C_m\, m \operatorname{sh} mb\right\} \sin mx$$
$$+ \sum_{n'} (-1)^{n'} m' \left\{A'_{m'} (m'x \operatorname{sh} m'x + \operatorname{ch} m'x) + B'_{m'} (m'x \operatorname{ch} m'x + \operatorname{sh} m'x)\right.$$
$$\left. + C'_{m'}\, m' \operatorname{sh} m'x + D'_{m'}\, m' \operatorname{ch} m'x\right\} = 0, \qquad \cdots\cdots(76)$$

From (70), (71),

$$\sum_m m \left\{B_m (my \operatorname{sh} my + 2 \operatorname{ch} my) + C_m\, m \operatorname{ch} my\right\}$$
$$- \sum_{m'} m'^2 C'_{m'} \cos m'y = 0, \qquad \cdots\cdots(77)$$

and

$$A'_{m'} + m' D'_{m'} = 0. \qquad \cdots\cdots(78)$$

From (72),

$$m = \frac{(2n+1)\pi}{2a}, \qquad [n = 0, 1, 2, \cdots] \cdots\cdots(79)$$

and

$$A'_{m'}\, a \operatorname{ch} m'a + B'_{m'}\, a \operatorname{sh} m'a + C'_{m'} \operatorname{ch} m'a + D'_{m'} \operatorname{sh} m'a = 0. \qquad \cdots\cdots(80)$$

Proceeding in the same manner as in the preceding case, we get

$$B_m = (-1)^n \frac{c}{m^2 \operatorname{ch} mb}, \quad \cdots\cdots\cdots(81)$$

$$\frac{m^2 b}{\operatorname{ch} mb} B_m + (-1)^{n'} \frac{2m'}{a} \Big\{ B'_{m'} \frac{(-1)^n m'^2}{m^2+m'^2} \Big(a \operatorname{sh} m'a + \frac{2m^2}{m'(m^2+m'^2)} \operatorname{ch} m'a \Big)$$

$$+ C'_{m'} (-1)^n \frac{m'^2}{m^2+m'^2} \operatorname{ch} m'a + D'_{m'} \frac{m'^2}{m^2+m'^2} \Big(\frac{2mm'}{m^2+m'^2}$$

$$- (-1)^n \frac{m^2-m'^2}{m^2+m'^2} \operatorname{sh} m'a - (-1)^n m'a \operatorname{ch} m'a \Big) \Big\} = 0,$$

$$\cdots\cdots\cdots(82)$$

$$(-1)^{n'} \frac{2m}{b} \frac{m^2-m'^2}{(m^2+m'^2)^2} \operatorname{ch} mb \, B_m + m' C'_{m'} = 0, \quad \cdots\cdots\cdots(83)$$

$$a \operatorname{sh} m'a \, B'_{m'} + \operatorname{ch} m'a \, C'_{m'} + (\operatorname{sh} m'a - m'a \operatorname{ch} m'a) D'_{m'} = 0. \cdots\cdots\cdots(84)$$

From these, we may construct the determinants below:

$$\vartheta = \begin{vmatrix} (-1)^n m \operatorname{ch} m'a, & m'(m'-(-1)^n m \operatorname{sh} m'a) \\ a \operatorname{sh} m'a, & \operatorname{sh} m'a - m'a \operatorname{ch} m'a \end{vmatrix}, \quad \cdots\cdots\cdots(83)$$

$$\vartheta' = \begin{vmatrix} \dfrac{b}{\operatorname{ch} mb}, & (-1)^{n+n'} \dfrac{4m'^2}{a(m^2+m'^2)^2} \operatorname{ch} m'a, & 0 \\ (-1)^n \dfrac{2m}{b} \dfrac{m^2-m'^2}{(m^2+m'^2)^2} \operatorname{ch} mb, & 0, & m' \\ 0, & a \operatorname{sh} m'a, & \operatorname{ch} m'a \end{vmatrix}. \quad \cdots\cdots\cdots(84)$$

Then the vertical displacement on $x = 0$ is expressed, when $\lambda = \infty$, in the form:

$$u = \frac{ca}{8\mu} \sum_{n'} \sum_{n} \frac{(-1)^{n+n'}}{\operatorname{ch} mb} \times \frac{(m^2+m'^2)^2}{mm'^2} \frac{\vartheta'}{\vartheta} \cos m'y. \quad \cdots\cdots\cdots(85)$$

The compiled result of this equation is shewn in Fig. 7.

Though the magnitude of f which should be induced at the surface of contact has not been calculated for the sake of simplicity, we can easily anticipate that f takes very small value in the present case. The sliding of the surface of the contact at the superficial portion of the

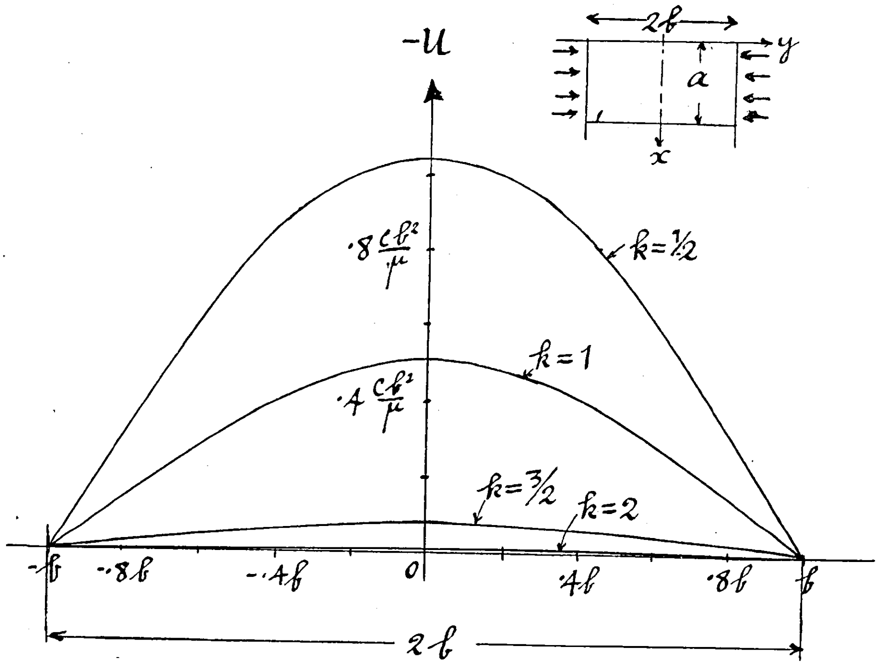

Fig. 7.

ground is very possible, because the location of the disturbing force extends to the vicinity of the surface where the limiting value of the sliding resistance is small on account of the low hydrostatic pressure. In the preceding cases the sliding resistance near the surface is, too, small, but the disturbing forces do not act directly on that surface region.

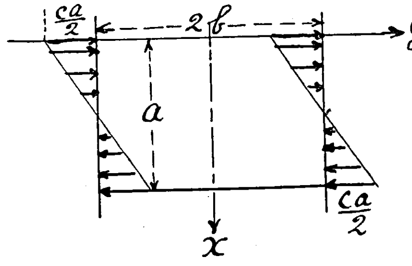

Fig. 8.

6. When the block is subjected to a varying horizontal force, the types of the Airy's function and of the complementary function are the same as those in (39) and (40). The equations of the displacement and of the stresses have similar forms, excepting some particular terms. The correct forms of them will be expressed by

$$2\mu u = \sum_m \left\{ A_m \left(my \cdot \text{ch}\, my + \frac{\lambda+2\mu}{\lambda+\mu} \text{sh}\, my \right) + D_m\, m\, \text{sh}\, my \right\} \sin mx$$

$$+ \sum_{m'} \left\{ A'_{m'} \left(-m'x\, \text{sh}\, m'x + \frac{\mu}{\lambda+\mu} \text{ch}\, m'x \right) \right.$$

$$+ B'_{m'} \left(-m'x\, \text{ch}\, m'x + \frac{\mu}{\lambda+\mu} \text{sh}\, m'x \right)$$

$$\left. - C'_{m'}\, m'\, \text{sh}\, m'x - D'_{m'}\, m'\, \text{ch}\, m'x \right\} \sin m'y, \quad \cdots\cdots(86)$$

$$X_x = \sum_m m \left\{ A_m (my\, \text{ch}\, my + 2\, \text{sh}\, my) + D_m\, m\, \text{sh}\, my \right\} \cos mx$$

$$- \sum_{m'} m'^2 \left\{ A'_{m'}\, x\, \text{ch}\, m'x + B'_{m'}\, x\, \text{sh}\, m'x \right.$$

$$\left. + C'_{m'}\, \text{ch}\, m'x + D'_{m'}\, \text{sh}\, m'x \right\} \sin m'y,$$

$$\cdots\cdots\cdots(87)$$

$$Y = - \sum_m m^2 \left\{ A_m\, y\, \text{ch}\, my + D_m\, \text{sh}\, my \right\} \cos mx$$

$$+ \sum_{m'} m' \left\{ A'_{m'} (m'x\, \text{ch}\, m'x + 2\, \text{sh}\, m'x) + B'_{m'} (m'x\, \text{sh}\, m'x + 2\, \text{ch}\, m'x) \right.$$

$$\left. + C'_{m'}\, m'\, \text{ch}\, m'x + D'_{m'}\, m'\, \text{sh}\, m'x \right\} \sin m'y,$$

$$\cdots\cdots\cdots(88)$$

$$X_y = - \sum_m m \left\{ A_m (my\, \text{sh}\, my + \text{ch}\, my) + D_m\, m\, \text{ch}\, my \right\} \sin mx$$

$$+ \sum_{m'} m' \left\{ A'_{m'} (m'x\, \text{sh}\, m'x + \text{ch}\, m'x) + B'_{m'} (m'x\, \text{ch}\, m'x + \text{sh}\, m'x) \right.$$

$$\left. + C'_{m'}\, m'\, \text{sh}\, m'x + D'_{m'}\, m'\, \text{ch}\, m'x \right\} \cos m'y.$$

$$\cdots\cdots\cdots(89)$$

Now we have the boundary conditions:

$$y = \pm b, \qquad u = 0, \quad \cdots\cdots\cdots(90)$$

$$\left. \begin{array}{ll} y = +b, & X_y = c\left(x - \dfrac{a}{2}\right), \\ y = -b, & Y_y = -c\left(x - \dfrac{a}{2}\right), \end{array} \right\} \quad \cdots\cdots\cdots(91)$$

$$y = \pm b, \qquad X_y = f\frac{\lambda}{\lambda+2\mu}\rho g x, \qquad \cdots\cdots(92)$$

$$x = 0, \qquad X_x = X_y = 0, \qquad \cdots\cdots(93), (94)$$

$$x = a, \qquad X_x = 0. \qquad \cdots\cdots(95)$$

Substituting from (86), (87), (88), (89), in (90)—(95), we find

$$m' = \frac{n'\pi}{b}, \qquad [n' = 1, 2, 3\cdots] \cdots\cdots(96)$$

$$A_m\left(mb \operatorname{ch} mb + \frac{\lambda+2\mu}{\lambda+\mu}\operatorname{sh} mb\right) + D_m\, m \operatorname{sh} mb = 0, \quad \cdots\cdots(97)$$

$$-\sum_m m^2\left\{A_m b \operatorname{ch} mb + D_m \operatorname{sh} mb\right\}\cos mx = c\left(x - \frac{a}{2}\right), \quad \cdots\cdots(98)$$

$$-\sum_m m\left\{A_m (mb \operatorname{sh} ma + \operatorname{ch} mb) + D_m\, m \operatorname{ch} mb\right\}\sin mx$$
$$+\sum_{n'}(-1)^{n'} m'\left\{A'_{m'}(m'x \operatorname{sh} m'x + \operatorname{ch} m'x) + B'_{m'}(m'x \operatorname{ch} m'x + \operatorname{sh} m'x)\right.$$
$$\left. + C'_{m'}\, m' \operatorname{sh} m'x + D'_{m'}\, m' \operatorname{ch} m'x\right\} = 0,$$
$$\cdots\cdots(99)$$

$$\sum_m m\left\{A_m(my \operatorname{ch} my + 2 \operatorname{sh} my) + D_m\, m \operatorname{sh} my\right\}$$
$$-\sum_{m'} m'^2 C'_{m'} \sin m'y = 0, \quad \cdots\cdots(100)$$

$$A'_{m'} + m' D'_{m'} = 0, \quad \cdots\cdots(101)$$

$$m = \frac{(2n+1)\pi}{2a}, \quad [n = 0, 1, 2, \cdots] \quad \cdots\cdots(102)$$

$$A'_{m'}\, a \operatorname{ch} m'a + B'_{m'}\, a \operatorname{sh} m'a + C'_{m'} \operatorname{ch} m'a + D'_{m'} \operatorname{sh} m'a = 0. \cdots\cdots(103)$$

Applying the method of Fourier's analysis on (98), (99), (100) and making use of (96), (97), (101), (102), (103), we get the relations:

$$A_m\left(mb \operatorname{ch} mb + \frac{\lambda+2\mu}{\lambda+\mu}\operatorname{sh} mb\right) + D_m\, m \operatorname{sh} mb = 0, \cdots\cdots(104)$$

$$m^2\left\{A_m b \operatorname{ch} mb + D_m \operatorname{sh} mb\right\} + \frac{2c}{a}\left(\frac{(-1)^n a}{2m} - \frac{1}{m^2}\right) = 0, \quad \cdots\cdots\cdots(105)$$

$$-m\left\{A_m (mb \operatorname{sh} mb + \operatorname{ch} mb) + D_m\, m \operatorname{ch} mb\right\}$$

$$+(-1)^{n'}\frac{2m'}{a}\Bigg[A'_{m'}\frac{m'}{m^2+m'^2}\Bigg((-1)^n\, m'a \operatorname{ch} m'a$$

$$+(-1)^n\frac{2m^2}{m^2+m'^2}\operatorname{sh} m'a + \frac{m(m^2-m'^2)}{m'(m^2+m'^2)}\Bigg)$$

$$+B'_{m'}\frac{(-1)^n\, m'}{m^2+m'^2}\left(m'a \operatorname{sh} m'a + \frac{2m^2}{m^2+m'^2}\operatorname{ch} m'a\right)$$

$$+C'_{m'}\frac{(-1)^n\, m'^2}{m^2+m'^2}\operatorname{ch} m'a + D'_{m'}\frac{m'^2}{m^2+m'^2}\left((-1)^n \operatorname{sh} m'a + \frac{m}{m'}\right)\Bigg] = 0,$$

$$\cdots\cdots\cdots(106)$$

$$\frac{2m}{b}\Bigg[A_m\frac{(-1)^{n'}\, mm'}{m^2+m'^2}\left(b \operatorname{ch} mb + \frac{2m'^2}{m(m^2+m'^2)}\operatorname{sh} mb\right)$$

$$+(-1)^{n'} D_m\frac{mm'}{m^2+m'^2}\operatorname{sh} mb\Bigg] - m'^2 C'_{m'} = 0, \cdots\cdots\cdots(107)$$

$$A'_{m'} + m' D'_{m'} = 0, \quad \cdots\cdots\cdots(108)$$

$$A'_{m'}\, a \operatorname{ch} m'a + B'_{m'}\, a \operatorname{sh} m'a + C'_{m'} \operatorname{ch} m'a + D'_{m'} \operatorname{sh} m'a = 0. \cdots\cdots(109)$$

From these, we find the surface displacement, when $\lambda = \infty$, in the following form:

$$u = -\frac{c}{2\mu}\sum_m \sum_{m'} (-1)^{n'}\left(\frac{a(-1)^n}{2} - \frac{1}{m}\right)\frac{m^2+m'^2}{m'^3\, m^3 \operatorname{sh}^2 mb} \times \frac{\vartheta'}{\vartheta}\sin m'y,$$

$$\cdots\cdots\cdots(110)$$

where

$$\vartheta = \begin{vmatrix} (-1)^n m'a \,\mathrm{ch}\, m'a + (-1)^n \dfrac{2m^2}{m^2+m'^2} \,\mathrm{sh}\, m'a + \dfrac{m(m^2-m'^2)}{m'(m^2+m'^2)}, & & \\ (-1)^n \left(m'a \,\mathrm{sh}\, m'a + \dfrac{2m^2}{m^2+m'^2} \,\mathrm{ch}\, m'a \right), & m'\left((-1)^n \,\mathrm{sh}\, m'a + \dfrac{m}{m'} \right) & \\ 1, & 0, & m' \\ a \,\mathrm{ch}\, m'a, & a \,\mathrm{sh}\, m'a, & \mathrm{sh}\, m'a \end{vmatrix}, \qquad \text{..........................(111)}$$

$$\vartheta' = \begin{vmatrix} (mb \,\mathrm{ch}\, mb + \mathrm{sh}\, mb), & m \,\mathrm{sh}\, mb, & 0, & 0 \\ -m(mb \,\mathrm{sh}\, mb + \mathrm{ch}\, mb), & -m^2 \,\mathrm{ch}\, mb, & (-1)^{n+n'} \dfrac{2m'^2}{a(m^2+m'^2)} \left(m'a \,\mathrm{sh}\, m'a + \dfrac{2m^2}{m^2+m'^2} \,\mathrm{ch}\, m'a \right), & (-1)^{n+n'} \dfrac{2m'^3}{a(m^2+m'^2)} \,\mathrm{ch}\, m'a \\ (-1)^{n'} \dfrac{2mm'(m^2-m'^2)}{b(m^2+m'^2)^2} \,\mathrm{sh}\, mb, & 0, & 0, & -m'^2 \\ 0, & 0, & a \,\mathrm{sh}\, m'a, & \mathrm{ch}\, m'a \end{vmatrix}. \qquad \text{..........................(112)}$$

The compiled result of (108) is shown in Fig. 9.

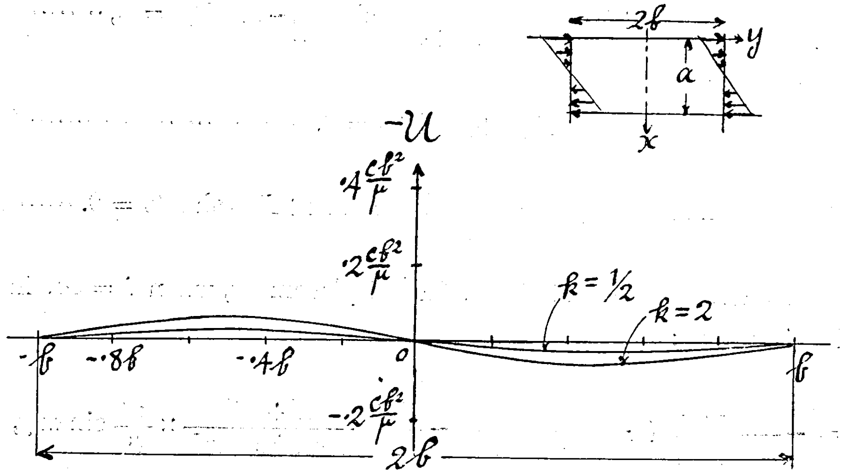

Fig. 9.

7. Now, we may combine two results in Sections 5 and 6. We can then obtain the problem of uniformly increasing compression or tension acting at the vertical wall of the block. The solutions thus obtained should satisfy the boundary conditions of the forms:

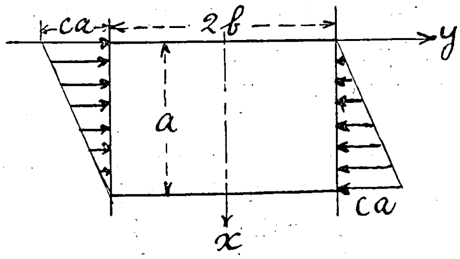

Fig. 10.

$$y=\pm b,\qquad u=0,\cdots\cdots\cdots\cdots\cdots(113)$$

$$\left.\begin{array}{ll} y=+b, & Y_y=cx, \\ y=-b, & Y_y=c(a-x), \end{array}\right\}\cdots\cdots\cdots\cdots(114)$$

$$y=\pm b,\qquad X_y \doteqdot f\frac{\lambda}{\lambda+2\mu}\rho g x,\cdots\cdots\cdots\cdots(115)$$

$$x=0,\qquad X_x=X_y=0,\cdots\cdots\cdots\cdots(116),(117)$$

$$x=a,\qquad X_x=0.\cdots\cdots\cdots\cdots(118)$$

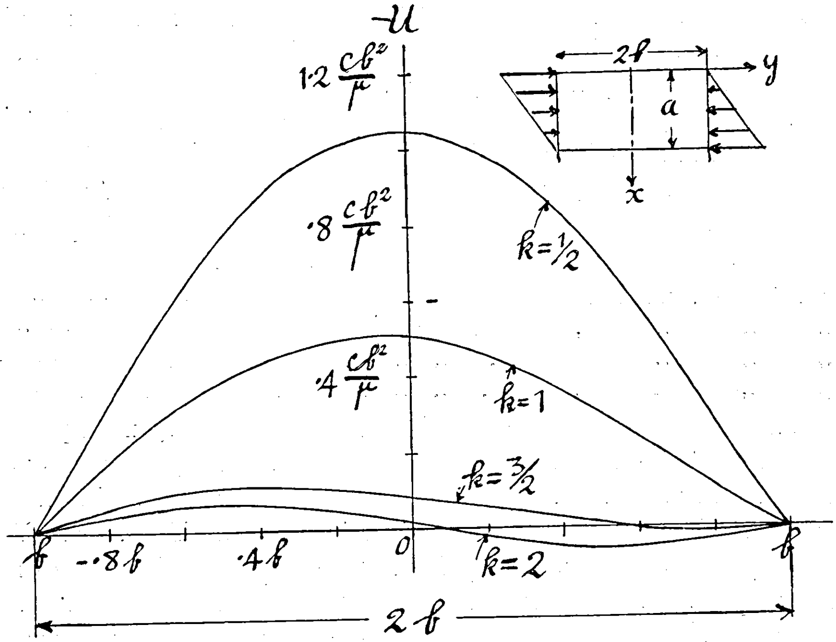

Fig. 11.

The result is plotted in Fig. 11. The general tendency which can be known from Fig. 7, 9, 11 may have some importance on the geophysical as well as on the geological problems. As already cited, the faulting of the surface of the earth crust seems to be accumulated only in the neighbourhood of the surface. The phenomena of the quick faulting, which occurs at the earthquake movement, are nearly the surface action of the earthcrust. Even the cases of the Great Nôbi Earthquake or the Great Kwantô Earthquake, where large areas of the earth surface are damaged, can be thought to be the effect of the acting forces accumulated in the vicinity of the surface of a relatively vast extent. We have no intention to think that the forces cannot act in the deep portion of the earth. The forces cannot give their influence on the sliding of the crust at such a depth. The ordinary depth of the focus of the seismic origin may be rather shallow in the actual sense. Even though the co-existence of the deepness of the focus of the earthquake and the shallowness of the fault is permissible to a certain extent, the occurrence of the deep seated earthquake is yet wonderful. The mechanism of such a fact will be determined in the course of the development of the seismological science. Again, the evidence of great faultings which are observed in the geological manner may have been caused at the time when the part of the crust was resident at a relatively shallow depth of the ground.

8. We may ascertain that our present conclusion is valid in the actual deformation of the land surface in the Tango District. Very fortunately we have a few days before acquired the result of revision of the levellings at that district carried out by the Land Survey Department. Four series of the compared results are given below:

Level change occurred between 1888 and April-May of 1927 ··························0–I
,, ,, ,, ,, April-May of 1927 and June-July of 1927 ·········I–II
,, ,, ,, ,, ,, ,, ,, ,, and March-April of 1928······I–III
,, ,, ,, ,, ,, ,, ,, ,, and August-Sept. of 1929 ···I–IV

Taking several portions of the district which have the most plausible appearance of the mosaic block as already shown by some student and connecting the extreme points of each block by a straight line, we have obtained the diagrams shown in Figs. 12–19. The marks of the type ● represent these extreme points. The horizontal lines are drawn at the respective heights which represent the level changes of all the bench marks distributed between the extreme bench marks: the intersection points of the respective horizontal lines with the straight line connecting the extreme bench marks are shown successively by ▮, ○, △, □, ×. In these ▮ corresponds to the bench mark which is nearest to the left extreme mark.

If the blocks move vertically like a mosaic work as shown by the student already cited, the horizontal distribution of ▮, ○, △, □, × should be equal in proportion to that of the corresponding bench marks in the actual map projected on a straight line connecting two points of the extreme bench marks in the same map. Even a fractional shift, however, of these points between the extreme points is to tell us the fact that the displacement of the land surface is curved. We will easily see that this condition is not quite satified almost in all cases.

Moreover, the incoincidence of each series of the marks such as ▮, ○, △, □, × with respect to the vertical line in Fig. 12–19 shows that the successive level changes are not only of a mosaic type, but also of much irregular procedure. Again, we will see in the figures that some points reside outside the extreme points. These facts oblige us to know that the so-called blocks cannot behave as a mosaic work.

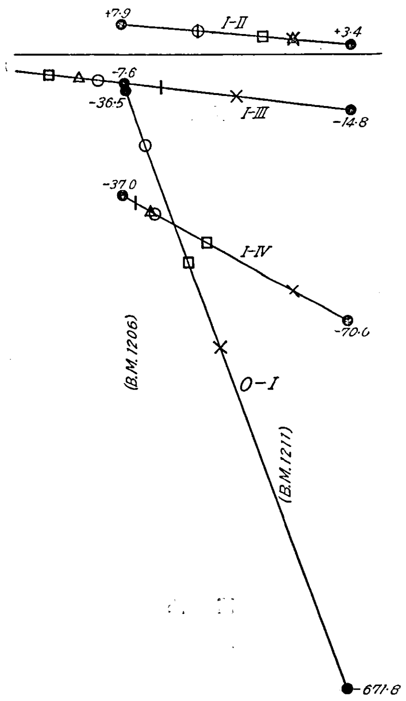

Fig. 12.

The other levelling portions of Tango-district which have not been touched in the present analysis from the start are curved in more complex forms so that we have not given any addition to such cases.

Except a few faulting lines caused by the earthquake, even the diagramatic steps of semi-discontinuous portions are not so large compared with and rather comparable to the general displacement. The faults which are caused by the earthquake seem, as already cited, to be of a superficial nature and not to be the traces of the surfaces of the contact of the blocks in the form of a mosaic work. We have the idea that the deformations of the crust, besides the faulting, are all of the superficial nature; the mathematical mechanics of this will appear more fully in our future paper.

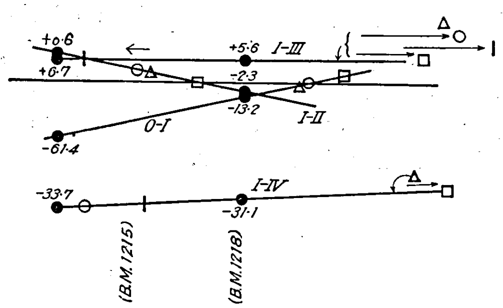

Fig. 13.

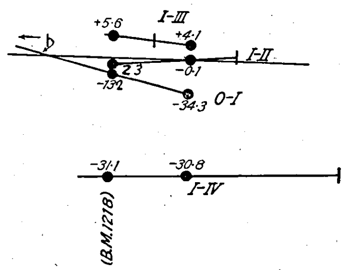

Fig. 14.

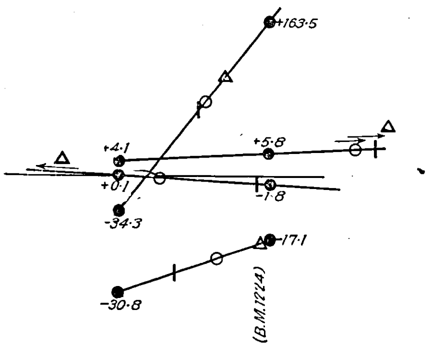

Fig. 15.

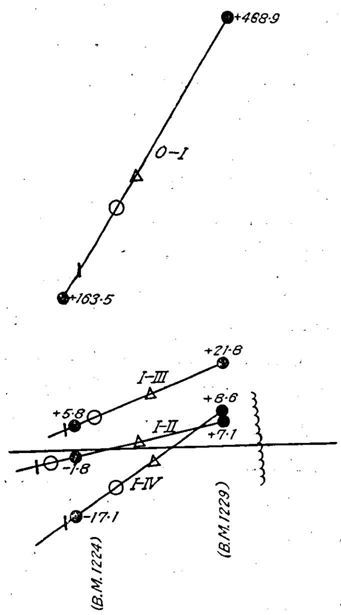

Fig. 16.

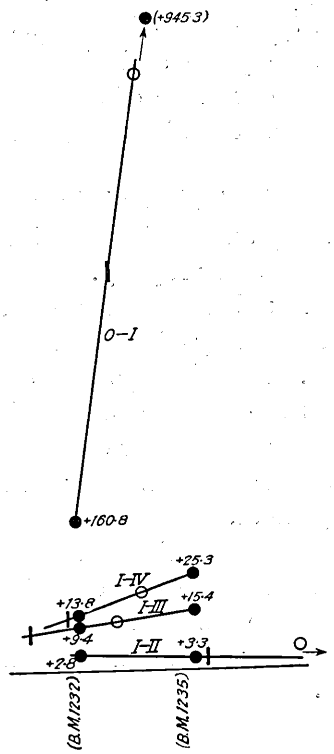

Fig. 17.

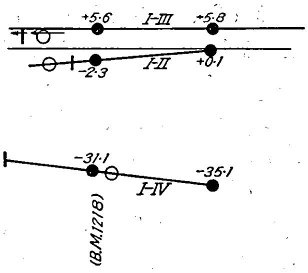

Fig. 18.

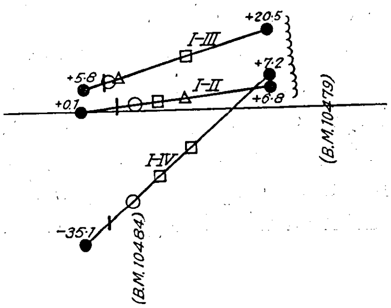

Fig. 19.

Résumé.

9. We may now complete this paper with some notice on the general results which have been obtained from this mathematical analysis.

1. The bodily movements of very rigid blocks of the earth crust are impossible. The idea of a mosaic work is not applicable to the actual earth crust where the elasticities or plasticities are very like to or less than that of the actual rocks and the solid friction at the surface of the separation of blocks (if pre-existing) is very large in gravitating masses.

2. The deformation of the surface of the earth crust must be curved in general. In particular cases of the boundary conditons or of the heterogeneous distribution of the rocks of various rigidities, the surface displacement may have certain discontinuous characters combined with the general curved flexural forms.

3. At the action of any extraordinary pressure unbalanced isostatically at the bottom of the so-called blocks, the sliding of the surface of the contact of these blocks can hardly take place and the surface displacement is very small and curved. The curvature of this surface movement partially conforms with the mode of the distribution of the unbalanced pressure at the bottom.

4. As the ratio of the depth to the breadth of a block is increased, the surface displacement diminishes anormally and the impossibility of the sliding of the surface of the contact increases very rapidly.

5. When the block is subjected to the unbalanced force acting horizontally at the surface of the contact, the sliding of the fault in the very vicinity of the surface becomes possible. The deep portion of the ground is not yet capable of sliding. Thus the evidence of the faults which are observed at the earthquake motion or in the slow deformation of the crust are of a superficial nature at the ground. The surface displacement in this case, too, is curved in general.

6. In the analysis of the data of the actual deformation, at the Tango District, we can easily find that the general deformation of the surface is curved in general and in very special cases of the observed results, some discontinuous displacements, which are probably of the surface nature, are combined with the general deformations.

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2. 地殼の群塊運動の可能度に就いて

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地殼の變形が寄木細工の様であり、各の地塊は殆んど完全な剛體として動く様なものであるかどうかは多くの人々の興味を惹く所であらう。著者の一人は大正十三年頃別の意味に於ける地塊を力學的に考へた事もあるが、其後多くの人々が地塊といふ事をしきりに提案し、遂には本邦の如き地體の唯一の構造とまでも想像されて居るかの様に見えたので、著者等は其重大さを感じ、こゝに改めて斯る事實が許せる事かどうかを吟味する事にした。

研究の方法は二次元の問題を導入してエヤリー函數を應用し、普通の彈性率を有する固體が重力に支配されて固體の表面に固體面摩擦のある事を注目して計算を施した。

研究の主要な結果を列擧すれば,

1. 普通地塊と考へて居る様な固體では寄木細工の様な運動は不可能である。即ち斯る大きなものでは彈性は通常のものであるのに重力の爲固體面の摩擦力が非常な影響を與へる爲めである。

2. たとひ地塊の様なものを假定しても地表面の變形は一般に曲面狀である事がわかる。極めて特別な場合、即ち剛性率の甚しく異なる岩石塊が混入する時は其近所だけに稍不連續に見える歪みがあり、又地表等の境界條件が著しく特定の時は、表面の附近に限り上述の曲面狀變形に混合して不連續變形の存在する事がある。
3. 地塊の下部に不平衡壓力が働く時は地塊面の斷層の辷り等は殆ど不可能の事柄となる。又表面の一般變形は不平衡壓力の分布と一致せる傾向を持つには持つが其變位は極めて小さい。
4. 地塊の深さが其幅に比較して大きくなるに從ひ表面の變位は急激に少くなる。又斷層の辷りは一層困難となる。
5. 地塊が水平の不平衡力を受ける時は斷層の辷りは比較的容易になる。しかしこの辷りは表面に極く近い部分のみであり、少し深くなればやはり辷り得なくなる。即ち一般に斷層は極めて表面的のものである事が判明する。地表面の變形はこの場合でも勿論曲面狀のものを主として含む。
6. 實際の測量の結果に就て著者等が行つた分析に照しても極めて特別な場合を除いては、一般に上述の形成にあり、殊に表面の變形が著しく曲面狀をなす事が認められる。