4. *Experimental Investigations of the Deformation of Sand Mass. Part III.**

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In the previous reports, the results of some preliminary experiments were described, in which a horizontal layer of sand contained in a rectangular vessel was subjected to a deformation such as to develop a series of step-faults, by horizontally receding or pressing one of the vertical side walls. Since the ordinary photographic camera was used in those experiments for photographing the successive stages of deformation, it was difficult, as already mentioned in the first report, to follow the process of deformation of a given sand layer by sufficiently close steps, with sufficiently small interval of time between the successive photograms. To avoid this difficulty, a kinematographical camera[2] was employed for the further experiments which we are going to report in the following.

The other apparatuses used in the present experiments are the same as those described in the first report. The same two kinds of sand as described in the second report were used, i.e. the finer and the coarser, both in the two different states of packing, i.e. close and loose. The velocity of motion of the moving wall was chosen at 0.02 or 0.08 cm./sec. As far as the present results show, we can find no systematic difference of the results due to the difference of the velocity of this order, so that we will make no distinction between the two cases in the following description. The successive photograms were taken at a constant interval corresponding to each 2 mm. of the displacement of the moving wall.

An enlarged print was taken of each photographic film obtained and subjected to examinations in the manner which will be described below.

The two cases of receding and pressing the movable side wall were respectively repeated for the verification of the previous results. The following four cases were treated separately:

1. Finer sand, closely packed: F.C.
2. Finer sand, loosely packed: F.L.

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* For the sake of future reference, the first report, this *Bull.*, 4 (1928), 33, and the second one, ibid., 6 (1929), 109, will be called Part I and II respectively.

1) Ica Kinamo was employed.

3. Coarser sand, closely packed: C.C.
4. Coarser sand, loosely packed: C.L.

For the sake of brevity we will denote these cases by the abbreviation given above to the right of the lines. For each of these cases, two or three series of the experiments were repeated for the purpose of control. The individual results of the repeated experiments for the same case varied quantitatively within a sensible range. This is unavoidable from the nature of the phenomena in which some instabitity plays an essential rôle and a slight heterogeneity in the state of packing or an accidental irregularity in the motion of the moving wall may bring about a finite difference in the results. In order to obtain any quantative average result, a great number of experiments must be repeated which would require a considerable expenditure of time and labour. Still, in the results of the present experiments, there are some essential features characteristic to different cases which may be distinctly observed in spite of these statistical variations. In the following we will take only one example for each of the above four cases and will describe those features regarded qualitatively invariant for the respective case considered.

(I) Receding Wall.

In the previous report of the experiments with the receding wall,[2] the form of the slip line was expressed by a simple algebraic curve which seemed to fit the observed curve, except near the upper and the lower ends. At the lower end of the curve the formula fails, as it makes the curve tangent to the bed plate, while the actual curve is by no means of such a form. The use of the formula was made merely with the purpose of comparing the relative inclination and curvature of different curves by comparing the coefficient k and the index n respectively.

For examining the results of the present experiments, we tried another method of reduction which seemed to suit our purpose better than the previous one.

In most cases, the slip line is nearly straight in its upper part down to about one half or more of its depth so that we may define a straight line touching or coinciding nearly with that part (Fig. 1, AB). On the other hand, a finite portion of the slip line at its lowest extremity may also generally be regarded as approximately straight so that we may always draw a definite tangent at this lower branch (Fig. 1, CD). It cannot be denied

2) We avoid the use of the words, "active" and "passive pressure" which are in common use among civil engineers.

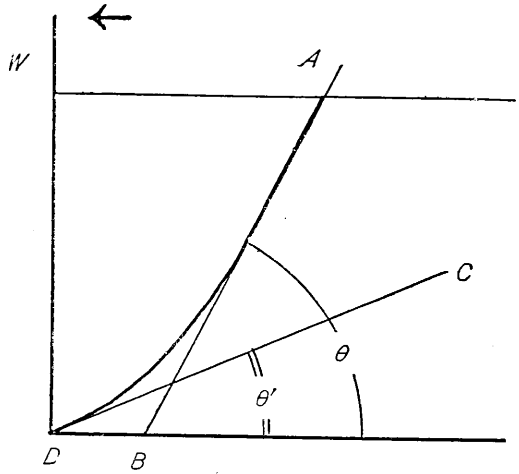

Fig. 1.

that in the case of the latter tangent, the error in determining its direction is sensibly larger than in the case of the upper tangent, but as we take a larger number of successive photograms, the general mode of its variation

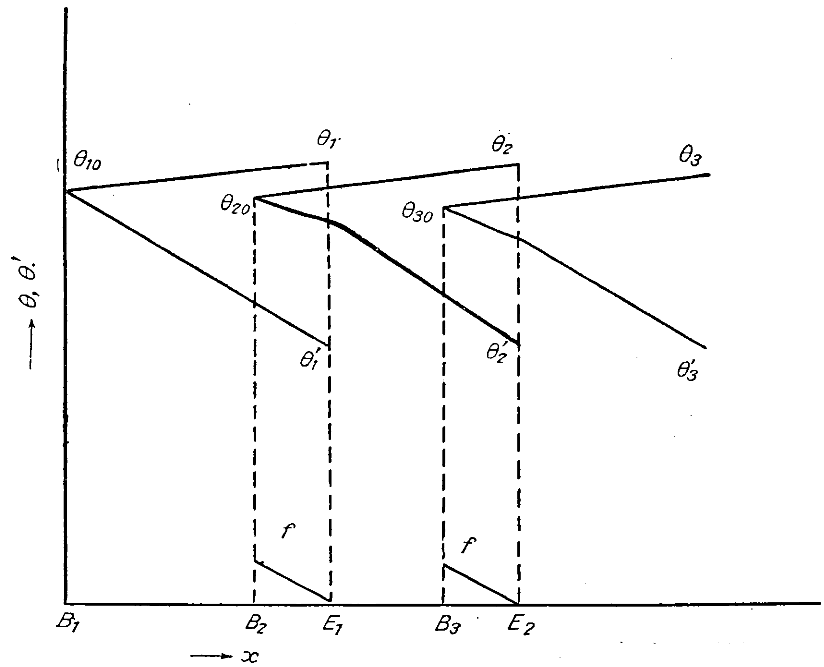

Fig. 2.

during the motion of the wall may be determined with a degree of accuracy just wanted for the present purpose. As shown in the figure the angle of inclination of the two tangent will be denoted by θ and θ' respectively.

The variation of the form of the slip line during the receding motion of the wall may then be conveniently illustrated by a digram in which the abscissa stands for the displacement, x, of the moving wall from its initial position at rest and the ordinates gives the value of θ and θ' respectively. Plotting the two angles in the same diagram we obtain two branches of curves, the upper one corresponding to the upper tangent and the lower to the lower (Fig. 2 is a schematical representation of such diagram).

As already mentioned in Part I, it may occur in some cases that two slip lines coexist and are simultaneously[3] active. In such a case, two pairs

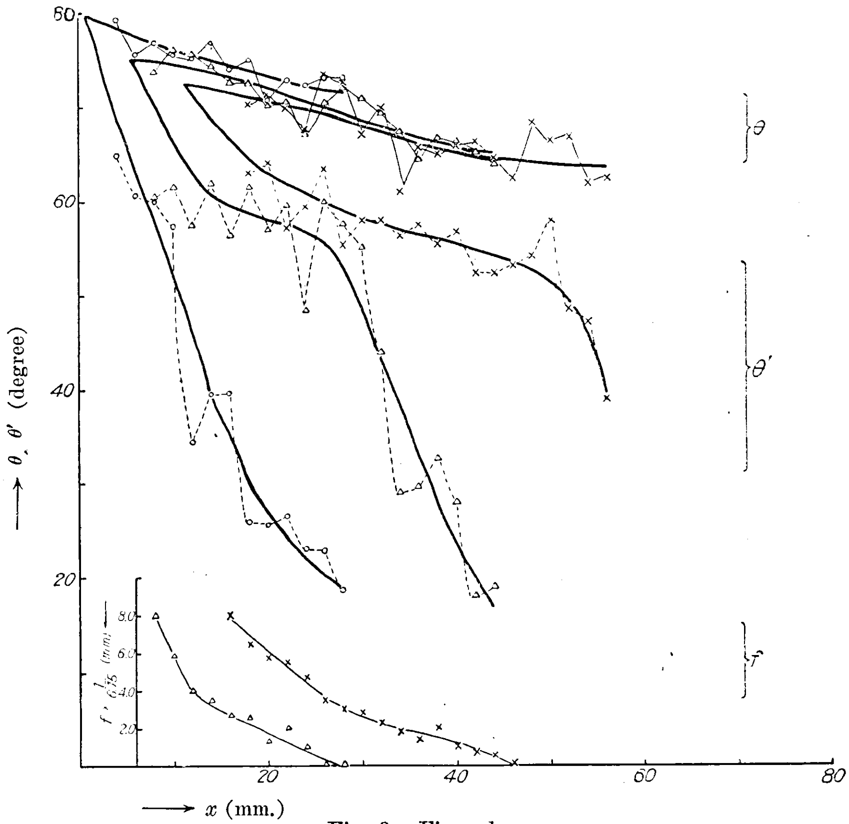

Fig. 3. Fine, close.

3) Or possibly alternatively with short irregular periods; at present, this point cannot be decided.

of θ and θ' (i.e., for example, θ_1, θ_1', θ_2 and θ_2'), are obtained as shown in Fig. 2 between $B_2 E_1$ or $B_3 E_2$. In some cases, especially in the case of close packing, the foot of the second (i.e. later developed) slip line does not pass the foot of the moving wall, but starts from a point of the wall at some height, say f, from the bottom (Fig. 15). In such a case, the variation of f with x will be given in the same diagram as f-curve below θ-and θ'-curves.

At the initial stage the slip line is approximately of a straight line so that θ-and θ'-curves intersect with each other at $x=0$,[4] whence the difference $\theta-\theta'$ increases with x showing the increasing curvature of the slip line. The angle between θ-and θ'-curves on the diagram is, therefore, a measure of the rate at which the curvature is increasing with x.

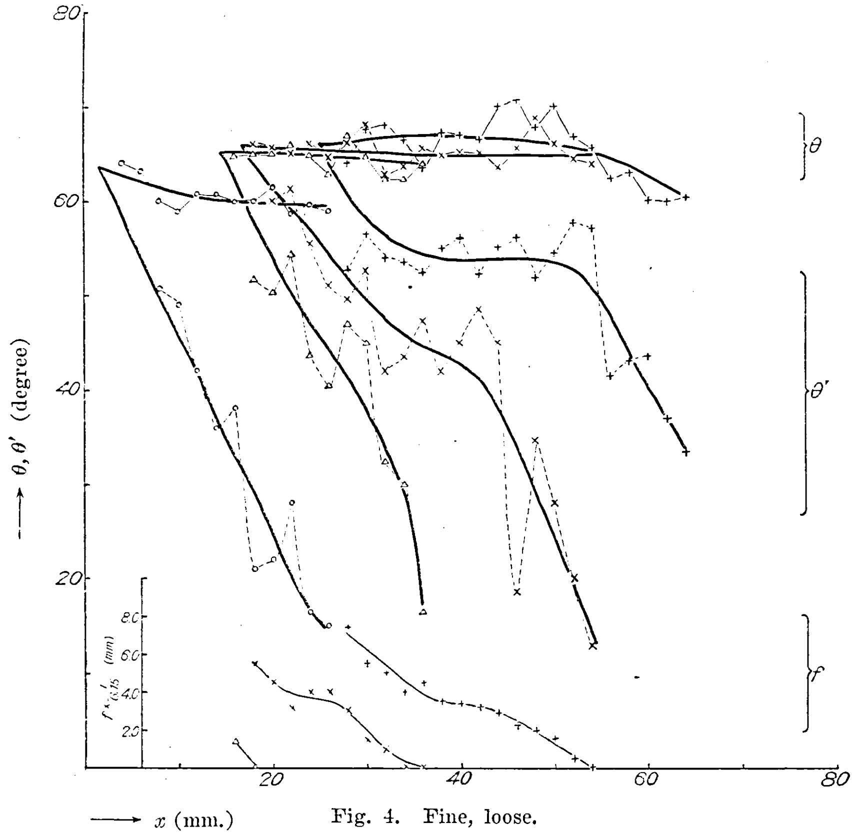

Fig. 4. Fine, loose.

4) The theory of earth pressure gives $\theta=\theta'=\frac{\pi}{4}+\frac{\rho}{2}$ where ρ is the angle of friction.

With these explanations kept in mind, the different characteristics of the different cases will be easily understood from Figs. 3, 4, 5 and 6 which

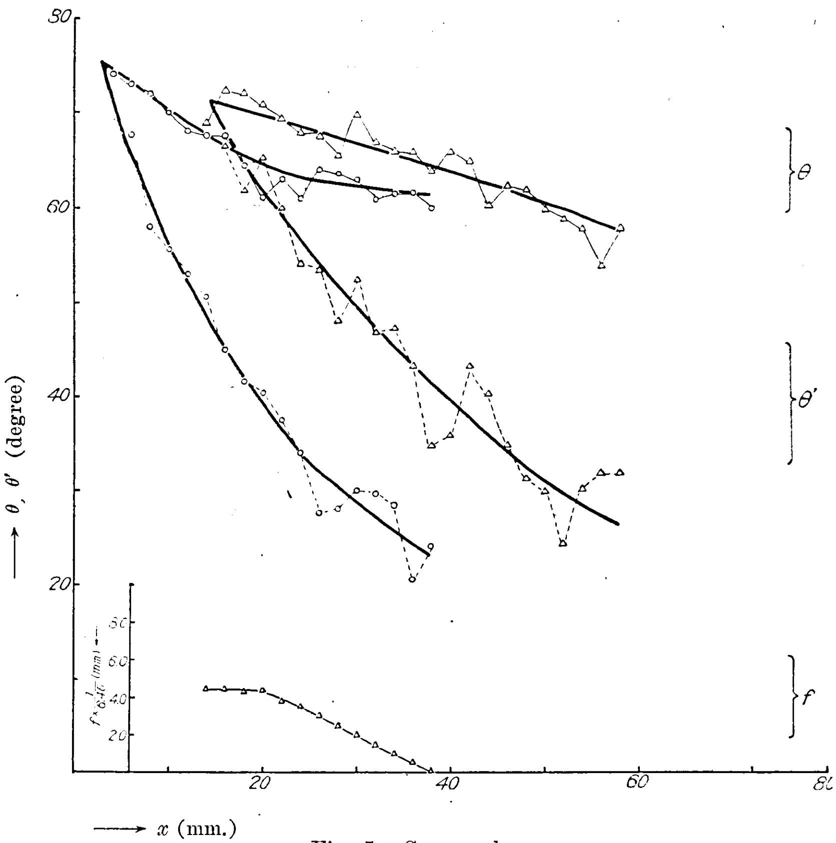

Fig. 5. Coarse, close.

respectively represent the above four cases. In these figures, the actually observed values of the angles are plotted by different marks corresponding to different slip lines and connected by thin full and dotted lines. Besides, the thick lines are drawn to show the gross features of the variations which are to be compared with Fig. 2 above explained.

From these figures, we may point out the following facts:

(1) In the case of close packing (Fig. 3 and 5), the initial value of $\theta=\theta'$ decreases successively for the successive slip lines, while in the case of loose packing it generally increases though not conspicuously. This fact may be understood if we assume that θ is generally greater for the closer packed state than for the looser one and that the sand mass initially in the

closest state is gradually loosened by the disturbance, whereas the loosest state may tend to settle down in some measure on account of the disturbance.

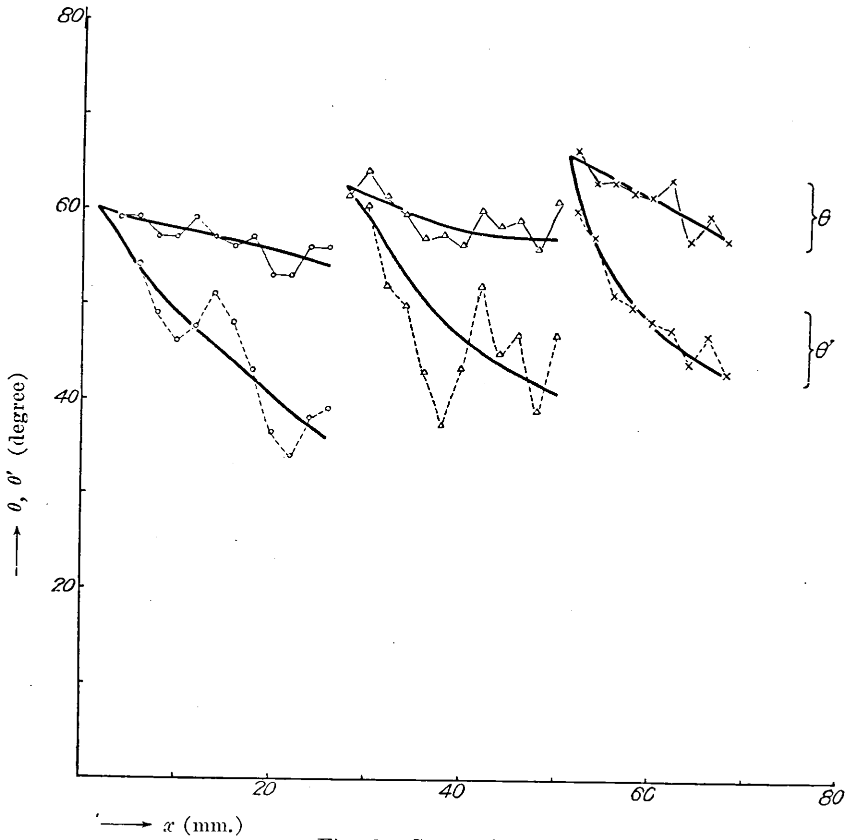

Fig. 6. Coarse, loose.

(2) The occurrence of a pair of simultaneously active slip lines is common in the three cases, F.C., F.L. and C.C., but absent in the case of C.L., i.e. the coarser sand in loosely packed state. As the development of a new slip line corresponds to the formation of a new crack in the case of an ordinary solid body, we may say that in the case of C.L. the sand between the wall and the existing first slip line behaves most "fluidous"[5] compared with the other cases, or on the other hand it may also be said to be greatest

5) In other words, in C.L. state the entire mass is beset with an infinite number of infinitesimal slip lines, while, in the other cases, the shearing deformation has a tendency to be concentrated into a small number of slip lines with finite slips.

in strength, though it may sound paradoxical. The mechanism by which the simultaneous slip line may be produced will be discussed in a later section.

(3) In the case of the finer sand, i.e. F.C. and F.L. (Fig. 3 and 4), it will be generally seen that as long as the two slip lines are simultaneously active, the earlier, i.e. the lower nth. slip line is rapidly decreasing θ_n' and thus increasing $\theta_n-\theta_n'$, while $\theta_{n+1}-\theta_{n+1}'$ for the secondary or upper $(n+1)$th. simultaneous slip line varies only slowly with x. As soon as the primary slip line is stopped, the value of $d(\theta_{n+1}-\theta_{n+1}')/dx$ for the new slip line is suddenly increased nearly up to the amount formerly shown by the first slip line.

(4) In the case of C.C. (Fig. 5), the absolute value of $d\theta'/dx$ is generally small compared with the cases F.C. and F.L. and the coexisting two slip lines behave quite similar to each other.

(II) Pressing Wall.

Preliminary experiments of the case in which the movable side wall is horizontally pressed upon the vertical lateral side face of the sand layer have been described in Part II, in which the angles θ and ω (Fig. 7) giving the inclinations of the two slip lines ab and bc were measured and their

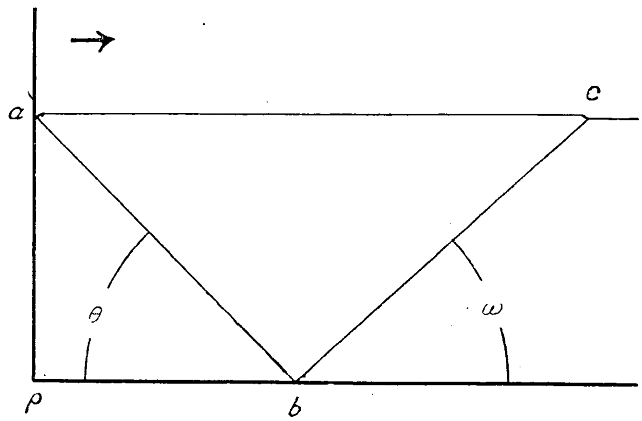

Fig. 7.

variations were followed as functions of the displacement, x, of the moving wall. The same two angles were measured also in the present experiments and plotted in Figs. 8, 9, 10 and 11.

The line bc (Fig. 7) is sharply defined in photograms and there is no ambiguity in determining ω. On the other hand, the region near the line ab is really to be considered as run through by a bundle of slip lines, as

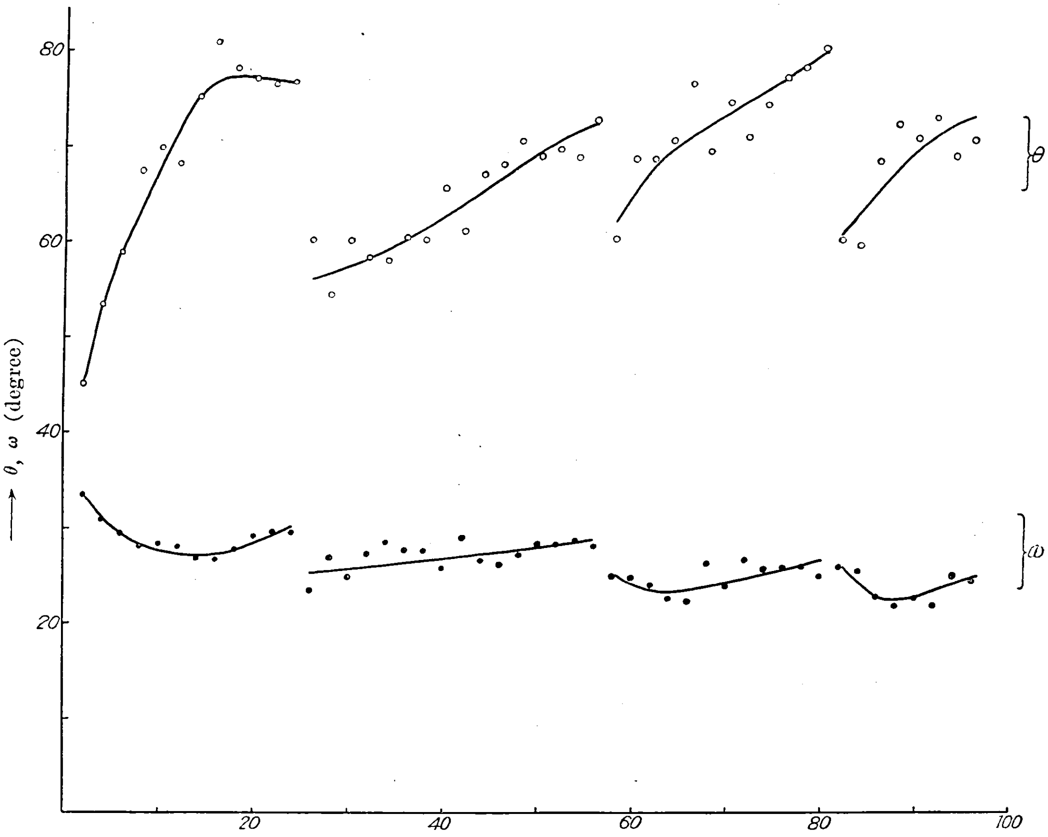

Fig. 8. Fine, close.

it is this portion which is undergoing the most conspicuous deformation, as already mentioned in Part II. Defining, however, the line *ab* as a straight line joining the foot of the line *bc* to the point *a*, the angle θ is determined and this line *ab* will nearly coincide with the boundary of the wedge-shaped sand mass *apb* moving with the moving wall, as if it were a rigid body fixed to the wall.

Fig. 12 is a schematical diagram showing the general feature of variation of θ and ω with x. When the wall begins to move on, i.e. at $x=0$, it seems that the initial values of θ and ω are nearly equal to each other. This fact is difficult to be directly verified from the photograms, but may be seen by the extrapolation of θ-and ω-curves. Besides, it is just what may be expected from the theory of plastic deformation[6] in general.

6) See the discussion in the later section.

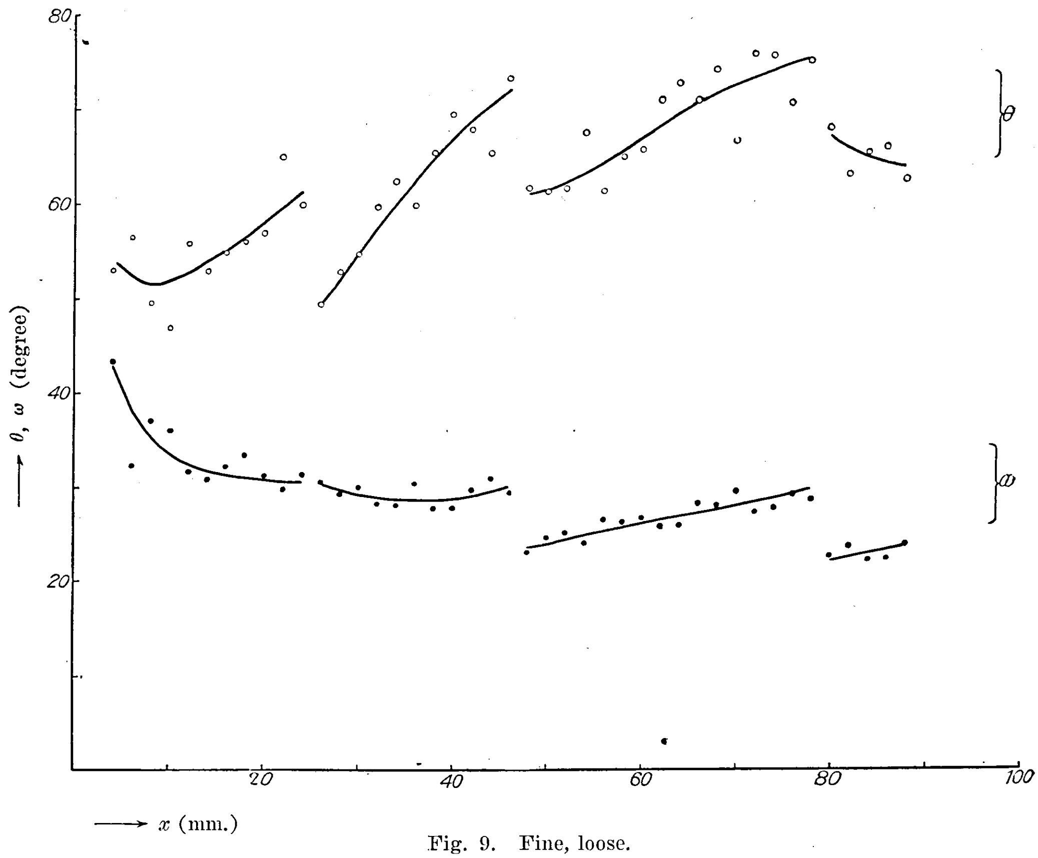

Fig. 9. Fine, loose.

At the first stage of the increase of x, ω decreases while θ increases. As x increases ω attains a minimum and, thence, $d\omega/dx$ changes its sign, while $d\theta/dx$ remains generally positive. As x increases further up to a critical point, $x=x_1$ say, θ has attained its greatest value θ_{m1} and the second slip line shoots forth, of which the initial values, θ_{02} and ω_{02} in Fig. 12, are generally less than the respective final values θ_{m1} and ω_{m1} for the first slip line. For the further increase of x, both the rates $d\theta/dx$ and $d\omega/dx$ are positive. At $x=x_2$, the discontinuity again occurs at which the greatest values θ_{m2} and ω_{m2} are reached and the third slip line appears with their initial inclinations θ_{03} and ω_{03} respectively less than θ_{m2} and ω_{m2}.

Referring to Fig. 8, 9, 10 and 11, it will be seen that the limiting values θ_0 and θ_m generally increase with the order numbers of the slip lines. On the other hand, the variation of ω_0 and ω_m with the number of the slip lines

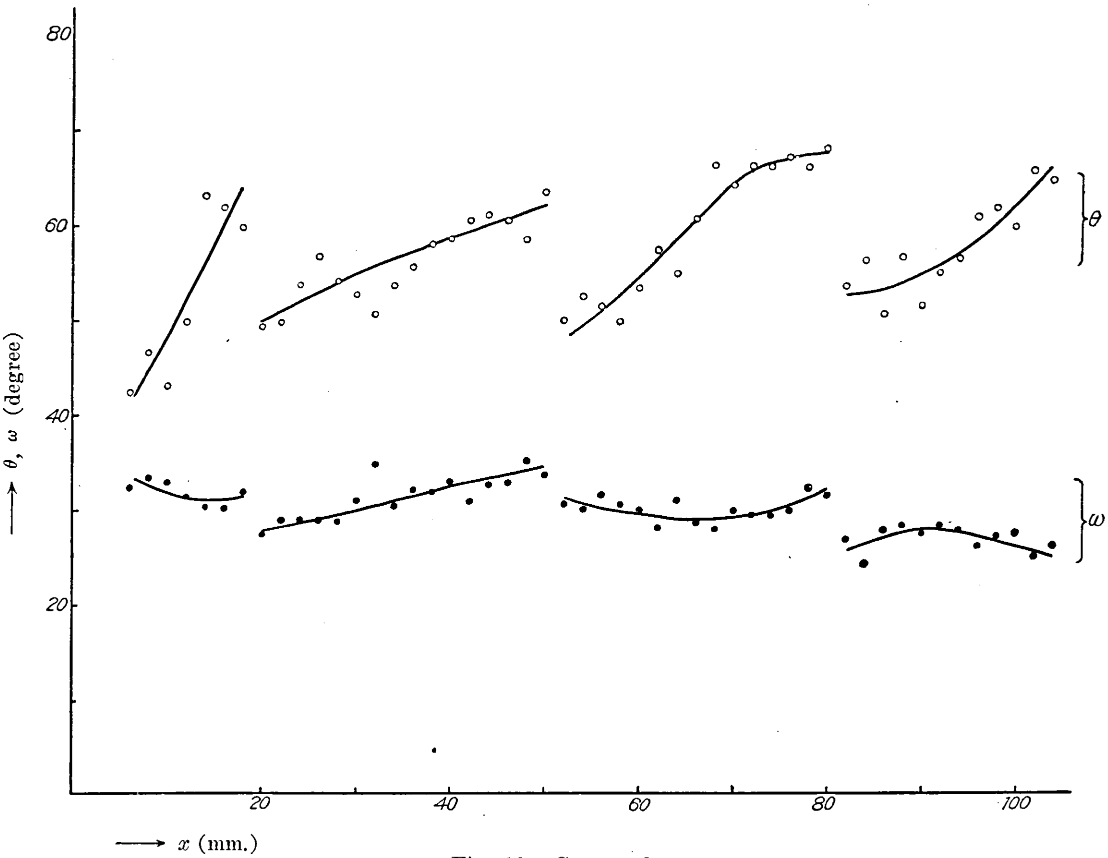

Fig. 10. Coarse, close.

is not remarkable, though a general tendency of gradual decrease may be perceived.

Thus, the qualitative characteristics of θ-and ω-curves are almost the same for the four different cases as far as above pointed out. Examining the four cases separately we may remark the following points.

(1) Comparing Fig. 8 with Fig. 10, or Fig. 9 with Fig. 11, it is interesting to observe that θ is generally greater for the finer sand than for the coarser one, while ω is smaller for F sand than for C sand. Moreover, the sum $\theta+\omega$ varies within a range nearly similar for all of the four states, i.e. not much dependent on the grain size and the state of packing. Fig. 13 shows the variations of $\theta+\omega$ for the four cases. This angle may also be considered to corresponds to the angle between the two intersecting slip planes in the lowest layer, which is a quantity defining the nature of a

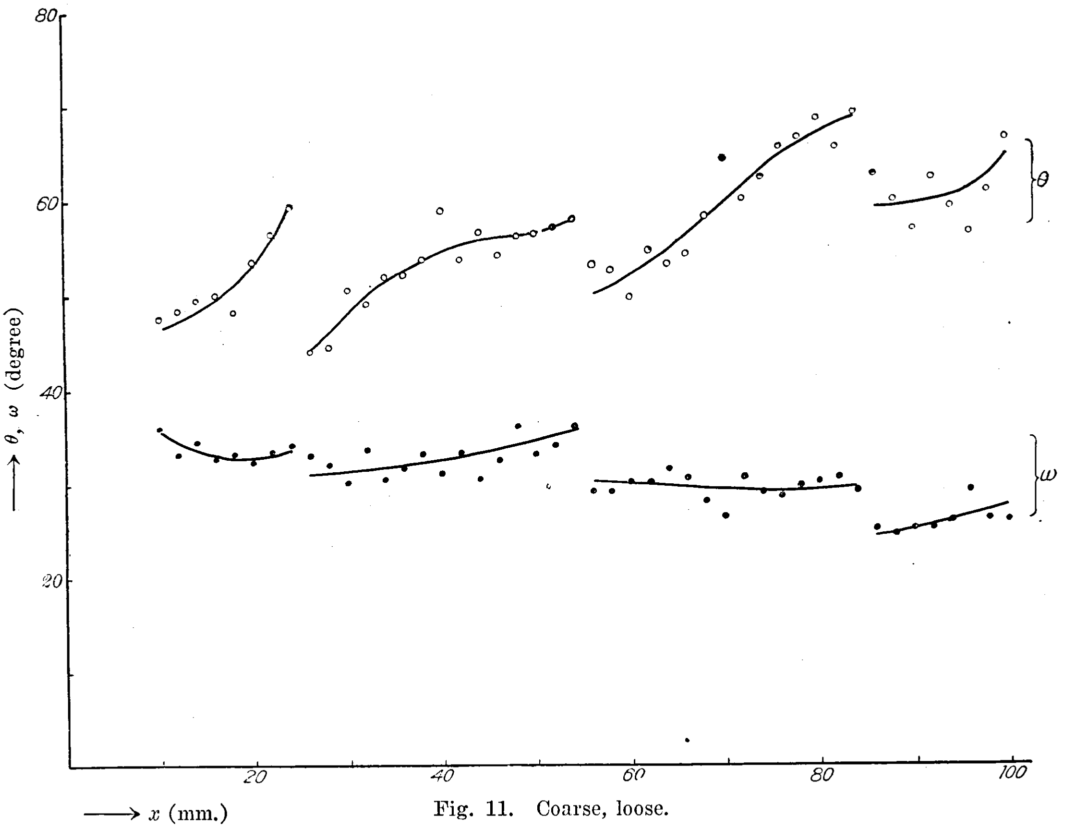

Fig. 11. Coarse, loose.

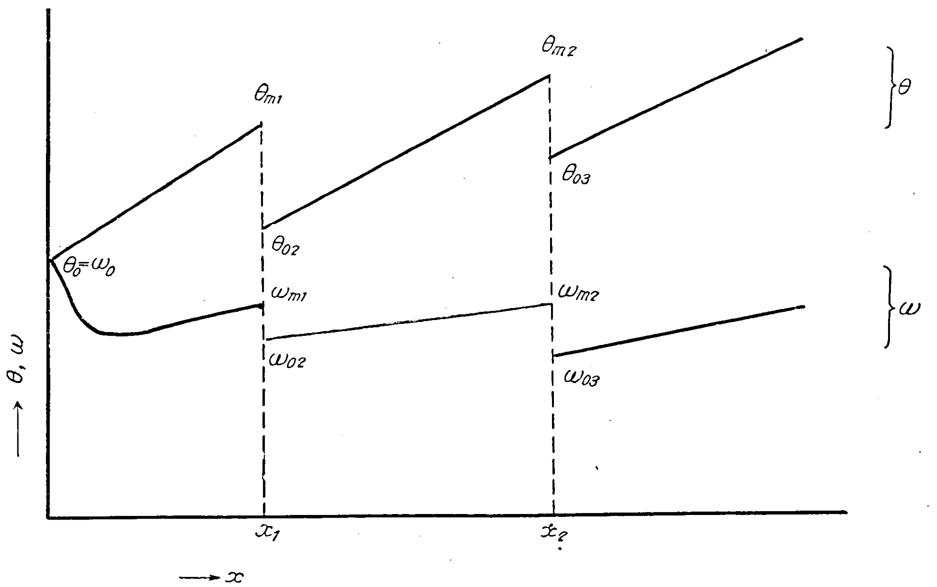

Fig. 12.

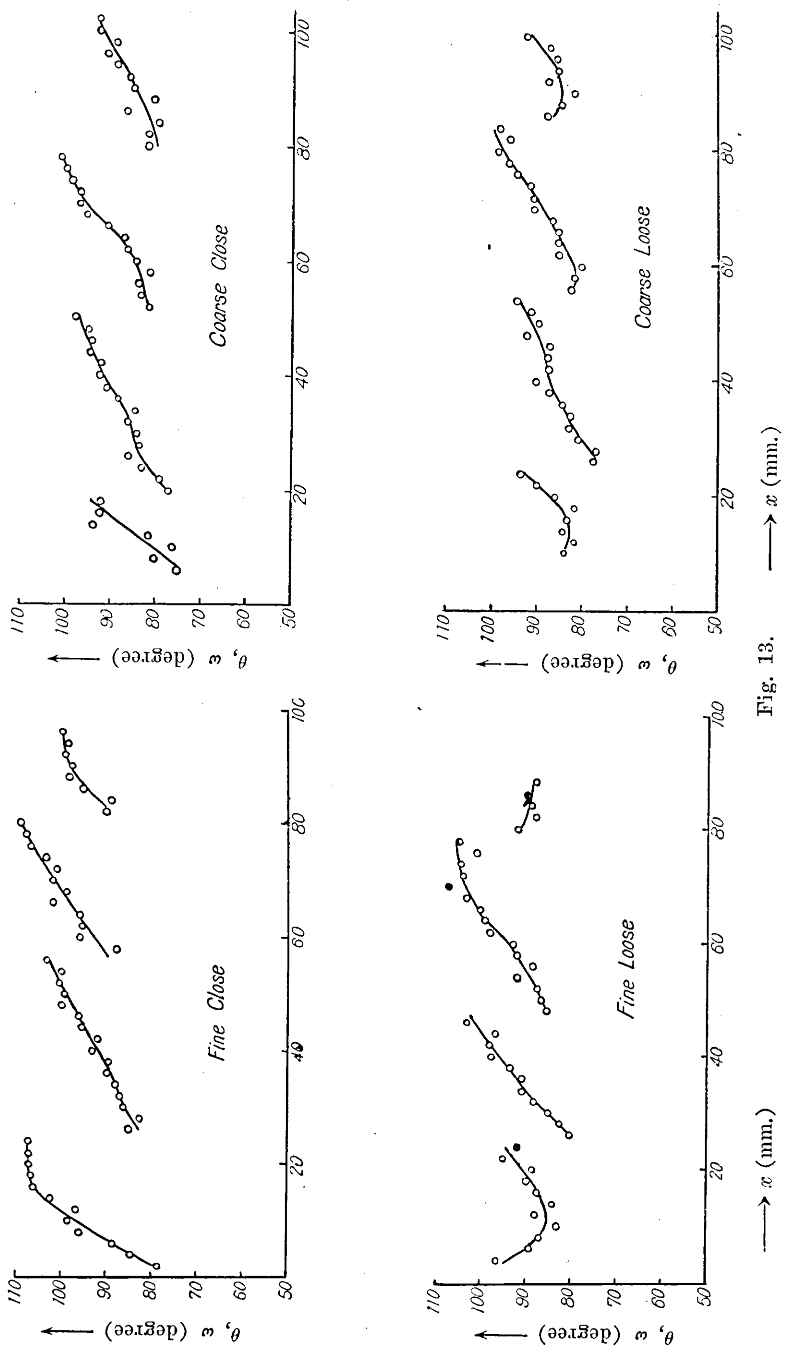
Coarse Close
Coarse Loose
Fine Close
Fine Loose
θ, ω (degree)
x (mm.)

Fig. 13.

plastic body. The physical meaning of the variation of $\theta+\omega$ will be explained later.

(2) Comparing again Fig. 8 with Fig. 9, or Fig. 10 with Fig. 11, it will be seen that the difference due to the state of packing expresses itself mainly in the mode of variation of θ and ω in the initial stage of the first slip plane. Thus, the initial value $\theta_0=\omega_0$ for $x=0$ is less for the closely packed state than for the loosely packed. Besides, as the consequence of the above fact, the initial value of $d\theta/dx$ is larger for C-state than for L-state, while the initial value of $|d\omega/dx|$ is greater for L-state than for C-state. According to the theory of plastic deformation, the initial angle $\theta_0=\omega_0$ corresponds, as will be explained later, to the angle between the slip plane and the major axis of the stress ellipse which decreases with the increasing angle of friction in the case of sand mass. As the value of this angle, ρ, for C-state is greater than that for L-state, the observed difference of θ_0 and ω_0 for the two states may easily be understood. It may be said, therefore, that the loosely packed sand is, at least apparently, transformed into a more closely packed state with the increases of x, which may mean that the disturbances due to the deformation is effective in rapidly settling down some of the most unstable state of packing.

(3) The difference between the initial values of $\theta_0=\omega_0$ for F-sand and C-sand is not conspicuous, for both the states of packing. This seems to imply that the angle ρ for the similar state of packing does not much depend on the grain size.

(III) Discussion.

In the previous reports, the preliminary results of the experiments were discussed in a provisory manner, merely with the purpose of facilitating the description of the general features of the phenomena observed. Though the results of the present experiments are still neither sufficiently systematic nor accurate to allow us any quantitative analysis, it seems proper to attempt at this stage a general review of the observed phenomena under the light of some mechanical considerations.

Since the classical treatise by Coulomb, numerous mechanical theories regarding the plastic deformation of sandy or earthy materials have already been developed, especially connected with the practical problem of earth pressure in civil engineerings and, consequently, these theories are little concerned with the further progress of deformation after the first slip has set in.

The same may be said with respect to the theory of plastic deformation of solid materials, in which the interest is similarly concentrated on the very initial stage of break-down. When the rupture has once commenced, the further mode of destruction is of little interest from the engineer's practical point of view. For the geologists and geophysicists, however, it is of prime importance to follow the entire history of the plastic earth crust through its incessant series of deformations, folding and faulting. For this reason model experiments of the kind as is here made have been repeatedly attempted in most various ways, chiefly by geologists such as Daubrée, Pfaff, Meunier, Paulke, Willis etc. In most of these cases, the physical conditions of the experiments are too much complicated to be subjected to a mechanical analysis. Our present investigations may, we hope, be regarded as taking something like a middle route between the widely diverging lines of research taken respectively by engineers and geologists, though it is only a first small step which has thus far been taken towards this direction.

The existing theories of earth pressure and plastic deformation[7] are exclusively concerned with statical problems. Though our cases are also nearly statical, in the sense that the kinetic energy plays no important rôle, they are still much complicated by the fact that the stress distribution in the deforming mass varies continually with the progress of deformation which is in its turn governed by the said distribution. It seems, therefore, difficult to make any direct application of the elaborate analytical theories of Prandtl, Hencky, Reissner, Kármán, etc. to the present case. At the present stage, however, the consideration of the distribution of stress ellipses among the sand mass as indicated by the observed slip lines seems to be just suitable for the general discussion of the qualitative results as hitherto observed. In the following we will mainly follow the method given in H. Krey's "Erddruck, Erdwiderstand," third edition, 1926.

In a sand mass which is characterized by a given angle of friction, ρ, a limiting value, n, is determined of the ratio, $p/q=r$ say, of the major axis p to the minor axis q of the stress ellipse, to which the ellipsoid reduces in our present cases, by a relation

$$\operatorname{tg} \rho=\frac{n-1}{2\sqrt{n}}.$$

Thus, when the ratio r increases and reaches the critical value n, the slip will take place along a slip line (plane) making an angle κ with the major axis such that

7) A concise summary of these theories has recently be given by Nadai in Geiger and Scheel's "Handbuch der Physik." 4, and also in his book "Der bildsame Zustand der Werkstoffe," 1927.

$$\operatorname{tg} \kappa = \frac{1}{\sqrt{n}}.$$

Thence, if ρ is given for the given sand, we may at once find the general orientation of the stress ellipse along any observed slip line, though some complication will occur near the solid walls.

In the vicinity of a wall, the friction between the wall and sand must be taken into account in which case we may, after Krey, distinguish four cases, according as the modes of the relative motion of the sand and wall. In our present case we may simplify the matter by the assumption that the angle of friction between the sand and wall is equal to ρ, as is approximately true. In such a case, the angle between the sliding wall and the major axis of the ellipse is just equal to the angle κ above given. Besides, the slip line running into the sand mass from the wall will be inclined to the wall by an angle 2κ, which is bisected by the major axis of the ellipse. The direction of the stress acting upon the slip plane is naturally inclined by ρ to the normal of the plane. The two cases, therefore, chiefly come here under consideration, according to the relative sense of motion on the two sides of the slip line:

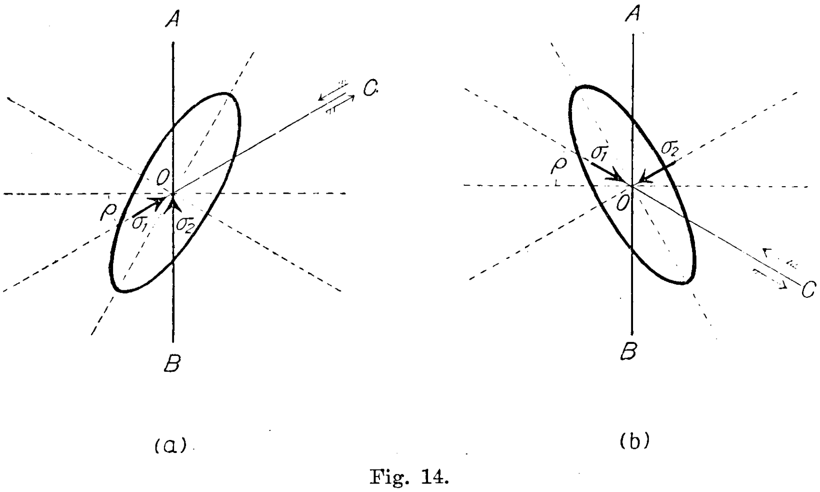

Fig. 14.

(I) If, referring to Fig. 14*a*, the sand mass on the right side of the receding vertical wall AB is considered sliding down along it, the pressure from the wall, σ_1 in Fig., must be directed from left down to right up, inclined by ρ to the horizontal.

(II) On the other hand, if the sand is sliding up along the pressing wall, the case is such as shown in Fig. 14*b*.

In each of these two cases, there is another possible slip line shown by OC running into the sand mass. In the cases of our experiments, the relative motion of sand on both sides of this slip plane OC will be such as shown by the pair of winged arrows and the direction of the pressure upon the slip line will be represented by the thick arrow σ_2. The cases of Fig. 14*a* and *b* will correspond respectively to the cases of receding and pressing wall.

With the above considerations kept in mind, we will attempt some discussions of the results described in the foregoing, for the two cases separately.

(1) *The case of receding wall.*

Referring to Figs. 3, 4, 5 and 6, it was found that the slip line observed is generally curved and while the angle of inclination θ of the upper branch varies only slightly, the inclination θ' of the lower part rapidly decreases with the displacement x of the moving wall.

Assuming roughly $\rho=30°$,[8] corresponding to $n=3$ and $\kappa=30°$, the fact that $\theta \fallingdotseq 60°$ will mean that the major axis of the stress ellipse is directed nearly vertical in the region near the free surface of the sand layer remote from the moving wall (Fig. 15). On the other hand, if we find $\theta' \fallingdotseq 30$, it will show that the state of stress at the foot of the moving wall is just corresponding to the case represented by Fig. 14*a* above.

In the first outset of deformation, we may assume that the stress ellipse is everywhere nearly circular. With the infinitesimal receding motion of the wall, the horizontal diameter of the circular ellipse will be reduced by some amount, depending on the position of the point considered. In any case, we may expect the initial orientation of the major axis to be more or less vertical. With the increasing deformation and the beginning of slip along the wall, the condition near the moving wall will approach the state of Fig. 14*a*, while the region remote from the wall will preserve the initial state with the vertical ellipse, except the possible change due to the change in the value of ρ, i.e. of θ, which depends on the state of packing and therefore may be affected by the deformation. The observed decrease of θ with the increase of x in the case of the closely packed sand (Figs. 3 and 5) may chiefly be due to this latter cause. The assumption $\rho=30°$ will give the ultimate value of $\theta'=30°$ which is roughly verified by the experiments.

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8) The actual value of ρ determined from the natural slope varies within a range 30°–40°, according to the states of packing; the corresponding range for $\pi/4+\rho/2$ is 60°–65°. The actual effective range within the sand mass seems to be much greater.

Referring to Fig. 15, the region on the right side of the first slip line I, AD, is behaving nearly as a rigid or elastic body; in other words, the axial ratio r of the stress ellipse is less than the critical value $n=3$, while on the left side of AD the ratio is equal or greater than n. As the motion of the wall is continued, we may suppose that the axial ratio r in the region

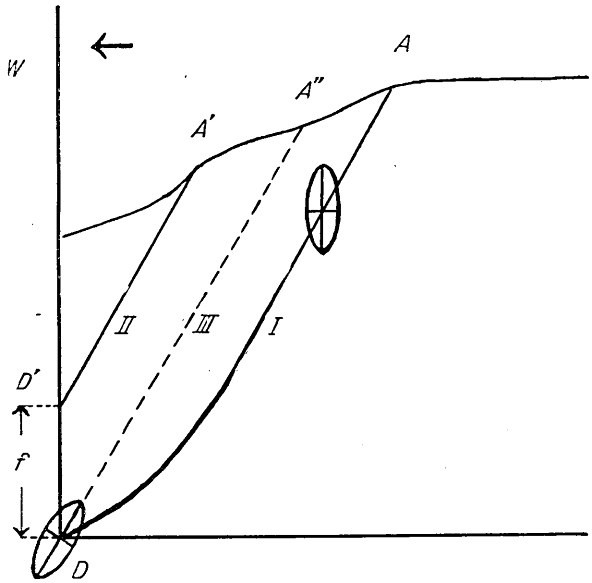

Fig. 15.

ADW (Fig. 15) is constantly tending to recover its initial value 1. This is facilitated by the fact that the slip line I once started to activity has a tendency to remain as such, because the value of ρ in the much disturbed layer along, and near, the line I will be decidedly decreased and, therefore, the shearing stress in the region to the left of AD may be gradually relieved, except near the moving vertical wall. Thus, we may expect that, somewhere in the midway between AD and WD, the ratio r may eventually fall back just below n. In this case the right side of this portion will regain the condition $r < n$ and become "solidified", so to speak, while on the left side, r will be everywhere $> n$. Thus, a new slip line, II in Fig. 15, will appear as a necessary consequence.

The fact that $d\theta/dx$ for this new slip line is small may be explained if we consider that at the foot of this secondary slip line, D′, the downward motion of sand must be continuous along the wall and thence just near the wall the condition of Fig. 14a is not quite fulfilled.

In Fig. 3, 4 and 5, the variation of the height f of the point D′ is plotted at the bottom of each diagram. It will be seen from the value of

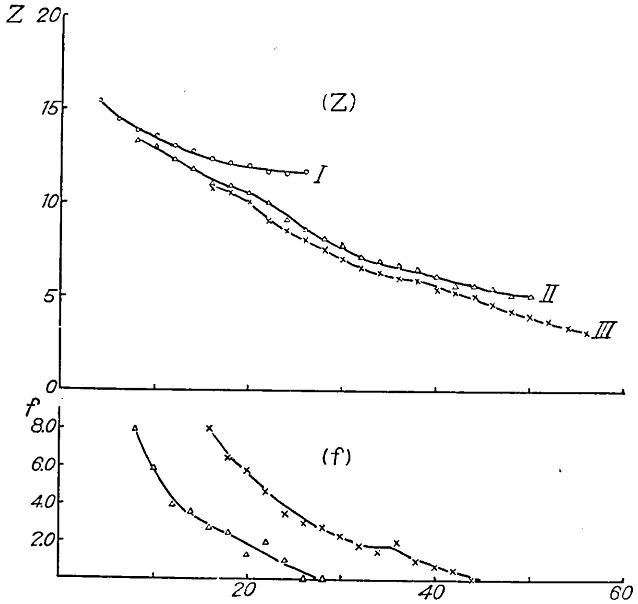

Fig. 16. Z is measured for the uppermost mark line;
I, in front of the first slip line,
II, „ second „ „ ,
III, „ third „ „ ,
Z and f are given in mm. measured on photograms which is 1/6.15 natural size.

df/dx that the downward motion is decidedly slower than if the line A′D′ be fixed in space and the wall only be receding. The line A′D′ seems to retain its position rather relative to the sand mass ADD′A′ which is sliding as a whole along the slip line I.

Fig. 16 shows the variation of the height, f, of the foot, D′, of the second slip line, II, (Fig. 15), compared with the variation of the height, Z, of the horizontal mark line of white sand in the region between the two slip lines I and II. The two curves, f and Z, show a nearly similar trend. As Z is, however, measured at the middle point of the segment of the white mark line cut off between the two slip lines, it may be understood that f-curve must be a little steeper than Z-curve, if the second slip line is nearly fixed with respect to the sand mass gliding along I, though possessing a small relative downward velocity relative to it.

Referring again to Fig. 15, when the point D′ approaches D, its motion is slackened as may be seen from f-curve of Fig. 16. When D′ coincides with D, the first slip line I decreases in its activity and II is now ready to take the place of I.

In the case of the finer sand, it occurs sometimes that the second slip line takes from the beginning such a form as shown by III in Fig. 15, i.e. $f=0$. Even in such a case, the two slip lines I and III may be simultaneously active for a certain interval of time.

All these phenomena seem to be conveniently explained referring to the schematical diagram Fig. 17. Taking a horizontal layer of sand in its initial

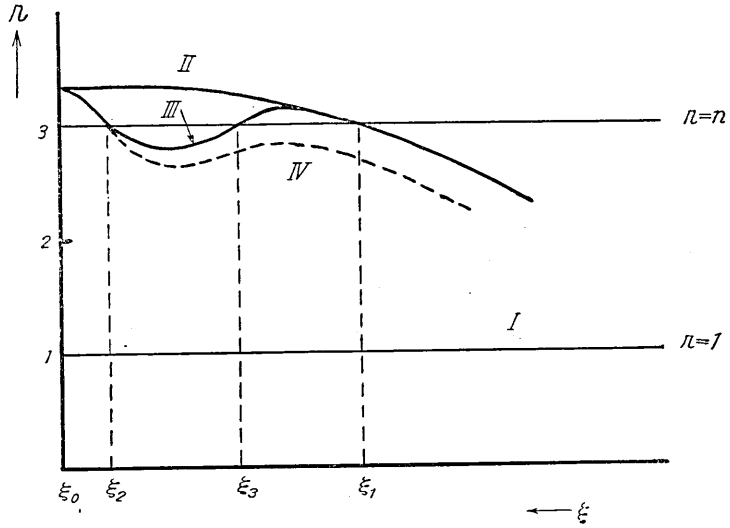

Fig. 17.

undisturbed state we denote the position of any elementary portion of sand by a *parameter* ξ which is the initial distance of the element from a fixed point remote from the moving wall. Let this parameter define the said element relative to the entire mass, regardless of its actual distance from the origin during the progress of deformation. Our ultimate problem will be to give the coordinates of the element as a function of ξ and t, which we will, however, not venture in the present paper.

For every value of ξ we may consider associated a definite value of the axial ratio r of the stress ellipse which is a function of ξ and the time t, or the displacement of the moving wall x. Besides, if the height of the initial layer be taken as another parameter, η, we have

$$r=F\ (\xi,\ \eta,\ x).$$

The same may be said with regard to the angle of inclination of the major axis of the ellipse.

Referring to Fig. 17, we will consider the variation of r as the function of ξ only, taking as η constant. For a given postition x of the moving wall, we will have a corresponding $(r,\ \xi)$ curve. In the initial undisturbed state r will be nearly equal to 1 for all values of ξ, so that $(r,\ \xi)$ curve is given by the straight line I parallel to ξ-axis. When the receding wall is set in motion, $(r,\ \xi)$ curve will tend to take such a form as shown by II. If this curve cuts the line $r=n$ at $\xi=\xi_1$, a discontinuous slip line will be developed at ξ_1 and the region between the wall, $\xi=\xi_0$ and ξ_1 will be subjected to plastic shearing, while the mass beyond this point will remain nearly rigid or elastic. By the later process of deformation $(r,\ \xi)$ curve will be subjected to an irregular fluctuation due to the heterogeneity in sand and the irregularity of motion due to it. As the portion in the midway between ξ_0 and ξ_1 has a tendency to settle down, as already mentioned,[9] $(r,\ \xi)$ curve may tend to assume such a form as shown by III (Fig. 17). Then, a new slip line will start at $\xi=\xi_2$ and the portion between ξ_2 and ξ_3 will behave nearly as rigid. Later, $(r,\ \xi)$ will take a form such as shown by IV (Fig. 17) in which case the first slip line at ξ_1 will stop, and so on.

The hypothetical variation of $(r,\ \xi)$ curve with the increase of x as above depicted is just what we may expect in the actual case, if we consider that by the initial displacement of the wall the horizontal component of stress is released by an amount the greater the nearer the moving wall, while on the other hand the moving sand has a tendency to recover the hydrostatic state as soon as the disturbance subsides. If we consider, moreover, the constraint introduced by the friction of the bed plate of the vessel, we may conclude that the point ξ_1 will appear the nearer the moving wall the lower the layer considered. This latter point will be probably important in ensuring the stability, i.e. the tendency to remain as such, of the slip line as a whole.

The theory of infinitesimal slip has really nothing to do with the development of a *stable* line of discontinuity with a *finite* slip as is here mainly in question, but gives only a swarm or net-work of mutually intersecting system of possible slip lines. In order to explain the actual phenomena as are here observed, it is, therefore, necessary to consider some process which may tend to concentrate the theoretical swarm of the in-

9) I.e., as an element with a given ξ is increasing its distance from the moving wall.

finitesimal slip lines into a definite slip line, or lines, with finite slips. The idea expressed in the above Fig. 17 seems to throw some light on this point. The said concentration implies the existence of some factor of instability which is afforded by the fact that ρ or n is sensibly reduced below its normal value in the region adjoining the slip plane. Referring to Fig. 17 we may say that at ξ_1 and ξ_2 the curve for n will not be given by the straight line I, but will show a sensible depression near these points; in other words, this region will approach the fluidous state compared with the other parts. The other parts will thus be partly relieved from the shearing stress and tend to reduce the ratio r; moreover, even a slight elevation of n-curve at these parts is probable, which will enhance the settling down of these parts.

In the case of the coarser sand in loose state, the said depression of n-curve near the slip line may be expected to be less accentuated than in the other three cases, since the initial state is already sufficiently loose and the corresponding n is near its lower limit.

Another condition for a stable finite slip line is that it will start from a singular point, i.e. a point at which the boundary condition is discontinuous. This fact has rarely been emphasized, but is of vital importance in the case of finite slip. At the foot of the wall, in the case of receding wall, both p and q tend to zero and r becomes indeterminate, or, in other words, the fluctuation of r is very large so that any slip line must pass this point.

(2) The case of pressing wall.

In the case of pressing wall, we have seen that the initial value of θ and ω are nearly equal. This seems to mean that the major axis of the stress ellipse is everywhere horizontal and $\theta=\omega=\rho$. This is just what we may expect from the theory of plastic deformation. The case is, indeed, somewhat analogous to a special case of the problems treated by Prandtl[10] and others with respect to the "special" plastic body, for which Mohr's "Grenzkurve" is a pair of straight lines parallel to σ-axis. It is, indeed, very interesting and instructive to compare the case of the photograms of slip bands developed in paraffin prism, as given in Nadai's book (p. 82 and 83) above cited, with the present case. In the present case of sand, Mohr's limiting curves is not parallel but inclined to σ axis by ρ, so that the ortho-

10) For the literature see Nadai, *loc. cit.* For example, compare the case of a prismatic body pressed upon one of the lateral face, *ZS. f. angew. Math. Phys.*, 1 (1921), 15–20, Fig. 2 and 7.

gonal net of slip lines in the former case will appear transformed into another net with the angle of intersection equal to $\pi/2-\rho$. Another point of essential difference is that in the present case the bed plate of the vessel may

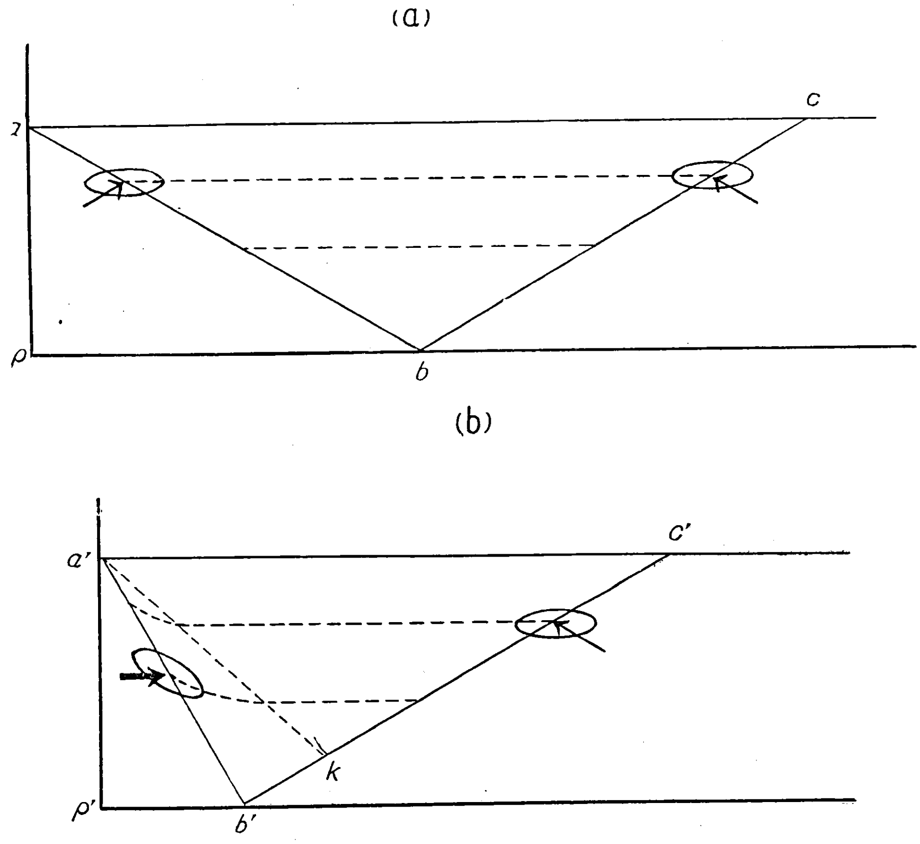

Fig. 18.

play an important rôle, while in the case of the paraffin prism such a boundary condition is absent. Since, however, the bed plate is situated in the regions where the plastic limit is not attained, i.e. on one side in the region at rest and on the other in the part moving as a rigid wedge, the comparison with the pressed paraffin prism is not quite illegitimate and may be freely utilized in our discussion as long as the qualitative features are in question.

In the present case, a sensible slip may also take place between the moving wall and the sand particles, especially near the upper part. In this case, the state will approach that given by Fig. 14*b* and there θ will be again equal to ρ. This circumstance may be significant in starting the initial slip line from the top of the wall.

However, the fact that the portion $a'p'b'$ (Fig. 18b) is behaving approximately as a rigid body, while the angle θ is rapidly increasing with x, shows that the horizontal pressure from the wall is being applied as such upon the slip plane $a'b'$. This will mean that, in the case $\rho=30°$, the major axis of the ellipse at the slip line is inclined to the horizontal by 30° and thus $\theta=60°$. This gives the ultimate value of θ to which it approaches with the increasing rigidity of the portion $a'p'b'$. The trajectory of the major axis will be somewhat as shown by the dotted line in Fig. 18b.

The region on the right side of the line $a'b'$ is subjected to the most conspicuous deformation, as mentioned in Part II and there may be brought about a similar variation as illustrated in Fig. 17 and thereby give an opportunity for starting the second slip line somewhere about $a'k$ (Fig. 18b).

In Fig. 13, a characteristic variations have been shown of the sum $\theta+\omega$ which varies within nearly the same range of 20° for the different cases. This range may probably correspond to the range of the sectorial segment within which the theoretical slip lines consist of divergent straight lines and logarithmic spirals.[11] Such a segment could, indeed, be formed when the direction of pressure from the pressing wall relative to the moving sand mass deviates from the horizontal, as may be expected to be the case in our dynamical case. If such be actually the case, we may expect that the observed slip line will be curved in a form of logarithmic spiral somewhere near the bed plate. This cannot be verified directly from the photogram on account of the irregularity observed in this region probably due to the glass wall, but at least made probable by the curvature of the streaks on the photogram marked by the moving sand grains. As this range of θ is then determined by the value of ρ in the most disturbed state of sand, it is natural that the value $\theta+\omega$ is not much different for the different states.

It may be remarked that in the case of pressing wall, the top of the moving wall is the singular point, as mentioned above, from which the actual slip line must start.

Summarising, thus far, we have seen that the process of step-faulting seems to be determined chiefly by the following three conditions:

(a) The space variation, and its time variation, of the stress ellipse

11) See Prandtl, *loc. cit.*

within the sand mass as determined by the motion of the prime mover, i.e. the moving wall, when the stable, existing slip planes are given as boundary conditions.

(b) The existence of a critical value n of the axial ratio of the ellipse which is a function of the degree or rather the rate of deformation, its value decreasing with the increasing rate of deformation and tending to its maximum normal value when the rate of deformation falls under a certain value.

(c) Existence of a discontinuity of boundary condition, which determines the location of the slip plane.

Taking the existing slip line as if it were a rigid boundary, the condition (a) may probably be followed in some simple cases by treating the moving sand mass as a kind of special viscous fluid[12] and thus anticipate in some measure the position of the next slip plane, taking (c) into consideration. The stability of the possible slip line will then be judged after (b) by a consideration similar to that described above referring to Fig. 17.

The above considerations may at least suggest a way of future formulation of the allied problem regarding the dynamics of moving granular mass.

The condition (b) will be rather accentuated if the cohesion between the sand grains comes into play beside the friction.

In the case of the earth crust the condition (b) may also be considered to hold, as it is known that the fragments of a fractured rock may again form a compact mass under a state in which the principal stresses are sufficiently large and the axial ratio of the ellipse is subjected to a suitable cycle of variation. Besides, a mode of variation of the plastic limit with the variation of the rate of deformation, similar to that considered above with respect to sand, may also be supposed in the case of the earth crust, since heat is generated by shearing and its rate of generation will increase with the rate of shearing, and the resulting temperature rise may tend to bring Mohr's curve nearer to σ-axis.

These considerations seem to justify the use of sand in the model experiments in preference to the other elastic or plastic materials such as mostly used by the previous investigators. In short, we may say that a sand mass is more dynamically similar to the earth crust than the other substance. The similitude holds also with regard to the effect of gravity playing an important part in both cases in determining the increase of the

12) The fact that a granular mass once set in deforming motion behaves more like a fluid than a plastic solid is well known, though the law of internal friction might differ considerably.

principal stresses with the depth, while in the laboratory experiments with elastic materials the gravity is utterly negligible. Both in the cases of sand and the earth crust, the absolute magnitude of stress is given by the gravity while the effect of the deforming mechanism is mainly to affect the shape of the stress ellipse.

Mohr's curve defining the plastic limit is of course different for the two cases compared. It must not, however, be overlooked that there is an essential point in which the two cases resemble to each other. Geotectonic features of the earth surface suggest that the earth crust is easily ruptured by tension. This is also natural, if we consider that the crust is run through by numerous cracks and fissures, as may be illustrated by Nasu's investigation, cited later, and also by Ch. Tsuboi's investigation[13] of the block structure of Tango Districts. Mohr's curve, therefore, may be assumed to lie almost exclusively on the negative side of σ, as in the case of sand mass. Ignoring, therefore, the quantitative difference for the present qualitative comparison, we may say that the sand model experiment in laboratory deals with the part of Mohr's diagram near the origin of (σ, τ) while the phenomena in the earth crust is related to the part of the same diagram very remote from τ-axis.

In passing, it may be remarked that the formation of the quasiperiodic series of slip planes in overstrained metallic crystals may be explained by a similar mechanism subjected to the three conditions (a) (b) and (c) above mentioned. When the average state of stress reaches the critical state, the actual stress at individual points will be subjected to natural fluctuation which will be quasiperiodic with respect to space and time. This fluctuation may become unstable on account of the circumstance analogous to (b) and the deformation will be concentrated to a finite number of slip bands while the remaining parts are released from strain. The periodic interval of the bands will probably be determined by the degree of heterogeneity in the molecular lattice and also by the irregularity of the boundary surface, which give the required singular points.

(IV) Geophysical Applications.

Some of the possible geophysical applications of the results of our present series of experiments have already been enumerated in the previous report, Part II.

Assuming the force applied to the earth crust on account of geological

13) CHUJI TSUBOI, *Proc. Imp. Acad.*, **4** (1928), 529; this Bull., **6** (1929), 71; also in the present Vol. of Bull.

disturbances to be chiefly horizontal, we may predict in some measure the probable angle of inclination of the fault planes if we could assume the proper value of ρ or n, or the form of Mohr's curve, and vice versa. Or, in some case, we may perhaps judge whether it is the case of receding or pressing the prime mover, provided the latter, i.e. the virtual "moving wall" may be located from some other considerations. The same may also be judged from the observed order of sequence of seismic activity among a set of parallel earthquake zones, if such could be previously located by a large number of observations, as was made by Mr. N. Nasu[14] in the case of Tango Districts. In his case, however, the intervals between the successive "slip planes" seem to be of an order of a few km., and, moreover, the angle between the intersecting slip planes is mostly near a right angle so that we may assume here the case of ordinary plastic deformation analogous to that of paraffin prism above cited. A more proper field of application of the present results seems to lie in the domain of geological phenomena of much larger scale. One of the most beautiful examples of a periodic series of fault lines may be seen in the geotectonic map of Korea given in Prof. Koto's[15] classical paper on the orographical features of Korean Peninsula. Another remarkable example is afforded by Prof. Yamasaki's investigation[16] on the typical block structure of Bôsô Districts. In most of these examples, the cases seem to correspond to our case of the receding wall, and they suggest that the boundary between the land mass and the oceanic bed is receding towards the ocean. This inference, if correct, may support

14) N. Nasu, *Proc. Imp. Acad.*, **4** (1928), 378. After this paper was read in the Meeting of the Institute, Mr. Nasu informed me of an interesting fact that, in Kwanto region, there is a set of parallel seismic zones which become successively active, i.e. among which the activity is transferred from one zone to the next one and so on. In the same Meeting, he read a very interesting paper on the three-dimensional distribution of the epicentres in Tango Districts. The distribution along a system of intersecting "slip planes" is quite conspicuous. Under the light of the considerations given in our present paper, his results vividy reveal the actual mechanism of earthquakes in the said district, in such a manner that we may now clearly trace the space distribution of stress ellipses as well as that of the plastic properties of the crust in defferent regions. The location of the net-work of the slip planes agrees well with that which we may just expect from the results of triangulations published by the Land Survey Department (to be published later in this Bulletin). We will return to the subject in some future. In passing, it seems to the authors that the most *direct* way of answering the question of earthquake forewarning is to measure the state of *existing* strain of the crust by triangulations and levellings and review the results under the light of the theory of plasticity.

15) B. Koto., *Jour. Coll. Sci., Tokyo,* **19** (1903), Art. 1.

16) N. Yamasaki, *Report of E.I.C.,* **100** B (1925), 11; *Journ. Fac. Sci., Tokyo,* [ii] **2**, (1926), 77.

Wegener's view and also the authors' hypothesis[17] on the origin of the Japanese Arc.[18] In our present case, however, the friction of the bed plate of the containing vessel introduces a boundary condition which cannot generally hold in the case of the earth crust, as already emphatically stated in Part II. Further discussion of the geophysical application may, therefore, be postponed until the experiment with a viscous substratum which we are now planning, has made some progress.

SUMMARY.

(1) The experiments similar to those described in the previous two reports are repeated somewhat more systematically, using a kinematographical camera for following successive stages of deformation with a sufficiently close step.

(2) Two kinds of sand were investigated, each in two different states of packing, respectively for the cases of receding as well as pressing the side wall of the containing vessel.

(3) In general, the closest packing tends to be loosened, and the loosest state to be settled down, towards a certain normal middle state, during the course of deformation.

(4) The difference of grain size expresses itself mainly in the results of experiment as the effects of the corresponding difference in the mode of variation of packing during the course of deformation, but not as those due to any essential difference in the angle of friction.

(5) The tendency to form simultaneously active slip lines is most remarkable in the case of the finer sand, while it is absent in the case of the coaser sand in loose packing. It may be said that the latter is more fluidous while the former is more brittle.

(6) The results of the experiments thus far made are reviewed under the light of the mechanical theory of plastic deformation, by considering the distribution of the stress ellipse and its variation due to the deformation. The circumstances[19] determining the formation of step faults are enumerated, and a way is thereby suggested which may be taken for a future analytical formulation of the allied problems.

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17) T. TERADA, *Rep. E.I.C.*, **100** B (1925), 63; this *Bull.*, **5** (1928), 21.

18) There are also some examples in which the case of pressing wall is applicable especially in cases of local disturbances; for example, the case of Tango Districts above cited in ft. nt. p. 91 and also the case of Kii districts recently investigated by Imamura *Proc. Imp. Acad.*, **5** (1929), 161.

19) I.e., the conditions (a), (b) and (c) in p. 89.

(7) Some features of dynamical similitude between the experimental sand layer and the earth crust is pointed out with respect to the following relations:

(*a*) The stress ellipses are mainly determined by gravity in both cases. Elasticity is negligible.

(*b*) Mohr's diagrams are of qualitatively similar character for the two cases.

(*c*) The mode of variation of the plastic property as the function of the rate of deformation may probably be similar in either case. This mode is essential in determining the formation of step fault.

(8) Some geophysical applications are mentioned.

(9) Analogy of the case of slip bands in deformed metallic crystals is again cited.

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4. 砂層の崩壞に關する實驗(第三報)

寺 田 寅 彦
宮 部 直 巳

前二回の報告に誌したと同樣に、砂層の横壁を後退或は前進させて斷層を生ぜしめる實驗を、今度は活動寫眞器械を用ゐて、前よりは稍系統的に繰返した。其結果によつて從前の結果を確めると同時に、又、砂粒の大小及其の充塡粗密度の差異に因る異同を、前よりも詳しく又確に知る事が出來た。

Stress ellipse の考を用ゐて、此等の實驗の場合に於ける砂層の變形破壞の機巧を考察した結果として、砂の摩擦角が砂の充塡度により又其運動狀態によつて色々に變る事が、現在の場合に重要な決定因子であるといふ考へを確かめ、其考へから階段的斷層の發生する經路を幾分明にする事が出來た。

又、此等の考察の結果から、地殼と砂層との力學的肖似が偶然でなく本質的のものであるといふ事を一般 plastic deformation の理論に照して指摘した。

終りに、此等實驗の結果を地球物理學上の現象に應用するに際して注意すべき二三の點を擧げておいた。