23. *Propagation of Love-waves on a Spherical Surface and Allied Problems.*

By **Katsutada SEZAWA**,

Earthquake Research Institute.

1. Quite recently Dr. H. Nakano[1] gave a problem of Love-waves in cylindrical coordinates and pointed out that there is a clear distinction between cylindrical Rayleigh-waves and cylindrical Love-waves. The problem has also attracted my attention since the occasion[2] of studying the propagation of Rayleigh-waves having azimuthal distribution, because of the fact that the second constituent of the general expressions of the displacement obtained at that time, namely (u_2, v_2, w_2), both in two-dimensions and on a spherical surface had no any significance on Rayleigh-waves but the forms of u_2, v_2, w_2 resembled those of the distortional waves.

As highly appreciated by European seismologists,[3] Love-waves are much important, indeed, on the problem of the surface waves which are propagated to very long distance. From this consideration I have taken the case of Love-waves on a spherical surface and for the point very near the epicentre I have taken into account of the depth of the origin. The azimuthal distribution is not, of course, to be taken uniform.

2. We shall take the origin of the disturbance to be at the portion of the axis, $\theta=0$, of the spherical coordinates, and suppose that the waves are transmitted in the direction of the colatitude. The radius of the free surface is to be a, while the thickness of the surface layer is to be taken $a-b$. Let u_2, v_2, w_2 be radial, colatitudinal and azimuthal components of the dis-

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1) H. NAKANO, "Love-wave in Cylindrical Coordinates," *Geophys. Mag.*, 8 (1929).

Added Nov. 1929.—I regret that I have been informed of Dr. NAKANO's death on Oct. 3, 1929. He is one of the mathematical seismologists to whom I have paid many respects. I have never expected that this paper would become my condolence on his death.

2) K. SEZAWA, "Propagation of Rayleigh-waves having a Certain Azimuthal Distribution of Displacements," *Proc. Imp. Acad.*, 4 (1928), No. 6, 267; and also "Further Studies on Rayleigh-waves having Some Azimuthal Distribution," *Bull. Earthq. Res. Inst.*, 6 (1929), 1.

3) B. GUTENBERG, H. JEFFREYS and similar eminent seismologists.

placement which are of no use on the problem of Rayleigh-waves, then we have from my preceding paper[4] that

$$u_2 = 0,$$

$$v_2 = \frac{m}{n(n+1)} C_{mn} \frac{J_{n+\frac{1}{2}}(kr)}{\sqrt{r}} \left\{ \frac{P_n^m(\cos\theta)}{\sin\theta} + \alpha_n^m \frac{Q_n^m(\cos\theta)}{\sin\theta} \right\} \begin{matrix} \cos \\ \sin \end{matrix} m\phi \, e^{i\sigma t},$$

$$w_2 = -\frac{1}{n(n+1)} C_{mn} \frac{J_{n+\frac{1}{2}}(kr)}{\sqrt{r}} \left\{ \frac{dP_n^m(\cos\theta)}{d\theta} + \alpha_n^m \frac{dQ_n^m(\cos\theta)}{d\theta} \right\} \begin{matrix} \sin \\ -\cos \end{matrix} m\phi \, e^{i\sigma t},$$

$$\dots\dots\dots\dots\dots (1)$$

where $k^2 = \rho\sigma^2/\mu$, in which ρ is the density and μ is the modulus of the rigidity. α_n^m is a number, whose value is constant for a given n and m and which is to be so adjusted that $P_n^m(\cos\theta)$ and $Q_n^m(\cos\theta)$ may have enveloping curves in common, excepting near the antipode and the origin. We take that the above expression (1) applies to the inner core of the sphere. For the outer layer we may write the expressions of the displacement without any difficulty such that

$$u_2' = 0,$$

$$v_2' = \frac{m}{n(n+1)} \left\{ A_{mn} \frac{J_{n+\frac{1}{2}}(k'r)}{\sqrt{r}} + B_{mn} \frac{Y_{n+\frac{1}{2}}(k'r)}{\sqrt{r}} \right\} \left\{ \frac{P_n^m(\cos\theta)}{\sin\theta} \right.$$
$$\left. + \alpha_n^m \frac{Q_n^m(\cos\theta)}{\sin\theta} \right\} \begin{matrix} \cos \\ \sin \end{matrix} m\phi \, e^{i\sigma t},$$

$$w_2' = -\frac{1}{n(n+1)} \left\{ A_{mn} \frac{J_{n+\frac{1}{2}}(k'r)}{\sqrt{r}} + B_{mn} \frac{Y_{n+\frac{1}{2}}(k'r)}{\sqrt{r}} \right\} \left\{ \frac{dP_n^m(\cos\theta)}{d\theta} \right.$$
$$\left. + \alpha_n^m \frac{dQ_n^m(\cos\theta)}{d\theta} \right\} \begin{matrix} \sin \\ -\cos \end{matrix} m\phi \, e^{i\sigma t},$$

$$\dots\dots\dots\dots\dots\dots (2)$$

where $k'^2 = \rho'\sigma^2/\mu'$, in which ρ', μ' are the density and the rigidity of the outer medium. When $\mu'/\rho' < \mu/\rho$, we find at the vicinity of $b < r < a$,

$$\frac{n+\frac{1}{2}}{kr} (= \cosh\tau) > 1, \qquad \frac{n+\frac{1}{2}}{k'r} (= \cos\tau') < 1. \dots\dots\dots\dots (3)$$

In applying the formulae (1) and (2), we should exclude the epicentral and the antipodal regions from the consideration because of the divergence of Q_n^m-functions at such regions.

As the boundary conditions, we should put

4) K. SEZAWA, *Bull. Earthq. Res. Inst.*, 6 (1929), 14.

$$\frac{\partial}{\partial r}\frac{v'}{r}=\frac{\partial}{\partial r}\frac{w'}{r}=0, \qquad [r=a] \dots\dots\dots\dots\dots (4)$$

$$\left.\begin{array}{l} \mu'\dfrac{\partial}{\partial r}\dfrac{v'}{r}=\mu\dfrac{\partial}{\partial r}\dfrac{v}{r}, \\[2mm] \mu'\dfrac{\partial}{\partial r}\dfrac{w'}{r}=\mu\dfrac{\partial}{\partial r}\dfrac{w}{r}, \\[2mm] v'=v, \\[2mm] w'=w. \end{array}\right\} \quad [r=b] \dots\dots\dots\dots\dots (5)$$

From (4) and (5), we obtain the equation to determine the velocity of propagation as in the following form:

$$\begin{vmatrix} \dfrac{\partial}{\partial a}\dfrac{J_{n+\frac{1}{2}}(k'a)}{a^{\frac{3}{2}}}, & \dfrac{\partial}{\partial a}\dfrac{Y_{n+\frac{1}{2}}(k'a)}{a^{\frac{3}{2}}}, & 0 \\[4mm] \dfrac{\mu'}{\mu}\dfrac{\partial}{\partial b}\dfrac{J_{n+\frac{1}{2}}(k'b)}{b^{\frac{3}{2}}}, & \dfrac{\mu'}{\mu}\dfrac{\partial}{\partial b}\dfrac{Y_{n+\frac{1}{2}}(k'b)}{b^{\frac{3}{2}}}, & \dfrac{\partial}{\partial b}\dfrac{J_{n+\frac{1}{2}}(kb)}{b^{\frac{3}{2}}} \\[4mm] J_{n+\frac{1}{2}}(k'b), & Y_{n+\frac{1}{2}}(k'b), & J_{n+\frac{1}{2}}(kb) \end{vmatrix} =0. \dots\dots (6)$$

To solve this equation, we take the asymptotic expansion due to P. Debye, namely

$$J_p(x)=-\frac{i}{2\pi}\,e^{x\,(\tau\,\mathrm{ch}\,\tau-\mathrm{sh}\,\tau)}\sum_{n=0}^{n=m}(-1)^n\,B_n\,(\tau)\,\frac{\Gamma\!\left(n+\dfrac{1}{2}\right)}{\left(\dfrac{x}{2}\,\mathrm{sh}\,\tau\right)^{n+\frac{1}{2}}},$$

$$(m:\text{finite limiting value.})$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

where $\mathrm{ch}\,\tau > 1$ and

$$B_0(\tau)=1, \quad B_1(\tau)=\frac{1}{8}-\frac{5}{24}\,\mathrm{cth}^2\,\tau, \quad B_2(\tau)=\frac{3}{128}-\frac{7}{576}\,\mathrm{cth}^2\,\tau+\frac{385}{3456}\,\mathrm{cth}^4\,\tau,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots ,$$

$$\dots\dots\dots\dots\dots\dots (8)$$

while

$$J_p(x)=\frac{1}{\pi}\sum_{n=0}^{n=m}A_n\,(\tau)\,\frac{\Gamma\!\left(n+\dfrac{1}{2}\right)}{\left(\dfrac{x}{2}\,\sin\tau\right)^{n+\frac{1}{2}}}\cos\left\{x\,(\sin\tau-\tau\cos\tau)-(2n+1)\,\frac{\pi}{4}\right\},$$

$$Y_p(x) = \frac{1}{\pi} \sum_{n=0}^{n=m} A_n(\tau) \frac{\Gamma\left(n + \frac{1}{2}\right)}{\left(\frac{x}{2}\sin\tau\right)^{n+\frac{1}{2}}} \sin\left\{x(\sin\tau - \tau\cos\tau) - (2n+1)\frac{\pi}{4}\right\}.$$

$$\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

where $\cos\tau < 1$ and

$$A_0 = 1, \quad A_1 = \frac{1}{8} + \frac{4}{24}\cot^2\tau, \quad A_2 = \frac{3}{128} + \frac{7}{576}\cot^2\tau + \frac{385}{3456}\cot^4\tau, \left.\right\} \dots (10)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Taking each first term of these asymptotic expansions, we get

$$\frac{\mu' k'(\sin\tau' - \tau'\cos\tau')}{\mu k(\tau\,\mathrm{ch}\,\tau - \mathrm{sh}\,\tau)} = \cot k'\{a(\sin\tau_1' - \tau_1'\cos\tau_1') - b(\sin\tau' - \tau'\cos\tau')\},$$

$$\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

where

$$\cosh\tau = \frac{n + \frac{1}{2}}{kb}, \qquad \cos\tau' = \frac{n + \frac{1}{2}}{k'b}, \qquad \cos\tau_1' = \frac{n + \frac{1}{2}}{k'a}, \dots (12)$$

in which $2\pi/k$ is the wave length of Love-waves.

When the radius of the sphere is very large compared with the thickness of the surface layer, we may put

$$\sin\tau_1' \longrightarrow \sin\tau', \dots\dots\dots\dots\dots\dots (13)$$

so that we obtain in place of (11)

$$\frac{\mu' k'(\sin\tau' - \tau'\cot\tau')}{\mu k(\tau\,\mathrm{ch}\,\tau - \mathrm{sh}\,\tau)} = \cot\eta k'(\sin\tau' - \tau'\cos\tau')\dots\dots\dots (14)$$

This expression is quite equivalent to Love's formula[5] to determine the velocity of the propagation of the long waves of the plane type, since the exponents in the expressions of displacements (v_2, w_2) and (v_3, w_3) in the present case are

$$rk(\tau\,\mathrm{ch}\,\tau - \mathrm{sh}\,\tau); \quad rk'(\sin\tau' - \tau'\cos\tau') \dots\dots\dots\dots (15)$$

respectively, which may be taken as the exponents of the factor defining the vertical variation of the displacements in the ordinary Love-waves.

The velocity of the propagation, thus, ranges between those of both dis-

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5) Cf. Love, *Some Problems of Geodynamics* (1911), 162.

tortional waves peculiar to the upper and inner medium. The expressions
of the displacements are given (1) and (2).

From the expressions of (1), (2), (14), we may easily arrive at the
conclusion that, besides the ordinary transverse component of the displace-
ment, Love-waves having azimuthal distribution on a spherical surface have
large co-latitudinal component of displacement, and that this component
cannot disappear as the waves proceed towards the equatorial circle. It
appears that even at the equatorial circle such long Love-waves as affected
by the curvature of the earth have the longitudinal and transverse com-
ponents of comparable magnitudes. The vertical component is not existent
as at the plane surface.

3. Though in the preceding section we have studied the propagation
of Love-waves on a spherical surface, the vibrations at the epicentral and
antipodal regions have been excluded. To get the solutions of the problems
at these regions, we should take the case of a plane surface and suppose that
the waves are causd by the disturbance at a certain point in the solid body.
Hence we will treat of a simplest case where Love-waves are excited due to
the oscillations of a point in the interior of the solid body. In this section
we shall study the excitation of Love-waves due to a disturbing origin at the
bottom medium.

We shall take the boundary surface between the upper layer and the
bottom medium to be $z=0$ and also that the positive sense is directed down-
wards. When the axis of each element of a multiplet is vertical, the primary
waves generated from this multiplet can be easily written by

$$2\varpi_z = e^{i\sigma t} \sum_{p=0}^{p=2m} \frac{(-1)^p}{m!} \left[l_p \frac{\partial}{\partial x} + m_p \frac{\partial}{\partial y} \right]^m \frac{\partial^n}{\partial z^n} f(x, y)\, ds_1^m\, ds_2^n, \ldots \ldots (16)$$

in which $f(x, y)$ is the mean strength of the source at the point (x, y), $2m$
is the number of the sides of the equiangular polygon made of the horizontal
elements of the multiplet, l_p, m_p are the direction-cosines of the angular point
p of the equiangular polygon, and n is the degree of the vertical elements.
ds_1 and ds_2 are the radial distance of the equiangular point and the vertical
length of the multiplet. The sign of the strength of each source at the
equiangular point of the polygon changes alternately.

If the source is of a simple-harmonic type, we may write

$$f(x, y) = \frac{e^{-ij\sqrt{r^2 + (z-\xi)^2}}}{\sqrt{r^2 + (z-\xi)^2}}, \ldots \ldots \ldots \ldots \ldots \ldots (17)$$

where ξ is the depth of the origin below the surface $z=0$ and $j^2 = \rho \sigma^2 / \mu$.
Now by means of the formula

$$\frac{e^{-ij\sqrt{r^2+(z-\xi)^2}}}{\sqrt{r^2+(z-\xi)^2}}=\int_0^\infty \frac{e^{-\beta(\xi-z)}}{\beta} J_0(kr)\,kdk, \qquad \begin{bmatrix} \beta^2=k^2-j^2, \\ j^2=\dfrac{\rho\sigma^2}{\sigma}, \end{bmatrix} \;\ldots\;(18)$$
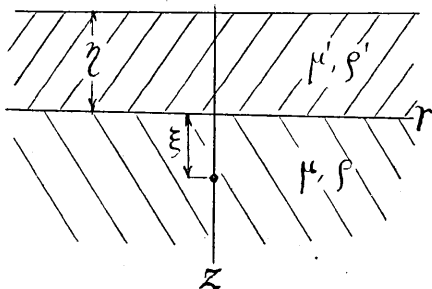
and omitting the constant factor $e^{i\sigma t}ds_1{}^m ds_2{}^m$, we get $[0<z<\xi]$

$$2\varpi_z=\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega\int_0^\infty e^{-\beta(\xi-z)} J_m(kr)\,\beta^{n-1} k^{m+1}\,dk, \quad [m>2]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

with the exceptions that

$$2\varpi_z=\int_0^\infty e^{-\beta(\xi-z)} J_0(kr)\,\beta^{n-1} k\,dk, \qquad\qquad [m=0]\ldots\ldots(20)$$

$$2\varpi_z=-2\cos\omega\int_0^\infty e^{-\beta(\xi-z)} J_0(kr)\,\beta^{n-1} k^2\,dk. \quad [m=1]\ldots\ldots(21)$$

These rotations ϖ_z are always accompanied by the corresponding ϖ_r and ϖ_ω, as the displacements of the material point necessitate some relations among these rotations. · The relations are as in the forms:

$$2\varpi_r=\frac{1}{r}\frac{\partial w}{\partial z}-\frac{\partial v}{\partial z}, \quad 2\varpi_\omega=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}, \quad 2\varpi_z=\frac{1}{r}\left(\frac{\partial(rv)}{\partial r}-\frac{\partial u}{\partial\theta}\right), \;\ldots(22)$$

where u, v, w are the radial, azimuthal, vertical components of displacement. We can, thus, write down the expressions:[6]

$$2\varpi_r=\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\times\int_0^\infty e^{-\beta(\xi-z)}\frac{\partial J_m(kr)}{\partial r}\beta^n k^{m-1}\,dk, \qquad [m>2]\ldots(23)$$

$$2\varpi_r=\int_{0.}^\infty e^{-\beta(\xi-z)}\frac{\partial J_0(kr)}{\partial r}\beta^n k^{-1}\,dk, \qquad\qquad [m=0]\ldots(24)$$

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6) Here use has been made of the result of my calculation on Rayleigh-waves, *Bull. Earthq. Res. Inst.*, **6** (1929), 4–5.

$$2\varpi_r = -2\cos\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{\partial J_0(kr)}{\partial r}\beta^n\,dk, \qquad [m=1]\ldots(25)$$

and

$$2\varpi_\omega = -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)m\sin\omega$$
$$\times \int_0^\infty e^{-\beta(\xi-z)} \frac{J_m(kr)}{r}\beta^n k^{m-1}\,dk, \quad [m>2]\ldots(26)$$

$$2\varpi_\omega = 0, \qquad\qquad\qquad\qquad\qquad\qquad [m=0]\ldots(27)$$

$$2\varpi_\omega = 2\sin\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{J_1(kr)}{r}\beta^n k\,dk. \qquad [m=1]\ldots(28)$$

The expressions of displacement corresponding to these ϖ_z, ϖ_r, ϖ_ω can be written by

$$u = -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)m\sin m\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{J_m(kr)}{r}\beta^{n-1} k^{m-1}\,dk,$$

$$v = -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{\partial J_m(kr)}{\partial r}\beta^{n-1} k^{m-1}\,dk, \quad\left.\right\}[m>2]$$

$$w = 0,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

$$u = 0,$$
$$v = -\int_0^\infty e^{-\beta(\xi-z)} \frac{\partial J_0(kr)}{\partial r}\beta^{n-1} k^{-1}\,dk, \quad\left.\right\}[m=0]\ldots\ldots\ldots(30)$$
$$w = 0,$$

$$u = 2\sin\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{J_1(kr)}{r}\beta^{n-1}\,dk,$$
$$v = 2\cos\omega \int_0^\infty e^{-\beta(\xi-z)} \frac{\partial J_1(kr)}{\partial r}\beta^{n-1}\,dk, \quad\left.\right\}[m=1]\ldots\ldots\ldots(31)$$
$$w = 0.$$

The reflected waves at the surface $z=0$ can be denoted by considering the image point ($r=0$, $z=-\xi$) in the forms:

$$2\varpi_z' = \frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega$$
$$\times \int_0^\infty A_m\, e^{-\beta(\xi+z)} J_m(kr)\beta^{n-1} k^{m+1}\,dk,$$

$$2\varpi_r' = -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega$$
$$\times \int_0^\infty A_m\, e^{-\beta(\xi+z)} \frac{\partial J_m(kr)}{\partial r}\beta^n k^{m-1}\,dk,$$

$$2_{\varpi_\omega}' = \frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)m\sin m\omega$$

$$\times \int_0^\infty A_m\, e^{-\beta(\xi+z)}\frac{J_m(kr)}{r}\beta^n\, k^{m-1}\, dk,$$

$$w' = -\frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)m\sin m\omega$$

$$\times \int_0^\infty A_m\, e^{-\beta(\xi+z)}\frac{J_m(kr)}{r}\beta^{n-1}\, k^{m-1}\, dk,$$

$$v' = -\frac{2(-n)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\times \int_0^\infty A_m\, e^{-\beta(\xi+z)}\frac{\partial J_m(kr)}{\partial r}\beta^{n-1}\, k^{m-1}\, dk,$$

$$w' = 0,$$

$$\left.\begin{array}{c}\end{array}\right\}[m>2]\ldots(32)$$

where A_m is an unknown factor to be determined presently. The special cases of $m=0$ and $m=1$ can be written down similarly at once.

In the stratum of the upper layer the waves are reflected multiply and in effect they may be imagined to be equivalent to the stationary surface waves, the expressions of them being written by

$$2_{\varpi_z}'' = \frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\times \int_0^\infty (B_m\cos\beta' z + C_m\sin\beta' z)\, e^{-\beta\xi}J_m(kr)\beta^{n-1}\, k^{m+1}dk,$$

$$2_{\varpi_r}'' = -\frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\times \int_0^\infty (B_m\sin\beta' z - C_m\cos\beta' z)\, e^{-\beta\xi}\frac{\partial J_m(kr)}{\partial r}\beta^n\, k^{m-1}\, dk,$$

$$2_{\varpi_\omega}'' = \frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)m\sin m\omega$$

$$\times \int_0^\infty (B_m\sin\beta' z - C_m\cos\beta' z)\, e^{-\beta\xi}\frac{J_m(kr)}{r}\beta^n\, k^{m-1}\, dk,$$

$$w'' = -\frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)m\sin m\omega$$

$$\times \int_0^\infty (B_m\cos\beta' z + C_m\sin\beta' z)\, e^{-\beta\xi}\frac{J_m(kr)}{r}\beta^{n-1}\, k^{m-1}\, dk,$$

$$v'' = -\frac{2(-1)^m}{m!}\left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\left.\begin{array}{c}\end{array}\right\}[m>2]\ (33)$$

$$\times \int_0^\infty (B_m \cos \beta' z + C_m \sin \beta' z)\, e^{-\beta \xi} \frac{\partial J_m(kr)}{\partial r} \beta^{n-1} k^{m-1}\, dk, \Bigg]$$

$$w'' = 0.$$

The special cases of $m=0$ and $m=1$ can be obtained in a similar manner. In the equations (33), it should be remembered that

$$\beta'^2 = j'^2 - k^2, \qquad j'^2 = \frac{\rho' \sigma^2}{\mu'}, \quad \dots\dots\dots\dots\dots (34)$$

where μ', ρ' are the rigidity and the density of the upper layer.

Now we have the boundary conditions:

$$\left.\begin{aligned}
u + w' &= u'', \\
v + v' &= v'', \\
\mu \frac{\partial}{\partial z}(v + v') + \mu' \frac{\partial}{\partial z} v'', \\
\mu \frac{\partial}{\partial z}(u + w') &= \mu' \frac{\partial}{\partial z} w'',
\end{aligned}\right\} [z=0] \quad \dots\dots\dots\dots (35)$$

$$\frac{\partial v''}{\partial z} = 0, \quad \frac{\partial w''}{\partial z} = 0, \qquad [z=-\eta] \dots\dots\dots\dots (36)$$

where η is the thickness of the layer.

Substituting from (29), (32), (33) in (35) and (36), we get

$$\left.\begin{aligned}
(1 + A_m) &= B_m, \\
\mu \beta (1 - A_m) &= \mu' \beta' C_m, \\
B_m \cos \beta' \eta + C_m \sin \beta' \eta &= 0,
\end{aligned}\right\} \dots\dots\dots\dots (37)$$

from which we find

$$\left.\begin{aligned}
A_m &= \frac{\dfrac{\mu \beta}{\mu' \beta'} \tan \beta' \eta + 1}{\dfrac{\mu \beta}{\mu' \beta'} \tan \beta' \eta - 1}, \\[2em]
B_m &= \frac{2 \dfrac{\mu \beta}{\mu' \beta'} \tan \beta' \eta}{\dfrac{\mu \beta}{\mu' \beta'} \tan \beta' \eta - 1}, \\[2em]
C_m &= \frac{-2 \dfrac{\mu \beta}{\mu' \beta'}}{\dfrac{\mu \beta}{\mu' \beta'} \tan \beta' \eta - 1}.
\end{aligned}\right\} \dots\dots\dots\dots (38)$$

The reflected waves in the lower medium is therefore given by

$$
\left.
\begin{aligned}
u' &= -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)m\sin m\omega \\
&\quad\times\int_0^\infty \frac{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta+1\right)}{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta-1\right)}e^{-\beta(\xi+z)}\frac{J_m(kr)}{r}\beta^{n-1}k^{m-1}\,dk, \\
v' &= -\frac{2(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega \\
&\quad\times\int_0^\infty \frac{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta+1\right)}{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta-1\right)}e^{-\beta(\xi+z)}\frac{\partial J_m(kr)}{\partial r}\beta^{n-1}k^{m-1}\,dk, \\
w' &= 0,
\end{aligned}
\right\}[m>2]\ldots(39)
$$

$$
\left.
\begin{aligned}
w' &= 0, \\
v' &= -\int_0^\infty \frac{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta+1\right)}{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta-1\right)}e^{-\beta(\xi+z)}\frac{\partial J_0(kr)}{\partial r}\beta^{n-1}k^{-1}\,dk, \\
w' &= 0,
\end{aligned}
\right\}[m=0]\ldots(40)
$$

$$
\left.
\begin{aligned}
u' &= 2\sin\omega\int_0^\infty \frac{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta+1\right)}{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta-1\right)}e^{-\beta(\xi+1)}\frac{J_1(kr)}{r}\beta^{n-1}\,dk, \\
v' &= 2\cos\omega\int_0^\infty \frac{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta+1\right)}{\left(\dfrac{\mu\beta}{\mu'\beta'}\tan\beta'\eta-1\right)}e^{-\beta(\xi+z)}\frac{\partial J_1(kr)}{\partial r}\beta^{n-1}\,dk, \\
w' &= 0.
\end{aligned}
\right\}[m=1]\ldots(41)
$$

Again, the displacement at the surface layer can be expressed in the forms:

$$
\begin{aligned}
u'' &= -\frac{4(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)m\sin m\omega \\
&\quad\times\int_0^\infty \frac{\sin\beta'(\eta-z)}{\left(\sin\beta'\eta-\dfrac{\mu\beta}{\mu'\beta'}\cos\beta'\eta\right)}e^{-\beta\xi}\frac{J_m(kr)}{r}\beta^{n-1}k^{m-1}\,dk,
\end{aligned}
$$

$$v'' = -\frac{4(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega$$

$$\left.+\int_0^\infty \frac{\sin\beta'(\eta-z)}{\left(\sin\beta'\eta-\frac{\mu\beta}{\mu'\beta'}\cos\beta'\eta\right)}\,e^{-\beta\xi}\frac{\partial J_m(kr)}{\partial r}\beta^{n-1}k^{m-1}dk,\right\}[m>2]\ .\ .\ (42)$$

$$w''=0,$$

$$w''=0,$$

$$\left.v''=-2\int_0^\infty \frac{\sin\beta'(\eta-z)}{\left(\sin\beta'\eta-\frac{\mu\beta}{\mu'\beta'}\cos\beta'\eta\right)}\,e^{-\beta\xi}\frac{J_0(kr)}{r}\beta^{n-1}k^{-1}dk,\right\}[m=0]\ .\ .\ (43)$$

$$w''=0,$$

$$u''=4\sin\omega\int_0^\infty \frac{\sin\beta'(\eta-z)}{\left(\sin\beta'\eta-\frac{\nu\beta}{\mu'\beta'}\cos\beta'\eta\right)}\,e^{-\beta\xi}\frac{\partial J_1(kr)}{\partial r}\beta^{n-1}dk,$$

$$\left.v''=4\cos\omega\int_0^\infty \frac{\sin\beta'(\eta-z)}{\left(\sin\beta'\eta-\frac{\mu\beta}{\mu'\beta'}\cos\beta'\eta\right)}\,e^{-\beta\xi}\frac{J_1(kr)}{r}\beta^{n-1}dk,\right\}[m=1]\ .\ .\ (44)$$

$$w''=0.$$

To evaluate the integrals in the above expressions, we should apply the formula:

$$J_m(kr)=\frac{2}{\pi}\int_0^\infty \sin\left(kr\,\mathrm{ch}\,f-\frac{m\pi}{2}\right)\cos mf\,df.\ \ .\ \ldots\ldots\ldots(45)$$

The final results of the evaluation of the surface displacement are written by

$$u''=\frac{8\pi(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)m\sin m\omega\,\frac{\sin\beta_1'\eta}{F'(\kappa)}\,e^{-\beta_1\xi}\beta_1^{n-1}\kappa^{m-1}\frac{Y_m(\kappa r)}{r}$$

$$+\text{asymptotically vanishing terms,}$$

$$v''=\frac{8\pi(-1)^m}{m!}\left(1-\cos^2\frac{\pi}{m}\right)\cos m\omega\frac{\sin\beta_1'\eta}{F'(\kappa)}\,e^{-\beta_1\xi}\beta_1^{n-1}\kappa^{m-1}\frac{\partial Y_m(\kappa r)}{\partial r}$$

$$+\text{asymptotically vanishing terms,}$$

$$w''=0.$$

$$[m>2]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(46)$$

In the above expressions κ is the real, positive root of

$$F(k) = \sin \beta' \eta - \frac{\mu \beta}{\mu' \beta'} \cos \beta' \eta = 0, \ldots\ldots\ldots\ldots (47)$$

where $\beta' = \sqrt{j'^2 - k^2}$ and $\beta = \sqrt{k^2 - j^2}$; while β_1 and β_1' are the values of β and β' corresponding to κ.

The special cases of $m = 0$ and 1 can be at once written down by putting m equal to 0 and 1 respectively and afterwards replacing $(1 - \cos^2 \pi/m)$ by $\frac{1}{2}$ and unity in each case.

As in the case of Rayleigh-waves, the propagated Love-waves can be expressed by

$$
\left.
\begin{aligned}
w'' &\approx \frac{4\pi (-1)^m}{m!} \left(1 - \cos^2 \frac{\pi}{m}\right) m \, \sin m\omega \, \frac{\sin \beta_1' \eta}{F''(\kappa)} \\
&\qquad\qquad \times e^{-\beta_1 \xi} \beta_1^{n-1} \kappa^{m-1} \frac{H_m^{(2)}(\kappa r)}{r} e^{i\sigma t}, \\
v'' &\approx \frac{4\pi (-1)^m}{m!} \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega \, \frac{\sin \beta_1' \eta}{F'(\kappa)} \\
&\qquad\qquad \times e^{-\beta_1 \xi} \beta_1^{n-1} \kappa^{m-1} \frac{\partial H_m^{(2)}(\kappa r)}{\partial r} e^{i\sigma t}, \\
w'' &= 0.
\end{aligned}
\right\} [m > 2] \ldots (48)
$$

$$
\left.
\begin{aligned}
w'' &= 0, \\
v'' &\approx 2\pi \frac{\sin \beta_1' \eta}{F''(\kappa)} e^{-\beta_1 \xi} \beta_1^{n-1} \kappa^{-1} \frac{\partial H_0(\kappa r)}{\partial r} e^{i\sigma t}, \\
w'' &= 0.
\end{aligned}
\right\} [m = 0] \ldots (49)
$$

$$
\left.
\begin{aligned}
w'' &\approx - \frac{4\pi \sin \omega \sin \beta_1' \eta}{F'(\kappa)} e^{-\beta_1 \xi} \beta_1^{n-1} \frac{H_1(\kappa r)}{r} e^{i\sigma t}, \\
v'' &\approx - \frac{4\pi \cos \omega \sin \beta_1' \eta}{F'(\kappa)} e^{-\beta_1 \xi} \beta_1^{n-1} \frac{\partial H_1(\kappa r)}{\partial r} e^{i\sigma t}, \\
w'' &= 0.
\end{aligned}
\right\} [m = 1] \ldots (50)
$$

It is important to notice that $F(\kappa) = 0$ is the equation to determine the velocity of Love-waves and hence the excited surface waves are transmitted with such a velocity. As this velocity depends on the wave-length, the dispersion of waves is quite possible. Again, in all cases the vertical component of the displacement does not appear, while the displacement component in the direction of the propagation of the waves is consistent in the neighbourhood of the epicentre. This component, however, quickly disappears as the distance from the epicentre is increased. The fact, that

the surface displacement decreases at the rate of $e^{-\beta_1 \xi}$ as the depth of the origin is increased, is not without significance in the sense that the transverse surface waves have a large amplitude compared with that of Rayleigh-waves whose amplitudes are propotional to $e^{-\alpha_1 \xi}$.

4. We now proceed to the case where the origin of the disturbance is situated at a point in the upper layer.
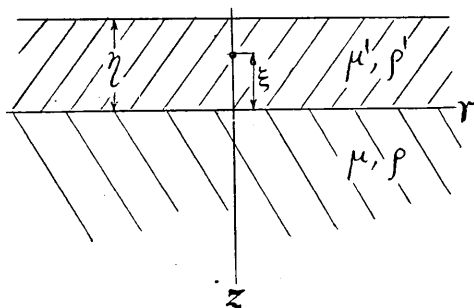
If the source is of a simple harmonic type, we write

$$f(x, y) = \frac{e^{-if'\sqrt{r^2+(z+\xi)^2}}}{\sqrt{r^2+(z+\xi)^2}}, \qquad \ldots\ldots\ldots\ldots\ldots (51)$$

so that the original disturbance is written by

$$
\left.
\begin{aligned}
2\varpi_z &= \frac{2(-1)^{m+n}}{m!} i^n \left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega \\
&\quad \times \int_0^\infty e^{-i\beta'(z+\xi)} J_m(kr)\,\beta'^{n-1}k^{m+1}\,dk, \quad [(\xi+z)>0] \\
&= \frac{2(-1)^{m+n}}{m!} i^n \left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega \\
&\quad \times \int_0^\infty e^{i\beta'(z+\xi)} J_m(kr)\,\beta'^{n-1}k^{m+1}\,dk, \quad [(\xi+z)<0]
\end{aligned}
\right\}[m>2]\ldots(52)
$$

in which $e^{i\sigma t}\,ds_1^m\,ds_2^n$ is omitted as in the preceding case.

The corresponding values of ϖ_r, ϖ_ω, u, v, w are denoted by

$$
\left.
\begin{aligned}
2\varpi_r &= -2(-1)^{m+n} i^{n+1}\left(1 - \cos^2\frac{2\pi}{m}\right)\cos m\omega \\
&\quad \times \int_0^\infty e^{-i\beta'(z+\xi)} \frac{\partial J_m(kr)}{\partial r}\,\beta'^n k^{m-1}\,dk, \quad [(\xi+z)>0] \\
&= 2(-1)^{m+n} i^{n+1}\left(1 - \cos^2\frac{\pi}{m}\right)\cos m\omega \\
&\quad \times \int_0^\infty e^{i\beta'(z+\xi)} \frac{\partial J_m(kr)}{\partial r}\,\beta'^n k^{m-1}\,dk, \quad [(\xi+z)<0]
\end{aligned}
\right\}[m>2]\ldots(53)
$$

$$2\varpi_\omega = 2(-1)^{m+n} i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty e^{-i\beta'(z+\xi)} \frac{J_m(kr)}{r} \beta'^n k^{m-1} dk, \quad [(\xi+z)>0]$$

$$= -2(-1)^{m+n} i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty e^{i\beta'(z+\xi)} \frac{J_m(kr)}{r} \beta'^n k^{m-1} dk, \quad [(\xi+z)<0]$$

$$\left. \right\} [m>2] \ . \ . (54)$$

$$u = -2(-1)^{m+n} i^n \left(1 - \cos^2 \frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty e^{-i\beta'(z+\xi)} \frac{J_m(kr)}{r} \beta'^{n-1} k^{m-1} dk, \quad [(\xi+z)>0]$$

$$= -2(-1)^{m+n} i^n \left(1 - \cos^2 \frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty e^{i\beta'(z+\xi)} \frac{J_m(kr)}{r} \beta'^{n-1} k^{m-1} dk, \quad [(\xi+z)<0]$$

$$\left. \right\} [m>2] \ . \ . (55)$$

$$v = -2(-1)^{m+n} i^n \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty e^{-i\beta'(z+\xi)} \frac{\partial J_m(kr)}{\partial r} \beta'^{n-1} k^{m-1} dk, \quad [(\xi+z)>0]$$

$$= -2(-1)^{m+n} i^n \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty e^{i\beta'(z+\xi)} \frac{\partial J_m(kr)}{\partial r} \beta'^{n-1} k^{m-1} dk, \quad [(\xi+z)<0]$$

$$\left. \right\} [m>2] \ . \ . (56)$$

$$w = 0.$$

The case of $m=0$ and 1 can be obtained by putting m equal to 0 and 1 respectively and afterwards replacing $(1-\cos^2 \pi/m)$ by $\frac{1}{2}$ and unity in each case.

The stationary oscillations of the layer may be expressed in the forms:

$$2\varpi_z' = 2(-1)^m i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty (A_m e^{i\beta'z} + B_m e^{-i\beta'z}) J_m(kr) \beta'^{n-1} k^{m+1} dk,$$

$$2\varpi_r' = 2(-1)^m i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty (A_m e^{i\beta'z} - B_m e^{-i\beta'z}) \frac{\partial J_m(kr)}{\partial r} \beta'^n k^{m-1} dk,$$

$$2\varpi_\omega' = -2(-1)^m\, i^{n+1} \left(1-\cos^2\frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty (A_m\, e^{i\beta' z} - B_m\, e^{-i\beta' z}) \frac{J_m(kr)}{r} \beta'^n\, k^{m-1}\, dk,$$

$$u' = -2(-1)^m\, i^n \left(1-\cos^2\frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty (A_m\, e^{i\beta' z} + B_m\, e^{-i\beta' z}) \frac{J_m(kr)}{r} \beta'^{n-1}\, k^{m-1}\, dk,$$

$$v' = -2(-1)^m\, i^n \left(1-\cos^2\frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty (A_m\, e^{i\beta' z} + B_m\, e^{-i\beta' z}) \frac{\partial J_m(kr)}{\partial r} \beta'^{n-1}\, k^{m-1}\, dk,$$

$$w' = 0,$$

$$\left. \begin{array}{l} [z<0] \\ [m>2] \end{array} \right. \cdots (57)$$

and similar expressions for the case $m=0$ and $m=1$.

The bottom medium should make some oscillations in accordance with the disturbances in the superficial layer. The expressions of the motion of the bottom layer are thus given by

$$2\varpi_z'' = 2(-1)^m\, i^{n+1} \left(1-\cos^2\frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty C_m\, e^{-\beta z} J_m(kr)\, \beta'^{n-1}\, k^{m+1}\, dk,$$

$$2\varpi_r'' = 2(-1)^m\, i^{n+1} \left(1-\cos^2\frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty C_m\, e^{-\beta z} \frac{\partial J_m(kr)}{\partial r} \beta'^n\, k^{m-1}\, dk,$$

$$2\varpi_\omega'' = 2(-1)^m\, i^{n+1} \left(1-\cos^2\frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty C_m\, e^{-\beta z} \frac{J_m(kr)}{r} \beta'^n\, k^{m-1}\, dk,$$

$$u'' = -2(-1)^m\, i^n \left(1-\cos^2\frac{\pi}{m}\right) m \sin m\omega$$

$$\times \int_0^\infty C_m\, e^{-\beta z} \frac{J_m(kr)}{r} \beta'^{n-1}\, k^{m-1}\, dk,$$

$$v'' = -2(-1)^m\, i^n \left(1-\cos^2\frac{\pi}{m}\right) \cos m\omega$$

$$\left. \begin{array}{l} [z<0] \\ [m>2] \end{array} \right. \cdots (58)$$

$$\times \int_0^\infty C_m\, e^{-\beta z}\, \frac{\partial J_m(kr)}{\partial r}\, \beta'^{n-1}\, k^{m-1}\, dk, \qquad \Bigg]$$

$$w''=0,$$

together with similar expressions for the cases $m=0$ and $m=1$.

At the boundary $z=0$, we should have

$$u+u'=u'', \qquad\qquad v+v'=v'',$$

$$\mu'\,\frac{\partial}{\partial z}\,(v+v')=\mu\,\frac{\partial}{\partial z}\,v'', \qquad \mu'\,\frac{\partial}{\partial z}\,(u+u')=\mu\,\frac{\partial}{\partial z}\,u''. \Bigg\} \cdots\cdots (59)$$

At the free surface $z=-\eta$,

$$\frac{\partial}{\partial z}\,(v+v')=0, \qquad \frac{\partial}{\partial z}\,(u+u')=0, \ldots\ldots\ldots\ldots (60)$$

Substituting from (52), (53), (54), (55), (56), (57), (58) in (59), (60), we get

$$\begin{aligned}
&(-1)^n\, e^{-i\beta'\xi}+A_m+B_m=C_m, \\
&\mu'\,\beta'[(-1)^{n+1}\,i\,e^{-i\beta'\xi}+i(A_m-B_m)]=-\mu\beta C_m, \\
&(-1)^n\, e^{i\beta'(\xi-\eta)}+A_m\,e^{-i\beta'\eta}-B_m\,e^{i\beta'\eta}=0,
\end{aligned} \Bigg\} \ldots\ldots\ldots (61)$$

from which we obtain

$$\begin{aligned}
A_m&=-(-1)^n\, \frac{\left\{e^{i\beta'(\xi-\eta)}+e^{-i\beta'(\xi-\eta)}\right\}\left(1-i\dfrac{\mu'\beta'}{\mu\beta}\right)}{\left(1+i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{i\beta'\eta}+\left(1-i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{-i\beta'\eta}}, \\[2ex]
B_m&=(-1)^n\, \frac{\left(1+i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{i\beta'(\xi-\eta)}-\left(1-i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{-i\beta'(\xi+\eta)}}{\left(1+i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{i\beta'\eta}+\left(1-i\dfrac{\mu'\beta'}{\mu\beta}\right)e^{-i\beta'\eta}}.
\end{aligned} \Bigg\} \ldots (62)$$

The displacements at the free surface are now written in the following forms:

$$u=-4(-1)^{m+n}\,i^{n+1}\left(1-\cos^2\frac{\pi}{m}\right)m\,\sin m\omega$$

$$\times \int_0^\infty \frac{\left(\cos\beta'\xi+\dfrac{\mu\beta}{\mu'\beta'}\sin\beta'\xi\right)}{\sin\beta'\eta-\dfrac{\mu\beta}{\mu'\beta'}\cos\beta'\eta}\,\frac{J_m(kr)}{r}\,\beta'^{n-1}\,k^{m-1}\,dk,$$

$$v = -4(-1)^{m+n}\, i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \int_0^\infty \frac{\left(\cos \beta' \xi + \dfrac{\mu\beta}{\mu'\beta'} \sin \beta' \xi\right)}{\sin \beta' \eta - \dfrac{\mu\beta}{\mu'\beta'} \cos \beta' \eta} \frac{\partial J_m(kr)}{\partial r} \beta'^{n-1} k^{m-1}\, dk.$$

$$[m > 2,\ z = -\eta]$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdots\cdots (63)$$

Integrating these in the same manner as that of the preceding section, we get finally

$$u \approx 8\pi (-1)^{m+n}\, i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) m \sin m\omega$$

$$\times \frac{\cos \beta_1' \xi + \dfrac{\mu\beta_1}{\mu'\beta_1'} \sin \beta_1' \xi}{F'(\kappa)} \beta_1'^{n-1} \kappa^{m-1} \frac{H_m^{(2)}(\kappa r)}{r}$$

$$v \approx 8\pi (-1)^{m+n}\, i^{n+1} \left(1 - \cos^2 \frac{\pi}{m}\right) \cos m\omega$$

$$\times \frac{\cos \beta_1' \xi + \dfrac{\mu\beta_1}{\mu'\beta_1'} \sin \beta_1' \xi}{F'(\kappa)} \beta_1'^{n-1} \kappa^{m-1} \frac{\partial H_m^{(2)}(\kappa r)}{\partial r}$$

$$w = 0.$$

$$[m > 2,\ z = -\eta]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (64)$$

In the above expressions κ is the real and positive root of

$$F(k) = \sin \beta' \eta - \frac{\mu\beta}{\mu'\beta'} \cos \beta\eta = 0, \cdots\cdots\cdots\cdots\cdots (65)$$

where $\beta' = \sqrt{j'^2 - k^2}$ and $\beta = \sqrt{k^2 - j^2}$, while β_1 and β_1' are the values of β and β' corresponding to κ.

The special cases of $m = 0$ and $m = 1$ can be obtained by the same operations as before.

It is worth noticing that in this case the surface displacement does not decrease even if the depth of the origin is increased, but the displacement varies as certain harmonic functions of the depth. The other natures are quite similar to those of the preceding case where the origin of the disturbance is resident at the bottom medium.

5. We have now arrived at the results which have some importance on the, practical seismology. We may enumerate them in the following conclusive forms:

1. Long Love-waves having azimuthal distribution on a spherical surface have the large colatitudinal component of displacement, besides the ordinary azimuthal component, and this colatitudinal component cannot disappear even at the equatorial circle.

2. The vertical component is not existent from the start.

3. The velocity of the propagation of Love-waves on a spherical surface is approximately equal to that on a plane surface, even though the waves are relatively long.

4. The dispersion of Love-waves is possible also on a spherical surface.

5. The growth of Love-waves towards the antipode is also possible.

6. The azimuthal variation of displacement of Love-waves is maintained towards the antipode.

7. In considering the neighbourhood of the origin, the longitudinal component of displacement of the short waves becomes quiescent as the waves are propagated towards infinity. When the waves are generated from an internal source, the displacement at the free surface conspires with the modes of the oscillations at the origin.

8. In this case the azimuthal variation of the azimuthal displacement at the surface conforms for all radial distances with the type of the oscillations of the internal source.

9. When the source is resident at the bottom medium, the amplitude of the surface waves decreases with the depth of the source in the bottom medium at the rate of $e^{-\beta_1 \xi}$. It is interesting to compare this with the case of Rayleigh-waves in which the amplitude of the waves is proportional to $e^{-\alpha_1 \xi} (\alpha_1 > \beta_1)$.

10. When the source is situated at the superficial layer, the amplitude of the surface waves varies as some harmonic functions of ξ.

In conclusion I have to express my sincerest thank to Mr. G. Nishimura who has assisted me in preparing this paper.

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23. 球面上に於けるラブ波の傳播とこれに關聯する一二の問題

地震研究所　妹　澤　克　惟

　ラブ波の問題は數學的には餘り簡單過ぎて興味が薄く、又地體構造の上からは常に表面層が必要な爲めに人爲的の嫌ひがある。しかし其結果が驗震上の事實に極めてよく符合する爲、歐洲の地震學者に可なり信頼され且つ種々の場合に應用されて居る。殊に地球を一周しても勢力を失はぬ樣な

ラブ型の長波は地球内部の構造を究める爲めに尊重され過ぎる傾向がある。しかしながら球面上を傳播するラブ波の問題は未だ解かれずに殘されて居る。殊に其が方向性を持つ場合は猶更省みられぬ樣である。私は此等の場合を力學的に研究し、尚震源に極く近い場所では一般解法が反て不正確になる爲に別に考へ、且つ震源の位置も考慮に入れて算出して見た。

　地震學的に重要な結果を摘錄して見ると、

1.　方向性を有するラブ波は球面上で進行方向の變位を持ち、震波赤道に到つても其變位を消滅させる事が出來ぬ。

2.　地表に直角な變位は絕對に存在することがない。

3.　波長が相當長い場合でも球面上でのラブ波の速度は平面上の場合と大した變りがない。

4.　球面上でもラブ波の分散が可能な事は勿論とする。

5.　球體の對蹠點に進むに從つて變位が再び增大する事も當然な事ながら認められる。

6.　ラブ波の方向性は對蹠點に到る迄保持される。

7.　震源附近を考へて見る時、地表上の變位が內部の震源の型と或程度迄一致する。

8.　橫變位の方向性のみはすべての距離に於て內部の震源の型と一致する。

9.　震源が下層の媒體中にある時は表面波の振幅は $e^{-\beta_1 \xi}$ に比例して少くなる。レーレー波の $e^{-\alpha_1 \xi}$ に比較して面白い對照を造る。

10.　震源が上層中にある時は震源の深さの影響は全く別の形式となる。