A Graphical Determination of the Position of the Hypocentre of an Earthquake and the Velocity of the Propagation of the Seismic Waves.

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地震の震源の位置と地震波の傳波速度とを求める圖法

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P 地震波と S 地震波との差波に速度 k を假定して、三點觀測から震源の位置を求める圖法は、大森教授を初として、色々の人によつて色々の方法が考案された。然し地震波の速度は何處ででも同一ではない、地震波の傳波速度を假定しないで震源の位置を求める圖法は既に和達氏、那須氏等に依つて考案されたものがある。本論文のものも其の樣な圖法の一新考案である。此の圖法は、三點觀測から求めた震央の位置が k を變へると直線の軌跡を畫き其の直線は其の三觀測點を頂點とする三角形の外心を通ると云ふ性質を應用したものである。此の方法の長所は作圖が容易であること、手早く震源の位置が求められる事、一地方に起る數多くの地震、例へば或る地震の餘震の震源を求める場合に他の方法より一層便利な事等である。

其の樣な場合の一つの例として、丹後大地震の餘震に就いて震源の位置と地下十五粁までの深さに對する震波の速度が求められた。

There are different methods for the determination of the position of of the epicentre of an earthquake from the seismometrical data obtained with the earthquake. One of the method is originally due to the late Professor F. Omori(1) and is based on his well-known empirical formula

$$\text{(epicentral distance in km.)} = 7.42 \times \text{(duration of the preliminary tremors in sec.)}$$

If the duration of the preliminary tremors of an earthquake observed at three seismological stations are known, we can determine the position of its epicentre by this formula. Different modifications of Omori's formula

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(1) Journ. Coll. Sci. Tokio XI 147 (1899).

have hitherto been suggested by many, of whom we may specially mention the names of S. Kusakabe, Seam. Nakamura, K. Hasegawa, K. Wadati, etc. In all of these methods, however, it has been necessary to assume the difference of the velocities of the compressional P and the distorsional S seismic waves, what is the same thing to assume the velocity of the differential P-S wave.

With the gradual increase of the number of seismological stations in this country, we are enabled to observe often an earthquake of a small intensity of which the seismometrical data from more than three stations are available. It becomes thus possible to determine the position of the hypocentre of the earthquake out of these data without any assumption concerning the velocity of the P-S wave as was formerly necessary. So far as the author is aware of, there have been devised two methods to effect this. One of these methods is that based upon the geometrical theorem due to Appolonius that the locus of the point whose distances from two given points remain in a given constant ratio is a sphere whose centre lies on the straight line joining the two given points. This method was adopted by K. Wadati[(2)] in his investigation of "deep earthquakes." Essentially the same method was also used by the present author for finding the coefficient in so-called Omori's formula which fits well for the earthquakes occuring in the Kwantô district. This method has, however, several disadvantages in its practice, especially when the duration of the preliminary tremors observed at any two stations happens to be nearly equal. In such a case, the sphere of constant ratio becomes very large in its diameter and tends to a plane, thus the sphere becomes practically difficult to be drawn. Moreover, the duration of the preliminary tremors measured on seismograms being limited in its accuracy to the order of 0.1 sec., large errors are unavoidable in the determination of the epicentre in the above case.

The other method consists in finding a coefficient k in Omori's formula for which two epicentres obtained with different combinations of three stations may fall as closely as possible. The method was adopted by

(2) Geophys. Mag. I 162 (1928).

N. Nasu[3] in his determination of the coefficient of Omori's formula for the after-shocks of the Great Tango Earthquake of 1927. This method is of a tentative nature and seems to be rather troublesome in its procedure though it has its own merit in that the errors in the determination of the epicentre is comparatively small.

The method which the present author is going to suggest in this paper owes, indeed, its origine to Nasu's method and is so devised as to be effected in a possibly short time without any tentative assignment of the value of k as was necessary in the case of Nasu's method.

Let $A\ B\ C$ be three seismological stations at which the observed durations of preliminary tremors are $t_1\ t_2$ and t_3 respectively. (Fig. 1). Draw three circles of radii $kt_1\ kt_2$ and kt_3 with their centres at A, B, C respectively. Join the intersections of the circles A and B, B and C. The intersection D of these two straight lines gives the position of the epicentre of the earthquake. When the constant k is varied successively and continuously the point D will describe a path of straight line. The proof of this theorem is as follows:—

Let E be the intersection of $\overline{AB}$ with the line which joins the intersections of the two circles A and B. Then we have

$$\overline{AE}^2-\overline{BE}^2=k^2(t_1^2-t_2^2),$$

$$\overline{AE}+\overline{BE}=\overline{AB}=a,$$

where a is the distance between A and B.

$$\overline{AE}-\overline{BE}=k^2\left(\frac{t_1^2-t_2^2}{a}\right),$$

$$\overline{AE}=k^2\left(\frac{t_1^2-t_2^2}{2a}\right)+\frac{a}{2}=Mk^2+N,\ \dots\dots\dots\dots\dots\ (1)$$

where M and N are constants for a given earthquake. Similarly, we have

$$\overline{BF}=k^2\left(\frac{t_2^2-t_3^3}{2b}\right)+\frac{b}{2}=M'k^2+N',\ \dots\dots\dots\dots\dots\ (2)$$

where F is the foot of the perpendicular from D to $\overline{BC}$, b the distance between B and C and M' and N' are constants.

(3) Proc. Imp. Acad. IV 378 (1928).

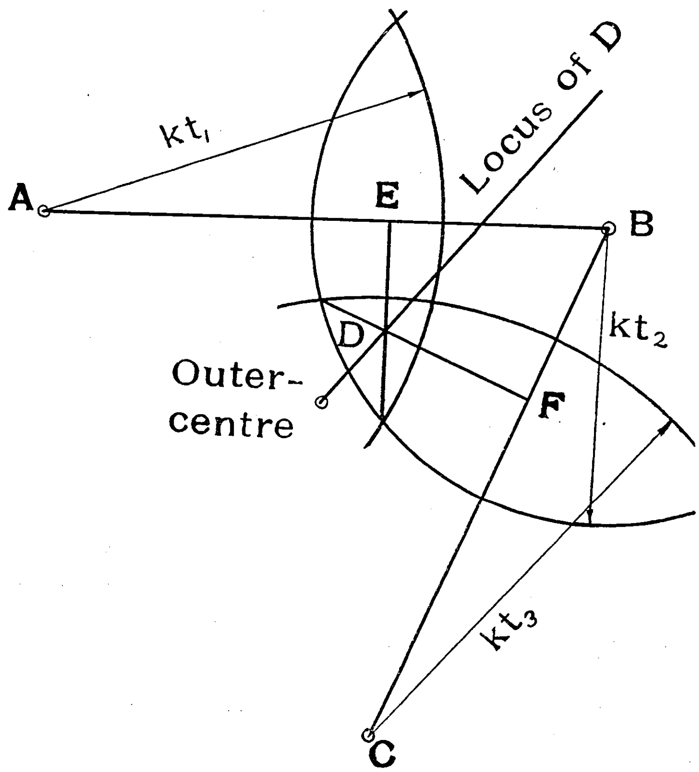

Fig. 1.

As can readily be seen from (1) and (2), the differential coefficient $\frac{d\,\overline{AE}}{d\,\overline{AF}}$ is a constant, so that the locus of D is a straight line. If $k=0$, then $\overline{AB}=a/2$ and $\overline{BF}=b/2$, so that it is seen that this straight line passes through the outer-centre of the triangle ABC.

This geometrical property can readily be utilised in the determination of the epicentre of an earthquake in the following manner. Take two different combinations of three seismological stations out of the four given stations and find the points D's in both combinations for an arbitrary value of k, say $k=8$. Join the points D's thus obtained to the corresponding outer-centres of the triangles. The intersection of these two straight lines gives the position of the epicentre. That the two lines will really intersect under the ground can easily be seen if we notice that the two of the four stations are common in both combinations. The value of the parameter k which correspond to this intersection can be readily obtained because the distance of a point on the locus of D from the outer-centre of the triangle is proportional to k^2 as is seen in (1), and the distance of the point corresponding to $k=8$ from the outer-centre of the triangle is known. The focal depth can be obtained without difficulty from the distance of the epicentre from any one of the stations and the coefficient k obtained above.

From an example illustrated in Fig. 2, the practice of the method may be understood. The earthquake taken in the example in one of the aftershocks of the Great Tango Earthquake of 1927. The data of the preliminary tremors of the earthquake is as follows:

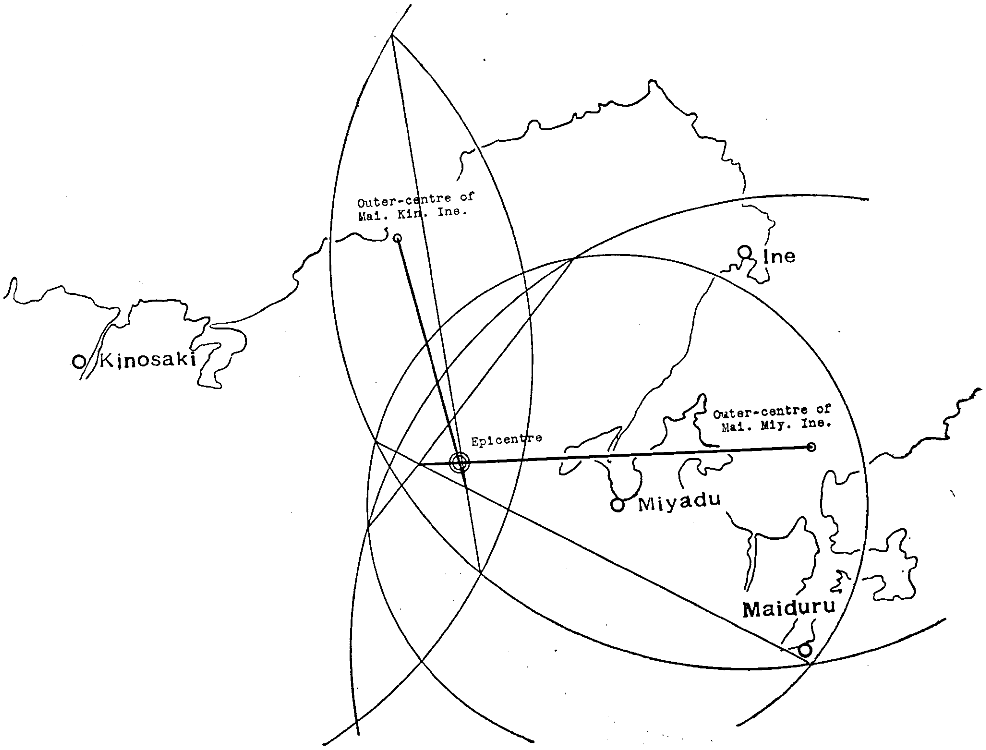

Fig. 2.

Date	Preliminary tremors observed at			
	Maiduru	Kinosaki	Ine	Miyadu
III 19 5^{h} 17^{m}	3.6^{s}	3.6^{s}	3.3^{s}	2.0^{s}

In the method proposed in the above, the velolcity of propagation of seismic waves has been assumed to be invariable with depth, which is of course an undue assumption. Consider a virtual ray of seismic waves which travels from the focus to a seismological station in a straight line with a uniform velocity and let its velocity be such that it starts the focus and arrives at the station simultaneously with the real wave. Assuming the velocity of the real wave to be a function of the depth only, expressed by the equation

$$\frac{1}{v}=n=a+bh,$$

we will examine in what degree the velocity of the virtual wave varies with epicentral distances. In the following calculations, the curvature of the earth is neglected. We have

$$\varDelta_h=\alpha\int_0^h\frac{dh}{\sqrt{n^2-\alpha^2}},$$

$$T_h=\frac{n^2dh}{\sqrt{n^2-\alpha^2}},$$

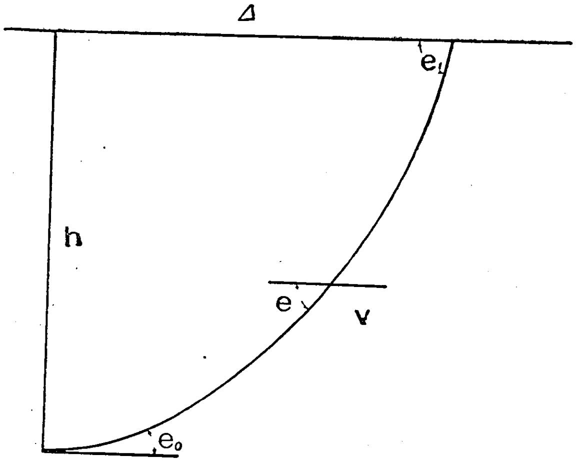

Fig. 3.

where Δ_h is the distance of the epicentre from a station,

T_h is the time of travel of the wave from the focus to the station,

$\alpha = n \cos e =$ a constant for a definite ray,

$n = \dfrac{1}{v}$

e is the angle which the ray of the wave makes with the horizontal plane at the depth h,

e_0 is the value of e at the focus.

After the integration we have

$$\Delta_h = \frac{\alpha}{b} \frac{1}{0.4343} \log_{10} \frac{\sqrt{n^2-\alpha^2}+n}{\sqrt{a^2-\alpha^2}+a},$$

$$T_h = \frac{n}{2b}\sqrt{n^2-\alpha^2} + \frac{\alpha\Delta}{2} - \frac{a}{2b}\sqrt{a^2-\alpha^2}.$$

The velocity of the virtual wave which is equal to $\dfrac{\sqrt{\Delta_h^2+h^2}}{T_h}$ is given in the following table. In the table it was taken

$$a = 0.154, \qquad b = 0.002, \qquad h = 20 \text{ km}.$$

(which gives for $h=0$, $v=6.49$; for $h=20$, $v=8.77$; a rather remarkable change)

e_0	0°	15°	30°	45°	60°	75°	90°
Δ	—	33.43	22.81	15.42	9.51	4.56	0
T	—	5.16	4.05	3.37	2.96	2.74	2.68
Velocity of virtual ray	—	7.56	7.54	7.50	7.48	7.47	7.46

As is seen in the above table, the velocity of the virtual wave does not vary much so far as the epicentral distance is not very large compared with the depth of the focus. Therefore, we are justified to use the straight path and the mean velocity of the virtual wave instead of the curved path and varying velocity without introducing sensible errors.

The hypocentre of an earthquake is of course not a mere geometrical point, but must be a domain with some wide extension. The true form of the hypocentral region, however, remains in obscurity in our present knowledge of earthquake. It is only convention that we give the nearest point of the hypocentral region from a station the name of hypocentre.

In the course of the present study the writer has met with some earthquakes for which no definite focus can be obtained by the present method. This may sometimes be due to the broad extension of the hypocentral region and sometimes due to the mis-recording of the duration of preliminary tremors or more probably due to the fact that in the meizo-seismal region a bodily movement of the earth-crust is superposed on the ordinary seismic waves.

According to the results of the calculations the errors involved in the determination of the hypocentre by the present method becomes large when the epicentre lies outside of the network of the seismological stations. Therefore, we must select the stations in such a way that the epicentre of an earthquake falls inside of the network of the stations.

The present method may be adopted with special advantage when we want to determine the hypocentres of a number of earthquakes occuring in a definite earthquakes district. As an example of the practice of this method in such a case, the results of the study made upon the after-shocks of the Great Tango Earthquake are given in the following table. The earthquake taken in the present study are those observed during the period from 13th. March to 31st. August of 1927, with their amplitude greater than 100μ. The data of the duration of the preliminary tremors were placed at the writer's disposal through the kindness of N. Nasu and K. Wadati. In the last two columns of the table are given the depth of focus and the coefficient k as obtained by the present method.

Date				Preliminary tremors abserved at					k	Depth
				Maiduru	Kinosaki	Ine	Miyadu	Toyooka		
		h	m	s	s	s	s	s		km.
3	12	15	28	3.0	4.1		1.8	3.6	7.6	10.3
3	13	10	20	5.4	3.4	4.2		3.4		
		13	37	3.7	3.3	2.9	2.4	2.6	8.5	15.5
		16	06	3.2	3.6		1.8	3.0	7.6	5.7
		20	34	6.7	3.1	4.7		3.2		
3	14	12	49	4.1	3.1	3.4	2.4	2.6	7.6	10.5
3	15	13	34	3.4	3.4	3.3	2.0	2.4	8.2	12.2
3	18	21	47	3.1	4.3	3.7	3.1	3.8	11.4	32.0

(to be continued)

(continued)

Date				Preliminary tremors observed at					k	Depth
				Maiduru	Kinosaki	Ine	Miyadu	Toyooka		
		h	m	s	s	s	s	s		km.
3	19	05	17	3.6	3.6	3.3	2.0	2.9	7.6	10.8
		15	50	7.4	3.8	5.7	6.1	4.6		
3	20	10	54	4.2	3.4	2.8		3.7	7.6	6.3
3	22	03	16	6.5	3.3	4.8		2.2		
		12	57	3.7	3.3	3.2	2.5	3.2	8.9	18.4
3	23	22	50	3.7	3.5	2.8	2.4	3.2	8.7	16.5
3	24	08	41	3.5	3.8	3.2	3.1	3.4	12.1	36.6
3	25	00	34		2.9	4.2	3.1	3.4	6.6	7.8
		19	13	4.4	2.7	3.4	2.6	3.0	7.7	6.5
3	29	05	06	5.1	2.6	3.9	2.4	2.7	6.9	5.7
3	30	22	30	3.0	5.3	3.1	3.4	3.4		
4	01	06	07	2.6	4.8	3.6		3.8		
		09	49	2.5	4.5	3.6		4.2		
		11	42	4.2	2.7	4.2		2.2	7.7	6.8
		18	23	2.3	4.9	2.9	1.7	3.9	8.1	13.4
4	03	02	07	2.2	5.1	3.7	1.4	4.2	7.1	6.0
4	12	09	33	5.0	2.2	4.1	3.3	2.6	7.9	11.7
4	15	10	49	5.0	2.3	4.3		3.0		
4	16	22	34	4.5	2.8	4.2	3.1	2.6	8.2	17.5
4	17	15	44	3.7	4.2	3.5	1.4		6.4	1.7
4	20	05	49	2.1			1.5	3.9		
4	28	08	37	7.5	4.0	5.3	6.0	4.2		
5	03	11	48	4.5	2.4	4.3		3.7		
5	23	01	30	4.7	3.2	3.4				
5	24	21	08	6.0	3.2	4.7	3.7	3.4	6.1	9.4
5	29	01	30	6.1	1.1	5.0	2.3	2.9		
		04	32	5.9	1.1	5.6	3.6	1.6		
6	06	09	12	6.0	2.6	5.0	2.6	3.9		
6	11	03	35	5.0	3.4	4.3		3.5	9.0	27.2
6	17	18	30	3.4	3.6	3.1	1.4	3.2	7.2	2.7
6	19	18	33	2.9	4.5	3.6	1.3	3.6	6.7	3.4
6	24	19	42	6.0	2.9	3.8	4.2		8.1	4.6
7	07	05	24	5.1	2.7	4.1	3.8	3.1	9.6	23.0
8	05	11	44	4.3	3.0	3.3	2.4	2.9	7.3	7.9
		21	40	5.1	2.6	4.4	4.6	3.1		
8	07	00	20	3.4	3.3	3.4	2.2	2.4	8.7	15.0
8	16	23	25	8.5	9.1	11.0		8.1		
8	19	12	45	5.4	2.7	4.6	5.0	3.6		
8	30	23	03	4.0	3.2	3.7		3.7		

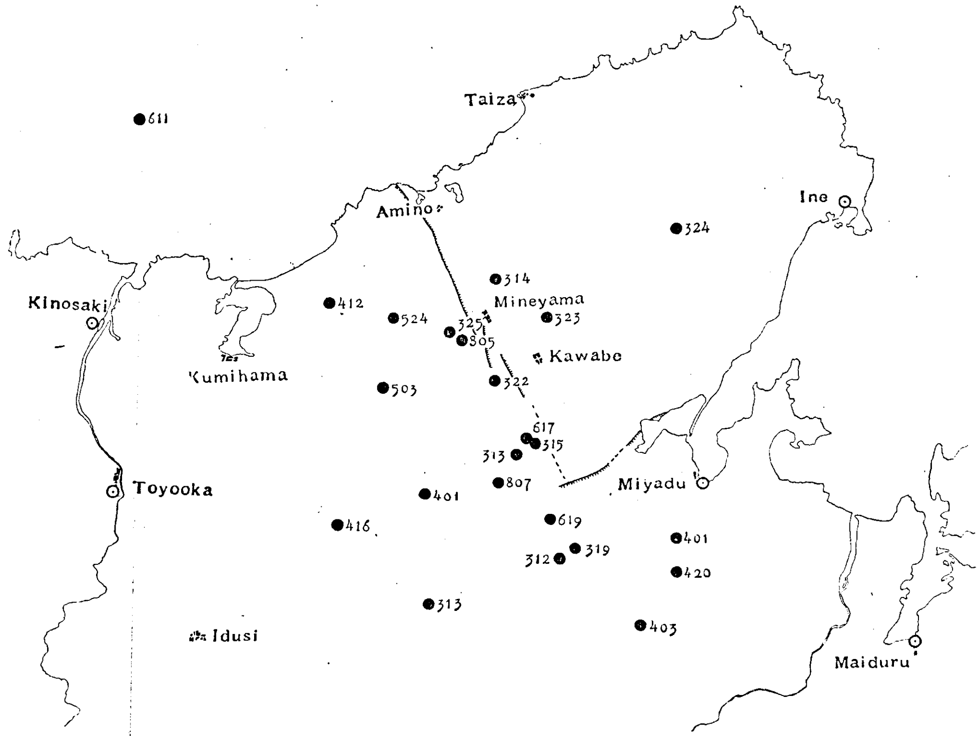

Fig. 4.

The geographical distribution of the epicentres found by the present method is shown in Fig. 4. The epicentres are distributed rather densely in two regions, the one in the west side of the Gomura fault and the other in the south side of the Yamada fault, diverging westward and southward respectively. In the Tango Peninsula proper which is distinctly separated from the main land by these two faults, only three epicentres are to be seen. This may be accounted for by assuming that the land block composing the mass of the Tango Peninsula is very rigid and stable in itself, while its adjacent blocks are movable and deformable, frequently giving rise to after-shocks in their contact planes or in the interior of themselves. With this connection, Professor Ishimoto's[4] observation of the tilting of the ground of the Tango District may be referred to. According to his

(4) Bull. Eqk. R. I. IV 203 (1928).

results, the tilting motion of the ground preceding an after-shock is often observed at Miyadu which lies on one of the movable blocks, while at Kawabe which lies on the stable mass of the Tango Peninsula no such tilting is observed. All of these facts seems to be in good accordance with each other.

In Fig. 5, is given the relation between the coefficient k and the depth

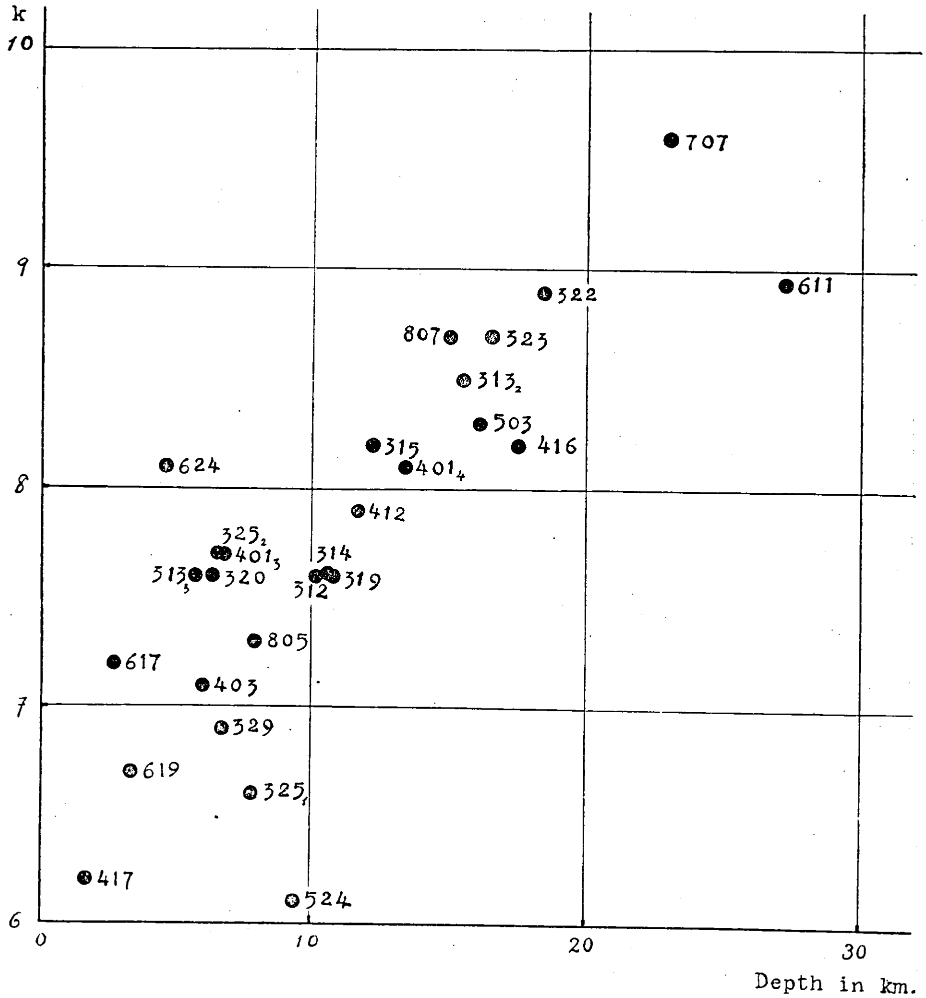

Fig. 5.

The numbers 315, 807 show that the earthquakes corresponding to those points ocurred on the 15th, of the third month of 1927 and on the 7th of the eighth month.

of the focus. A remarkable increase of k with depth may be seen. Generally speaking, errors introduced in this method is of the sense to make k increase with depth. For all that the fact that the values of k found from independent earthquakes has a tendency to lie on a rapidly ascending straight line seems to be due to something real.

Protting the depth of the hypocentres against the corresponding value of depth/k we have a graph shown in Fig. 6.

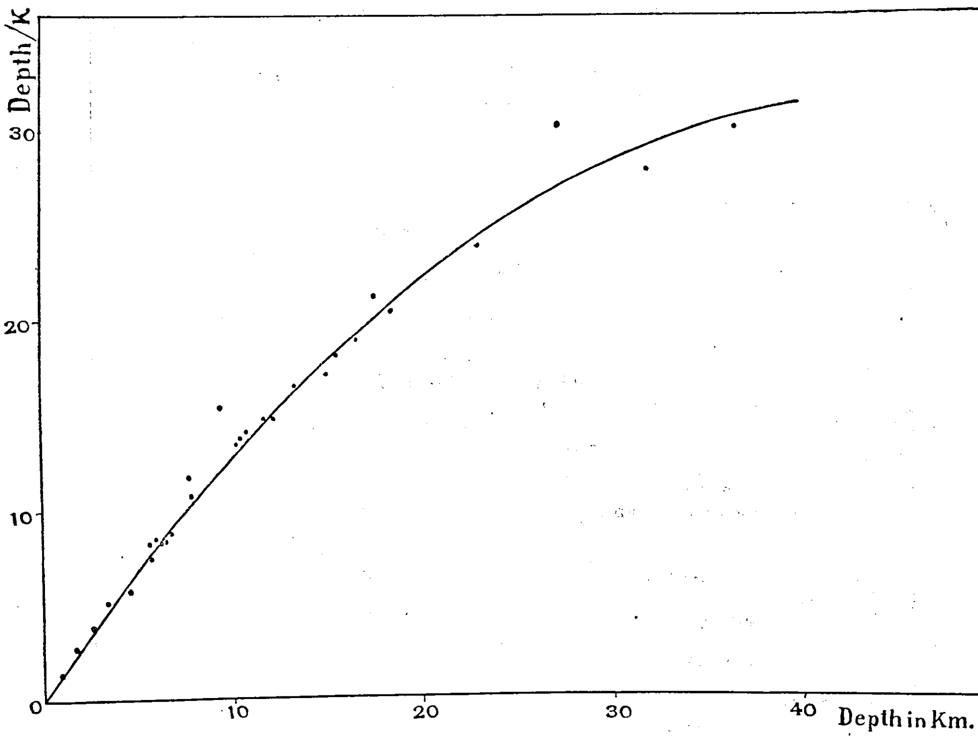

Fig. 6.

The value of depth/k gives the time of travel of seismic waves from the hypocentre of an earthquake to the epicentre. The inclination of the tangent at any point of the curve gives the velocity of the wave at the corresponding depth. Assuming the curve to be expressed by a form

$$\text{depth}/k = t = Ax + Bx^2,$$

where x being the depth in km., the constants A and B were determined by the method of least squares out of the data amounting to 28 in number. The result is

$$t = 0.1437x - 0.001628x^2.$$

For the determination of the coefficient of the terms of the order higher than two the number of data is not sufficiently large. Moreover, for the depth more than about 15 or 20 km. the above expression for t cannot be very much relied upon, as the points to determine the coefficient is too rare and scattered widely in that region. From the above expression, we have

$$\frac{dt}{dx} = \frac{1}{v} = 0.144 - 0.00326\,x,$$

$$v = 6.95\,(1 + 0.0226\,x),$$

where v gives the velocity of the differential P-S wave. If Poisson's ratio of the crust is taken to be 0.27, the velocities of the longitudinal and transversal wave will be

$$v_p = 6.06\,(1 + 0.0226\,x),$$

$$v_s = 3.40\,(1 + 0.0226\,x).$$

When these results are compared with those obtained by Wiehert, Gutenberg and others, we notice that the velocity of the seismic waves obtained in the present study increases so rapidly with depth that it surpasses the values given by them at a depth of between 15 and 20 km. This is, however, of no wonder because the values obtained by Gutenberg and others do not hold so well for the superficial layers of the earth within the depth of 50 km., while the values obtained by the present author is valid only for the layers within a depth of some 15 or 20 km. If we assume with T. Matsuzawa,(5) the existence of a surface of discontinuity at the depth of 15 km. and the velocity of the wave in the deeper layers to be given by the results of Gutenberg and others, we can connect, if we want, our present values to theirs. But we need not make such unnatural connections. It is rather a wonder to obtain the same value for the velocity of the seismic waves in the surface layer for entirely different districts whose geological constructions are very different from each other, even though they may be the same in the deeper layers.

(5) Bull. Eqk. R. I. V 1 (1928).

In conclusion, the present writer wishes to express his most cordial thanks to Professors M. Isimoto and T. Terada and to Dr. C. Tsuboi for their valuable advices and encouragements. His best thanks are also due to Drs. N. Nasu and K. Wadati who kindly placed the valuable data concerning the after-shocks of the Tango Great Earthquake at the author's disposal.

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