14. *On the Form of Volcanos.*

By **Torahiko TERADA.**

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(1) A volcano, in a purely mechanical point of view of a civil engineer, is a more or less irregular conical heap of rocky mass resting upon the ground, which we call the earth crust, and it may be his proper concern to inquire if the ground could endure or eventually yield to the superincumbent load of this gigantic masonry of the nature.

On the other hand, it is a noteworthy fact that the most volcanos, at least of this country, are surrounded by a conspicuous zone of depression of the surface crust. If such a tendency is universal, we are lead to the following alternatives, i.e. either the volcanic eruption is liable to occur in the zone of depression, or it is generally followed by a depression of the surrounding region. Nor is it improbable that both the factors may be cooperating in determining the characteristic topographical feature mentioned. For answering this question, many investigations will be wanted from geological, topographical, geophysical as well as mechanical sides. It is the last mentioned stand point which is taken by the present author in the following note. He will be much obliged by any criticism and information, especially from the side of geologists.

An actual example of the volcanic eruption which was accompanied with a general depression of the neighbouring region is afforded by the eruption of Sakurazima in 1914.[1] The very fact that this volcano is surrounded by a remarkable circumferential zone of depression below the sea level, taken together with the recent revelation of the existing activity makes us suspect that the mountain might have been repeating the similar process of eruption and depression through the geological ages since its first birth.

On the other hand, the heights of the most volcanos appear to be of the same order of magnitude, even when the differences are taken between the levels of the summits and the bottoms of the circumferential zones of depression (see Table I, 8th. column). As it seems difficult to

1) F. OMORI, *Bull. E.I.C.*, 8, No. 2 (1926), No. 4 (1920). CH. TSUBOI, *Bull. E.R.I.*, 7 (1926), 103.

assume that a single volcano may appear isostatically compensated,[2] the above fact may more plausibly be assumed as indicating an upper limit of the height of any volcano which may be sustained by the underlying crust.

The elasto-statical problem regarding the stability of a mountain has been already treated by such authorities as G. H. Darwin and A. E. H. Love. Prof. Nagaoka also treated some cases of the problem and compared his results with experiments.[3] It seems, therefore, interesting to regard the matter from a somewhat different side, i.e. with reference to the recent theories of plastic deformations, or those of the earth pressure. In a previous paper, the author pointed out some essential analogies between the sandy or earthy mass and the earth crust with regard to the plastic deformation of these matters, so that it will not be without some interest to treat a volcano as a load upon a sandy material.

(2) In order to examine and compare the geometrical forms of different volcanos with regard to the above point of view, it is necessary to take a procedure which may reduce the actual irregular form of a volcano and its surrounding region into its simplest essential outline. The method adopted here for this purpose is as follows.

The topographical maps in 1/50,000 scale, issued by the Military Land Survey Department, were utilized for the material of the investigation. On the summit of the volcano to be examined a point was chosen such that it may suitably represent the vertex of the ideal conical mountain. From this point, straight lines were drawn radiating towards the sixteen directions. The profile curve of the vertical section for each of these directions was constructed and thence the mean profile of these sixteen sections was calculated and plotted in a diagram as shown in Fig. 1. In carrying out the above method, the height for every kilometre of the distance from the vertex was read off from the map, and after obtaining the mean profile it was smoothed by calculating the means for the successive 5 km. intervals.[4]

2) In the case of a perfect compensation, the depression of the circumferential zone will be comparatively larger than the elevation of the summit. Such a form of volcano, however, has rarely been observed. An example which may suggest an approach to such a case is given by Lake Tôya (Fig. 7). At the centre of this lake an island is situated concentrically, of which the summit is of nearly equal height as the surrounding region, whereas the surrounding lake bottom is below the sea level.

3) His paper was communicated to the Colloquium Meeting of the Inst.; not yet published.

4) The laborious works of map-reading and calculation were mostly made by late Mr. S. Higasi, to whom the author feels much obliged. The later part of the works was carried out by Mr. Kawasima to whom the best thanks of the author are due.

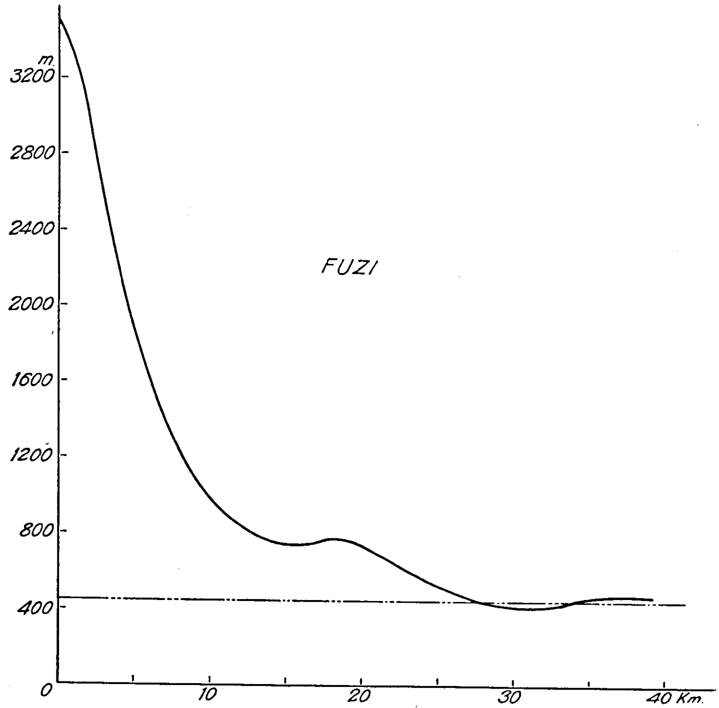


Fig. 1_1. Horizontal chain line gives the mean height Z_m.

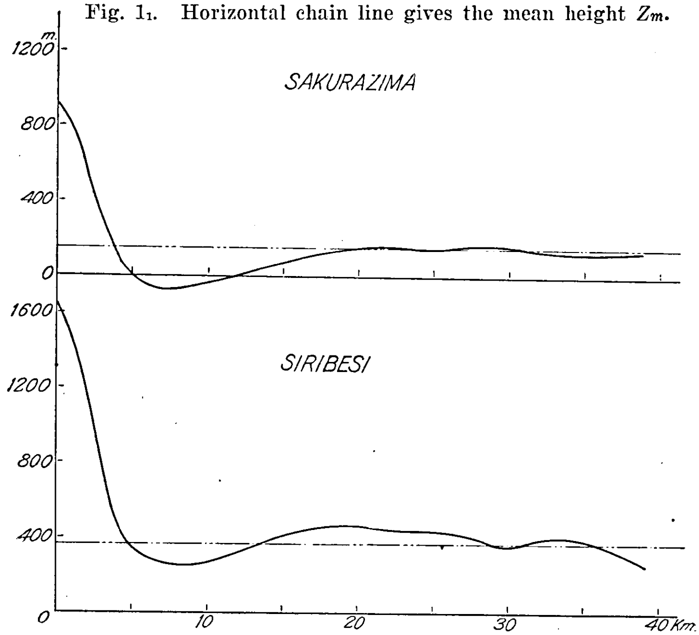


Fig. 1_2.

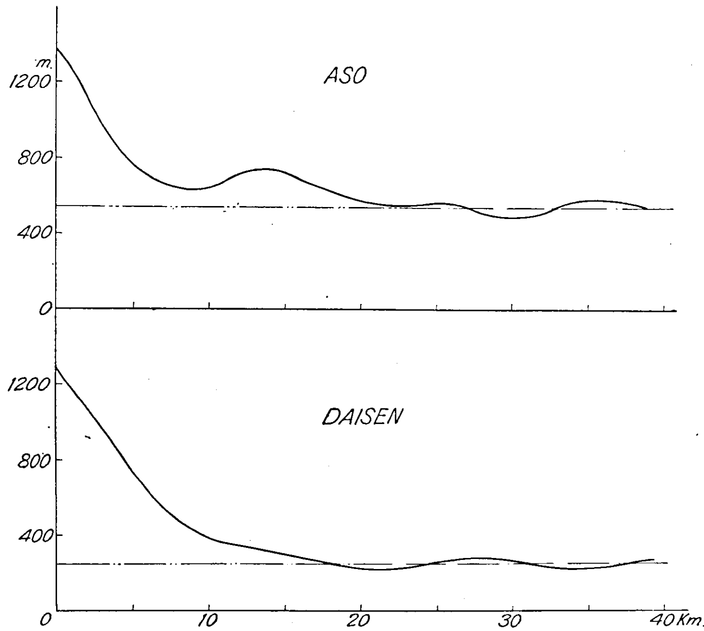

m.
1200
800
400
0
ASO
1200
800
400
0
DAISEN
10
20
30
40 Km.

Fig. 1_3.

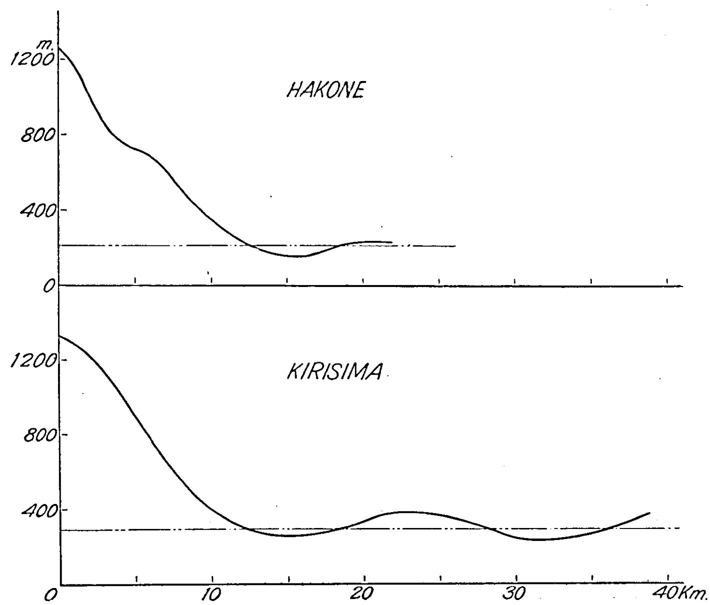
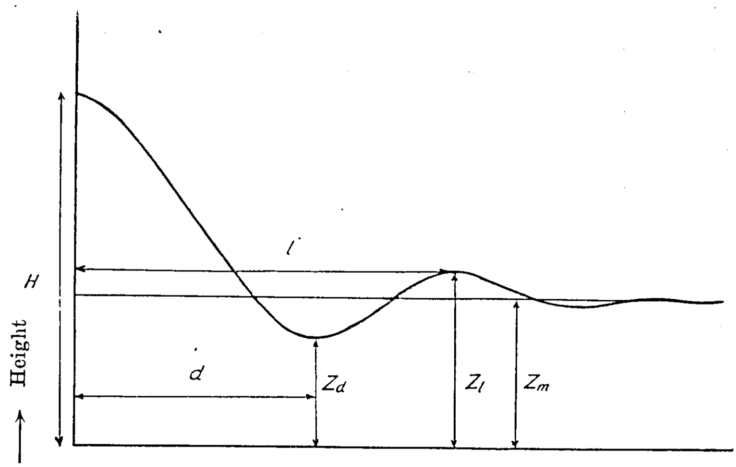

m.
1200
800
400
0
HAKONE
1200
800
400
0
KIRISIMA
10
20
30
40 Km.

Fig. 1_4.

By the above method, any irregularities which may be expressed as a periodic function of the azimuth are nearly eliminated and the zonal term will be made apparent. It may happen that the volcano is actually situated near the bottom of a conspicuous depression of the crust which existed before or formed after the genesis of the volcano. If, however, the mean profile show a marked narrow zone of depression just at the foot of the cone, while the land beyond it is generally of a uniform level, it will be plausible to assume some kind of genetical connection between the depression and the mountain, instead of regarding the phenomena as due to a mere chance; especially so, when the different volcanos have this characteristic form of profile in common.

It will be noticed that the irregularities due to the differences of the geological structure and the degree of erosion in different azimuth are also smoothed out by the above procedure and the purely mechanical feature, if such may exist, will be exposed in daylight.



⟶ Distance from the summit.

Fig. 2.

After examining Fig. 1, we may assume the typical form of a volcano to be represented schematically by Fig. 2, referring to which we define the heights H, Z_d, Z_l, Z_m and the distances d and l as the quantities suitable for specifying the characteristic features of the individual cases. Z_m is the mean height taken over the region lying within the zone 25–40 km. from the summit and may be assumed free from the disturbance due to the mountain, if such may exist near the foot.

TABLE I.

Heights H''', Z_d, Z_l and Z_m in m.; distance d and l in km.

Volcano	H'''	d	l	Z_d	Z_l	Z_m	$H'''-Z_d$	Z_l-Z_d	$H'''-Z_m$	Z_l-Z_n	$\frac{H'''-Z_d}{d}$	$\frac{Z_l-Z_d}{H'''-Z_d}$	$\frac{Z_l-Z_m}{H'''-Z_m}$
Huzi* (a)	3516	16	18·5	720	790	458	2796	70	3058	332	175	0·022	0·109
,, (b)	,,	31·5	36·5	400	500	,,	3116	100	,,	42	99	0·032	0·014
Siribesi	1653	8	19	240	480	364	1413	240	1287	116	177	0·170	0·090
Sakurazima	913	7	21	−80	180	151	993	260	762	29	142	0·262	0·038
Kirisima	1329	15	22	260	390	294	1069	130	1035	96	71	0·122	0·093
Aso	1377	9	13·5	620	760	546	759	140	831	214	84	0·185	0·258
Daisen	1169	21	27·5	220	280	250	949	60	919	30	45	0·063	0·033
Hakone	1274	16	20	160	230	217	1114	70	1057	13	70	0·063	0·012
Akagi	1313	13	(20)	540	(540)	—	773	(70)	(628)	—	59	(0·091)	—
Haruna	1154	12	(21)	400	(590)	—	754	(70)	(359)	—	63	(0·093)	—

* For Huzi the two depressions are taken, one (a) at the Five-Lakes Zone and the other (b) at the base of the mountain.

Referring to Table I, it will be seen that Z_m varies from 150 m., for Sakurazima, to 550 m., for Aso, generally increasing with the distance from the sea coast. Again, it will be noticed that generally $Z_l > Z_m$, while Z_a is sometimes $>$ and sometimes $< Z_m$.

Instead of taking the actual height of the summit, H, for the comparison intended, the mean height H''' was taken of the heights at the three distances 0, 0.5 and 1 km. from the summit, in order to smooth out the minor accidental irregularity near the summit. The values of H''' are given in Table I. It may be observed that H''' shows a tendency to increase with Z_m, though no simple quantitative relation cannot be established from the scanty data. It may again be remarked that Huzi appears exceptional in its height whether we take H''' or $H'''-Z_a$ for comparison.

(3) Suppose now that the volcano was heaped up on a flat ground with the height Z_m, until the substratum failed and the depression took place. In some cases, the entire cone may be depressd as shown by I in

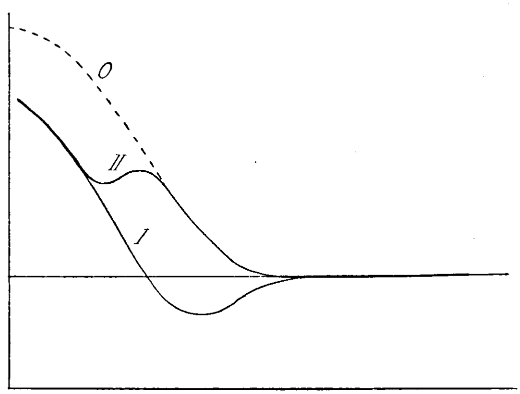


Fig. 3.

Fig. 3. Or, the depression may take place as shown by II in the same figure. Sakurazima and Huzi may be taken respectively as the representatives of the cases I and II.

In any case, we may roughly assume $H'''-Z_a$ as giving the limiting height of the volcano.[5] Table I will show that this height varies within a range of about 800–1400 m. excepting Huzi which appears abnormal in any respect.

The plastic problem for the case when a semi-infinite body is subjected

5) This assumption may be justified if we realize that the actual slope near the foot of a volcano is very small.

to a uniform surface loading over a circular area was treated by Hencky.[6] If σ_s be the maximum pressure sustained and σ_c is the strength for compression we have

$$\sigma_s = 3\cdot33\ \sigma_c.$$

In the case of the two dimensional problem treated by Prandtl,[7]

$$\sigma_s = 2\cdot57\ \sigma_c.$$

In the present case, the pressure at the base of the mountain is not uniform, but must decrease with the increasing distance from the centre. We assume, however, the uniformity of the pressure for the present, which may be allowed if only the order of magnitude of the strength is in question. For the same reason, we take roughly

$$\sigma_s = 3\ \sigma_c$$

in the round number.

Taking the mountain as a cone with the height h cm. the mean pressure σ at the base is $h\rho/3$ gr./cm^2., where ρ is the density. Assuming $\rho=3$, we obtain simply $\sigma=h$. If σ be given in kg/cm^2. and h in km., we have $\sigma=100h$. Hence,

$$\sigma_c = \frac{\sigma_s}{3} = \frac{\sigma}{3} \fallingdotseq 33\ h\ \ \text{kg/cm}^2.$$

Assuming $h = H''' - Z_a$, its value is mostly of the order of 1 km. Thus we have

$$\sigma_c \fallingdotseq 33\ \ \text{kg/cm}^2.$$

According to the data adopted commonly, σ_c for compression test of different rocks such as granite, marble, sandstone, etc. varies within a range 2000–3000 kg/cm^2. On the other hand, the corresponding value of σ_c for the surface loading of a loose sand layer was determined roughly by pressing upon a circular area of 5 cm. diameter and found to be of the order of about 0.05 kg/cm^2. Thence, we may say that the strength of the surface crust of the earth is of the order of 1/100 that of a compact rock piece and 1000 times that of a loose sand layer regarded as a kind of soft rock.

(4) Reissner[8] treated the problem of surface loading of a sand layer, for the case when one side of a straight line upon the surface is loaded

6) H. HENCKY, *ZS. f. angew. Math. u. Mech.*, **3** (1923), 241.
7) L. PRANDTL, *ibid.*, **1** (1921), 15.
8) REISSNER, *Proc. 1st. Intern. Congr. f. Appl. Mech.*, 1924, 297.

uniformly by p_1 while the other side is counterpoised by another uniform pressure p_2 ($< p_1$) such that the yielding is just prevented. For the condition of equilibrium, he obtained

$$\frac{p_2}{p_1}=\frac{1-\sin\rho}{1+\sin\rho}\,\epsilon^{-\pi\mathrm{tg}\rho},$$

where ρ is the angle of repose of the sand mass. Fig. 4 shows an approximate form of the curve given by $y=p_2/p_1$, $x=\rho$.

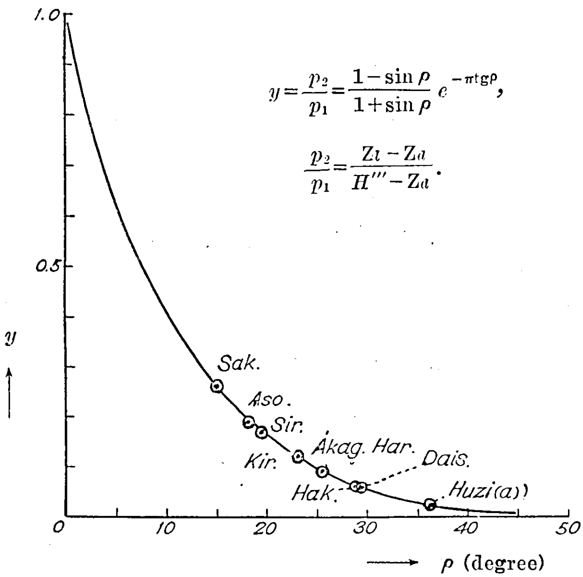


Fig. 4.

Suppose that the earth crust, considered as a sandy mass, has yielded under the load of volcano and is just sustained in equilibrium by the load of the surrounding region upheaved as the result of the deformation. In that case we may assume very roughly

$$\frac{p_2}{p_1}=\frac{Z_l-Z_a}{H'''-Z_a}.$$

The values of this ratio for different mountains are given in Table I. From these values of the ratio we may find the corresponding values of ρ from the curve in Fig. 4, or we may arrange the different volcanos upon the curve of the Figure. It will be seen that for Sakurazima, Siribesiyama, Kirisima and Aso, the values of ρ are generally much smaller than for the ordinary sand mass.

If we take

$$\frac{p_2}{p_1}=\frac{Z_l-Z_m}{H'''-Z_m},$$

the results are somewhat modified though not essentially. At any rate, it may be of some interest that the ratio of the pressures turns out to be of a right order of magnitude.

(5) According to the above hypothesis, Z_l-Z_d, which we denote by ζ, gives a measure of the plastic depression. A question may arise whether the depression occurred suddenly as in the case of a brittle material, or gradually as in the case of a plastic substance. Let us assume

$$\zeta=\zeta_0(1-e^{-\lambda t})\,e^{-kt},$$

where t is the time and ζ_0, λ, k are constants. The last factor is taken to account for the erosion.

On the other hand, put for the measure of the mountain slope

$$\xi=\frac{H'''-Z_d}{d}=\xi_0\,e^{-kt}.$$

Eliminating t, we obtain

$$\frac{\zeta}{\zeta_0}=\left\{1-\left(\frac{\xi}{\xi_0}\right)^{\mu}\right\}\frac{\xi}{\xi_0},$$

where $\mu=\lambda/k$.

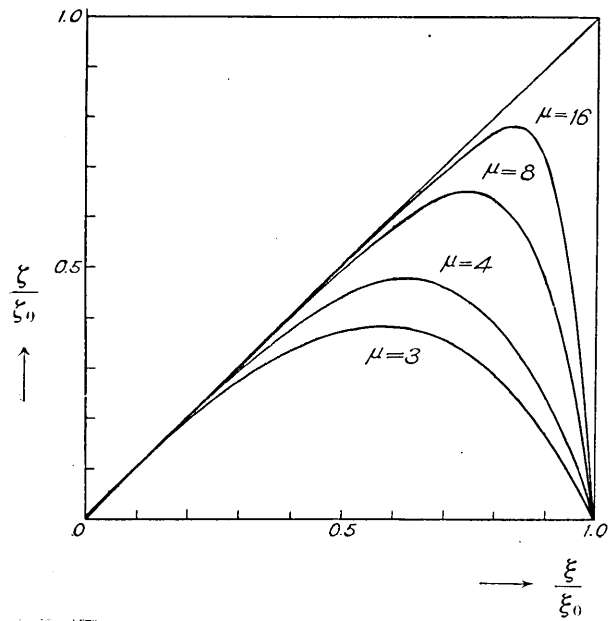


Fig. 5.

Fig. 5 shows (ζ/ζ_0)-(ξ/ξ_0) curve for different values of μ. In comparing this with Fig. 6 in which the actual relations between ζ and ξ are plotted, we may assume the value of ξ for Huzi(a) and Siribesiyama to be very near the maximum possible value ξ_0. In that case, we may infer that

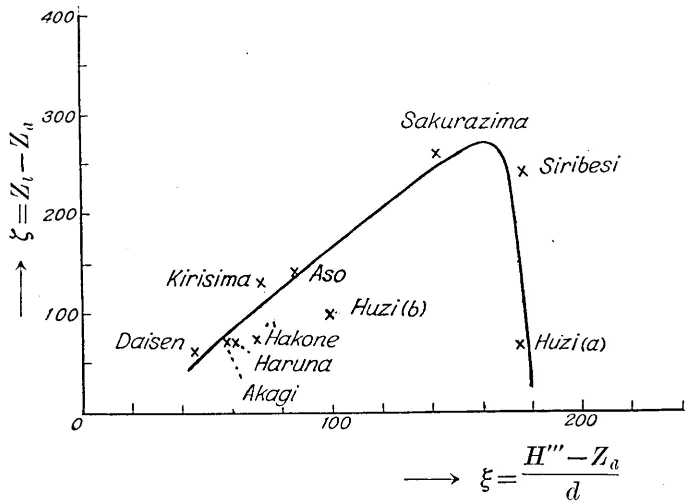


Fig. 6.

μ is rather large. This will mean that the depression took place rather rapidly compared with the progress of erosion. It may also appear that the Lake Zone of Mt. Huzi has not yet completed its depressing movement in these days and may still be depressed by a sensible amount in an indefinite future.

Apart from such a hypothesis, it will be noticed that Huzi is an exceptional phenomenon even with respect to the above relation between ζ and ξ.

(6) The exceptional height of Huzi may also be explained by assuming that the strength of the crust varies widely with the geological ages and that the age of Huzi is decidedly different from the ages of the other volcanos here compared. Huzi is, indeed, considered by many geologists as a young volcano. The assumption that in the age when the volcanic activity was at its height, the strength of the crust was much smaller than in the later period, is also plausible.

In the above the slope ξ was taken as a measure of the age of the mountain which is given by

$$t = \log \frac{\xi_0}{\xi}.$$

It appears (Fig. 6), therefore, according to this hypothetical measure, that Huzi (a), i.e. its Lake Zone, and Siribesiyama are the youngest. Daisen appears oldest.[9] Aso and Kirisima may be said to be of a middle age, while Sakurazima appears a little younger.

(7) For the geophysical significance of the circumferential depression of a volcano, the following alternative may perhaps be suggested. Suppose that a vertical cylindrical column of the earth crust, with the radius r_1 and the height equal to the thickness of the crust, is subjected to a change of density say from ρ to $\rho-\Delta_1\rho$, by a chemical transformation or any other cause. In expanding, the cylindrical column will press upon the surrounding crust and consequently the density of a concentric cylinder with the internal and external radii r_1 and r_2 respectively, will be change from ρ to $\rho+\Delta_2\rho$. If the isostatic compensation be effected, the ratio of the elevation of the central cylinder to the depression of the surrounding one will be equal to the ratio $\Delta_1\rho$ to $\Delta_2\rho$.

It r_1 expands to r_1+s, we have

$$\Delta_1\rho=\frac{2s}{r_1},$$

$$\Delta_2\rho=\frac{2r_1s}{r_2^2-r_1^2},$$

$$\frac{\Delta_1\rho}{\Delta_2\rho}=\left(\frac{r_2}{r_1}\right)^2-1.$$

We may take $r_1=\alpha d$ where $\alpha<1$ and $r_2=\beta h$ where $\beta>1$. On the other hand, we may assume roughly

$$\frac{\Delta_1\rho}{\Delta_2\rho}=\frac{H'''-Z_m}{Z_m-Z_a}.$$

Hence, we must have

$$\frac{H'''-Z_m}{Z_m-Z_a}=\left(\frac{\beta l}{\alpha d}\right)^2-1.$$

The assumption is, however, not valid even qualiltatively, since Z_m-Z_a is negative for the most volcanos. Even if we take Z_l-Z_a, which is positive, instead of Z_m-Z_a, the relation as above deduced does not hold in the least.

(8) If in the above assumption we take s negative, then Δ's will

9) Daisen belongs to the toroide-type, according to Schneider's classification, while the others are conides. In the present discussion, however, the difference of the type may be put out of account when only the order of magnitude of a physical constant of rock material is concerned.

become also negative. Such a case may possibly occur, in which case we will expect a central depression surrounded by a zone of elevation. It may, therefore, be suspected that some of the lakes such as Towada or Tazawa might have been produced by a similar process. The mean profile of the region with the lake as the centre was constructed in the same manner as in the case of volcanos. The results are shown in Fig. 7. It

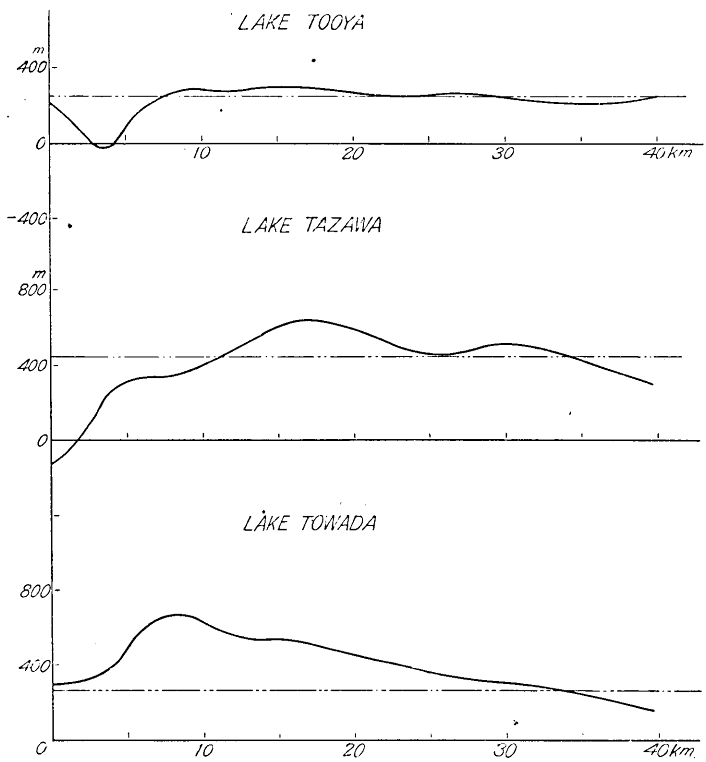


Fig. 7_1. ——- - -—— Mean Height Z_m.

will be seen that the profile curves may be compared with the inverted profiles of some volcanos, the circumferential elevation of the formers corresponding to the zonal depressions of the latters. The zones of elevation in these examples seems, however, to be of too wide extents to be explained by the above assumption. It seems improbable that the general elevation is merely a secondary phenomenon caused by the central depression as the primary one. Here, the alternative hypotheses examined in the cases of volcanos seem to fail. There remains a possibility that a local transformation of an extended magma field, resulting in a considerable change in the density of a nuclear spot may be associated with a density change in opposite sense in the surrounding region of wide extent.

It will of course be premature to try to decide among these various possibilities on the basis of the simple mechanical considerations as above expounded. The discussions may, however, at least be of some use in suggesting a further line of investigation regarding the allied problems.

(9) Lastly, we will take the case of the twin volcanos Akagi and Haruna. The similar mean profile was constructed for each of the pair, as given in Fig. 8. In both the cases, the ground rises continually from

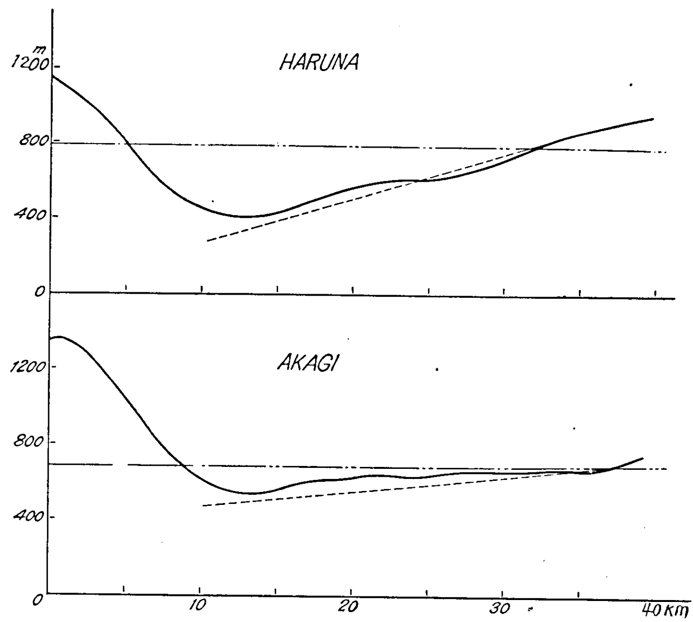


Fig. 8. —·—·— Mean Height Z_m.

about 15 km. from the centre, i.e. the summit, to a distance greater than 40 km. This means that the volcano is situated at the bottom of a conspicuous depression area of large extent. Drawing the dotted line in the Figure as an ideal profile of the general depression, the difference of the curves in full line and dotted line is found to show the characteristic form given by Fig. 2. Estimating the values of d, l, Z_a and Z_l from this difference curve, as given in Table I enclosed in bracket, the position of the mountain in Fig. 6 was found. It will be seen that the "age" turns out rather old. The values of ρ in Fig. 4 fall also near those for Daisen and Hakone.

Compare again the profiles of these two mountains with the inverted profiles of the lakes given in Fig. 7. An essential similarity of feature can scarcely be overlooked. It strongly suggests the presence of some magmatic process which operated in opposite senses may give rise to such a pair of profiles which are mutually in inverted sense as in the case of a mirror image. This again leads to the consideration expounded at the end of § (8), regarding to a special type of magmatic differentiation.

(10) From all what has been said in the preceding paragraphs, we are lead to the following summary:

(A) The circumferential zone of depression at the foot of some valcanos may probably be due to the yielding of the earth crust by the load of the mountain. This suggest us a possible way of the "hardness test" of the earth crust.

(B) Some volcano in the middle of a large area of depression, on the one hand, and some lake situated at the centre of an extensive area of elevation, on the other hand, seem to be genetically correlated to each other in opposite senses.

The theoretical discussions above given may appear rather speculative and grossly qualitative for some of the readers, but may contain, I hope, some germs of what might in some future be developed into a more comprehensive theory of the crustal deformation at large.

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14. 火 山 の 形

寺 田 寅 彦

火山を取巻く凹地帯は、火山の荷重によつてプラスティックな地盤が陥没したものと假定すれば、地盤の堅さがどの位になるかといふことを、Hencky の理論を應用して計算すると、普通岩石試片と土壌の類との中間の強さが算出される。又砂の場合の Reissner の理論を應用すると、其の摩擦角の範圍は色々の山について 15°-35° 位となる。