13. *Periodic Rayleigh-waves caused by an Arbitrary Disturbance.*

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A few years ago I studied an important problem[1] of the dispersion of Rayleigh-waves on the surface of a stratified elastic body. The problem gave merely the theory of the dispersion of an aggregate of trains of harmonic waves of an infinite extent, but did not involve the propagation of a train of waves of a finite extent. Although some cases of the transmission of a train of Rayleigh-waves of a limited extent in a dispersive medium have already been given in one of my papers,[2] yet the law of dispersion which was assumed in that paper was very simple and hence the results are far from being fulfilled in the actual waves at the ground.

It has in recent times been questioned by many seismologists that the regularity of the periods of the pulsatory motion of the ground may depend on the time intervals of the disturbances or on the restitutive nature of the ground itself. Prof. B. Gutenberg[3] and others pointed out that the continuous actions of the breach and the frost are the causal disturbances of the unceasing pulsations. Dr. K. Wadati ascribed in a former time the periodic pulsatory motion of the land surface to the effect of the free oscillations of the upper layer of that ground, though he does not seem now to care for such a problem. These theories will apparently account for the periodic motion of all the pulsation and, indeed, for each of the original disturbances the idea of Prof. Gutenberg seems to be most excellent; yet the explanation of the ordinary pulsatory motion can be made in another way. We know that the continuous periodic pulsation can be observed even beyond a certain distance from the shore or even at a season which is free from the frost. The superficial layer as in Dr. Wadati's sense is effective for the periodic oscil-

1) "Dispersion of Elastic Waves propagated on the Surface of Stratified Bodies & etc.", *Bull. Earthq. Res. Inst.*, 3 (1927), 1–18.

2) "On the Propagation of the Leading and Trailing Parts of a Train of Elastic Waves," *Bull. Earthq. Res. Inst.*, 4 (1928), 107–122.

3) B. GUTENBERG, *Zeits. f. Geophys.* 3 (1927) and a number of his other papers.

lations under the condition that the rigidity of the bottom medium is extremely great in comparison with that of the layer, so that the arbitrary disturbance can be multiply reflected at the upper as well as at the lower boundaries and therefore the apparently periodic motion will be felt at the surface. This phenomena which takes place at such an extreme case is no more than the free oscillations of the vertical type of the layer and gives no explanation to the periodic waves propagated on the surface of the body.

To understand such a nature of the propagated waves, it seems to be best to take some dispersive character of the waves caused by the outer disturbances. The postulation of the superficial layer is not meaningless in the direction that the dispersive character of the waves depends on such a layer. The formulation of the dispersion of waves propagated in a solid body is seriously complicated. Even if we could assume the law of the dispersion, such law would not apply with exactness to the solid of a certain geometrical form, as in the case of a stratified body. The difficulties, however, can be avoided by assuming that in the case of the surface waves the velocity of the phase of the harmonic waves is equivalent to that of Rayleigh-waves on a stratified body and that other treatments are quite analogous to those of the ordinary surface waves. This attempt is, of course, a first approximation; but, as far as the neighbourhood of the upper surface is concerned, the method seems to be rather correct than otherwise methods are employed.

Apart from the above problem, there is another fact to be noticed. The long trailing train of waves of the harmonic type which follows the main tremors may have some connection with the preceding problem. Though the periods of the oscillations are quite different from those of the ordinary pulsations, the character may be thought to be entirely similar; and it may be assumed in this case that the effective layer to cause the dispersion is of a larger scale, besides the path of the transmission of the waves which is long enough to excite the secondary or the sub-secondary trailing trains of waves.

The formula of the phase velocity which may be imagined from the results of my calculation in a preceding paper[1] is too complicated due to the inflexion nature of the velocity curves to apply the method of integration on such a complex formula. Owing to this reason, we shall assume that the velocity of the propagation of the phase of waves depends on the wave length to the degree of the square. The more complicated cases will be left to some other occasion.

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1) *Loc cit.*, 193.

1. We may first consider the two-dimensional problem of Rayleigh-type waves. The axis of x being taken on the undisturbed surface, the axis of z is drawn vertically downwards. If we assume the effective elastic constants λ', μ' to be functions of the wave length such that

$$\left.\begin{aligned} \lambda' &= \lambda\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}, \\ \mu' &= \mu\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}, \end{aligned}\right\} \qquad (1)$$

in which L, H are the wave length and the effective depth of the layer and c_1, c_2 are dispersion constants, then the equations of motion of a semi-infinite body may be written in the forms:

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2} &= (\lambda+2\mu)\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\Delta}{\partial x} \\ &\qquad +2\mu\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\varpi}{\partial z}, \\ \rho\frac{\partial^2 w}{\partial t^2} &= (\lambda+2\mu)\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\Delta}{\partial z} \\ &\qquad -2\mu\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\varpi}{\partial x}, \end{aligned}\right\} \qquad (2)$$

where

$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}, \qquad 2\varpi=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}, \qquad (3)$$

and u, w are the components of displacement in x- and z-directions.

From (2) and (3), we get

$$\left.\begin{aligned} \Delta &= A_1 e^{-\alpha z+ikx}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}pt, \\ 2\varpi &= -iB_1 e^{-\beta z+ikx}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}pt, \end{aligned}\right\} \qquad (4)$$

in which

$$k^2-h^2=\alpha^2, \qquad k^2-j^2=\beta^2, \qquad (5)$$

$$\left.\begin{aligned} h^2 &= \frac{\rho p^2}{\lambda+2\mu}\Big/\left(1+\frac{2\pi c_1}{kH}+\frac{4\pi^2 c_2^2}{k^2H^2}\right), \\ j^2 &= \frac{\rho p^2}{\mu}\Big/\left(1+\frac{2\pi c_1}{kH}+\frac{4\pi^2 c_2}{k^2H^2}\right), \end{aligned}\right\} \qquad (6)$$

where $2\pi/k$ is taken in place of L.

The displacements corresponding to (4) can be easily written by

$$\left.\begin{aligned} u&=-i\left(\frac{kA_1}{h^2}e^{-\alpha z}+\frac{\beta B_1}{j^2}e^{-\beta z}\right)e^{ikx}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}pt,\\ w&=\left(\frac{\alpha A_1}{h^2}e^{-\alpha z}+\frac{kB_1}{j^2}e^{-\beta z}\right)e^{ikx}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}pt, \end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(7)$$

At the surface, $z=0$, we should have

$$\lambda'\Delta+2\mu'\frac{\partial w}{\partial z}=0,\qquad \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=0,\quad\cdots\cdots\cdots\cdots(8)$$

from which we get

$$\frac{B_1}{j^2}\Big/\frac{A_1}{h^2}=\frac{-2k\alpha}{2k^2-j^2},\quad\cdots\cdots\cdots\cdots(9)$$

and

$$(2k^2-j^2)^2-4k^2\alpha\beta=0.\quad\cdots\cdots\cdots\cdots(10)$$

The typical solutions at $z=0$ are thus written by

$$\left.\begin{aligned} u&=-i\cos\left\{\frac{jV_2}{\kappa}\sqrt{k^2+\frac{2\pi c_1}{H}k+\left(\frac{2\pi c_2}{H}\right)^2}\,t\right\}\frac{\kappa(2\kappa^2-j^2-2\alpha_1\beta_1)}{j^2\alpha_1}e^{ikx},\\ w&=\cos\left\{\frac{jV_2}{\kappa}\sqrt{k^2+\frac{2\pi c_1}{H}k+\left(\frac{2\pi c_2}{H}\right)^2}\,t\right\}e^{ikx}, \end{aligned}\right\}\cdots(11)^{4)}$$

where κ is the real positive root of the equation (10) and α_1, β_1 are the corresponding values of α, β, while V_2 is the velocity of the propagation of certain distortional waves of the form $\sqrt{\mu/\rho}$.

To generalise (10), we use Fourier's double integral theorem, the generalised forms of u, w being as follows:

$$\begin{aligned} u=&\frac{iq}{2\pi}\int_{-\infty}^{\infty}\cos\left\{l\sqrt{k^2+mk+n^2}\,t\right\}dk\int_{-\infty}^{\infty}f(\sigma)\,e^{ik(x-\sigma)}d\sigma\\ &+\frac{iq}{2\pi}\int_{-\infty}^{\infty}\frac{\sin\{l\sqrt{k^2+mk+n^2}\,t\}\,dk}{l\sqrt{k^2+mk+n^2}}\int_{-\infty}^{\infty}F(\sigma)\,e^{ik(x-\sigma)}d\sigma,\quad\cdots\cdots(12) \end{aligned}$$

$$\begin{aligned} w=&\frac{1}{2\pi}\int_{-\infty}^{\infty}\cos\left\{l\sqrt{k^2+mk+n^2}\,t\right\}dk\int_{-\infty}^{\infty}f(\sigma)\,e^{ik(x-\sigma)}d\sigma\\ &+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\sin\{l\sqrt{k^2+mk+n^2}\,t\}\,dk}{l\sqrt{k^2+mk+n^2}}\int_{-\infty}^{\infty}F(\sigma)\,e^{ik(x-\sigma)}d\sigma,\quad\cdots\cdots(13) \end{aligned}$$

for the initial conditions that $w=f(x)$ and $\partial w/\partial t=F(x)$ at $z=0$.

4) These expressions are most suitable for the condition that the surface is always free from the stress.

In (12) and (13) we have used the notations such that

$$\left.\begin{aligned} q&=\frac{\kappa(2\kappa^2-j^2-2\alpha_1\beta_1)}{j^2\alpha_1} \qquad (=\text{constant for any } p), \\ l&=\frac{jV_2}{\kappa}, \qquad m=\frac{2\pi c_1}{H}, \qquad n=\frac{2\pi c_2}{H}. \end{aligned}\right\} \quad \ldots\ldots (14)$$

If the intial disturbance is given by

$$f(x)=\frac{A}{a}e^{-\frac{x^2}{a^2}}, \qquad F(x)=\frac{B}{b}e^{-\frac{x^2}{b^2}}, \quad \ldots\ldots (15)$$

then, by means of the formula

$$\int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{c^2}} e^{-ik\sigma}\, d\sigma=\sqrt{\pi}\, c\, e^{-\frac{c^2k^2}{4}}, \quad \ldots\ldots (16)$$

we get

$$\begin{aligned} u=&\frac{iAq}{2\sqrt{\pi}}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\left(1-\frac{a^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}t}{l\sqrt{k^2+mk+n^2}}e^{ikx}\,dk \\ &+\frac{iBq}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\left(1-\frac{b^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}t}{l\sqrt{k^2+mk+n^2}}e^{ikx}\,dk, \quad \ldots (17) \end{aligned}$$

$$\begin{aligned} w=&\frac{A}{2\sqrt{\pi}}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\left(1-\frac{a^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}t}{l\sqrt{k^2+mk+n^2}}e^{ikx}\,dk \\ &+\frac{B}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\left(1-\frac{b^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}t}{l\sqrt{k^2+mk+n^2}}e^{ikx}\,dk. \quad \ldots (18) \end{aligned}$$

When the disturbance is very sharp and is concentrated in the neighbourhood of the origin, a and b will be negligibly small and therefore it will be sufficient to take each first term of the brackets. Taking, thus, such an extreme case and putting $k+m/2=k'$ and $n^2-m^2/4=n'^2$, we obtain

$$\left.\begin{aligned} u=&\frac{iAq}{2\sqrt{\pi}}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}e^{i\left(k'-\frac{m}{2}\right)x}\,dk' \\ &+\frac{iBq}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}e^{i\left(k'-\frac{m}{2}\right)x}\,dk', \\ w=&\frac{A}{2\sqrt{\pi}}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}e^{i\left(k'-\frac{m}{2}\right)x}\,dk' \\ &+\frac{B}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}e^{i\left(k'-\frac{m}{2}\right)x}\,dk'. \end{aligned}\right\} \quad \ldots\ldots (19)$$

These may be transformed to

$$\left.\begin{aligned}
u &= \frac{iAq\,e^{-\frac{imx}{2}}}{\sqrt{\pi}}\frac{\partial}{\partial t}\int_0^\infty \frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}\cos k'x dk' \\
&\qquad + \frac{iBq\,e^{-\frac{imx}{2}}}{\sqrt{\pi}}\int_0^\infty \frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}\cos k'x dk', \\
w &= \frac{Ae^{-\frac{imx}{2}}}{\sqrt{\pi}}\frac{\partial}{\partial t}\int_0^\infty \frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}\cos k'x dk' \\
&\qquad + \frac{Be^{-\frac{imx}{2}}}{\sqrt{\pi}}\int_0^\infty \frac{\sin l\sqrt{k'^2+n'^2}\,t}{l\sqrt{k'^2+n'^2}}\cos k'x dk'.
\end{aligned}\right\} \quad \ldots\ldots(20)$$

But we know the expressions

$$\left.\begin{aligned}
\int_0^\infty \frac{\sin l\sqrt{k'^2+n'^2}\,t}{\sqrt{k'^2+n'^2}}\cos k'x dk' &= \frac{\pi}{2}J_0(n'\sqrt{l^2t^2-x^2}), \qquad [x^2<l^2t^2] \\
&= 0. \qquad [x^2>l^2t^2]
\end{aligned}\right\} \ldots(21)^{5)}$$

Hence, taking the real part, we have

$$\begin{aligned}
u &= -\frac{A\pi^{\frac{3}{2}}}{H}\sqrt{c_2^2-\frac{c_1^2}{4}}\,\frac{\frac{jV_2}{\kappa}t}{\sqrt{\left(\frac{jV_2}{\kappa}\right)^2t^2-x^2}}\,\frac{\kappa(2\kappa^2-j^2-2\alpha_1\beta_1)}{j^2\alpha_1}\sin\frac{2\pi c_1 x}{H} \\
&\qquad \times J_1\left\{\frac{2\pi}{H}\sqrt{\left(c_2^2-\frac{c_1^2}{4}\right)\left\{\left(\frac{jV_2}{\kappa}\right)^2t^2-x^2\right\}}\right\} \\
&\quad + \frac{B\sqrt{\pi}}{2}\,\frac{\kappa}{jV_2}\,\frac{\kappa(2\kappa^2-j^2-2\alpha_1\beta_1)}{j^2\alpha_1}\sin\frac{2\pi c_1 x}{H} \\
&\qquad \times J_0\left\{\frac{2\pi}{H}\sqrt{\left(c_2^2-\frac{c_1^2}{4}\right)\left\{\left(\frac{jV_2}{\kappa}\right)^2t^2-x^2\right\}}\right\}, \quad \left[\left(\frac{x}{t}\right)^2<\left(\frac{jV_2}{\kappa}\right)^2\right] \\
&= 0, \qquad \left[\left(\frac{x}{t}\right)^2>\left(\frac{jV_2}{\kappa}\right)^2\right]
\end{aligned}$$

$$\begin{aligned}
w &= -\frac{A\pi^{\frac{3}{2}}}{H}\sqrt{c_2^2-\frac{c_1^2}{4}}\,\frac{\frac{jV_2}{\kappa}t}{\sqrt{\left(\frac{jV_2}{\kappa}\right)^2t^2-x^2}}\cos\frac{2\pi c_1 x}{H} \\
&\qquad \times J_1\left\{\frac{2\pi}{H}\sqrt{\left(c_2^2-\frac{c_1^2}{4}\right)\left\{\left(\frac{jV_2}{\kappa}\right)^2t^2-x^2\right\}}\right\}
\end{aligned}$$

5) H. LAMB, *Proc. Math. Soc.*, London, [2], 7 (1907), 122; and also S. SANO, *Bull. Centr. Meteor. Obs.*, Japan, 2 (1913), 13.

$$+\frac{B\sqrt{\pi}}{2}\frac{\kappa}{jV_2}\cos\frac{2\pi c_1 x}{H}$$
$$\times J_0\left\{\frac{2\pi}{H}\sqrt{\left(c_2^2-\frac{c_1^2}{4}\right)\left\{\left(\frac{jV_2}{\kappa}\right)^2 t^2-x^2\right\}}\right\}, \quad \left[\left(\frac{x}{t}\right)^2<\left(\frac{jV_2}{\kappa}\right)^2\right]$$
$$=0. \quad \left[\left(\frac{x}{t}\right)^2>\left(\frac{jV_2}{\kappa}\right)^2\right]$$
$$\cdots\cdots\cdots\cdots(22)$$

Since jV_2/κ is the velocity of propagation of the ordinary Rayleigh-waves transmitted on the surface of a semi-infinite elastic body whose elastic constants are λ and μ, these results shew that the leading part of a train of waves is transmitted with the velocity of Rayleigh-waves. It appears, moreover, that, in spite of the concentrated and impulsive type of the original disturbance, the propagated waves are of the harmonic type with gradually increasing periods and decreasing amplitudes. This fact is in agreement with the nature of the trailing waves which follow the main shock of the earthquake movements.

It is easily seen from the forms of the equations (22) that, though the period increases with successive oscillations in a train, each period is of a certain order of length depending on the values of H, c_1 and c_2. This fact well accounts for the curious nature that the the pulsatory motions with a certain range of periods will frequently be obserbed at a given station.

It is also to be remarked that the factors $\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}\frac{2\pi c_1 x}{H}$ in the expression of (22) give us the abnormal regions of the earthquake movements at certain periodic distances from the epicentre. This depends on c_1, H and the epicentral distance x. Such a special nature disappears when c_1 is nil.

The above investigation is taken substantially from the forms of the

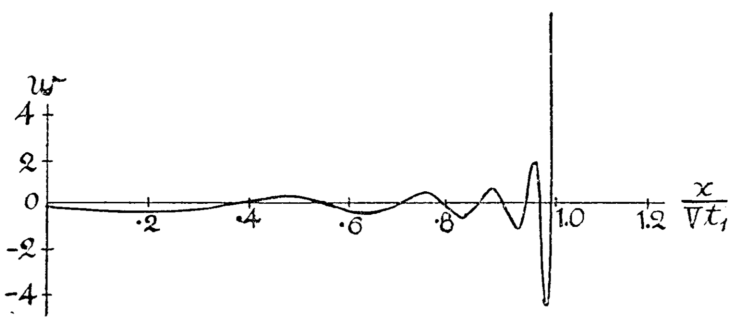

Fig. 1. Space-distribution of displacement at $t=t_1$ (Two-dimensions).

equations (22). To understand the nature more concretely, a few drawings have been illustrated: they will probably admit of a variety of

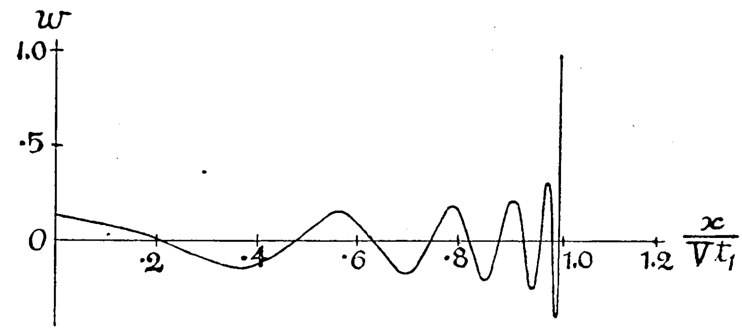

Fig. 2. Space-distribution of displacement at $t=t_1$ (Two-dimensions).

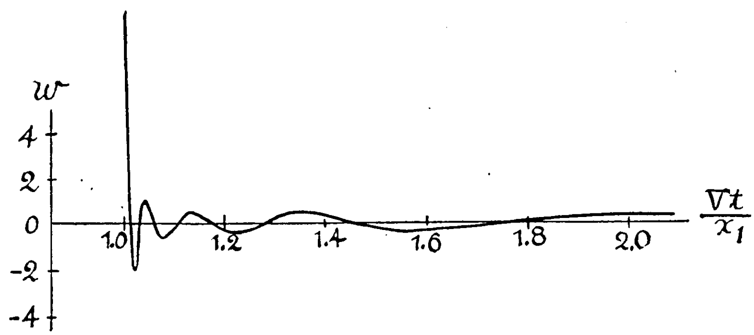

Fig. 3. Time-variation of displacement at $x=x_1$ (Two-dimensions).

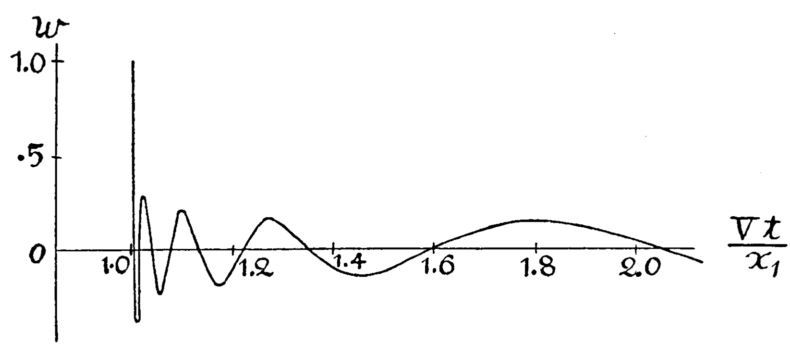

Fig. 4. Time-variation of displacement at $x=x_1$ (Two-dimensions).

interpretations. Fig. 1 is the distribution of the displacements in space at the time t_1 due to the initial surface displacement at $x=0$, while Fig. 2 is the similar distribution due to the initial surface velocity at the origin. Fig. 3 is the time variation of the displacement at the point x_1 due to the initial surface displacement at $x=0$, and Fig. 4 gives us the like problem due to the initial surface velocity at the origin. In these figures V is the velocity of certain Rayleigh-waves, i.e. $\frac{jV_2}{\kappa}$.

2. Secondly, we shall introduce the three-dimensional problem, which can be treated by a similar method as in the preceding section. The axis of r is taken on the free surface and u, w are the radial and the vertical components of displacement. The equatians of motion can be expressed by

$$\left.\begin{aligned}\rho\frac{\partial^2 u}{\partial t^2}&=(\lambda+2\mu)\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\Delta}{\partial r}\\&\qquad+2\mu\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\varpi}{\partial z}=0\\\rho\frac{\partial^2 w}{\partial t^2}&=(\lambda+2\mu)\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{\partial\Delta}{\partial z}\\&\qquad+2\mu\left\{1+c_1\frac{L}{H}+\left(c_2\frac{L}{H}\right)^2\right\}\frac{1}{r}\frac{\partial}{\partial r}(r\varpi)=0,\end{aligned}\right\}\quad\ldots\ldots(23)$$

where

$$\Delta=\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z},\qquad 2\varpi=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\quad\ldots\ldots\ldots\ldots\ldots(24)$$

From (23) and (24), we obtain

$$\left.\begin{aligned}\Delta&=A_1e^{-\alpha z}J_0(kr)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}pt,\\2\varpi&=B_1e^{-\beta z}J_1(kr)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}pt,\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots(25)$$

where

$$k^2=\alpha^2+h^2=\beta^2+j^2,\quad\ldots\ldots\ldots\ldots\ldots(26)$$

and h^2 and j^2 have the same meanings as those in (6).

The displacements corresponding to (25) can be expressed by

$$\left.\begin{aligned}u&=\left(\frac{kA_1}{h^2}e^{-\alpha z}+\frac{\beta B_1}{j^2}e^{-\beta z}\right)J_1(kr)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}pt,\\w&=\left(\frac{\alpha A_1}{h^2}e^{-\alpha z}+\frac{kB_1}{j^2}e^{-\beta z}\right)J_0(kr)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}pt.\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots(27)$$

As the boundary conditions we put

$$z=0, \qquad \lambda' \Delta + 2\mu' \frac{\partial w}{\partial z}=0, \qquad \frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}=0, \quad \ldots\ldots\ldots\ldots (28)$$

from which we get

$$\frac{B_1}{j^2}\Big/\frac{A_1}{h^2}=-\frac{2k\alpha}{2k^2-j^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (9')$$

$$(2k^2-j^2)^2-4k^2\alpha\beta=0. \quad \ldots\ldots\ldots\ldots\ldots\ldots (10')$$

The typical solutions at $z=0$ are written by

$$\left.\begin{aligned} u &= -\cos\left\{\frac{jV_2}{\kappa}\sqrt{k^2+\frac{2\pi c_1}{H}k+\left(\frac{2\pi c_2}{H}\right)^2}\,t\right\}\frac{\kappa\,(2\kappa^2-j^2-2\alpha_1\beta_1)}{j^2\alpha_1}J_1(kr), \\ w &= \cos\left\{\frac{jV_2}{\kappa}\sqrt{k^2+\frac{2\pi c_1}{H}k+\left(\frac{2\pi c_2}{H}\right)^2}\,t\right\}J_0(kr). \end{aligned}\right\} (29)^{5)}$$

Generalising these by means of Fourier's double integral, we find

$$\begin{aligned} u = &-q\int_0^\infty \cos\left\{l\sqrt{k^2+mk+n^2}\,t\right\}J_1(kr)\,kdk\int_0^\infty f(\sigma)\,J_0(k\sigma)\,\sigma d\sigma \\ &-q\int_0^\infty \frac{\sin\{l\sqrt{k^2+mk+n^2}\,t\}}{l\sqrt{k^2+mk+n^2}}J_1(kr)\,kdk\int_0^\infty F(\sigma)\,J_0(k\sigma)\,\sigma d\sigma, \quad \ldots\ldots (30) \end{aligned}$$

$$\begin{aligned} w = &\int_0^\infty \cos\left\{l\sqrt{k^2+mk+n^2}\,t\right\}J_0(kr)\,kdk\int_0^\infty f(\sigma)\,J_0(k\sigma)\,\sigma d\sigma \\ &+\int_0^\infty \frac{\sin\{l\sqrt{k^2+mk+n^2}\,t\}}{l\sqrt{k^2+mk+n^2}}J_0(kr)\,kdk\int_0^\infty F(\sigma)\,J_0(k\sigma)\,\sigma d\sigma, \quad \ldots\ldots (31) \end{aligned}$$

for the initial conditions that $w=f(r)$ and $\partial w/\partial t=F(r)$ at $z=0$.

If the initial disturbance is given by

$$f(r)=\frac{A}{a^2}e^{-\frac{r^2}{a^2}}, \qquad F(r)=\frac{B}{b^2}e^{-\frac{r^2}{b^2}}, \quad \ldots\ldots\ldots\ldots\ldots (32)$$

then, by means of the formula

$$\int_0^\infty e^{-\frac{\sigma^2}{c^2}}J_0(k\sigma)\,\sigma d\sigma=\frac{c^2}{2}e^{-\frac{k^2c^2}{4}}, \quad \ldots\ldots\ldots\ldots (33)$$

we obtain

$$u=-\frac{Aq}{2}\frac{\partial}{\partial t}\int_0^\infty\left(1-\frac{a^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}\,t}{l\sqrt{k^2+mk+n^2}}J_1(kr)\,kdk$$

5) These expressions are most suitable for the condition that the surface is always free from the stress.

$$-\frac{B}{2}\int_0^\infty\left(1-\frac{b^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}\,t}{l\sqrt{k^2+mk+n^2}}J_1(kr)\,kdk, \quad\ldots\ldots\ldots(34)$$

$$w=\frac{A}{2}\frac{\partial}{\partial t}\int_0^\infty\left(1-\frac{a^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}\,t}{l\sqrt{k^2+mk+n^2}}J_0(kr)\,kdk$$

$$+\frac{B}{2}\int_0^\infty\left(1-\frac{b^2k^2}{4}+\ldots\right)\frac{\sin l\sqrt{k^2+mk+n^2}\,t}{l\sqrt{k^2+mk+n^2}}J_0(kr)\,kdk. \quad\ldots\ldots\ldots(35)$$

Assuming that the initial disturbance is of the sharp nature and also that $c_1=0$ specially, we find the vertical displacement of the form:

$$w=\frac{A}{2}\frac{\partial}{\partial t}\int_0^\infty\frac{\sin l\sqrt{k^2+n^2}\,t}{l\sqrt{k^2+n^2}}J_0(kr)\,kdk$$

$$+\frac{B}{2}\int_0^\infty\frac{\sin l\sqrt{k^2+n^2}\,t}{l\sqrt{k^2+n^2}}J_0(kr)\,kdk. \quad\ldots\ldots\ldots(36)$$

If we put $k^2+n^2=k'^2$, then

$$w=\frac{A}{2l}\frac{\partial}{\partial t}\int_n^\infty\sin lk't J_0(\sqrt{k'^2-n^2}\,r)\,dk'$$

$$+\frac{B}{2l}\int_n^\infty\sin lk't J_0(\sqrt{k'^2-n^2}\,r)\,dk'. \quad\ldots\ldots\ldots(37)$$

Now we know

$$\int_n^\infty\sin lk't J_0(\sqrt{k'^2-n^2}\,r)\,dk'$$

$$=\int_{nr}^\infty\sin\frac{lt}{r}(k'r)\,J_0(\sqrt{(k'r)^2-(nr)^2})\frac{d(k'r)}{r}$$

$$\left.\begin{aligned}&=\frac{\cos\sqrt{\left(\frac{lt}{r}\right)^2-1}\;nr}{r\sqrt{\left(\frac{lt}{r}\right)^2-1}}, &&[(lt)^2>r^2]\\ &=0. &&[(lt)^2<r^2]\end{aligned}\right\} \quad\ldots\ldots\ldots(38)^{6)}$$

Hence we have

$$w=\frac{A}{2}\left(\frac{\kappa}{jV_2}\right)\frac{\partial}{\partial t}\frac{\cos\frac{2\pi c_2 x}{\Pi}\sqrt{\left(\frac{jV_2}{\kappa}\right)^2t^2-r^2}}{\sqrt{\left(\frac{jV_2}{\kappa}\right)^2t^2-r^2}}$$

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6) H. LAMB and S. SANO, *loc cit.*, 198.

$$+\frac{B}{2}\left(\frac{\kappa}{jV_2}\right)\frac{\cos\frac{2\pi\, c_2\, x}{H}\sqrt{\left(\frac{jV_2}{\kappa}\right)^2 t^2-r^2}}{\sqrt{\left(\frac{jV_2}{\kappa}\right)^2 t^2-r^2}}, \qquad \left[r^2<\left(\frac{jV_2}{\kappa}\right)^2 t^2\right]$$

$$=0. \qquad \left[r^2>\left(\frac{jV_2}{\kappa}\right)^2 t^2\right] \quad \ldots (39)$$

In this case, too, the leading part of the group is transmitted with the velocity jV_2/κ and the train which is caused by a single disturbance is also of a harmonic type with the amplitudes more quickly disappearing than in the case of a two-dimension. The nature of the successive periods and other characters are the same as those of the two-dimensional problem.

The annexed figures are illustrated as examples of the propagation in

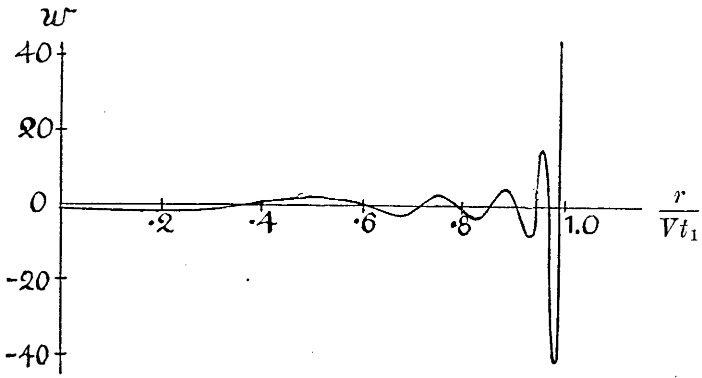

Fig. 5. Space-distribution of displacement at $t=t_1$ (Three-dimensions).

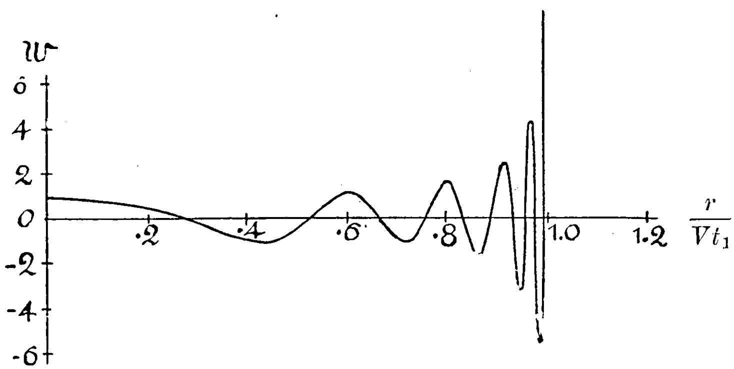

Fig. 6. Space-distribution of displacement at $t=t_1$ (Three-dimensions).

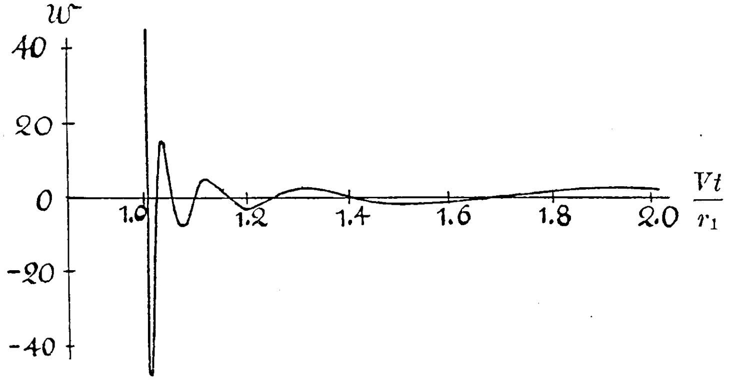

Fig. 7. Time-variation of displacement at $r=r_1$ (Three-dimensions).

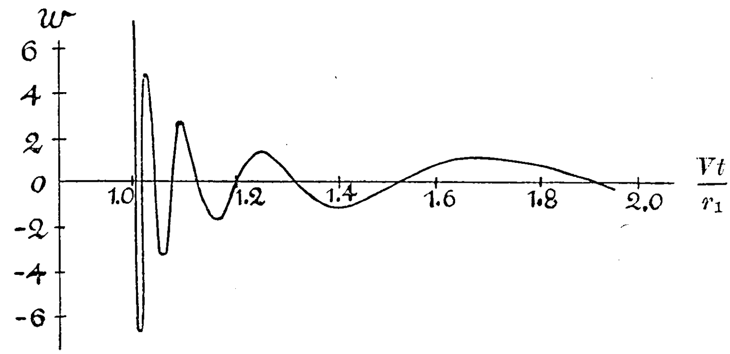

Fig. 8. Time-variation of displacement at $r=r_1$ (Three-dimensions).

three dimensions. Fig. 5 is the distribution of the displacements in space at the time t_1 due to the initial displacement at the origin, while Fig. 6 is the similar distribution due to the initial surface velocity. Fig. 7 is the time variation of the displacement at a certain point r_1 due to the initial surface displacement and Fig. 8 is that due to the initial surface velocity at the origin. In these V is written for jV_2/κ.

SUMMARY.

The principal results which have some importance on the seismology are enumerated as follows:

1. In spite of the application of a single disturbance at the origin of a semi-infinite body of a certain dispersive nature, the generated surface waves

are of a harmonic type.

2. The leading part of the train of these waves is propagated with the velocity of a special Rayleigh-type waves.

3. The periods of the successive oscillations of the harmonic displacements are of a gradually increasing nature.

4. The amplitudes of the successive oscillations are, in the case of two dimensions, of a gradually decreasing character, while in the three-dimensional problem they are more quickly decreasing.

5. The order of the length of the periods depends on the dispersive nature of the body, i.e. the elasticities, the effective thickness of the layer and some dispersive constants.

6. In certain dispersive waves there are abnormal regions of the earthquake movements at certain periodic distances from the epicentre.

In concluding this paper I wish to express my thanks to Mr. G. Nishimura for his kind assistance in preparing this paper.

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13. 任意の衝撃によつて起る週期的レーレー波

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半無限固體に或法則に從ふ波動分散性がある時、この固體面に勝手な撃力を加へても、週期的に繰替す調和波動が傳播されるといふ事柄がわかつた。

計算の方法は運動方程式の解を境界條件に適合させ、且つフーリエの積分によつて勝手な初動に對する一般化を行つただけに過ぎぬが、其結果は地震學上重要な種々の意味を持つ。地表面の微動や地震の主要動に續く振動などはこの問題に關聯して居る樣に思はれる。

研究の主な結果を摘錄すれば

1. 或波動分散性を有する半無限體上の一點に瞬時的外力を加へても發生する表面波動は調和波動となる。
2. この調和波動は群をなし、其首部は或レーレー波の速度を以て傳播される。
3. この調和波動の次々の週期は次第に長くなる傾向を持つ。
4. この調和波動の次々の振幅は二次元の問題では次第に減少し、三次元の場合には更に著しく減少して行く。
5. この調和波動の各週期の實際の長さは分散恒數及び彈性係數によつて決定される。
6. 或分散性波動では衝點から週期的の距離に異常震域が見出される。
