# Experimental Investigations of the Deformation of Sand Mass by Lateral Pressure.

(With an Appendix.)

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## 砂層の崩壞に關する實驗（第二報）

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前回の實驗では砂層の横壁を後退させて崖崩れに類する崩壞を生じるといふのであつたが、今度の實驗では、其横壁を前進させ、即ち砂層を壓しつけ、其れによつて生ずる變形を調べたのである。此場合も、前と同樣に、横壁の連續的な進行につれ、辷り面或は斷層が不連續な段階的に發生する。此の辷り面の變化の過程を稍詳しく記載して其の機巧に就て概略の説明を試みた。

又横壁を週期的に前進後退する時に生ずる變形をも調べた，壁の週期的運動によつて砂の變形が唯一方にのみ進行する點に特別の興味があるであらう。

地震の問題に、上記の結果が應用され得べき若干の要項を擧げておいた。

附錄として第一報の實驗に關する若干の考察を述べてある。

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In a previous paper,(1) some experiments were described in which a horizontal pile of sand contained in a rectangular vessel was made to form a kind of step-faults by receding one of the end walls of the vessel and it was shown that the formation of the periodic faults is determined by the existence of the upper and lower limits of the inclination of the slip-surface. The following is the results of further experiments carried out with the same apparatus,(2) but this time by pushing on the end wall instead

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(1) T. Terada and N. Miyabe, *Bull. Earthq. Res. Inst.*, 4 (1928), 33; the paper is written in Japanese, with an English Abstract in which the references are made to the text figures and Tables so that the foreign readers may follow the essential points there reported. Another abstract has also been given in *Proc. Imp. Acad.*, 3 (1927), 655, in which allusion is already made to an experiment of pressing the end wall.

(2) A photogram of the apparatus is reproduced in Fig. 2, Pl. IV of the paper cited; the movable side wall is 20 × 20 cm.$^2$

of receding it. Later on, some experiments were also made in which the movable end wall was subjected to an alternate periodic motion of a given amplitude.

## I. Formation of Periodic Thrust-Faults by Lateral Pressure.

The movable vertical end-wall of the rectangular vessel in which a horizontal layer of sand is deposited, is pressed on horizontally with a nearly uniform small velocity and the successive stages of deformation were photographed in five series. The first series consists of the photograms taken when the displacements, $d$, of the movable wall are respectively 0–0.2, 1.0–1.2, 2.0–2.2.... cm., the second series corresponds to $d=0.2$–$0.4$, 1.2–1.4,.... cm. and so on. For different series, the sand layer was deposited afresh, so that the five series do not represent a continuous motion-picture of a single layer of sand. We were compelled to take this procedure, as it was desirable to take photographs without stopping the moving wall, and with an ordinary photographic apparatus the time required for changing the photographic plate could not be made so short as to allow a series with a shorter step of $d$ than 1 cm.

As before, the exposure was made for an interval of time corresponding to 2 mm. displacement(1) of the end wall such that the traces of moving particles of sand could be clearly defined and the boundary between the moving and stationary masses could be sharply discerned. Horizontal lines of white sands were also used for indicating the deformation of the internal mass. The white sand was applied only near the glass side-wall, such that it cannot affect the property of the sand mass as a whole.

For the present experiments two kinds of sand were used, one finer and the other coarser. The coarser one is the same as employed in the previous experiment cited and consists of grains which passed a sieve of 1 mm.$^2$ mesh and was retained by another with 0.25 mm.$^2$ mesh. The finer one was obtained by washing out very fine powder from that portion which passed 0.25 mm.$^2$ sieve. The sand was well dried before using for the ex-

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(1) The velocity of motion of the pressing wall was about 0.02 cm./sec.

periment. The white index sand was also chosen such as to match the average grain size of the main mass.

Some of the photograms obtained are reproduced in Pl. I–III.

It will be seen that, also in the present case, the slip surfaces are produced in discrete steps, while the motion of the pressing wall is continuous.

*(A) Fine sand, closely packed* (Pl. I and III).

For the explanation of the process by which the successive slip surfaces are developed we will first take the case of the finer sand in closely packed state,(1) as in this case the phenomena take place in most typical and regular form.

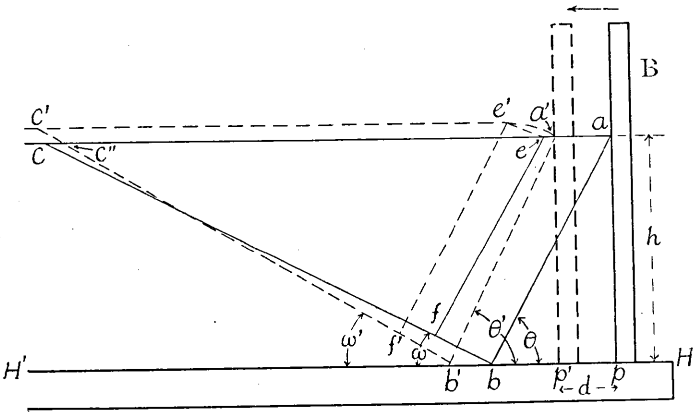

Fig. 1.

Referring to Fig. 1, $HH'$ represents the fixed bed plate of the containing vessel, $B$ the movable end-wall which is driven in the direction of the arrow, and $ac$ is the initial horizontal surface of the sand layer. When the wall $B$ is pushed in a slip surface $bc$ is formed immediately. The sand mass on the right side of $bc$ is set in general motion, while the mass on its left side remains quite unaffected. When the end-wall advances for a small finite distance the angle of inclination, $\omega$, of the slip line generally increases. In the meantime, the angle of inclination, $\theta$, of the line $ab$ is also increased so that the motion of the point $b$ does not keep pace with

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(1) In the present experiments, the closely packed state was obtained by depositing successively layers of small thickness which were pressed by a horizontal plate from above before laying a further layer.

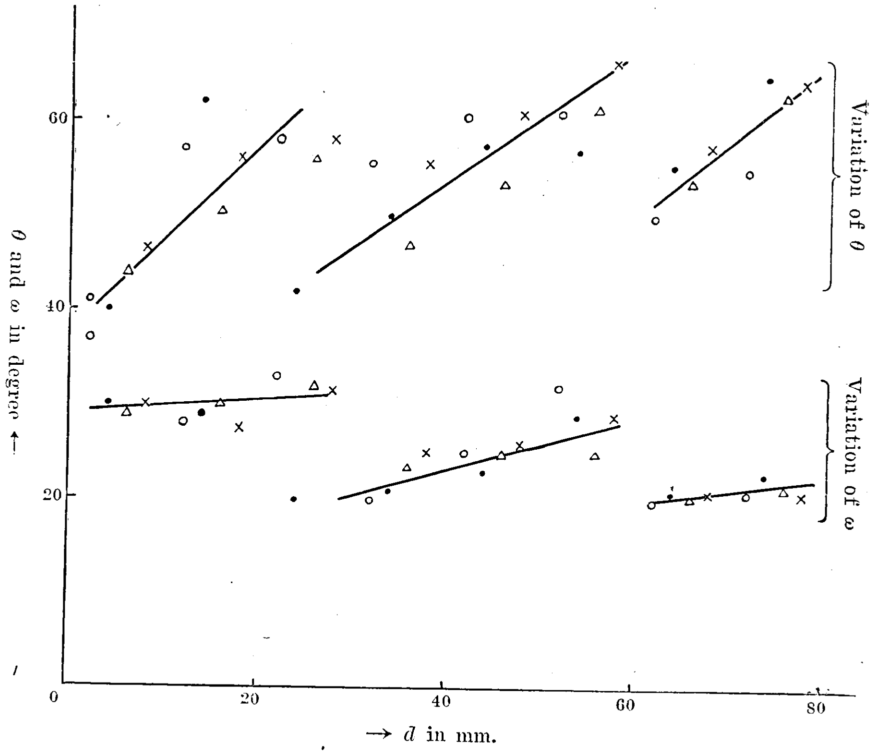

Fig. 2.

that of the wall *B*. Fig. 2 shows the modes of variation of $\theta$ and $\omega$, taking the displacement, *d*, of the wall *B* as abscissa. The diagram in full line is drawn schematically from the mean values for the five sets of experiment, as the fluctuation of the data among different series is considerable and the curve for each separate set cannot give any exact quantitative information except for the order of magnitude.

As may be judged from the photographic traces of sand particles, the portion of the sand mass contained within the triangular prism $a'p'b'$ (Fig. 1) is moving horizontally as if it were a rigid body fixed to the moving wall, and sliding on the bed plate $HH'$ against the friction upon it, thus consuming a part of the work done by the moving wall.

As already mentioned, the foot of the slip line, $b'$, is gradually overtook by the wall *B*, on account of the change in the angle $\theta$. Thus, corresponding to the displacement of the wall, $pp'=d$, the displacement of *b* is

$$\begin{aligned} bb' &= pp' + b'p' - bp \\ &= d + h(\cot\theta' - \cot\theta), \end{aligned}$$

where $h = ap$, and the change in $h$ is neglected. The second term is always negative as $\theta'$ increase with $d$. Similarly, $cc''$, the displacement of the point at which the slip line meets the free surface is

$$\begin{aligned} c''c &= bb' + h(\cot\omega' - \cot\omega) \\ &= d + h(\cot\theta' - \cot\theta + \cot\omega' - \cot\omega). \end{aligned}$$

As $\omega$ also increases with $d$, $cc''$ is still smaller than $bb'$. As a matter of fact, $cc''$ is generally negative, i.e. the point $c''$, and also the slip line as a whole, recedes as $B$ advances. This may be clearly seen on examining the series of photograms from the mode of the change of position of the slip line relative to the mass at rest. Fig. 3 will illustrate the successive positions of the slip lines in an actual experiment. This receding motion of the slip line is, however, generally small and, for a rough approximation, we may say that a portion of sand mass initially contained in a prismatic boundary $efc$, where $ef$ is an ideal plane, is pushed up along a fixed plane $fc$ to a final position $e'f'c'$, meanwhile the entire prism behaves approximately as a rigid body. On the other hand, the mass initially contained in $ab''bfe$ ($ab''$ is to be drawn parallel to $a'b'$ to meet the bed at $b''$) is deformed into a new shape $a'b'f'e'$.

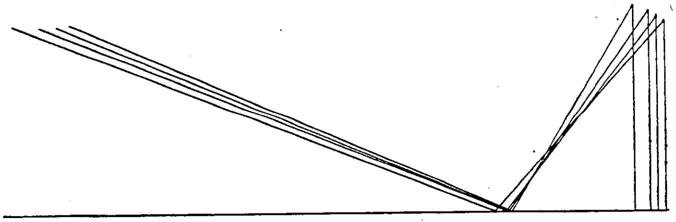

Fig. 3.

When the deformation is advanced for a certain extent, the sand on the sloping surface $e'a'$ will glide down and result in raising the point $a'$ above the initial level so that we must replace in the above expression $h'(\cot\theta' - h\cot\theta)$ for $h(\cot\theta' - \cot\theta)$ where $h' = a'p'$ is also a function of $d$, which could be calculated approximately if wanted.

Thus, a sensible deformation of sand mass is chiefly confined within the region $a'b'f'e'$ which is behaving like a plastic mass, while a thin layer along the slip line plays apparently a rôle of viscous lubricant, as far as a kinematical analogy is concerned.

When the motion of $B$ is still continued and accordingly the angles $\theta$ and $\omega$ are continually increasing as shown in Fig. 2 a critical stage is attained when further increases of the above angles are prohibited. At this stage a new set of the slip surfaces is started suddenly as shown in Fig. 4, in which $b'c'$ represents the slip line formerly active and $b_1c_1$ the new one. The subsequent course of deformation is essentially a repetition of what was described above. Thus, the angles $\theta_1$ and $\omega_1$ which now play the parts of $\theta$ and $\omega$, gradually increase with $d$. That a wedge-shaped portion gliding up along the nearly stationary line $b_1c_1$ is practically rigid may be seen from the fact that the conspicuous bend of the white index line marking the trace of the former slip line $b'c'$ is neatly preserved during the second step of deformation.

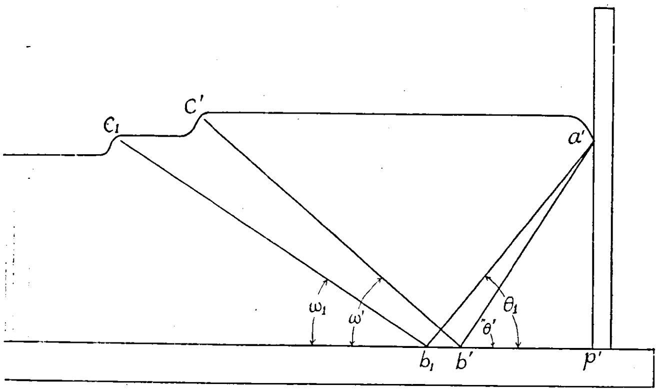

Fig. 4.

When the wall $B$ is further advanced a second critical stage is attained and then a third slip line $b_2c_2$ suddenly appears and so on.

Referring to Fig. 2, it may be remarked that the upper and lower limits of $\theta$ increase successively for the successive steps of formation of the new slip line. On the other hand, the corresponding limits for the angle $\omega$ gradually decrease for the successive steps, contrary to the corresponding variation of $\theta$. This will suggest already that the two angles are related with each other in a complementary manner and the simultaneous variation is due to the change in a certain common factor determining the inclinations of slip planes relative to the horizontal.

It will be seen that the length between the upper ends of the second and third slip line, $c'c_1$ (Fig. 4), is determined firstly by the possible range of variation of $\theta$ and secondly by the amount of increase of $h'$ during the second step. The latter factor is in its turn governed by the first factor. Hence, when the range of $\theta$ is small, the successive steps will occur in a rapid succession and the sets of slip lines will be closely spaced, as will be seen later in the case of coarser sand.

In some experiments with the same fine sand it occurs accidentally that, while the gliding is still going on along the slip line $b'c'$ (Fig. 5), some discontinuous deformation appears in front of the moving wall. It has been already mentioned that the mass between $a'b'$ and $e'f'$ (Fig. 1) is subjected to a general shearing, but in most cases the shearing is chiefly localized near the line $a'b'$ so that a sharp bend of the white index line may be observed along $a'b'$, which thus forms also a kind of slip line. Referring to Fig. 6, let $a'p'$ and $a''p''$ be two successive positions of the moving wall. Assuming $b'$ as approximately stationary and drawing $a'b''$ parallel to $a''b'$, it follows that the sand formerly contained in the prism $ab'b''$ has been deformed into the new prism $a''b'k$, such that the areas of the two triangles are equal

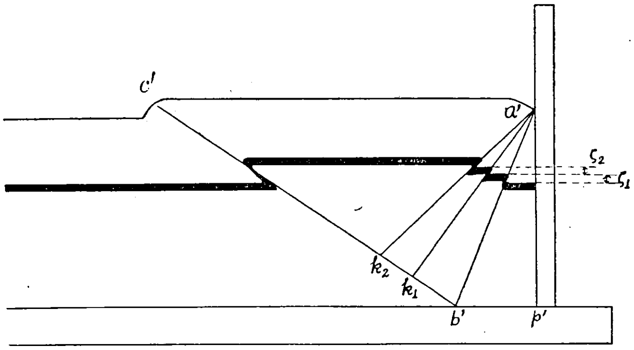

Fig. 5.

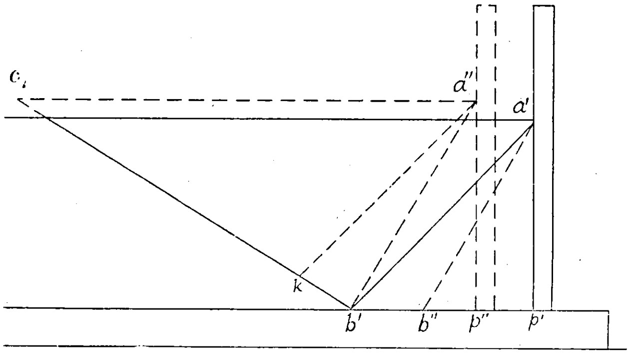

Fig. 6.

to each other. The slip line must have been displaced, in the meantime, relative to the sand mass from a position near $a''k$ to $a''b'$.(1) When such a transition of the slip line relative to sand takes place gradually we obtain a smooth bend of the index line as shown in Photo. 356, 361, etc. It happens, however, in some cases that while the slip line $a''b'$ is still active an auxiliary slip plane shoot out from $a''$ somewhere in the direction of $a''k$, i.e. in the position of the initial slip line. As may be confirmed by measuring the increases of the two step distances $\zeta_1$ and $\zeta_2$ in Fig. 5, we observe that the two slip lines $a''b'$ and $a''k$ are in action simultaneously, or at least alternately, during the motion of $B$. The slip line $a''k$ is, however, not able to break through $b'c'$. In some case, a third auxiliary slip line $a''k_2$ (Fig. 5) may appear, before the first step of deformation comes to an end. What causes such a variety is not quite clear, but it may be brought forth by different circumstances such as a heterogeneity in sand and an accidental fluctuation in the magnitude and direction of pressure from the moving wall. The latter fluctuation is unavoidable, since with the increase of the pressure, the iron rod pressing on the wall may sensibly yield to an accidental bending moment.

On account of the shearing motion, which deforms the prism $a'b'b''$ into $a''kb'$, the pressure transmitted from $B$ through the sand mass will be

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(1) It may be easily seen that $a''k$ as above defined is approximately coincident with the original position of $a'b'$ relative to sand.

weakened in magnitude and deviated in direction, as we go further in front. This effect will be of importance in determining the inclination of the slip plane as will be referred to later.

After the foregoing explanations, we may now attempt to discuss the mechanical process by which the sequence of the phenomena is brought about, though anything like a handy theory is at present difficult to propound and may better be reserved until more quantitative experimental data have been accumulated.

In the first outset of the motion of the pressing wall, when the trace of slip surface begins to appear, the active pressure on the slip surface will be the resultant of the component due to the weight of the overlying mass and the component due to the pressing wall. The latter will at first be small, so that the resultant pressure on the slip plane *ab* will be nearly vertical, and still more so on the surface *bc*. On this assumption we will expect that

$$\theta=\omega=\phi,$$

where $\phi$ is the angle of natural repose as given by

$$\tan\phi=\mu,$$

$\mu$ being the coefficient of friction. Unfortunately, it is difficult to determine the truely initial values of $\theta$ and $\omega$ with the present method of experiment. As far as may be judged from the photogram obtained, however, the above expectation is fulfilled in some measure by the fact that the difference of the initial values of the two angles, estimated from Fig. 2 by extrapolation, is small and falls within a few degrees which is of the order of magnitude of the experimental error.

As the moving wall is advanced the pressure on it will rapidly increase and tend to outweigh the effect of gravity. In the extreme case when the pressure becomes predominant the resultant pressure on the surface *a′b′* will become nearly horizontal. The statical relation in this case will be

$$\cot\theta=\mu=\tan\phi.$$

On the other hand, the pressure upon the slip plane *b′c′* remote from the pressing wall could not undergo the same variation as in the case of *a′b′*, because, as already mentioned, the pressure transmitted from the wall will be weakened and at the same time deflected due to the yielding of the intermediate mass. Hence, the increase of $\omega$ with the displacement of the wall will take place with much slower rate than that of $\theta$. This is actually observed in most cases. Another result to be expected is that near the end of each step, that is just before the start of the next discontinuous fault, the sum $\theta+\omega$ will be roughly equal but fall short a little of 90°. This is indeed the case, as may be seen from the second column of the annexed Table in which the values of $180^\circ-(\theta+\omega)$ for different cases with different sizes of grains and different states of packing are given for comparison.

| Sand | Fine Grain | | Coarse Grain | |
|---|---|---|---|---|
| Packing<br>No. of Slip. Surf. | Loose | Close | Loose | Close |
| I | 102 | 98 | 94 | 97 |
| II | 104 | 99 | 94 | 96 |
| III | 105 | 101 | 96 | 93 |
| IV | — | — | 100 | 96 |

The fact that the angle $180^\circ-(\theta+\omega)$ generally increases with the number of steps corresponds to the fact that the gradual increase of the average of limiting values of $\theta$ with the successive steps outweighs the corresponding decrease of $\omega$. As may be seen from Fig. 2 the limiting values of $\theta$ actually increases with the number of steps more rapidly than the corresponding value of $\omega$ decreases. The meaning of such a variation with the number of step may partly be sought in the change in the value of $\mu$ and partly also in the change in the mode of transmission of the active pressure on the wall $B$ due to the change in the boundary condition effected by the upheaval of conspicuous mound. A detailed discussion in this respect is at present difficult to be made.

When the slip line $a'b'$ has been brought to its extreme position as determined by the relation $\theta=\cot^{-1}\mu$, a further adjustment of the slip surface by its relative displacement through the sand mass is no more possible and the slip line becomes as it were clutched, so that a further movement is only possible by starting a fresh slip surface from the top of the wall. By a sudden start of the new slip, the active pressure upon it will be momentarily released to some extent, so that the conditions tend to fall back to the initial state, resulting in sudden decreases of $\theta$ and $\omega$.

*(B) Fine sand, loosely packed* (Pl. III).

The results for the case when the same fine sand as employed in the above experiment is in loosely packed state are qualitatively similar to the above case with close packing. For the loose mass, however, the angle $\theta$ is generally larger than for the closely packed sand. This may at least partly be explained by a difference in the coefficient of friction of which the dynamical value may depend on the initial state of packing more strikingly than in the case of the statical coefficient. On the other hand, the difference of $\omega$ for the two states of packing is not so marked as in the case of $\theta$. As far as may be judged from the present results, it seems that the limiting value of $\omega$ at the final stage of each discontinuous stage, i.e. before starting a new slip line, is generally a little greater in the case of loose packing than in the case of close packing. The cause of the difference may probably be sought chiefly in the difference of the mode of transmission of the applied pressure, though sufficient experimental data are wanting at present to enable us to enter into a detailed discussion on this point.

Generally speaking, the modes of deformation is much more irregular in the case of loose packing, as may be illustrated by irregular forms of the white index lines (for example, Pl. III, Photo. 419). The irregularity is due to the fact that the active slip line is subjected occasionally to an accidental abrupt change of position relative to the sand mass.

Another characteristics of loose mass is that the shearing deformation in front of the slip line $a'b'$ is not confined to a well defined quadratic prism as in the case of closely packed one, but is extended onwards such that it is generally difficult to mark off sharply a wedge-formed portion which is behaving apparently as a rigid body.

*(C) Coarser sand, closely packed* (Pl. II).

Compared with the finer sand in closely packed state, the value of $\theta$ is not sensibly different in the closely packed coarser sand as far as may be judged from the present data. This is plausible, as $\mu$ is chiefly determined by the material of the particles, but not so much by the grain size. On the other hand, the angle $\omega$ is generally greater for the coarser sand than for the finer. The factor considered in ($A$) regarding the transmission of pressure is probably here effective. It will be interesting to observe that the general mode of variation of $\omega$ for the closely packed coarser sand is nearly similar to that of loosely packed finer sand.

The most striking characteristics of the coarser sand is that the successive slip planes are more closely spaced than in the case of the finer sand. This may be explained at least partly by the difference in the rate of increase of pressure with the displacement of wall, i.e. a quantity corresponding to Young's modulus of an elastic body. A systematic experimental investigation of the relations between this factor and the grain size as well as the state of packing is necessary for a further discussion in this line.

*(D) Coarser sand, loosely packed* (Pl. III).

In the case of the loosely packed coarser sand, the characteristics of the coarser sand mentioned above in comparison with the finer sand, with respect to the case of close packing, is much more enhanced. The discontinuous steps in faulting take place in rapid succession such that each step is mostly difficult to be sharply discerned. The result is that instead of a regular step of white line with a number of horizontal segments (for example Photo. 376) we have a curved line disturbed with minute ripples of irregular zigzag indentations (Photo. 238). Even in this case, however, we may discern a small wedge-shaped portion at the top of the raised mass, which still preserves its initial configuration as a rigid mass. Hence, it will be interesting to regard the extended *portion* of sand with the inclined trends of the index line as taking up here the rôle played by the sharply defined slip *planes* in the case of fine sand.

Taken as a whole it will be remarked that the difference between the fine sand in closely packed state and the coarse sand in loosely packed state is somewhat similar to the difference between a *brittle* material and a *plastic* one, as far as the geometrical, and perhaps also the kinematical, relation of deformation is concerned. This point may throw some light on the problems of plastic deformation of ordinary substance.

## II. Deformation caused by Reciprocating Motion of Movable Wall.

In the next series of experiment, the movable wall *B* was subjected to a reciprocating motion with a range of motion of 0.6 cm. The velocity of motion was kept nearly constant at 0.02 cm./sec. as before. Pl. IV (*a*), (*b*), (*c*), (*d*), (*e*) show the typical modes of deformation produced in a fine sand layer by this treatment. (*a*) represents the state at the end of the first progressive motion of the wall *B*, where we may clearly discern the principal system of characteristic lines, corresponding to *a′b′*, *b′c′* and *e′f′* of Fig. 1. (*b*) was obtained after the end of the first retrograde motion of *B*, during which a wedge-shaped portion *aps* has slipped down along a slip plane running aslant from the foot of the wall, just as was shown in our previous report. Next, the reciprocating motion of *B* was repeated ten times of cycles and then (*c*) was obtained. It will be seen that the quasi-rigid portion on the right side of the left slip plane has been considerably raised on account of the purely periodic motion of *B*. In the meantime, the sand just in front of the wall *B* has been successively lowered in its level, as this portion is nearly fixed to *B* during the progressive motion and slides down by every retrograde motion. In (*c*) we may observe that a double step-fault is already developed in front of *B*. Besides, it will be noticed that the slanting free surface of sand on the right side of the photogram is nearly parallel to the slip line *bc* marking the boundary of the mass at rest.(1) Next, the wall *B* was moved in alternate directions for another ten times, after which the entire mass has been deformed in a form as shown by (*d*) of the Plate.

Referring to the photogram (*d*), it will be seen that the slip line *bc* has become completely inactive and a new set of slip lines, $\alpha\beta$ and $\beta\gamma$, has appeared which now plays the rôle formerly played by *ab* and *bc*. Here, the relation is apparently somewhat similar to an ideal case in which the line *bc* is replaced by a fixed bed plate and a moving piston or wall at *ab*

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(1) This means $\omega=\phi$, showing that the pressure transmitted from *B* is small in the region of this slip line.

is pushed in a direction parallel to the line *bc*, meanwhile the direction of gravity is nearly parallel to *ab*. According to the considerations above made on the relative direction of the slip line and the resultant pressure, it will be seen that, in the present case, the applied pressure of the wall is deflected upwards more or less parallel to the slanting free surface. Referring to the photogram, it will be interesting to observe that, in the region within the prism $ab\beta$, the white index line which has previously been everywhere horizontal, appears here nearly vertical. A comparison with (*c*) will show that this is the trace of a former slip line pushed inwards by the reciprocating motion.

On repeating the reciprocating motion for further ten times, the photogram (*e*) is obtained. Here, the slip lines $a\beta$ and $\beta\gamma$ are already stopped and a second set, $\alpha_1\beta_1$ and $\beta_1\gamma_1$ has been developed.

It is well known that there is a class of mechanism by which a progressive motion of a certain working part of it is produced by a purely reciprocating motion of another. The present case will afford another interesting example which promises to find an important application to some geophysical problem, as will be presently seen. The essential point of the mechanism consists in the existence of the two distinct slip planes which are called into play alternately by the alternate motion of the wall which is the prime mover.

## III. Applications to Geophysical Problems.

Different experiments have been made by geologists to imitate the deformations of the earth crust subjected to horizontal thrust. They are, however, mostly based on a tacit assumption that the crust behaves as a sort of an ordinary plastic body in small scale. It is, however, well known that in the problem of elastic deformation the effect of gravity, which is usually neglected in the cases of bodies of laboratory scale, gradually increase in its importance with the increase of the size of the body. On the other hand, the effective elastic strength of the earth crust as a whole will probably of a quite different order of magnitude compared with that of a small defectless test piece handled in laboratory, as the superficial crust is run across by numerous joints and fissures. Such a crust will, of course, behave as a more or less perfect elastic bodies towards a disturbance such as earthquake waves propagating through it. When, however, a deformation of vast scale and of lasting nature such as indicated by geological structures is concerned, the matter will be quite different and the crust will resemble in some measure an aggregate of discrete units, blocks or grains as the case may be. Each unit may be supposed practically rigid, while the connection between the consecutive units is elastically weak and held chiefly by a sort of friction. In other words, the crust will resemble a layer of sand in some respect, if the above supposition be justifiable.

We have seen from the results of our experiments that a sand mass may behave sometimes like a brittle solid body and sometimes as a plastic material, according to circumstances. At any rate, therefore, the use of sand as the material for experiments will include a wider possibility than experimenting with ordinary plastic mass. Thus, H. M. Cadell(1), succeeded early in imitating a characteristic geological structure in Scotland by the use of sand layer, which is difficult to obtain with an ordinary plastic material.

In view of these considerations, it will not be without interest to attempt here to apply the present results of experiment for explaining some geological or geophysical phenomena, of course with much reserve and precaution.

(*a*) We have seen that when the movable end-wall is receded or pushed on continuously a fault line is formed which remains active for a definite interval of time and then stops to make way for a second active fault which suddenly appears at a certain distance from the first. It is probable that some of the actual groups of parallel fault lines might have been formed by a similar process.

(*b*) When the end-wall of the experimental apparatus is moved to and fro alternately, two different fault lines become alternately active. It occurs in an earthquake district that two distinct epicentral zones alternately

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(1) H. M. Cadell, *Trans. Roy. Soc., Edinburgh*, **35** (1888).

reveal their activity, with a period equal to some of the meteorological periods. The two phenomena may have some feature in common with respect to their mechanisms.

(c) Mechanisms by which a periodic horizontal stress in the earth crust may produce a progressive upheaval or depression have sometimes been discussed among geophysists. We have seen above a rather striking example of such mechanisms, in which a reciprocating motion with a range of 6 mm. resulted in heaving up a mound of more than 20 mm. height. Though our case is not adequate(1) for an immediate application, the above will serve as a demonstration of possibility in this line.

(d) In the present experiment, the bed plate is fixed and practically rigid and thus introduces a boundary condition that cannot be realized in the case of the earth crust. The above experimental condition may, however, correspond to a geological case when a superficial layer of small elastic strength lying upon a more rigid bed is subjected to a lateral thrust. For imitating the entire crust, however, it is desirable to experiment with a sand layer floating upon a more plastic bed. We are now preparing for a next series of experiment in which a layer of "miduame" will form a yielding substratum.

Apart from these and other possible geophysical applications, the above results seems also to suggest many points which may throw some new light on the problems connected with the fracture of solid and plastic materials in general. A way is suggested of bringing the brittle, plastic and fluidous materials on a continuous scale of some property which may conveniently be expressed in terms of some definite factors corresponding to those factors essential in the case of our sand model. Before approaching this side of the problem we must wait long until sufficient experimental data have been accumulated under widely varied conditions.(2) Especially, the effect of cohesion between the sand particles is one of the most important factor to

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(1) Especially because here the friction against the bed plate is considerable which may be absent or small in the case of a "floating" crust.

(2) In the case of plastic deformation, there is, of course, no factor corresponding to gravity in the present case, but the mode of variation of stress with the progress of strain by yielding may in many respect be analogous to the present case.

be investigated; it may affect the modes of deformation in a profound manner.

Recently, we were enabled to begin a cinematographical investigation of the deformation of sand mass by which means a considerable saving in time is obtained. In the next report, we hope we will be able to give some account of the results obtained. As the above series of experiments with ordinary photographic camera has now come to an end, it was considered convenient to give here a provisional summary of the result thus far obtained. The discussions here made regarding the mechanical explanation of the phenomena is still of a tentative character which may want a considerable modification in the course of further investigations. The chief aim of these discussions was to suggest different theoretical as well as experimental problems which may be raised regarding the matter in question.

## APPENDIX.

### A Further Note on the Mechanism of Formation of Step-Faults Produced by Receding the Wall.

In the previous paper, we have seen that the formation of step-faults by receding the end wall of the containing vessel is determined by the existence of upper and lower limits of the angle of inclination of the slip plane. The variation of the angle of inclination was expressed in terms of the variation in the apparent coefficient of friction of sand in motion. It is, indeed, undeniable that the true coefficient of friction may differ sensibly in the case as is here concerned from the statical case when only the initial infinitesimal slip is in question. As will be seen from the considerations mentioned above in connection with the discussion of the results of the present experiments, there is, however, another important factor determining the angle of inclination of the slip plane, i.e. the variation in the effective pressure on the wall $B$ brought about by the motion of it.

According to the statical theory of earth pressure on retaining wall, the angle of inclination of the slip plane, $\theta_1$, will be given by

$$\tan\theta_1 = \frac{\sin\phi + \sin 45^\circ}{\cos\phi},$$

where $\phi$ is the angle of natural slope, and the coefficient of friction $\mu'$ on the wall is assumed to be equal to the coefficient $\mu$ on the sand.

If, however, $\mu' = 0$, the inclination is given by

$$\tan\theta_2 = \frac{\sin\phi + 1}{\cos\phi}.$$

On the other hand, if the pressure on the wall is annulled, the sand mass will glide freely on the slip plane so that the inclination will be given by

$$\tan\theta_3 = \tan\phi = \mu.$$

In the actual case with the sand in motion, the limiting angle $\theta_3$ can never be attained, as long as the motion of the receding wall is slow and uniform. When, however, the wedge-shaped mass in front of the wall is in continuous motion we may compare the case with the statical one by saying that the effective friction against the wall is diminished, as in the layer immediately in front of the wall the horizontal pressure is being partly released and thence the total friction against the wall is reduced. This will tend to make the angle of inclination approach the value $\theta_2$.

In the actual examples, we have $\phi$ approximately equal to 30°, whence

$$\theta_1 = 64^\circ 20', \qquad \theta_2 = 60^\circ.$$

The actual range of variation seems to concide roughly with that given by the above. Besides, the fact that the slip surface is generally curved and the inclination of its tangent decreases with the depth of the point along the slip line, may also be explained by considering that the lower layer is undergoing a relatively larger horizontal extension than the upper(1) and thence approaches more to the case of free fall.

We hope we will return to the discussion of these points when more sufficient data have been accumulated.

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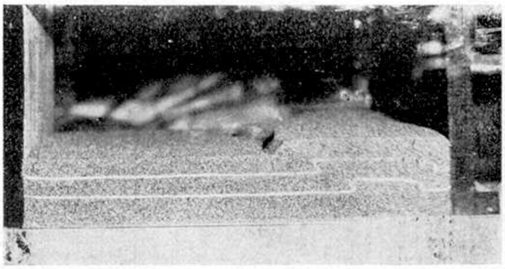
(1) Previous paper, *loc. cit.* p. 52–53, Table I and II.

[T. Terada and N. Miyabe.] [Bull. Eqk. Res. Inst., Vol. 6. Pl. I.]

Photo. No. 341

346

351



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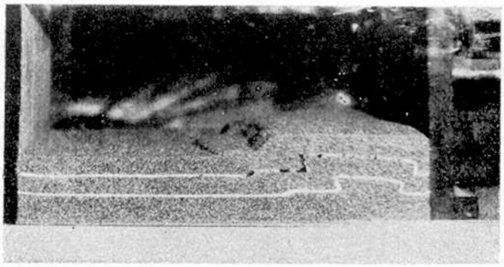

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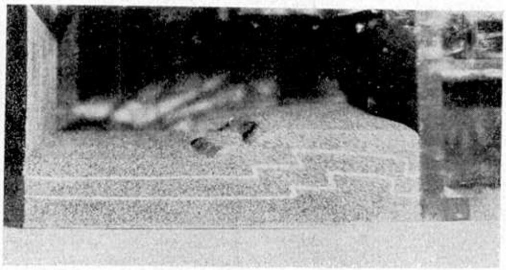

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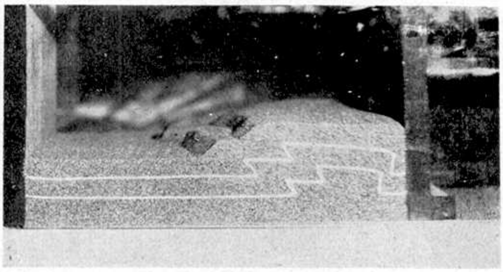

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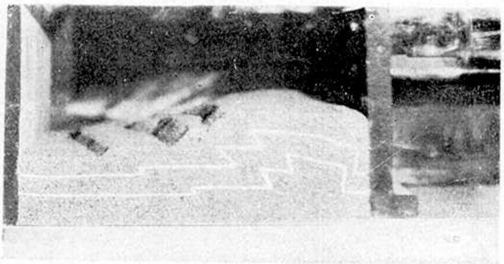

Finer Sand, Closely Packed.

（震研彙報、第六號、圖版、寺田、宮部）

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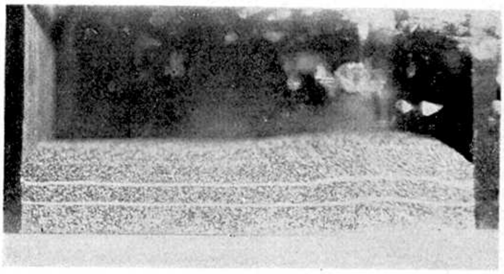

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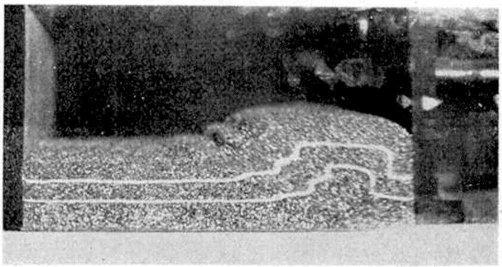

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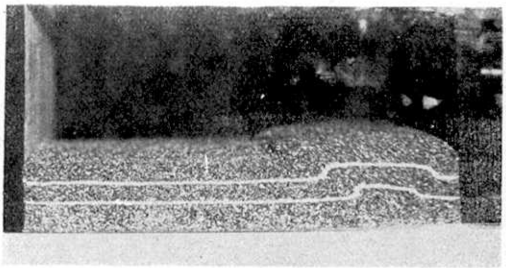

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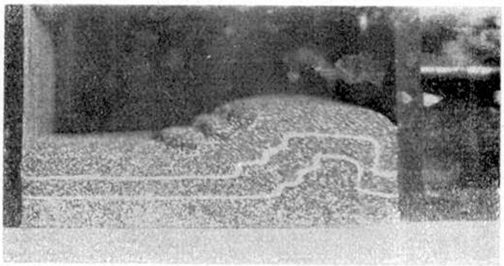

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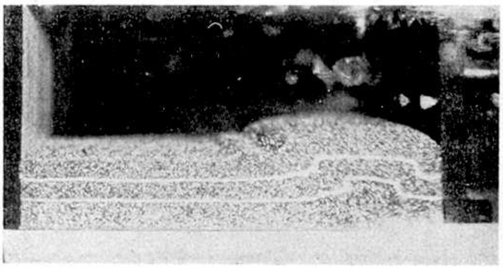

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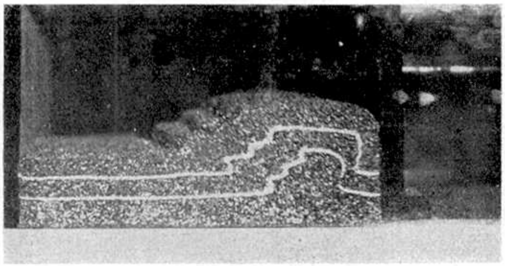

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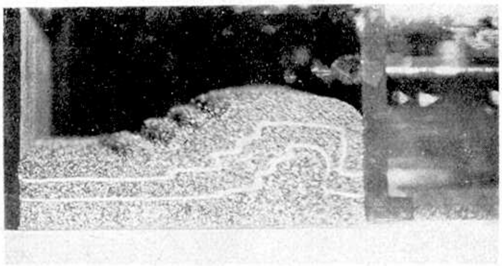
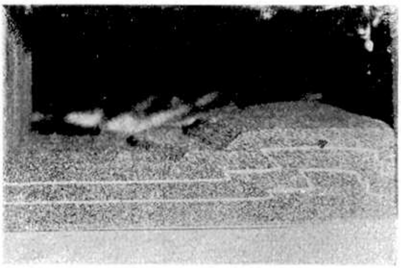


Coarser Sand, Closely Packed.

（震研彙報、第六號、圖版、寺田、宮部）

Photo. No. 368



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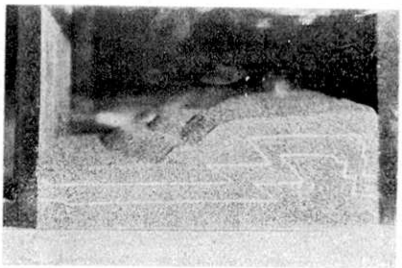
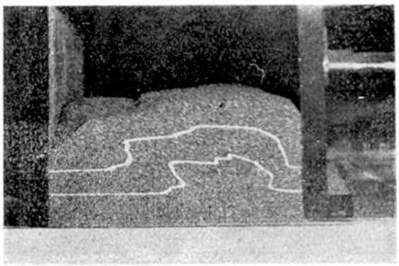


Finer Sand, Closely Packed.

419



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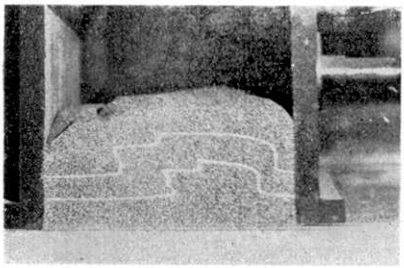

Finer Sand, Loosely Packed.

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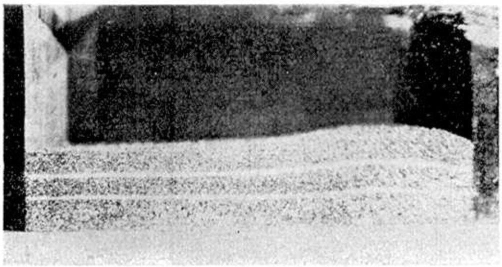

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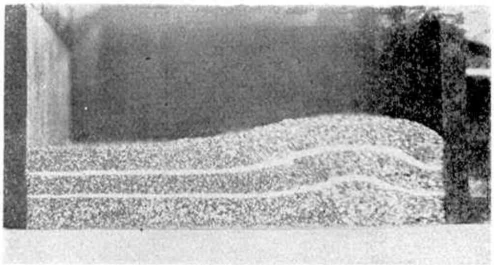

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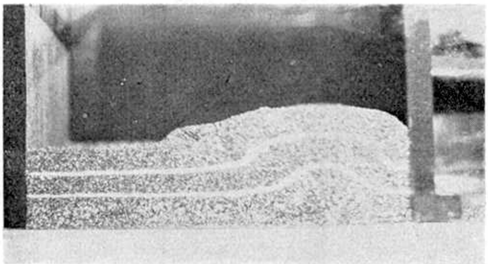

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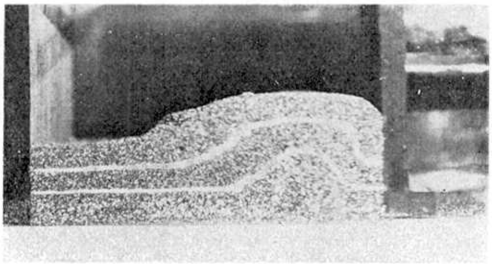
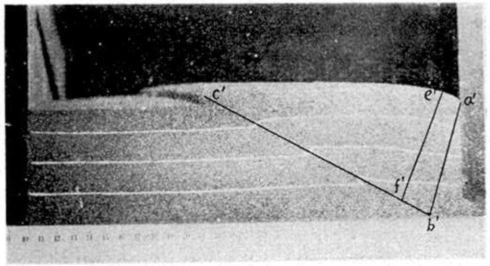


Coarser Sand, Loosely Packed.

（震研彙報、第六號、圖版、寺田、宮部）



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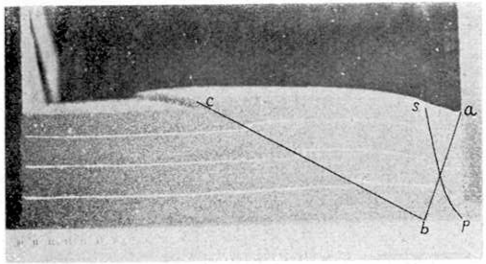

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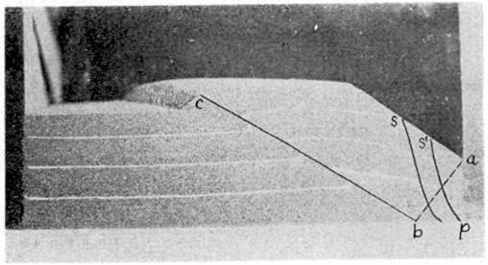

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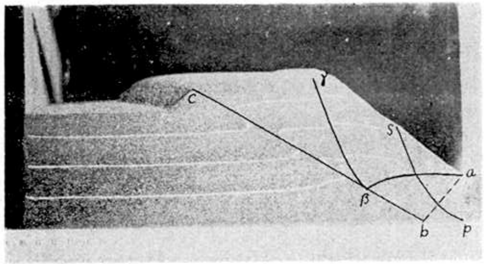

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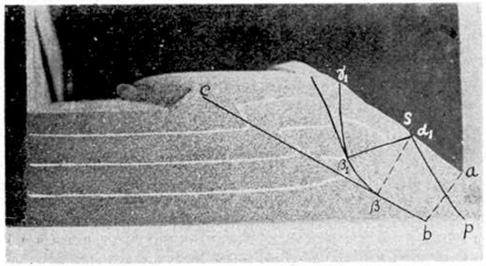

(*e*)

(*a*) First positive motion.
(*b*) First negative motion.
(*c*) After (*b*), 10 reciprocating motion.
(*d*) After (*c*) ,, ,,
(*e*) After (*d*) ,, ,,

（震研彙報、第六號、圖版、寺田、宮部）