Observation of Some Recent Earthquakes and their Time-Distance Curves.

(Part IV.)

By **Takeo MATUZAWA,**

Nukigaki (abstract in Japanese.)

(1) Zissai no Zisin no Baai ni *Rayleigh* oyobi *Love* no tonaeru yôna Hyômen ni tutawaru Nami no Kwansoku sareru koto wo tokiakasita.

(2) Korera no Nami wa aru *Dispersion* wo simesu. Sosite sono Zyôkyô wa Tairiku wo tutawaru mono to Taiheiyô wo tutawaru mono tode medatta Tigai ga aru.

(3) *Dispersion* no Seisitu wa Hyômen no sôzyô no Kôzô wo katei suru Koto ni yotte daitai setumei dekiru.

(4) Onoono Sô ni okeru *Distortion* no Nami no Hayasa wa sono-2 ni oite motometa mono to muzyun sinai.

(5) Sita no Bubun ni okeru Nami no Hayasa ga sidai ni kawaru Baai no Eikyô wo giron sita.

I. INTRODUCTION.

In the preceding papers attention was mainly confined to the dilatational and the distortional waves observed at stations within small epicentral distances. In this paper, problems relating to surface waves will mainly be treated. For this purpose study of remote earthquakes is necessary, for in cases of near earthquakes, separation of the surface waves from the distortional waves is not sufficient to admit of any definite conclusions being drawn.

The inference that the velocity of propagation of seismic waves may differ according to whether their path lies under the oceanic area or under the continental land dates back, as far as the present author is aware, to

1909, when the late Professor F. Omori(1) investigated the propagation of disturbance due to the Guatemala earthquake of April 19, 1902, and that of the Kangra (India) earthquake of April 4, 1905. His method of reduction was, however, somewhat different from that employed by modern seismology.

Recently many valuable papers in this direction have appeared from authors, such as E. Tams,(2) G. Angenheister,(3) S. W. Wisser,(4) B. Gutenberg,(5) W. Hiller(6) et. al. Results found by these authorities are not in all cases in agreement with each other. Some of them show almost no difference of velocity of propagation of surface waves along the continent from that along the oceanic area, whilst others show much difference, even as much as twenty percent. Such discrepancies seem to come from many apparent factors. Above all, the practical difficulty of identification of the waves seems to constitute the main ground for divergence of opinion.

The geographical position of Japan is very favourable for observation of purely trans-pacific seismic waves. The distance from Tôkyô to regions of great seismicity, as far as the Pacific Coast of South America, extends from zero to almost 18000 km. The only spot from whence seismic disturbances do not come, or seldom come, is in a small segment (about 20°) covering a part of Polynesia. Under these circumstances, typical examples of trans-pacific seismic waves have been obtained in the long series of observations made at the Seismological Institute, Imperial University of Tôkyô, especially by long period seismographs with proper period of 50 sec. or more. The data treated in this paper will accordingly not be confined to recent earthquakes.

II. REAL EXISTENCE OF TWO KINDS OF SURFACE WAVES.

From the theoretical point of view, Rayleigh wave can be propagated along a surface of a certain medium. In addition to this, waves of Love's

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(1) F. Omori, Bull. Imp. Earthq. Invest. Comm. **3** (1909) 61–68.

(2) E. Tams, Zentral Bl. f. Mineralogie usw. (1921).

(3) G. Angenheister, Nach. d. Kgl. Ges. d. Wissensch. Göttingen (1921) 113.

(4) S. W. Wisser, Kon. Magnet en Met. Obs. et Batavia (1921) 7.

(5) B. Gutenberg, Zeits. f. Geophys. **1** (1925) 94; Phys. Zeits. **25** (1924) 377 etc.

(6) W. Hiller, Beitr. z. Geophys. **17** (1927) 3.

type can also be propagated if the medium is stratified under the boundary surface. Whether or no such waves are really existent is a problem which can easily be solved by seismometry. And yet some physicists are often sceptical of the real existence of surface waves, perhaps from difficulty of accurate identification of waves. On the other hand, even among seismologists, there is a diversity of opinion on the separate existence of two kinds of surface waves, i.e. the Love's type and the Rayleigh's type owing to the lack of typical examples free from any ambiguity. Therefore, some typical seismograms will be reproduced here. (Figs. 1, 2 and 3)

Atacama Earthquake, 1922 Nov. 11, $4^h\,32^m$.
(Upper 3 lines: $\frac{E}{W}$ comp. Lower 3 lines: $\frac{S}{N}$ comp.)

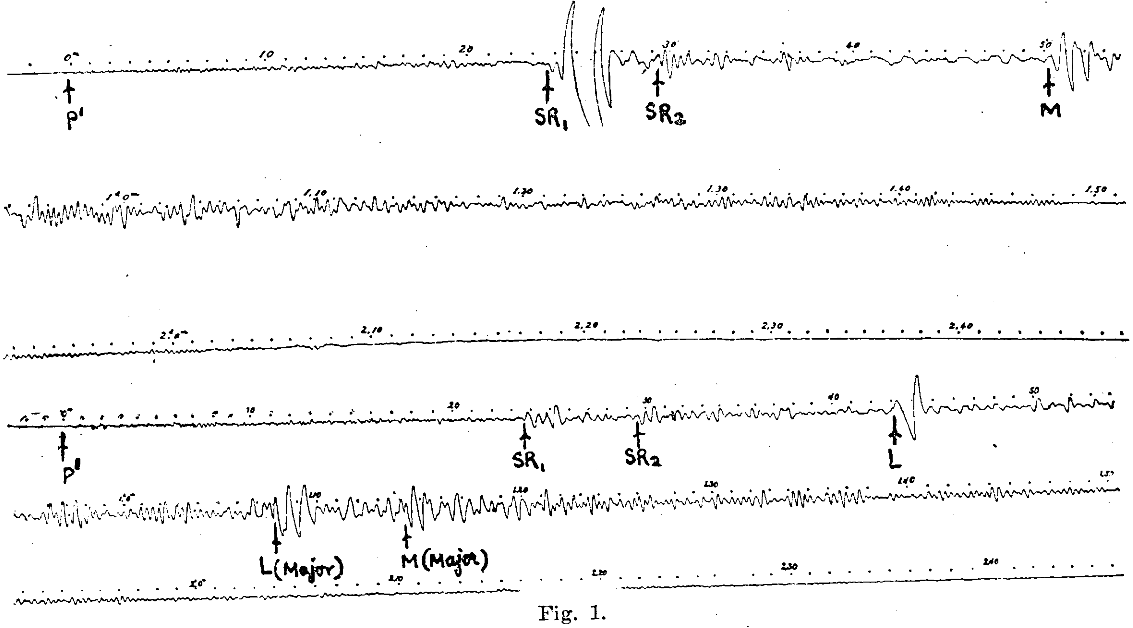


Fig. 1.

Earthquake, 1926 Oct. 26.
($\varphi=2°$ S, $\lambda=138°$ E)

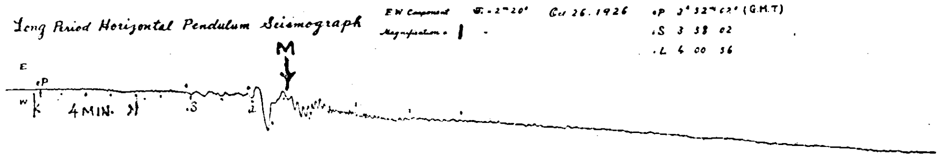
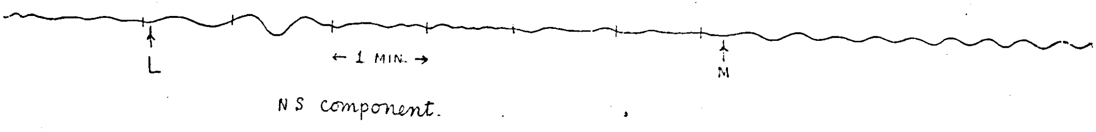


Fig. 2.

Earthquake, 1920 III 20, $18^h\,31^m$.
$\left(\begin{matrix}\varphi = 35^\circ.8S \\ \lambda = 109^\circ.4W\end{matrix}\right.$ Far South of Easter Islands)



Tonga Earthquake, 1921 II 27, $18^h\,23^m$.

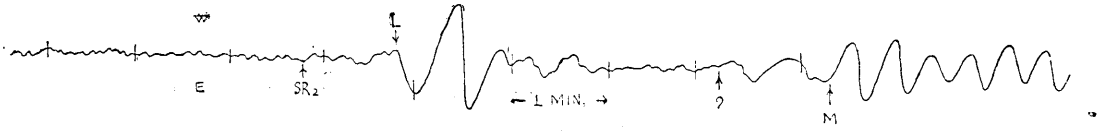


Fig. 3.

In each of the figures the phase denoted by L corresponds to the commencement of waves of Love's type, and the phase denoted by M corresponds to that of Rayleigh waves. The inference that each of these two phases of train of waves may belong to different kind of surface waves respectively can be easily seen from a study of their time-distance curves. (Fig. 4) The polarisation of waves in each phase is also remarkable. The amplitude of motion of the L phase is predominant, mainly in the direction perpendicular to that of the epicentre, though it is not the case, rigorously speaking, owing to many complicated factors. Moreover, the polarisation of the L and the M phase is nearly perpendicular to each other as can be seen in Fig. 1.

The period of motions of phase L is usually far longer than that of phase M. The wave form of phase M is somewhat regular and more coherent than that of phase L. In the case of transpacific waves, such characteristics are usually distinct. On the other hand, in the case of transcontinental waves, separation of the L and the M phase does not seem to be so distinct as in the former case. The L phase commences with gradually increasing amplitude and is superposed by a more or less regular, large M phase, as shown in fig. 25(1) of B. Galitzin's "Vorlesungen über Seismometrie." And yet the existence of two kinds of surface waves hardly admits of doubt.

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(1) B. Galitzin, Vorlesungen über Seismometrie, (1914) 110.

III. DATA OF EARTHQUAKES.

One good way of studying the mode of propagation of seismic waves is to compare seismograms of one and the same earthquake observed at different places. And yet to study many different cases in such a way is not an easy matter to put into practice. Because of this reason my seismometric study will be confined to that of seismograms obtained in our Institute, while reports from other stations will be referred to as a supplement. In the following list, the position of the epicentre and the time of occurrence of earthquakes are adopted from the International Seismological Summary, B. A. Seism. Reports and B. A. Seism. Circular, unless special remarks to the contrary are not added.

TABLE I-a.

(Transpacific)

Epicentre.	$\varDelta$ km.	Time of Occur. at the Epicentre.	Phase L			Phase M			Remark.
			Time of Comm.	Vel. km/sec.	Period Sec.	Time of Comm.	Vel. km/sec.	Period Sec.	
$46\overset{\circ}{.}0$ N 154.0 E	1650	1922 May 4 9^h 12^m 48^s				20^m 11^s	3.70	$\overset{s}{17.5}$	
48.0 N 157.0 E	1970	1921 May 21 22 25 42				35 23	3.39	12	
24° 33′N 122 25 E	2060	1922 Sept. 1 19 15 53	25^m 16^h	3.73	50	27 45	2.89	8	By the author.
24.8 N 122.4 E	2060	1910 Sept. 1 14 21 04				35 43	2.35	14	,,
52.5 N 157.5 E	2200	1922 March 4 13 7 34	16 51	4.02	32				
23.0 N 121.7 E	2240	1919 Dec. 19 20 37 24	46 31	4.08	40				
22.8 N 121.7 E	2260	1910 Sept. 1 0 44 45				59 07	2.62	14	
24.0 N 116.5 E	2586	1918 Feb. 13 6 7 10	17 50	4.03	55~40	20 21	3.27	20	
11.5 N 144.0 E	2730	1917 May 9 15 54 59	05 34	4.17		06 23	3.88	30	

(to be continued)

(continued)

Epicentre.	Δ km.	Time of Occur. at the Epicentre.	Phase L Time of Comm.	Phase L Vel. km/sec.	Phase L Period Sec.	Phase M Time of Comm.	Phase M Vel. km/sec.	Phase M Period Sec.	Remark.
8.0 N 128.0 E	3305	1921 Nov. 11 18 36 06	50 02	3.95	35	52 03	3.45	29	
Off the S coast of Mindanao	3500	1924 April 14 16 20 19	34 30	4.11	12~13	35 33	3.83	30	By the author.
49.5 N 178.0 W	3720	1913 March 31 3 40 48	56 55	3.85					
45.0 N 170.0 W	3950	1910 Sept. 9 1 13 00	29 28	4.00	16	32 39	3.35		
3.0 N 122.0 E	4070	1913 Jan. 11 13 17 02	33 25	4.14	15.6	35 31	3.67	24	
Off the NW coast of Celebes	4550	1917 Aug. 30		4.22	37		3.49	54	By the author.
5.0 S 154.0 E	4760	1913 May 30 11 46 46	12^h 04 47 (06 42	4.40 4.12	38 17)	08 30	3.68	24	( ), 2nd group.
54.5 N 160.0 W	4990	1917 May 31 8 47 20	9^h 07 42	4.07		09 28	3.76		
8.5 S 144.0 E	5100	1920 May 7 21 31 13	52 38	3.97	40	55 26	3.66	15	
10.0 S 156.0 E	5210	1926 Mar. 27 10 48 27	11^h 08 56	4.29	15	13 16	3.54	14 18	By the author.
Alaska	5300	1912 July 7 7 56 48	17 33	4.25	40				”
11.0 S 161.0 E	5650	1926 April 12 8 32 15	54 10	4.30	35	57 33	3.72	20	
Simalur, Nias.	5860	1907 Jan. 4 14 28 or so.		3.78	45		3.56	32	O is not certain exactly.
4.2 S 102.0 E	5930	1914 June 25 19 7 15	30 12	4.32	30	33 33	3.76	30	From data of Batavia.
11.4 S 111.5 E	6023	1921 Sept. 11 4 01 25	28 02	3.77	35	38 04	2.74	18	By the author.
13.5 S 167.0 E	6180	1920 May 20 7 25 46	49 53	4.27	37	52 31	3.85	18	
15.0 S 165.0 E	6330	1919 Aug. 31 17 20 34	44 54	4.33	28	47 10	3.97	30	
12.0 S 164.0 E	6780	1913 Oct. 14 7 53 14	8^h 34 07	4.42	50				By the author.

(to be continued)

(continued)

Epicentre.	Δ km.	Time of Occur. at the Epicentre.	Phase L			Phase M			Remark.
			Time of Comm.	Vel. km/sec.	Period Sec.	Time of Comm.	Vel. km/sec.	Period Sec.	
20.6 S 168.8 E	6970	1920 Sept. 20 14 38 50				15^{h} 10 12	3.70	21	
Tonga	7750	1917 June 26 5 49 29				6^{h} 20 57	3.84		By the author.
72.0 N 8.5 W	7790	1922 April 8 20 42 12	21^{h} 12 35	4.27	17	21 17	3.32	14	
22.5 S 173.5 W	8130	1913 June 26 4 57 01	5^{h} 26 39 (27 21	4.56 4.47	45 30)				
Kermadec	8400	1919 April 17 11 22 09	54 14	4.36	30	56 23	4.09	35	By the author.
40.15 N 117.10 W	8500	1915 Oct. 3 6 52 49	7^{h} 26 35	4.25	27	7^{h} 31 10	3.76	24	
29.2 S 177.0 W	8540	1917 May 1 18 26 46	58 25	4.50		19^{h} 05 01	3.72	19	
34° 39′N 135° 26 E	8580	1906 April 18		4.43	32		4.02	35	San Francisco.
40.9 S 177.1 E	9340	1917 Aug. 5 15 50 01	25 58	4.33	35∼42	29 55	3.90	35	By E.F. Pigot S.J.
58.0 S 159.0 E	10500	1924 June 26 1 37 10	17 22	4.35	60	22 24	3.87		By the author.
18.0 N 100.0 W	11340	1907 April 15 6 5 42		4.63	30		3.51 (3.29 (3.19	30 29) 24)	From catalogo de los temblores en Mexico (1909) 440.
17.0 N 100.0 W	11410	1911 Dec. 16 19 14 38	20^{h} 01 51	4.03	33				By the author.
14.5 N 91.0 W	12300	1917 June 8 0 51 28				1^{h} 48 21	3.61	23	
13° 46′N 90 50 W	12360	1915 Sept. 7 (1) 1 20 17	07 21	4.37		14 21	3.81		By the author.
19.0 N 80.0 W	12630	1917 Feb. 20 19 29 32	17 16	4.42	30	21 26 (27 40	4.07 3.63	24 20)	
14.5 N 86.0 W	12650	1921 March 28 7 49 20	37 54	4.35	60	44 34	3.82	30	
35.8 S 109.4 W	13920	1920 March 20 18 31 15	22 30	4.48	41∼34	28 40	4.04	29	See Fig. 3.

(to be continued)

(continued)

Epicentre.	Δ km.	Time of Occur. at the Epicentre.	Phase L			Phase M			Remark.
			Time of Comm.	Vel. km/sec.	Period Sec.	Time of Comm.	Vel. km/sec.	Period Sec.	
10° 30′N 66 54 W	14160	1900 Oct. 29 10 10 30	12^h 00 41	4.55		12^h 25 21	3.15		By F. Omori.(2)
3° 12′ S 82 50 W	14320	1922 Jan. 17(3) 3 50 24	4^h 41 33	4.65					
14.0 S 75.5 W	15790	1916 Octt 3 1 26 13	2^h 24 13	4.54		32 13	3.95		
18.0 S 73.0 W	16240	1913 Aug. 6 22 14 14				23^h 22 08	3.99	23	
16° 28′ S 71 30 W	16260	1922 Oct. 11 14 49 55	15^h 50 45	4.45	38	16^h 04 32	3.79	17	From data of La Paz.
26.5 S 70.5 W	16970	1918 Dec. 4 11 47 44	12^h 51 04	4.47	45~50	58 50	3.97	40~50	
28° 20′ S 71 0 W	17000	1922 Nov. 11 4 32 55	5^h 35 56	4.55	55	5^h 42 28	4.07	45	By Sieberg and Gutenberg.
23.0 S 66.0 W	17160	1916 Aug. 25 9 43 57	10^h 46 56	4.54	35~50	56 32	4.03	27	

(1) Seismograms of this earthquake are quite similar to those of the earthquake of Sept. 23, 1902. See F. Omori, Publ. Imp. Eqke. Invest. Comm. **21** (1905) 52.

(2) F. Omori, ibid. **13** (1903) 137.

(3) According to P. Byerly, Bull. Seism. Stations (Berkley) Vol. 2, No. 3, this earthquake consisted of three simultaneous earthquakes at Brasil ($\varphi=4°35'.2S$, $\lambda=63°56'.3W$), Venezuela ($\varphi=5°11'.5N$, $\lambda=66°45'.2W$) and Ecuador ($\varphi=3°12'.1S$, $\lambda=82°50'.2W$). Observed values at Tôkyô are consistent with Ecuador as the origin.

TABLE I-b.

(Transcontinental earthquakes)

Epicentre.	Δ km.	Time of Occur. at the Epicentre.	Phase L			Phase M			Remark.
			Time of Comm.	Vel. km/sec.	Period Sec.	Time of Comm.	Vel. km/sec.	Period Sec.	
35°.61 N 105.40 E	3090	1920 Dec. 16 12 05 50	20 09	3.60	27				By Y. Dammann.(1)
31.5 N 100.8 E	3560	1924 March 24 12 41 04	56 34	3.83	50	59 46	3.17	22	By the author.

(to be continued)

(continued)

Epicentre.	Δ km.	Time of Occur. at the Epicentre.	Phase L			Phase M			Remark.
			Time of Comm.	Vel. km/sec.	Period Sec.	Time of Comm.	Vel. km/sec.	Period Sec.	
26.0 N 100.0 E	3920	1925 March 16 14 41 54	59 14	3.77	38	15^h 02 29	3.18	14.5	By the author.
Maldive Islands	4570	1912 May 23 2 23 52	41 56	4.22	43	45 14 (47 09	3.59 3.27	21 13)	Along margin of Asia
42.75 N 77.10 E	5330	1911 Jan. 3 23 25 30	47 52	3.97	57				By the author.
31° 49′N 77 00 E	5720	1905 April 4 0 49 48		3.67	54		3.11	30	From F. Omori.(2)
38° 16′N 73 34 E	5760	1911 Feb. 18 18 40 59	19^h 07 05	3.68	38	12 59	3.00	22	
39° 40′N 67 30 E	6230	1907 Oct. 21 4 23 550		3.71	42		3.41		By the author.
39.6 N 41.5 E	8130	1924 Sept. 13 14 33 54	15^h 07 29	4.03	60	11 29	3.51	34	
44.0 N 25.0 E	8980	1913 June 14 9 23 21	10^h 08 28	4.15		17 05	3.42	22	
40.0 N 26.4 E	9190	1912 Aug. 9 1 28 551	06 03 (08 10	4.12 3.90	20 40)?	15 02	3.32	20	By the author.
38° 7½′N 18 35 E	10000	1908 Dec. 28 4 20 27		3.83	26		3.46 3.04	23 25	T.C. at Messina.

(1) Y. Dammann, le tremblement de terre du Kan-sou au 16 Dec. 1920. Publ. du Bureau Central Seismologique international, (1925).

(2) F. Omori, Publ. Eqke. Invest. Comm. **24** (1907) 6.

(3) This earthquake was discussed by H. Jeffreys, the Earth (1924) 170, as caused by a land slip at Pamir.

In preparing this table, identification of each phase was sometimes quite doubtful, as will be appreciated from the nature of seismograms. And yet for greater distances, say more than 8000 km., each phase of waves is resolved clearly without superposition of one another, as stated in the foregoing section. Another difficulty is the identification of the L phase at a locality of small epicentral distance and is due to the fact that the time of arrival of the SR phases (distortional waves reflected at the surface of the Earth) is not much different from that of the L phase. Thus, strictly speaking, the commencement of the L phase is quite liable to be mistaken. The time-distance relation of the SR phases, however, is tolerably definite owing to the small effect of dispersion, and the magnitude of amplitude of reflected waves, especially reflected more than twice, is usually small. Therefore, their identification is not always impossible, if the epicentre, the time of occurrence at the epicentre etc., of an earthquake can be determined from other data.

IV. TIME-DISTANCE RELATIONS.

As is apparent from Fig. 4, the lower boundary of the points representing the time-distance relation of the L phase is a straight line which is characterised by $\frac{\partial \Delta}{\partial T}=4.45$ km/sec. It must be remarked that this value is the same as that of the velocity of propagation of the S phase as obtained in part II of this paper. A similar phenomenon can be seen with regard to the M phase, especially for greater distances, and the straight line bordering the lower limit is characterised by $\frac{\partial \Delta}{\partial T}=3.9$ km/sec. The fact that the time-distance curves of these phases can be represented by straight lines shows that these phases can be interpreted as surface waves.

V. DISPERSION OF WAVES.

Dispersion of surface waves of Love's type, especially from the practical point of view, has been fully discussed by B. Gutenberg.(1) If the writer's understanding is correct, he has used many trains of waves in an earthquake from the consideration that such separation of train of waves might be due to dispersion. In the present paper, however, attention will be focussed on the commencement of each phase, for it is not certain that any train of waves appearing later is excited simultaneously at the origin of an earthquake. With these considerations in view, the velocity-period diagram has been constructed as shown in Fig. 5-a and 5-b.

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(1) B. Gutenberg, Phys. Zeits. **25** (1924) 377; **27** (1926) 111–114; Zeits. f. Geophys. **1** (1924–25) 94.

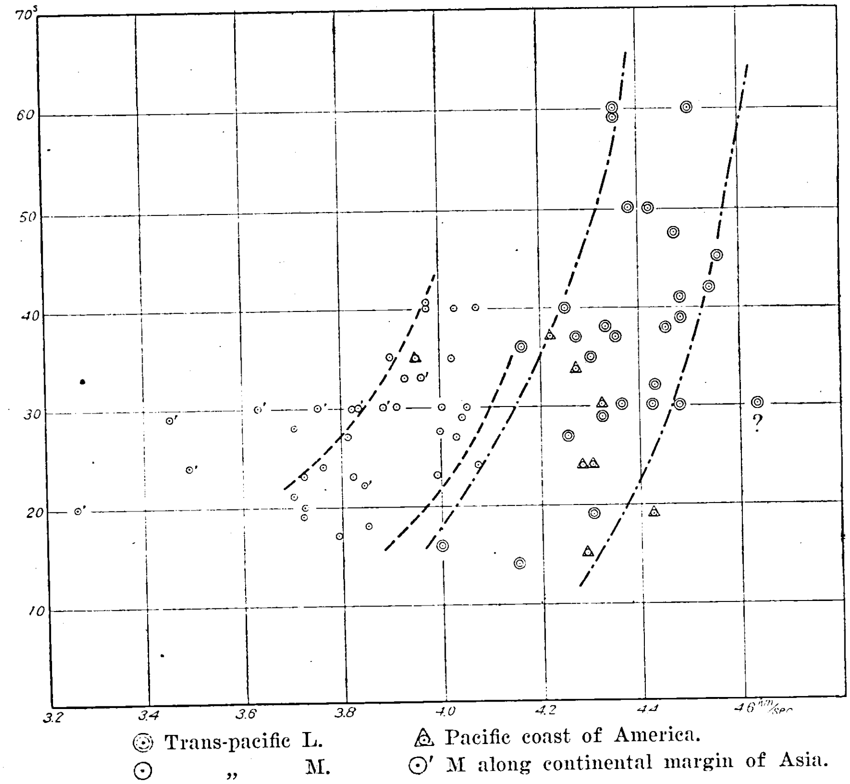

Fig. 5-a.

In Fig. 5-a, that of transpacific waves is shown, and in Fig. 5-b that of trans-continental waves is plotted. In both cases, dispersion of waves can be seen, though not very sharply, which is inevitable from the difficulty of accurate identification of waves. Moreover, it is almost certain that the mode of dispersion in the case of trans-pacific waves is quite different, quantitatively, from that of the trans-continental waves. For the same period of motion, trans-pacific waves are propagated much faster than the trans-continental waves. The same characteristic can be seen in the case of the M phase.

For example, for the M phase movements with period of about 15 sec., the velocity of the trans-continental waves is 3.1 km/sec. or so, and that of the transpacific waves is nearly 3.6 km/sec. These values are not in-

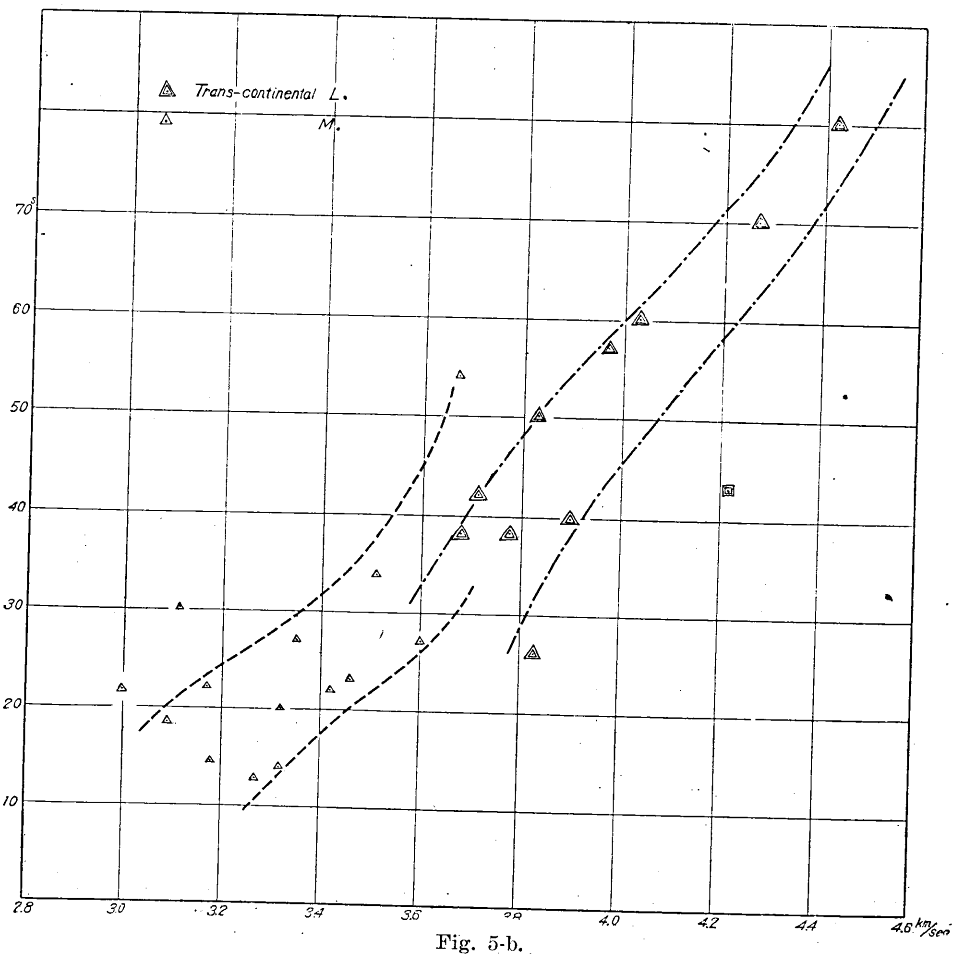

Fig. 5-b.

consistent with the result in connection with waves of Rayleigh's type (R–Wellen by his notation) obtained by W. Hiller.(2) It must be remarked that the word "Trans-continental" here used must not be interpreted literally, because Japan is bounded on all sides by sea.

VI. DISCUSSION OF THE RESULTS.

The facts observed with regard to the dispersion of surface waves stated above must now be examined in the light of the theory of elasticity.

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(2) W. Hiller, Beitr. z. Geophys. **17** (1927) 3.

Mathematical calculations covering several cases⁽¹⁾ have already been published in Proceedings of the Physico-Mathematical Society of Japan, 3rd Ser. Vol. 10 (1928) No. 3, though numerical values taken as examples differed from those obtained in this investigation. It can be seen that the observed relation between the velocity of propagation and the period of movements is explained, at least, qualitatively, from the elastic theory even in taking into account the double stratification. The author will not examine the effect of group velocity, since in this investigation the velocity of propogation of wave front is what is under consideration. Thus, it is quite untenable in this case to compare the result here obtained with the theoretical group velocity which is a formal consequence of wave superposition.

Effect of gradual change of medium.

In the above mentioned paper⁽²⁾ the author treated only cases of dispersion in stratified layers, each of which is of an uniform structure. In part I and part II, the first and the second layer can very well be regarded as uniform. The lower layer must, however, be regarded as having undergone gradual changes. Therefore, this effect must be verified. Verification will only be confined to the case of single stratification, for the object is to examine how the gradual change of medium affects the character of the period-velocity relation.

The equation of motion of love's type in a heterogeneous medium⁽³⁾ is given by, using customary notations, and assuming heterogenuity in the z direction only,

$$\rho\frac{\partial^2 v}{\partial t^2}=\mu\nabla^2 v+\frac{\partial\mu}{\partial z}\frac{\partial v}{\partial z}, \qquad u=0 \quad \text{and} \quad w=0.$$

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(1) The author came across a paper by Stoneley and others, published almost about the same time, in which similar subjects were discussed. (Geophys. Suppl. Month. Not. Astr. Soc. London **1** (1928) 521.)

(2) T. Matuzawa, Proc. Physic Mathm. Soc. Japan [ii] **10** (1928) No. 3.

(3) A similar case without stratification was already discussed by the late Prof. K. Aichi (Proc. Physic. Mathm. Soc. Japan [iii] **4** (1922) 137–142.) In his paper the last term of the equation of motion was unduly neglected.

By putting $v=v(z)\cos(fx+pt)$, where f and p are constants,

$$\mu\frac{d^2v}{dz^2}+(\rho p^2-\mu f^2)v+\frac{d\mu}{dz}\frac{dv}{dz}=0.$$

Again, by $v=\dfrac{V}{\sqrt{\mu}}$

$$\frac{d^2V}{dz^2}+IV=0,\quad \text{where}\quad I=b+\frac{1}{4}\frac{1}{\mu^2}\left(\frac{d\mu}{dz}\right)^2-\frac{1}{2\mu}\frac{d^2\mu}{dz^2}$$

by putting

$$b=\frac{\rho p^2}{\mu}-f^2.$$

Taking the z axis vertically upward and a point on the boundary surface of discontinuity as the origin and assuming linear relations such as $\mu=\mu_2-\beta z$ and $\sqrt{\dfrac{\mu}{\rho}}=v_0-\beta_1 z$, for discussion of small range of z the following approximation is permissible;

$$I=A+Bz,\quad \text{where}\quad A=\frac{p^2}{v_0^2}+\frac{\beta^2}{4\mu^2}-f^2\quad \text{and}\quad B=2\left(\frac{p^2\beta_1}{v_0^3}+\frac{\beta^2}{2\mu_2}\right).$$

Thus,

$$\frac{d^2V}{dz^2}+(A+Bz)V=0.$$

Substituting $t=A+Bz$

$$\frac{d^2V}{dt^2}+\frac{t}{B^2}V=0.$$

In the present case treating of surface waves, the case of negative value of t may be important. Thus the solution of the equation is given by

$$V=\sqrt{t}\left\{\mathfrak{a}H^{(1)}_{\frac{1}{3}}\left(\frac{2}{3B}t^{\frac{3}{2}}\right)+\mathfrak{b}H^{(2)}_{\frac{1}{3}}\left(\frac{2}{3B}t^{\frac{3}{2}}\right)\right\},$$

Accordingly

$$v=\frac{\sqrt{t}}{\sqrt{\mu_2+\beta\dfrac{A}{B}-\dfrac{\beta}{B}t}}\left\{\mathfrak{a}H^{(1)}_{\frac{1}{3}}+\mathfrak{b}H^{(2)}_{\frac{1}{3}}\right\},$$

where $\mathfrak{a}$ and $\mathfrak{b}$ are arbitrary constants.

For large absolute value of t, v must vanish. Thus, $\mathfrak{b}=0$.

$$\frac{dv}{dt}=\mathfrak{a}\left[\frac{1}{2}t^{-\frac{1}{2}}\left(\mu_2+\beta\frac{A}{B}-\frac{B}{B}t\right)^{-\frac{1}{2}}\left\{1+\frac{\beta}{B}t\left(\mu_2+\beta\frac{A}{B}-\frac{\beta}{B}t\right)^{-1}\right\}H^{(1)}_{\frac{1}{3}}\right.$$
$$\left.+\frac{t}{B}\left(\mu_2+\beta\frac{A}{B}-\frac{\beta}{B}t\right)^{-\frac{1}{2}}\frac{dH^{(1)}_{\frac{1}{3}}(x)}{dx}\right),\quad \text{where}\quad x=\frac{2}{3B}t^{\frac{3}{2}}.$$

In the next place, boundary conditions must be considered. As the xy plane is taken along the lower boundary of the uniform superficial layer, the value of z is positive in this domain. Quantities in this layer will be denoted by adding suffix (1) and those in the lower medium by suffix (2). The thickness of the layer will be noted by H.

In the superficial layer

$$v_1=(A_1\cos s_1 z+B_1\sin s_1 z)\cos(pt+fx).$$

Boundary conditions are as usual

$$\frac{\partial v_1}{\partial z}=0 \quad \text{at} \quad z=H$$

$$\left.\begin{aligned} & v_1=v_2 \\ & \mu_1\frac{\partial v_1}{\partial z}=\mu_2\frac{\partial v^2}{\partial z} \end{aligned}\right\} \quad \text{at} \quad z=0.$$

Taking into account that

$$\frac{dv_2}{dz}=B\frac{dv_2}{dl} \quad \text{and} \quad \frac{dH^{(1)}_{\frac{1}{3}}(x)}{dx}=\frac{1}{2}\left(H^{(1)}_{\frac{1}{3}-1}(x)-H^{(1)}_{\frac{1}{3}+1}(x)\right),$$

these conditions are

$$-A_1\sin s_1 H+B_1\cos s_1 H=0,$$

$$A_1=\sqrt{\frac{A}{\mu_2}}\left\{\alpha H^{(1)}_{\frac{1}{3}}\left(\frac{2}{3B}A^{\frac{3}{2}}\right)\right\},$$

and $$B_1=\frac{\mu_2 B}{\mu_1 s_1}\frac{\alpha}{2}\left\{\frac{1}{\sqrt{\mu_2 A}}\left(1+\frac{\beta}{B}\frac{A}{\mu_2}\right)H^{(1)}_{\frac{1}{3}}+\frac{A}{B\sqrt{\mu_2}}\left(H^{(1)}_{-\frac{2}{3}}-H^{(1)}_{\frac{4}{3}}\right)\right\}.$$

Eliminating A_1, B_1 and α

$$\tan s_1 H=\left\{\frac{1}{2}\frac{\mu_2 B}{\mu_1 s_1}\frac{1}{A}\left(1+\frac{\beta}{B}\frac{A}{\mu_2}\right)+\frac{1}{2}\frac{\mu_2\sqrt{A}}{\mu_1 s_1}H^{(1)}_{-\frac{2}{3}}-H^{(1)}_{\frac{4}{3}}\right)\Big/ H^{(1)}_{\frac{1}{3}}\right\}$$

in which the argument of function H is $\frac{2}{3B}A^{\frac{3}{2}}$.

In the case of a large absolute value of this argument, an asymptotic expansion of function H, that is $H^{(1)}_n(x)\approx\sqrt{\frac{2}{\pi x}}\,c^{\,i\frac{2n+1}{4}\pi}e^{ix}$ may be utilised. Thus, $(H^{(1)}_{-\frac{2}{3}}-H^{(1)}_{\frac{4}{3}}/H^{(1)}_{\frac{1}{3}}=-2i$.

Putting $$A=\frac{p^2}{v_0^2}+\frac{\beta^2}{2\mu_2^2}-f^2=-s_2^2 \quad \text{and} \quad B=2\left(\frac{p^2\beta_1}{v_0^3}+\frac{\beta^3}{4\mu_2^3}\right)$$

$$\tan s_1 H=\frac{\mu_2 s_2}{\mu_1 s_1}-\frac{1}{2}\frac{\mu_2 B}{\mu_1 s_1}+\frac{1}{2}\frac{\beta}{\mu_1 s_1}.$$

If the last two terms are neglected, the equation degenerates to that with regard to the uniform medium.

A numerical example will be given to show the order of magnitude of the effect.

Assuming that $H=50$ km., $v_1=3.7$ km/sec., $v_2=4.5$ km/sec., $\mu_1=41.07$ (km. g. s.), $\mu_2=68.85$ (km. g. s.), $\beta=0.01$ (km. g. s.) and $\beta_1=0.005$ (km. g. s.) the period-velocity relation can be determined. The coefficients β and β_1 are assumed, taking into account the result obtanied in the discussion of the P phase in part III. As the Poison's ratio in the earth's crust does not seem to undergo much change, such an assumption will not be purposeless in order to obtain an idea of the order of magnitude of the effect. In Fig. 6, the full curve denotes the relation in the case of the uniform layer. The crosses represent the relation when the underlying layer assumes a gradual change.

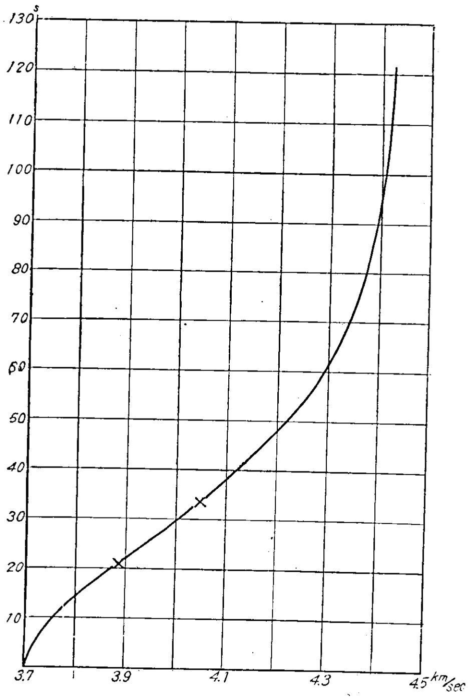


Fig. 6.

It may be seen from the figure that the effect of the gradual change of the medium is not so great, at least, when the wave length of the disturbance is not to great compared with the thickness of the layer. When the wave length becomes large, the argument of the Hankel's function $\frac{2}{3B}A^{\frac{3}{2}}$ becomes so small that the asymptotic expansion of it cannot be applied. In such cases, tables of $H^{(1)}_{\frac{1}{3}}$, $H^{(1)}_{\frac{4}{3}}$ and $H^{(1)}_{-\frac{2}{3}}$ are necessary for calculation, but no such tables being accessible, numerical calculation for such a case will not be attempted here.

VII. CONCLUSION.

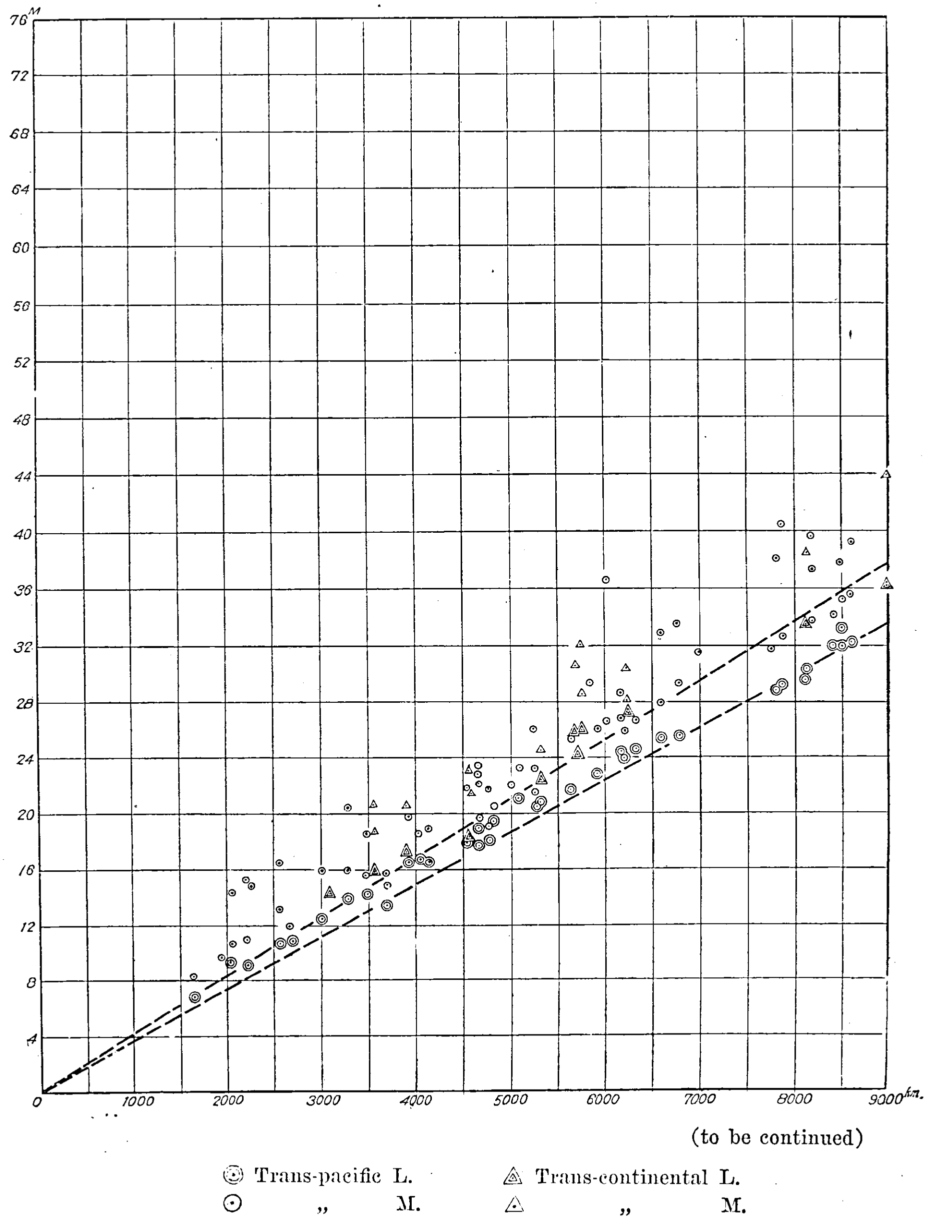
In this paper the real existence of two kinds of surface waves, that is, Rayleigh waves and waves of Love's type, has been affirmed from the seismometrical point of view. Especially in the case of trans-pacific waves from a remote origin, they can be distinctly identified. They both undergo dispersion of a certain character wich is to be expected from the theory of elasticity, qualitatively, at least. From the dispersion of waves across the Pacific it would seem justifiable to assume that the superficial earth's crust under the Pacific also is stratified.

Velocities of propagation of the S, S^* and $\bar{S}$ waves determined in part II are quite consistent with the mode of dispersion of the waves of Love's type here obtained. Under the Pacific, however, it seems that the layer in which the velocity of propagation of the distortional movement is 3.15 km/sec. may be absent. Comparison of Fig. 5-a with Fig. 6 will show that the thickness of the upper layer may be much less than 50 km. As the density of each layer is not known for certain, the author will not attempt to find, by arbitrarily adjusting the constants, a value for the thickness by means of which the observed dispersion might be explained in a plausible manner.

It is also remarkable to note that the dispersion of trans-pacific waves is different, quantitatively, from that of trans-continental waves, which difference would furnish plausible evidence for the existence of a different crustal stratification in both region, as has been suggested by some european writers.

Before concluding this paper the author takes the opportunity to express his cordial thanks to Professor A. Imamura for his kind advices and valuable suggestions.

July 17, 1928.



(to be continued)

Fig. 4-a.

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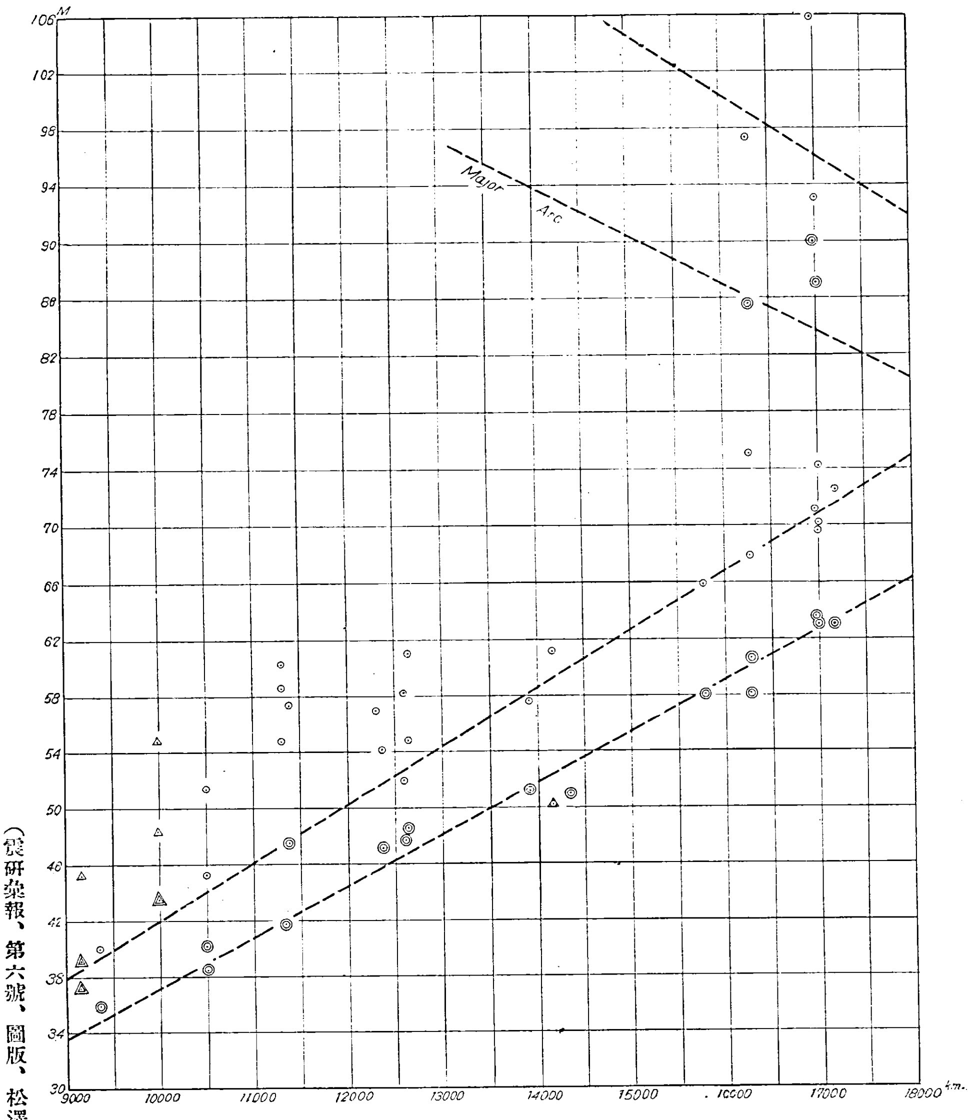

(continued)

Fig. 4-b.