24. *Observation on the Time Variation of the Second Space Derivatives of the Gravitational Potential. (Part I).*

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(Received Sept. 20, 1929.)

1. The gravitational state at a definite point on the earth does not remain absolutely constant with time. The intensity and direction of gravity are subject to periodic disturbances of astronomical origin. These effects are mainly due to the attractions of the moon, the sun and of the tide of sea-water produced by them. Schweydar¹⁾ made detailed investigations on these phenomena and detected experimentally the existence of these effects. We must notice further that the main part of the gravitational state at a point on the earth depends on the subterranean mass distribution in its neighbourhood which is not absolutely immobile at all from geophysical point of view. It will thus easily be accepted that if there are some such changes in the subterranean mass distribution at all, they will produce no less influences on the gravitational state at the neighbouring fields of the displaced mass. A few workers²⁾ already reported that they observed some secular changes in the intensity of gravity which they believed to be due to this latter cause. Some others,³⁾ however, did not accept these results as undoubtful and regarded these variations rather as due to some residual instrumental errors happened in the respective measurements of these workers. Whichever opinion may be justified in each respective case, it is of great

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1) W. SCHWEYDAR, *Veröffti. d. kön. preuss. geodät. Inst.*, **54** (1912).
„ *ibid.*, **59** (1914).
„ *ibid.* (Neue Folge), **38** (1921).

2) F. W. PFAFF, *ZS. deut. geol. Ges.*, **51** (1899), 125.
„ *Geogr. Jahresber.*, **15** (1903),.
S. G. BURRAD, *Phil. Trans. Roy Soc.*, **205** (1906), 289.
O. FISHER, *Amer. Journ. Sci.*, **21** (1906), 216.
K. R. KOCH, *Ann. d. Phys.*, **15** (1904), 146.
A. BORN, *Isostasie und Schweremessung*, Berlin (1923), 49.

3) F. R. HELMERT, *Enz. d. math. Wiss.*, Bd. VI, 1 B., Heft 2 (1910), 177.
O. HECHER and O. MEISSER, *Handb. d. phys. und tech. Mech.*, Bd. II (1928), 174.

importance to make further investigations into these lines and to decide whether or not such secular changes in gravitational states is actually taking place.

The present writer[4] has recently come across with an interesting fact that the gravitationally anomalous regions in this country are also the regions most frequented by earthquakes. This fact suggests that the geological movements taking place at present in a certain region are closely correlated to the special state of the gravitational fields in its environs. If it be allowed further that the geological and volcanic activities are connected with some macroscopic changes in the subterranean mass distribution of any mode, some changes in the gravitational state on the earth must be correspondingly expected especially in earthquake and volcanic regions as their surface manifestation. If there is such a phenomenon at all, this must be especially prevalent in this country in which the crustal strain in much concentrated.

Considering in this way, it seems probable to the writer that the gravitational state *is* actually subject to secular changes and that the only reason why this has not yet been established is that no observation has ever been made from the stand point developed in the above.

The continuous or at least successive observations on any of the quantities relating to the gravitational potential must be made for the object of obtaining some clue to the changes in the subterranean mass distribution. The observations of the intensity of gravity as well as the direction of gravity with respect to the reference ellipsoid may be ones of the methods which will meet this object. Small changes in these quantities are, however, generally hard to be observed. On the other hand, the second space derivatives of the gravitational potential can be determined with considerable accuracy. We can make comparatively easily an accurate continuous measurement of the latter quantities and detect changes in them if any. For these reasons, the continuous observations of the second space derivatives of the gravitational potential will match best for our present purpose. For a time since, the present writer has been engaged in the continuous observation of the second space derivatives with a specially deviced instrument. This instrument is nothing but an extremely sensitive torsion balance fixed in a definite azimuth. The observation was commenced only in the beginning of this year (1929), and the period of observation is not yet long enough for the writer to get some definite conclusions with regard to his primary object. Yet it seems worth while to describe the results of the observations

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4) C. Tsuboi, *Proc. Imp. Acad.*, 5 (1929), 326.

so far obtained because they will not only serve as an introductory report of the further investigations but will also suggest many interesting problems regarding the short period fluctuations in the value of the gravitational potential.

It is a great pleasure for the writer to record his appreciation of the helpful suggestions of Professor Torahiko Terada in this work.

2. The instrument used in the present observation is a modification of a torsion balance of which the theory of action was originally developed by Eötvös. It consists essentially of a horizontal beam suspended at its middle point from above by a fine fibre. The horizontal beam is loaded at its both extremities by equal spherical masses. The equilibrium of the hanging system will be attained when the torsional moment acting on the beam due to the slight curvature of the gravitational equipotential surface just balances with the restitutive force due to the rigidity of the fibre towards its untwisted position. Let ξ η ζ be three co-ordinate axes taken northwards, eastwards and downwards respectively. Let further V be the gravitational potential, τ the torsional constant of the fibre, θ its angular deviation from the untwisted position, K the moment of inertia of the beam, α the angle between the northward direction and the beam measured from the northward direction clockwise as seen from above. The conditions of equilibrium of the balance is expressed by,

$$\tau\theta = K\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right)\sin 2\alpha + 2K\frac{\partial^2 V}{\partial\xi\,\partial\eta}\cos 2\alpha \ldots\ldots\ldots\ldots (1)$$

Suppose some changes happen in the form of the equipotential surface and what is the same thing in either or both of the values of $\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right)$ and $\frac{\partial^2 V}{\partial\xi\,\partial\eta}$. In response to them, the corresponding change must take place in the value of θ. Thus we have,

$$\tau\Delta\theta = K\sin 2\alpha\Delta\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right) + 2K\cos 2\alpha\Delta\frac{\partial^2 V}{\partial\xi\,\partial\eta}$$

$$+2K\cos 2\alpha\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right)\Delta\alpha - 4K\sin 2\alpha\frac{\partial^2 V}{\partial\xi\,\partial\eta}\Delta\alpha, \ldots\ldots (2)$$

and since

$$\Delta\theta = \Delta\alpha, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\left\{\tau - 2K\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right)\cos 2\alpha + 4K\frac{\partial^2 V}{\partial\xi\,\partial\eta}\sin 2\alpha\right\}\Delta\theta$$

$$= K\sin 2\alpha\,\Delta\left(\frac{\partial^2 V}{\partial\eta^2} - \frac{\partial^2 V}{\partial\xi^2}\right) + 2K\cos 2\alpha\,\Delta\frac{\partial^2 V}{\partial\xi\,\partial\eta} \ldots\ldots\ldots\ldots (4)$$

$2K\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right)\cos 2\alpha$ and $4K\frac{\partial^2 V}{\partial \xi\,\partial \eta}\sin 2\alpha$ are négligible small against τ, therefore we have at last,

$$\Delta\theta=\frac{K}{\tau}\sin 2\alpha\Delta\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right)+\frac{2K}{\tau}\cos 2\alpha\Delta\frac{\partial^2 V}{\partial \xi\,\partial \eta}\dots\dots\dots\dots(5)$$

The equilibrium position of the hanging system is recorded by the ordinary lamp and mirror method on a photographic paper wound round on a revolving circular drum. If the distance between the mirror attached to the horizontal beam and the photographic paper is D and an angular deflection $\Delta\theta$ of the mirror causes a linear deflection Δn of the image of the light source on the recording paper, then

$$\frac{\Delta n}{2D}=\Delta\theta.\dots\dots\dots\dots\dots\dots\dots\dots\dots(6)$$

Substituting (6) in (5), we have,

$$\begin{aligned}\Delta n&=\frac{2DK}{\tau}\sin 2\alpha\Delta\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right)+\frac{4DK}{\tau}\cos 2\alpha\Delta\frac{\partial^2 V}{\partial \xi\,\partial \eta}\\&=A\Delta\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right)+B\Delta\frac{\partial^2 V}{\partial \xi\,\partial \eta}\end{aligned}$$

for a given value of α, where A and B are geometrical constants. If $\alpha=0$, then

$$\Delta n=B\Delta\frac{\partial^2 V}{\partial \xi\,\partial \eta},$$

and if $\alpha=\pi/4$, then

$$\Delta n=A\Delta\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right).$$

We can thus determine the change in $\frac{\partial^2 V}{\partial \xi\,\partial \eta}$ and $\left(\frac{\partial^2 V}{\partial \eta^2}-\frac{\partial^2 V}{\partial \xi^2}\right)$ separately by means of two instruments set in two different azimuths making an angle of $\pi/4$ with each other.

3. It is of great importance in this observation to secure the constancy of the instrumental constants as accurately as possible. The variation in the instrumental constants will produce the deviation of the records masking the true change in the gravitational fields which it is our primary object to observe. Of all the available materials to be used in these lines, the material of vitreous silica gives the most satisfactory results.⁵⁾ The expansion coefficient of this material is extraordinarily small. This material is further

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5) C. TSUBOI, *Proc. Imp. Acad.*, 4 (1928), 300.

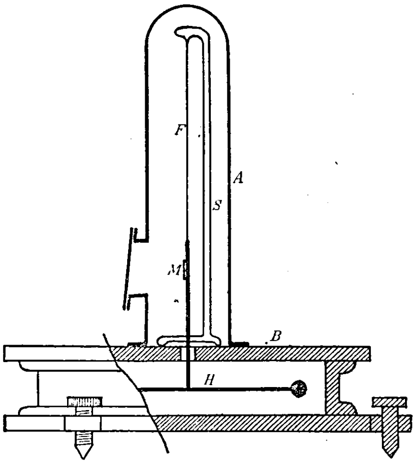


Fig. 1 a.

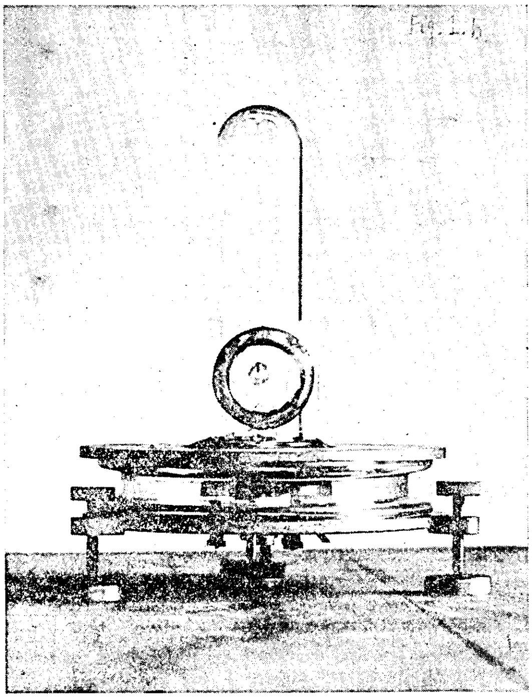


Fig. 1 b.

practically free from elastic hysterisis and can be drawn into an extremely fine fibre, its tenacity being sufficiently large for many purposes. The whole instrument used in this observation was therefore constructed of vitreous silica. Not only the torsional fibre but also the horizontal beam and the frame from which the suspension is to be hung were made of this material. The lower end of the suspension fibre is directly fused to the horizontal beam avoiding the use of any kind of the binding material. So is the upper end of the fibre to the instrumental frame. This is a very significant characteristic of this instrument. Binding materials often cause disturbances in the equilibrium position of the balance in an irregular manner. The working part of the instrument is mounted in a protection case in such a manner as is shown in Fig. 1.

The frame of the instruments is fastened to the brass plate B. The hanging beam H can revolve through full azimuth of 2π, and it reaches at its equilibrium position by itself without any external adjustment. Then the cover glass A is fixed to the plate B in such a position that the opening in A will come just in front of the reflecting surface of the mirror M attached to the hanging system. After this was done, the plate B to which the working part and the cover glass are now fixed is rotated until the direction of the beam takes the desired azimuthal angle with respect to the reference direction. The recording of the equilibrium position of the beam is effected by ordinary lamp and mirror method.

The length of the horizontal beam is about 13 cm., its mass 2 gr., and moment of intertia about 45 gr. cm.^{2} This beam is suspended from above by a quartz fibre of which the length is about 12 cm. and the diameter a few hundredths of millimeter. The distance between the mirror attached to the hanging beam and the photographic paper is about 100 cm. With these instrumental constants, it becomes,

$$\varDelta n = 8\times 10^{+9}\left[\varDelta\left(\frac{\partial^2 V}{\partial\eta^2}-\frac{\partial^2 V}{\partial\xi^2}\right)\sin 2\alpha+\varDelta\frac{\partial^2 V}{\partial\xi\,\partial\eta}\cos 2\alpha\right].$$

With this sensitivity, the change of the quantity in the square bracket by an amount of 1.25×10^{-10} will cause 1 cm. deflection of the image on the recording paper. This corresponds to a sensitivity about twenty times as much as the ordinary portable torsion balance.

4. The rigidity μ of the suspension fibre inevitably is affected by its temperature. The equilibrium position of the beam which depends on the rigidity of the fibre will thus be affected by temperature change. We have,

$$\frac{\varDelta\mu}{\mu}+\frac{\varDelta\theta}{\theta}=0$$

On the other hand,

$$\mu_t=\mu_{15}\{1+1{\cdot}2\times 10^{-4}(t-15)\},$$
$$\mu_0=\mu_{15}\{1-1{\cdot}2\times 10^{-4}\times 15\},$$
$$\mu_{30}=\mu_{15}\{1+1{\cdot}2\times 10^{-4}\times 15\},$$
$$\mu_{30}-\mu_0=3{\cdot}6\times 10^{-3}\mu_{15},$$
$$\mu=3\times 10^{11},$$
$$\frac{\varDelta\mu}{\mu}=1{\cdot}2\times 14^{-14}.$$

Therefore,

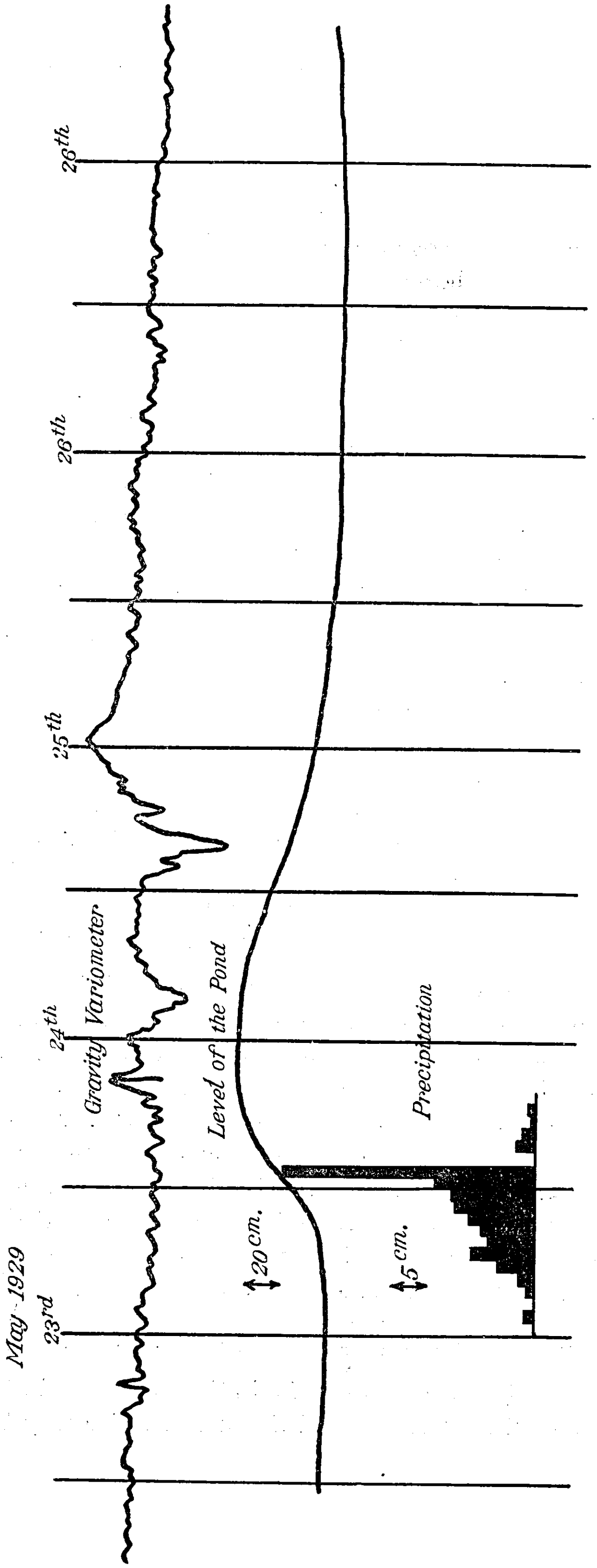
$$\frac{\varDelta\theta}{\theta}=-1{\cdot}2\times 10^{-14}.$$

We see therefore that even a temperature change of 30°C produces only a negligible effect on the equilibrium position of the beam.

The air inside of the vessel is not entirely free from convection and in fact the position of the image taken on the recording paper shows fluctuations within a small range, but this produces no serious objection in the present observation. It is true that defective construction of the torsion balance often causes some irregular fluctuations of the equilibrium position of the balance. Before beginning the observation, we must check the reliability of the records taken by this instrument. For this purpose, two instruments of similar dimension were placed side by side in the same azimuthal direction with respect to the reference direction. The simultaneous records of their equilibrium position were taken and compared. They are shown in Fig. 2 in Pl. XXXII. We see a close resemblance in the two records. Not only they resemble to each other in their general appearances but also in almost every details of minute fluctuations. Thus it is proved that the instruments were recording faithfully what was intended to be recorded without sensibly disturbed by external conditions.

5. We see, in the records of Fig. 2, daily variations of the equilibrium position of the images. They are no doubt due to the gravitational attraction of men who come in the building of our Institute every morning and go out every evening. This was ascertained as in Sundays and National Horidays when few workers come to the Institute, the equilibrium position remains fairly constant all day long. We see further minute fluctuations appearing simultaneously in both of the curves of Fig. 2, almost in the same form. Minute fluctuations in the day time may be explained as due to some external mass displacements, but we see also similar fluctuations even in the midnight. The origin of these disturbances is not yet certain.

6. The equilibrium position of the balance may obviously affected by the gravitational attraction of the underground water. Not so far from the Institute, there is a pond in the compound of our University. The instruments are almost just at the same level as the surface of the water of the pond. This was ascertained by means of precise levellings. A rain will raise the level of the pond as well as of the general underground water. This produces some deformation of the gravitational equipotential surface in the neighbourhood of the instrument. According to calculations, the effect of the rise of the level of the pond alone is very small, the rise of the level of the pond by 1 cm. producing a change $+5.5180\times10^{-12}$ on the term $\left(\frac{\partial^2 V}{\partial\eta^2}-\frac{\partial^2 V}{\partial\xi^2}\right)$ and -3.6469×10^{-12} on the term $\frac{\partial^2 V}{\partial\xi\,\partial\eta}$ of the observing point. The level of the pond may, however, be regarded as an indication of the level of the underground water, thus it may be expected that a correlation may be found in


May 1929
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Gravity Variometer
Level of the Pond
Precipitation
20 cm.
5 cm.

Fig. 3.

the level of the pond and the deviation of the equilibrium position of the records. This was actually the case. As a conspicuous example of this relation, a part of the records taken is reproduced in Fig. 3, with the height of the level of the pond and the hourly precipitations for comparison. It rained very hardly on May 23rd, 1929. The total precipitation was measured no less than 119.5 mm. including the maximum hourly precipitation of 35.5 mm.[6] With some lag of time from the rainfall, the level of the pond reached at its maximum. The equilibrium position of the balance began to fluctuate at about this time, continuing further about one day and a half. This is evidently due to general change in the condition of underground water.

7. In Fig. 4 is reproduced a part of the records obtained by this instrument. Through the period of observation, the equilibrium position of the balance was not fairly stationary. The effects of the underground water and others being taken out, we see still other slow variations remaining. They show no apparent correlation with meteorological and other common phenomena. It will need a fairly long continuous observation to find out the origin of these variations and to appreciate their full significances. The present paper must be regarded only as an introductory note for the further investigations.

(*To be continued.*)

24. 重力ポテンシヤルの空間二次微分係數の時間的變動に就いて

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重力野の狀況が、殊に我が國の樣な不安定な所では、永年變化をなすのではないであらうかと云ふ事を確める爲に始めた實測である。觀測に用ひた器械は重力偏差計の原理によつたもので、全部熔融シリカで製作した。精度は、普通の可搬重力偏差計の約 20 倍程のものである。先づ此の器械が信用すべき記錄を與へると云ふ事を確めた後、連續觀測を始めた。始めたのは今年一月であるから、未だ何等の結論は得られてゐない。只其の間に降雨による地下水の影響が重力野にも及ぶ事を確めた。又夜半にも短週期の重力野の變動が現れる事がある。地下水の影響と認められるものを除いても、かなり長い時間に亘る重力野の變動が見られる。之は何に原因するのかは今の所解つて居ない。

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[6] These data were given by Professor Takematsu Okada of the Central Meteorological Observatory, to whom the writer wishes to tender his sincere thanks.

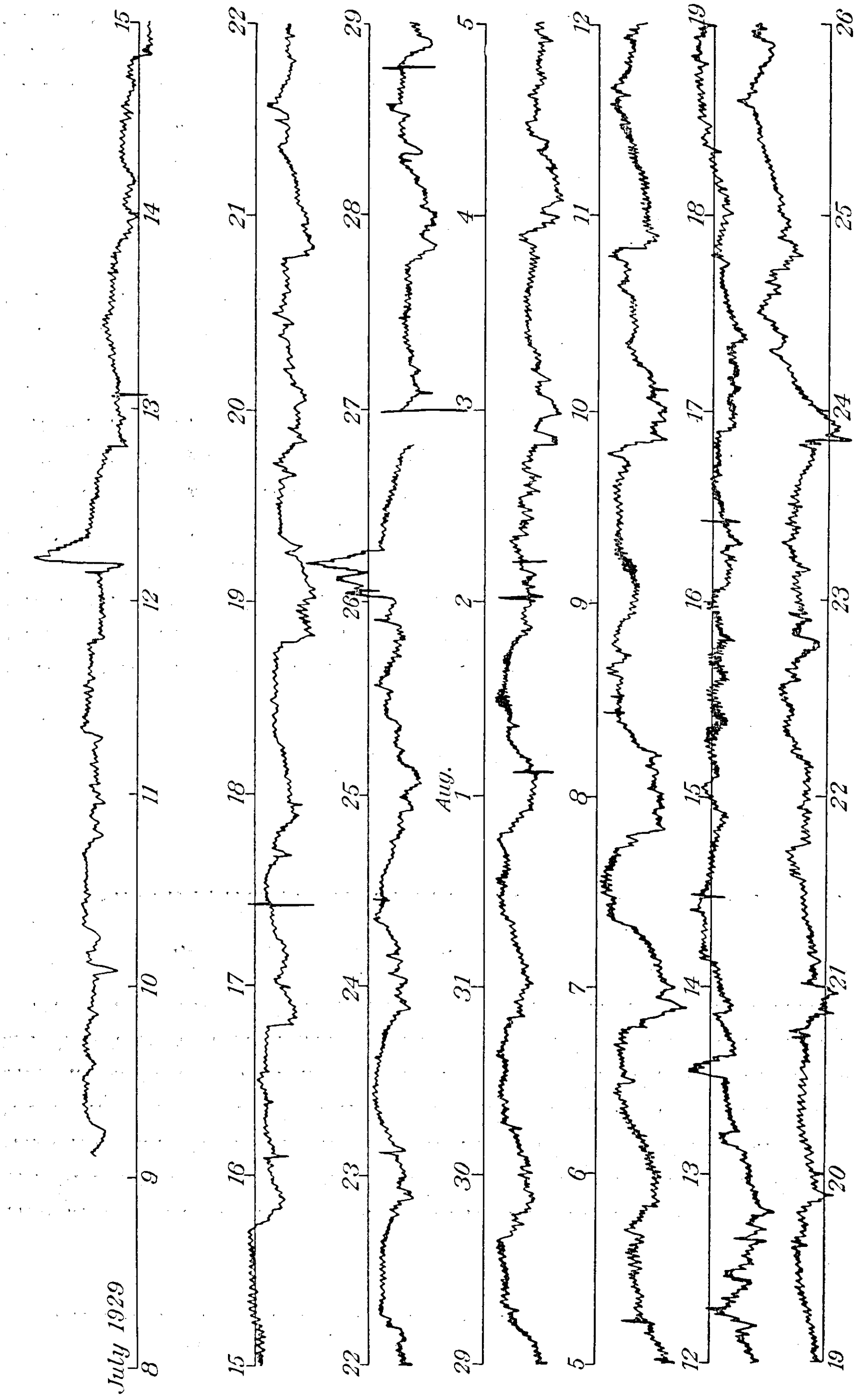
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Fig. 4.

（震研彙報、第七號、圖版、坪井）

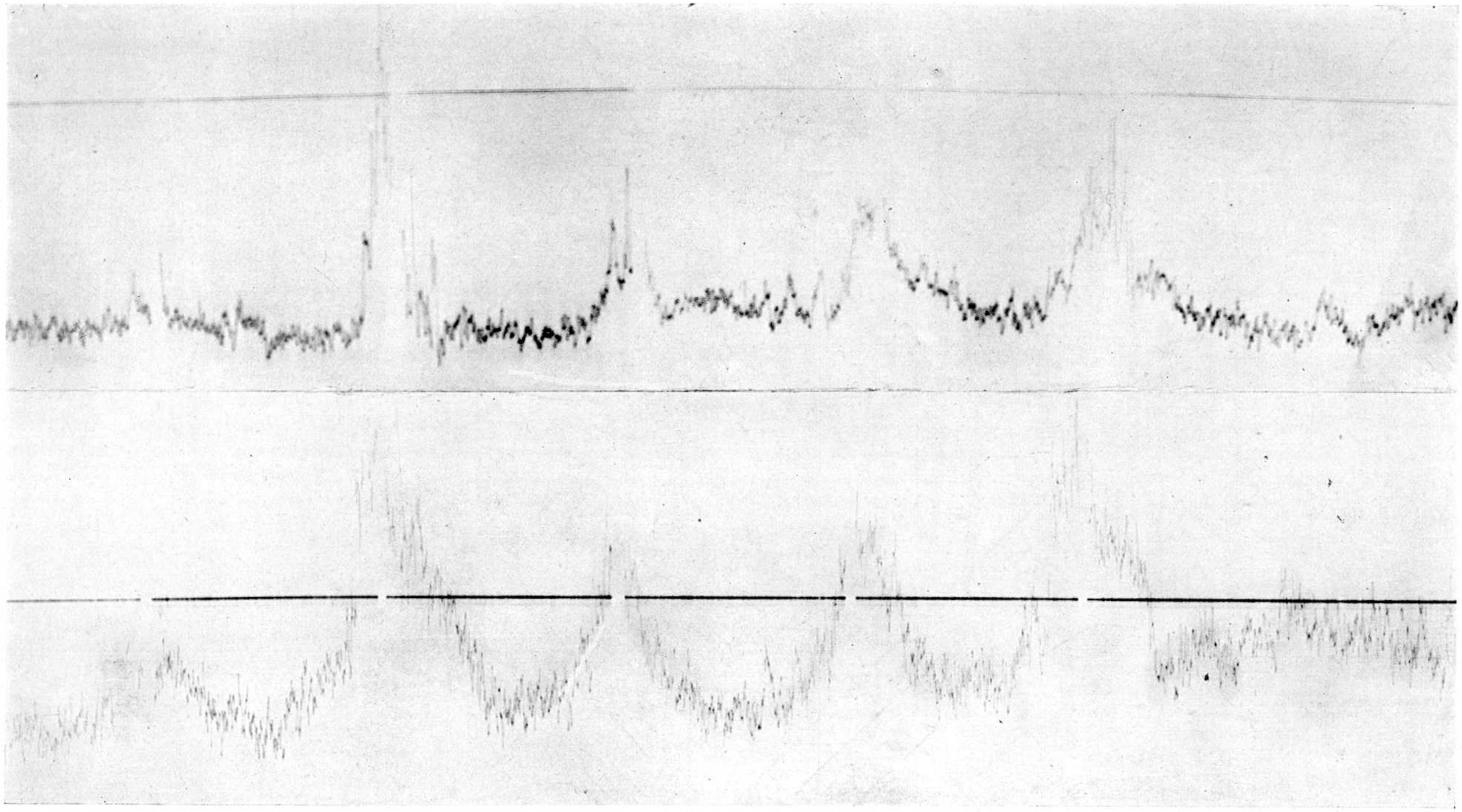

Fig. 2.