Experiments on the Effect of an Irregular Successsion of Impulses upon a Simple Vibrating System.

By

Torahiko TERADA and **Ukitirô NAKAYA.**

不規則なる衝撃群に因る振動系の運動に關する實驗

所　　員　寺　田　寅　彦

理化學研究所　中　谷　宇　吉　郎

摘　　要

一定の固有振動週期を有する振動系に、不規則な間隔をもつて、多數の衝撃が相次いで與へられた時に、其振動系が如何なる運動をするであらうかといふ事は、地震學上に於ても興味ある問題である。此問題に就て著者の一人は、十餘年前に、若干理論上の考察をして其結果を發表して置いた。今、此處では、此の問題に關する實驗を行つて、前に得た理論上の結果と比較し、又實驗的摸型によつて得られる運動を地震の運動等と比較して見たのである。

振動系としては振子を用ゐ、其れに、二つの互に反對な方向から散彈の流れを衝突させ、その衝撃によつて生ずる振子の運動を、地震計と同樣な裝置で自記させた。

此の運動の平均の運動エネルギーの數値を記象から求めるには、小野博士が地球磁場變化の度を表示する爲に用ゐた mean variability の方法を採用した。此の量が衝撃の每秒回數や、各衝撃の運動量や、又振子の減衰率等の變化によつて如何に變化するかを實驗し、其結果が理論から期待される關係と如何なる程度迄一致するかを試驗した。

振子の片側に每秒衝突する散彈の數 n が數百乃至數千といふ場合には、平均運動エネルギー I_0 と n との關係は全然理論と一致しない。I_0 と減衰率との關係は、質的には略一致するが、量的には一致しない。此れは n が大きいと振子の有效質量並に減衰率が著しく變るといふ事によつて說明される。併し、此の場合でも、散彈の運動量と I_0 との關係は略理論と一致する。

次に n が十或は二十位の場合に就て實驗して見ると、此場合には I_0 と n 及減衰率との關係は大體に於て理論の示す通りになる事が確かめられた。

以上の實驗で得た振子運動の記象は、此れを各種地震波の記象や又潮汐副振動其他種々な自然界の運動の記象と比較すると、殆んど區別の付かない程類似したものがある。此の事實は地震記象の意義を解釋する際にも、又地震計の作用を考察する際にも考慮しなければならないことであらうと考へられる。

In 1917, one of the present authors published a paper[1] on some theoretical considerations regarding the behaviour of a simple resonator acted upon by an irregular succession of discontinuous impulses. For the case when the duration of a single impulse is small compared with the intervals of successive impulses and a great number of such impulses takes place before the initial amplitude is sensibly damped, an expression for the mean kinetic energy of the resonator was derived. Thus, when the equation of motion for the natural vibration is

$$m\ddot{x}+f\dot{x}+kx=0$$

the mean kinetic energy is given by

$$I_0=\frac{a^2Q^2}{4m\alpha t_0}$$

where a is the amplitude of a single impulse considered as constant and t_0 the mean interval of the impulses. α is the measure of damping given by

$$\alpha=\frac{f}{2m}.$$

Q is a function of the natural frequency given by β where

$$\beta^2=\frac{k}{m}-\frac{f^2}{4m^2},$$

as well as of the duration τ of the single impulse, the function differing according to the form of the pulse.

The ideal case treated in the above theory may be applied to the case of a physical pendulum of which the suspended mass is subjected to an irregular succession of impacts by a stream of some granular substance such as shots or sand directed upon it. It is, however, diffiicult to see to what degree of approximation the assumption made for the ideal case is fulfilled in this actual case, especially when the number of grains colliding per unit time becomes large and the mutual independence of the elementary pulses cannot be insured. An experimental study of the problem seems, therefore, not without interest, especially in connection with some fundamental problem of seismology.

These considerations led the present authors to undertake a series of experiments in this line. Though the method and apparatus used are of a

(1) Proc. Tokyo Math.-Phys. Soc., [ii] **9** (1917), 142; the promised publication of the results of further investigation has been reserved on account of the lack of immediate application, as our conception on the nature of X-rays has since that time been greatly modified. For the problems as are concerned in the present paper, however, the result given in the paper cited seems to be sufficient for an approximate discussion.

provisory nature and the results obtained are still fragmentary, we may be allowed to give a brief account of them in the following.

The experiments were made in two series, the first with a large number of grains impinging on the pendulum in unit time and the second with a small number. Our descriptions of the experiments will therefore be conveniently divided into two parts.

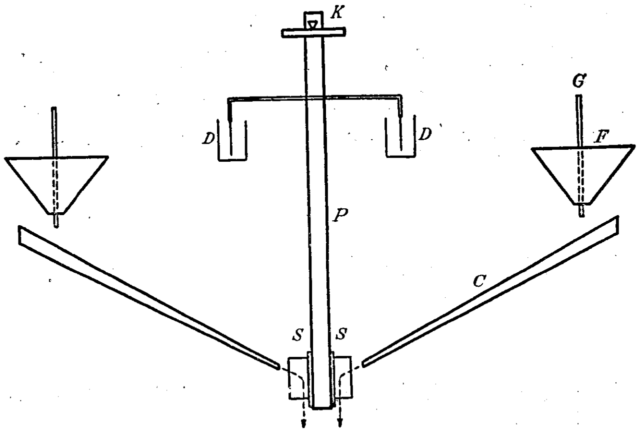
I. Experiments with Large Number of Impulses per Unit Time.

(a) *Apparatus.*

For a preliminary experiment, a physical pendulum consisting of a lead bob and a brass rod was subjected to impact of two streams of fine sand directed on two opposite sides of the bob and the motion of the rod was recorded on a smoked paper covering a rotating drum, by means of a light style attached to the rod. The amplitudes of irregular oscillations of the pendulum thus excited were too small to be subjected to the method of analysis intended. Stream of water was also tried instead of sand without success. Finally, we decided upon the use of ordinary gun shots as the most suitable material for our present purpose.

The pendulum No. I used in the earlier part of the first series of experiment was a simple brass rod suspended by an iron pin attached perpendicular to the rod near the upper end of the rod, the pin playing the rôle of a knife edge, resting upon a plane support. Near the lower end of the pendulum rod a pair of thin iron plate was attached in diametrally opposite positions for receiving the impacts and serving as the targets of the two streams of shots respectively. This plate was bent in a form of semi-cylinder, with its axis parallel to the rod and its concave side outward, in order to avoid the lateral scattering of the impinging shots. After a few trial experiments made with this pendulum, another pendulum No. II (Fig. 1) was constructed, for which a knife edge K was replaced for the pin of No. I. The natural period of the pendulum was 1.48 sec. A pair of liquid dampers D was also provided for, by means of which the rate of damping could be adjusted through a wide range by using different kinds of liquid, or by varying the dimension of the vane. The semi-cylindrical receivers or targets S were retained. The shots were fed from the funnel F and directed upon the receiver S by means of a conduit

Fig. 1.



K, Knife edge; *P*, pendulum rod; *D*, damper; *S*, semi-cylindrical receiver; *C*, conduit; *F*, funnel; *G*, glass rod.

C with a fixed inclination of about 22°. The velocity of the shots impinging upon *S* could be varied by displacing the funnel *F* along the length of the conduit. A difficulty was felt in sending out a nearly uniform stream of grains from the mouth of the funnel upon the conduit, as the grains are apt to stop their own outlet when the aperture is made too small and if the aperture be widened the flow became too large. This difficulty could be avoided by inserting a thin glass rod *G* in the funnel as shown in Fig. 1 and by giving it an oscillating motion in the vertical direction. The motion was given by hand with an average frequency of 3–6 vibrations per second. It was verified later by special experiments that the variation in the frequency of motion of the glass rod produces no sensible change in the numerical results of the experiments.

For recording the motion of the pendulum, a thin aluminium rod was attached to the pendulum rod *P* near *S*. The end of this aluminium rod carried a light style taken from a seismograph of Omori type, which recorded the motion upon the smoked paper covering a revolving drum, also borrowd from a seismograph.

The shots used were mainly of two kinds:

No.	Mass of a single grain
5	0.15 gr.
12	0.024 gr.

The shot No. 12 was used only in a few earlier experiments; in the later experiments, No. 5 was chiefly used.

(b) *Methods of Numerical Reduction of the Results of Experiments.*

In order to obtain an approximate measure of the magnitude of the impulse due to the collision of a single grain of shots, it is needed to determine the velocity with which it impinges upon the pendulum. As it suffices for the present purpose to know an approximate relative measure of the momentum of the grain, we determined the average velocity of the stream of grains at the outlet of the conduit by estimating the mean horizontal distance a traversed by the stream during its fall from the muzzle of the conduit to the floor below. If the height of the muzzle above the floor be h, the distance between the foot of the muzzle and the foot of the stream be a and the angle of inclination of the conduit be θ, the muzzle velocity v_0 will be given by

$$v_0^2 = \frac{g\, a^2}{2 \cos\ \theta\, (h \cos \theta - a \sin \theta)},$$

where g is the acceleration due to gravity. The horizontal component will be given by $v_0 \cos \theta$, if desired.

As already mentioned, the velocity was varied by varying the distance l between the outlet of the funnel supplying the shots into the head of the of the conduit and the muzzle of the conduit facing the pendulum target. The result of experiment showed that the velocity is approximately independent of the number of grains n flowing per unit time when the number is varied from 300 to 450 per sec. The relation between l and v_0 may be tabulated as follows:

l (cm.)	v_0(cm/sec.)
48	143
28	117
15	88

Though a more detailed investigation regarding the relation between l and v_0 will of itself form an interesting theme for some technical physicists, we were for the present satisfied with obtaining thus an approximate *relative* measure of the momentum transmitted to the pendulum by the impact of a single grain.

A discussion regarding the details of the actual mechanism of collision of a stream of grains upon a given form of the receiver such as here used is by no means an easy matter. We assume, therefore, simply that each grain transmits a momentum proportional to Mv_0 where M is the mass of a single grain, and proceed to see empirically in which way the motion of the pendulum is governed by Mv_0, n and the logarithmic decrement.

The damping ratio κ of the pendulum was measured by the ratio of the successive swings in alternate directions. It is related with α above defined and the natural period of oscillation T by the equation

$$\alpha T = 2 \log \kappa$$

The values of κ obtained in different sets of our experiments was 1.00, 1.15, 1.72, 10, 100 and ∞ respectively. The latter two numbers are of course very roughly estimated. For the aperiodic case, a mixture of water glass with water was used for the damping liquid instead of paraffin used in other cases.

For a measure of the mean kinetic energy of the pendulum excited by the impact of the stream of shots we adopted the "mean variability" of the displacement of the pendulum. The concept of the mean variability of a physical quantity y depending on another x was introduced by Dr. S. Ono[1] as a convenient measure of the activity of disturbance in the terrestrial magnetic field, or of any physical quantity subjected to a similar fluctuation. Thus, the variability of y referred to a definite interval of x, say l, is defined by the integral

$$V_l = \frac{1}{l} \int_0^l \left(\frac{dy}{dx} \right)^2 dx.$$

The mean value $\overline{V}$ of this quantity for different intervals throughout a certain range of x is called the mean variability of y in that range.

In the present case, taking y for the displacement of the pendulum as a funtion of time x, $\overline{V}$ is proportional to the mean kinetic energy, as the vibrating mass of the pendulum is kept constant.

In Ono's paper cited, a very simple method is given for evaluating the mean variability from the graph representing $y=f(x)$. A transparent paper or plate ruled with a number of parallel lines with a constant interval p is placed upon the graph such that the ruled lines are parallel to the axis of x. The number of the points of intersection of the curve with the equidistant parallel lines is counted, including the point at which the curve comes into contact tangentially with one of the parallel lines. Let this number be denoted by R for a range of x extending over a length of τ. The variability for the range τ is then given by the expression

(1) S. Ono, Ann. Rep. Centr. Met. Obs. Jap., Magnetic Observation for the Year 1916; Jap. Journ. Astr. Geoph., **3** (1925), 11.

$$\overline{V}=\frac{2\,R^2\,p^2}{\tau^2}.$$

The mean value of $\overline{V}$ for a number of intervals may be taken as the mean variability for those intervals. The numerical factor of the above expression may deviate from 2 in some extraordinary cases, but seems to be approximately equal to 2 in most cases of fluctuation in kindred natural phenomena.

In order to apply the above method in our case, we used a glass plate ruled with equidistant parallel lines with the interval $p=2$ mm. for the most cases except for a few records with small amplitude of vibration, in which case another plate with $p=1$ mm. was used. For a number of records, mean R per unit time was counted with each of the two plate and it was found that the product Rp is very nearly independent of p, though with a tendency to be slightly greater for the smaller p. Thus, for example, the ratios of two R's obtained with the two values of p were 2.14, 2.12 and 2.02 for the three records examined.

In order to obtain $\overline{V}$ for one second, we must count the actual number R for each second and calculate the mean value of R^2 for a number of data. As the calculation is not simple, we took the mean value of R per second from the record extending over several ten seconds and replaced $(\overline{R})^2$ for $\overline{R^2}$, as it may be shown that the two quantitities are approximately proportional in ordinary cases.[1] In any case, we may conveniently take $(\overline{R})^2$ obtained with a constant p as an approximate measure proportional to the mean kinetic energy of the pendulum, i. e. $I_0 \propto (\overline{R})^2$.

From (1), we have

$$I_0=C\,\frac{a^2}{\alpha\, t_0},$$

where C is a constant inversely proportional to the pendulum mass. If the number of impulses, or of the grains impinging per second be n, we have $t_0=1/n$. Since a is proportional to the mean velocity v_0 for a given mass of the single grain and α to $\log \kappa$ as already mentioned, we obtain

$$I_0=C'\,\frac{v_0^2\, n}{\log \kappa},$$

where C' is a constant proportional to the square of the mass of a single

(1) Usually $\overline{R^2} \doteqdot 2(\overline{R})^2$; See Ono's paper, loc. cit.

grain. This value of I_0 will be proportional to $(\bar{R})^2$ or $\bar{R}^2$ as above determined from the experimental records.[1]

The methods for evaluating v_0 and κ have already been described. n could be estimated approximately from the time required for streaming out a known total mass of shots and the mean mass of a single grain.

(c) *Results of Experiments.*

In the earlier part of the experiments some test has been made regarding the effect of the mass M of the single grain, using the shots No. 5 and 12. It was found, for example, that comparing the two cases with $M=0.15$, $n=700$ and $M=0.024$, $n=2000$, the disturbance of the pendulum is decidedly larger for the former case. The proportionality between $\bar{R}^2$ and $M^2 n$ could, however, not be confirmed. This experiment was, however, made before the final improved form of the apparatus was decided upon and wants a future revision.

In Table I we give a summary of the results of the experiments carried out with the apparatus described in a previous section, using various combinations of the values of n, κ and v_0 or Mv_0, available.

TABLE I.

Number of records used	κ	n	Mv_0	$\bar{R}$	$\bar{R}^2$
10	1.00	300	21	7.24	52.4
,,	,,	320	18	5.43	29.5
,,	,,	340	13	4.32	18.7
,,	,,	490	21	5.87	34.5
,,	,,	510	18	4.05	16.4
,,	,,	525	13	3.21	10.3
10	1.15	305	21	5.81	33.8
,,	,,	355	18	3.46	12.0
,,	,,	470	21	4.07	16.6
,,	,,	470	18	3.62	13.1
10	1.72	295	21	2.87	8.2
5	10	320	21	1.94	3.8
5	100	320	21	1.68	2.8
10	,,	470	21	1.90	3.6
,,	,,	620	21	1.51	2.3

(1) The theoretical case treated was that of one-sided impulses, i. e. a was all of the same sign, while in the case of our present experiment the shots were directed upon the pendulum from two opposite sides so that the impulses consisted of nearly equal numbers of positive and negative ones. It will, however, be evident that the expression for I_0 still holds for this case, provided we take for n the total number of the shots impinging on both sides of the pendulum.

The result will be seen to be far from agreeing with the thereotical relation. In the following we will, therefore, examine in what respect the discrepancy is brought about.

Firstly, we will examine the relation between R and κ. Arranging the data according to the magnitude of κ, we obtain Table II below.

TABLE II.

κ	$\log_{10}\kappa$	$\bar{R}^2$ $n \doteqdot 300$	$\bar{R}^2$ $n \doteqdot 500$
1.00	0.000	52.4	34.5
1.15	0.061	33.8	16.6
1.72	0.235	8.2	—
10.	1.00	3.8	—
100.	2.00	2.8	3.6

The relation is plotted in Fig. 2. It will be seen that the mean variability decreases with the increasing logarithmic decrement. The apparent deviation from the theoretical hyperbolic form may be understood if we consider that the effective magnitude of damping will be much greater when the pendulum is moving among the stream of granular mass than in the case of free vibration

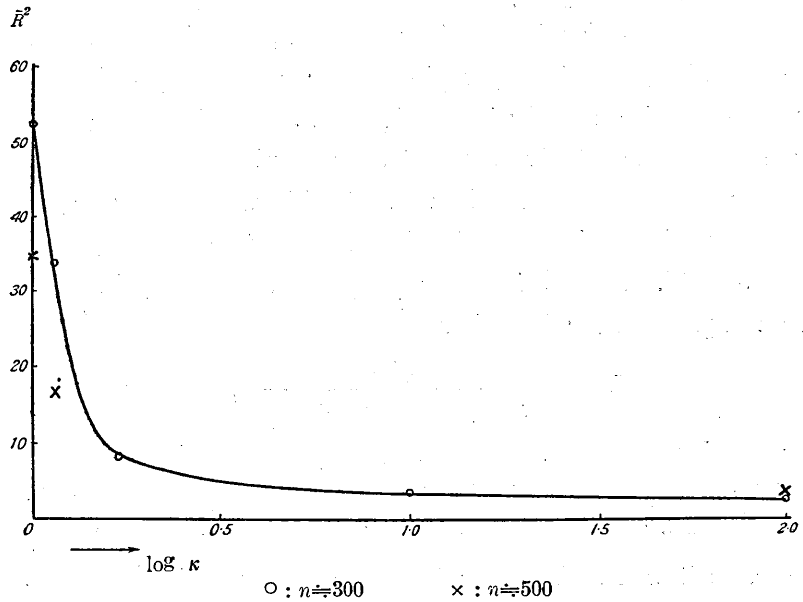


Fig. 2.

without the impinging shots and that the value of κ here used was, of course, determined for the latter case.

Next, the effect of varying n will be seen from Table III.

TABLE III.

κ	Mv_0	$\bar{R}$ $n \doteqdot 300$	$\bar{R}$ $n \doteqdot 500$	Ratio of $\bar{R}$ $(n \doteqdot 300)/(n \doteqdot 500)$
1.00	22	7.24	5.87	1.25
	18	5.43	4.05	1.34
	13	4.32	3.21	1.34
1.15	22	5.81	4.07	1.43
	18	3.46	3.62	0.96
100	22	3.39	3.80	0.89

Here, the most striking deviation from the theoretical relation is found. Thus, while a linear increase of the mean variability with increasing n is expected, the experimental results show an inverse relation, especially for the case of small damping. It is to be noted that for $\kappa=1.00$ the ratio of the variabilities for the two cases with $n=300$ and 500 is nearly independent of the value of Mv_0. For a large value of κ, the above ratio seems to decrease and becomes less than unity as expected from the theory. This relation may also be seen from Fig. 2. This discrepancy may also partly be explained by the increase of the effective damping with the increase of n, but it is evidently difficult to explain the above relation by replacing for κ some expression of the form $\kappa+f(n)$ where $f(n)$ is an algebraic function of n. Another cause may be sought in the increase of the effective mass of the pendulum with the increase of n. That such must be the case will be understood if we consider an extreme case in which the pendulum is placed in the midst of two colliding streams of a dense viscous fluid. Hence, it seems plausible to assume, in the case with large n, an expression for I_0 such as

$$I_0 = C'' \frac{v_0^2\, n}{[m+\phi(n)] \log [\kappa+f(n)]},$$

C'' being another constant. Though it may be possible to determine the functional forms of $f(n)$ and $\phi(n)$, if a sufficiently large number of systematic experiments be carried out, we have refrained for the present from proceeding any further, as it will lead too far from our proper subject at hand.

Lastly, the relation between Mv_0 and $\bar{R}$ will be shown by Table IV.

TABLE IV.

$\kappa=1.00$		
	Mv_0	$13:18:22=1:1.39:1.69$
$\bar{R}$	$n \doteqdot 300$	$4.32:5.43:7.24=1:1.25:1.67$
	$n \doteqdot 500$	$3.21:4.05:5.87=1:1.26:1.83$
$\kappa=1.15$		
	Mv_0	$18:22=1:1.22$
$\bar{R}$	$n \doteqdot 300$	$3.46:5.81=1:1.68$
	$n \doteqdot 500$	$3.62:4.07=1:1.12$

It will be seen that the two quantities are roughly proportional to each other at least for the case of smaller damping.

Thus far, we have seen that when the number of shots impinging on the pendulum is of the order of several hundreds to thousands per second the result of the simple theoretical calculation does not hold even approximately. The examination of the nature of discrepancy observed has shown that the swarm of grains seems to play a part of something like an atmosphere of very dense gas surrounding the vibrating mass, thus conspicuously affecting the effective mass as well as the damping of the system. In short, the assumption does not hold that each grain of shots acts as a discrete impulse independent of others. Thence, we proceeded to carry out another series of experiments with a decidedly smaller n than in the above, as will be described in the following.

II. EXPERIMENTS WITH SMALL NUMBER OF IMPULSES PER UNIT TIME.

In this second series of experiment n was to be varied within a range of small numbers between 5 and 20. In order to obtain suffiently large amplitudes of the pendulum it was necessary to use larger shots than in the previous experiments. The shots chosen were of 0.6 mm. diameter with the mass $M=1.24$ gr. The total mass of the shots used in a series of experiment was 2600 gr. so that the total number impinging upon one side of the pendulum was about 1050. The frequency n of the impulses was found by dividing this number by the number of seconds required for discharging this total mass.

As the grain diameter of the shots was large, it was found difficult to obtain a regular flow by means of the funnel used in the previous experiment so that a different method was devised for supplying a nearly uniform stream of shots. Referring to Pl. XII, Fig. 3 and 4, an endless band B of rubber is set in a slow uniform motion by means of the two cylinders C revolving on their parallel axes, over which the band plays the part of a machine belt. The motion is regulated by means of a motor M and a reducing gear R. The shots S which is supplied by hand to the rear end of the moving band, fall through the pair of symmetrically placed conduits D into the "distributor" R. The latter is a rectangular shallow box of which the bed plate is studded with nails at the net points of 1.5 cm. square mesh. This devise was introduced to disperse any coherent group of shots which might be formed during their passage through the conduit. The shots were then guided to the pendulum P by means of the short canal T of which the muzzle was directed to the receiver V of the semi-cylindrical form as in the previous experiment. The motion of the pendulum was then recorded on the smoked paper covering the drum M by means of the needle N. A shows the damper.

Two kinds of pendulums were used. One of them No. II was of a brass rod with a mass of about 1.05 kg. As the mass of the grain of shot increased, the transverse vibration of the pendulum rod became conspicuous and appeared superposed on the record. Hence, another pendulum No. III was made of iron tube with 1.65 kg. mass. The most experiments were made with this latter one. In the following the results of experiments carried out with the pendulum No. III will be given graphically by Fig. 5—8. The abscissae represent the numbers of shots impinging on each side of the pendulum

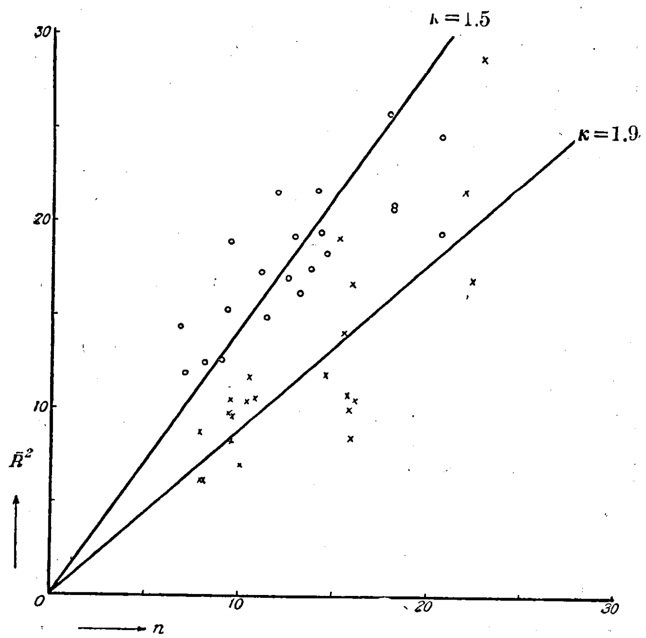


Fig. 5.

per second and the ordinates the quantities $(\bar{R})^2$ which is proportional to the mean variability or the mean kinetic energy. The points corresponding to the different values of κ are distinguished by the different marks o and ×.

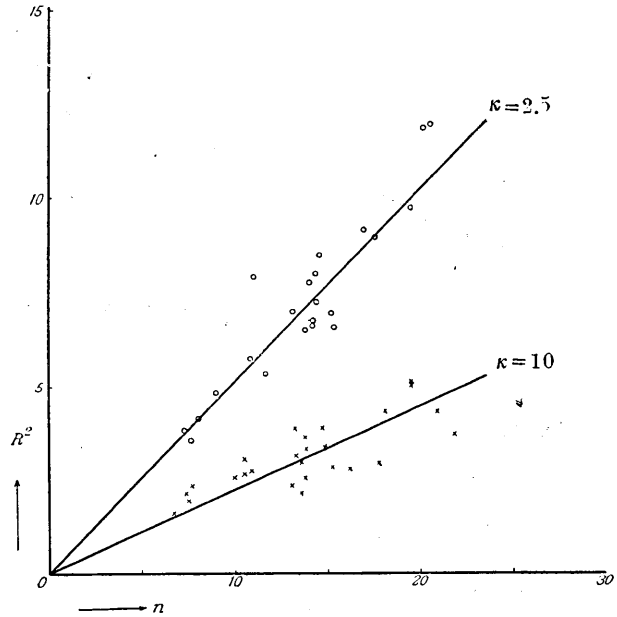

15
10
5
$\overline{R^2}$
0
10
20
30
n
κ=2.5
κ=10

Fig. 6.

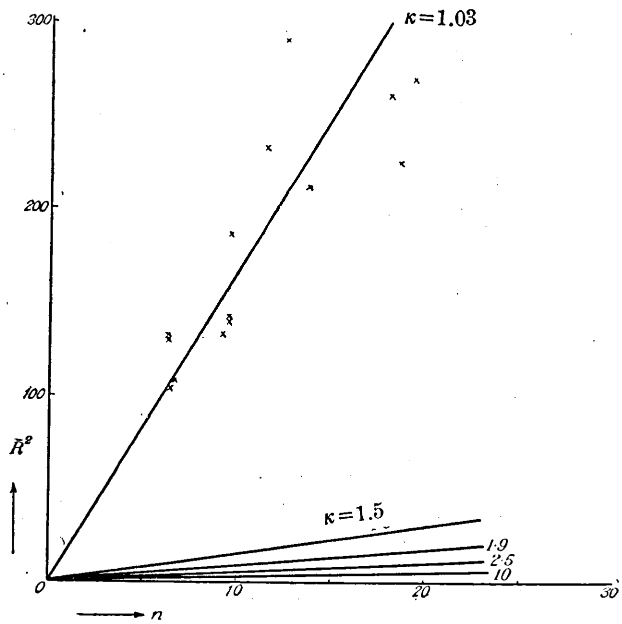

300
200
100
$\overline{R^2}$
0
10
20
30
n
κ=1.03
κ=1.5
1·9
2·5
10

Fig. 7.

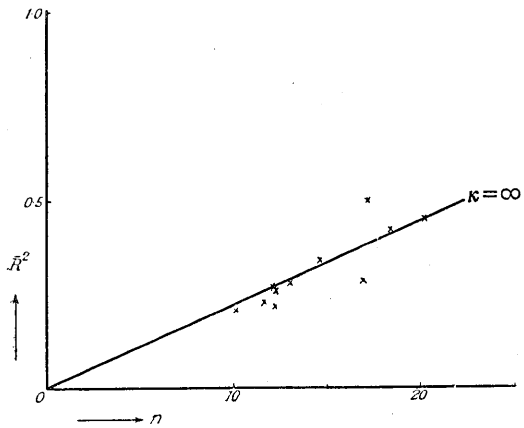


Fig. 8.

The straight lines are drawn by rough estimation. Though the points plotted are considerably scattered, it will be seen that the proportionality between n and $(\bar{R})^2$ holds approximately. From the above figures, the relation between the logarithmic decrement and $(\bar{R})^2$ may be obtained as shown in Fig. 9. An approximate hyperbolic trend of the curves for different values of n will be noted.

Thus the relation

$$\text{Mean variability} \propto \frac{n}{\log \kappa}$$

which does not hold in the case with large n, is found to hold approximately in the present case with small n.

In a series of experiment, another kind of shots receiver was used which was a hollow cylinder attached coaxially with the pendulum rod and provided with two lateral holes for receiving the stream of shots from the mouths of the canals T. The shots entering the holes and impinging on the pendulum target were reflected from it and collided upon the cylinder wall of the receiver imparting the components of momentum in the opposite direction. Fig. 10 shows the comparison of the results of the experiments with and without the cylindrical receiver. Inspite of the fact that, with the receiver, the effective value of n may be considered to be larger than without it, the actual value of the

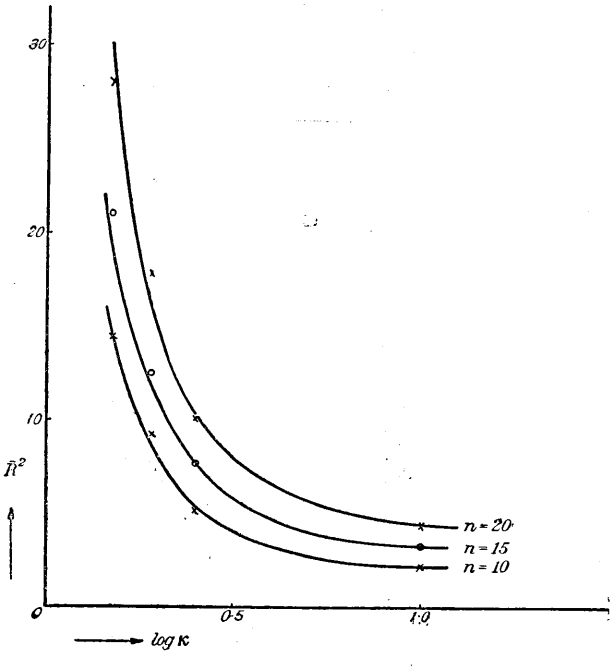


Fig. 9.

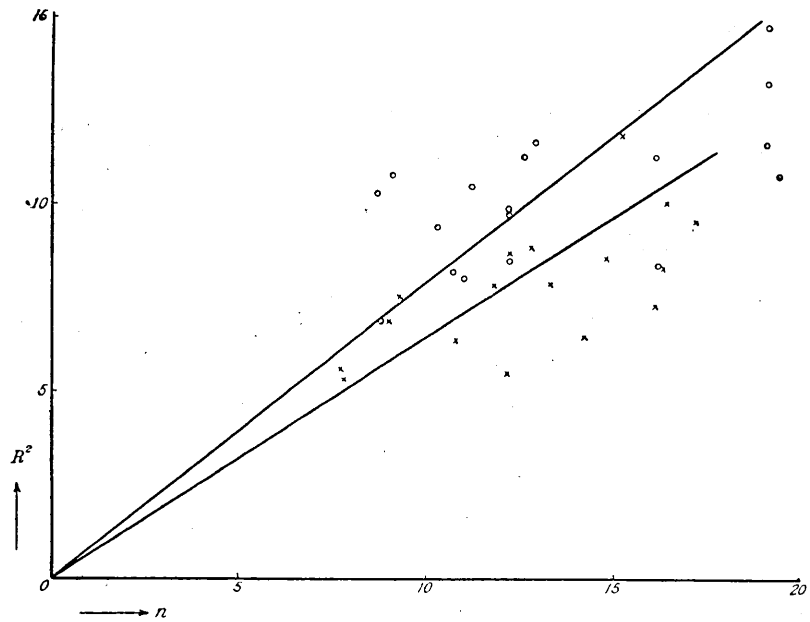


Fig. 10. o : without receiver. × : with ".

mean variability is decidedly smaller in the former case than in the latter. This fact may probably be explained partly by an increase in the effective mass of the pendulum and perhaps also partly by the presence of a regular time relation between the direct impact of a shot and the second opposite impact after its reflection.

III. Character of Motions.

Thus far, we have been concerned with the quantitative relation between the mean kinetic energy of the vibrating system and the quantities specifying the nature of the system as well as that of the exciting impulses. In the following, we will spend a few words on the general qualitative characters of motion of such system and discuss them in connection with those of some kindred motions found in the domain of seimology.

Pl. XIII, Fig. 11 illustrates some specimens of the records obtained with the pendulum No. III with different values of n and κ. It will be seen that among the irregular motions recorded the natural period of the system appears always conspicuous. This is evident even in the case with very large damping (Fig. 11, lowest graphs). Those who are familiar with the records of seismographs will easily perceive the most striking analogy of forms between the motions of the earth as recorded by the instruments and the motions of the pendulum here investigated. Some of the present records resemble the principal parts of the long waves of some distant earthquake, while the others reminds as of the microseismic vibration of the ground. Experienced seismologists may feel at a loss to decide which is the natural and which is the artificial record, if two suitable pieces of matching records be shown for connoisseuring.

In the other fields of the domain of geophysics there are also many kinds of motion resembling that of the pendulum here investigated. We may cite, among the others, the secondary undulation of tides appearing on mareograms, especially of stations situated in bays or gulfs(1) and also the "seiches" of some lakes.(2) These are also the cases in which some oscillators with definite natural periods are excited into irregular oscillations with fluctuating amplitudes. Our present investigation shows that such oscillations may be caused by an irregular succession of impulses, where no "resonance" in the ordinary sense of the word is possible.

Returning to our records shown in the above figures we may remark that besides the natural period of the pendulum a real "spectrum" of different periods is revealed, some of them greater and the others less than the natural one. If we pick up as is usual in the case of natural phenomena, a number of more or less regular trains of waves and determine their average periods, we will obtain a long list of different values. If we sort out the numbers thus obtained into groups by this or that choice we may obtain a number of "apparent proper periods" which in reality have nothing to do with the true natural periods of the vibrating system at hand.

These considerations will be sufficient for warning us againt drawing a hasty inference regarding the natural period from the results of an analysis of the kind above stated. For the discrimination of the true period some procedure is necessary. The periodogram method of Schuster may be one of the criterion, but it must fail when the natural period is gradually varying or when the damping is considerable. A practicable method in such case seems to obtain

(1) Many interesting examples of mareograms will be found in the paper "Secondary Undulations of Oceanic Tides" by K. Honda and others, Journ Coll. Sci., Tokyo, **24** (1908).

(2) See Journ. Coll. Sci., Tokyo, **28**, Art. 5 (1911).

a more or less complete spectrum of frequency curve from a sufficiently large number of records and search for the conspicuos frequency maxima. We must, however, be cautious in that case not to be deceived by the "periodicity" due to chance[1] of the frequency curve.

IV. Bearing of the Results of the Experiments on Seismometry.

Seismographs in common use are some or other kinds of oscillating system with definte natural periods and damping coefficients. The building within which the seismograph is installed may be another system. Again the portion of the earth crust may also have its own natural mode of vibration. If then a proper succession of a large number of discrete impulses be given to these systems the seismograph will probably record an irregular train of waves quite similar to those obtained in our experiment and thence also similar to actual seismograms. Here seems to arise some questions of fundamental importance on the true nature of the seismic waves as well as on the reliable method of seismometry.

Though the present investigation has led to no positive conclusion regarding these problems, the above may be of some significance in pointing out one of the lines of inquiry to be taken up seriously for the future development of seismology.

The experiments were carried out in the Labaratory of the Institute of Physical and Chemical Research, Tokyo. Our best thanks are due to Prof. Okochi, the Director of the Institute, for allowing us the use of the laboratory room and its installations necessary for the present experiments.

Summary.

A physical pendulum is excited into an irregular oscillation by the impact of two streams of shots impinging upon it from the opposite sides. The motion recorded is studied with respect to its mean kinetic energy as a function of the velocity and the number of the colliding shots and also of the logarithmic decrement of the pendulum. The results of experiments are compared and

(1) See T. Terada, Proc. Tokyo Math. Phys. Soc., 8 (1916), 492.

found approximately agreeing with those expected from the theory, in the case when the number of collision per second is small. When the number is very large some discrepancies are observed which may be explained by the increase in the effective mass as well as the effective damping of the pendulum on account of the swarm of shots surrounding the vibrating mass.

It is pointed out that the general formal characters of the motion of the pendulum here studied are essentially similar to that of some or other types of motion of the earth as recorded by seismographs. An important bearing of this fact upon the fundamental problems of seismology is briefly suggested.

Fig. 3.

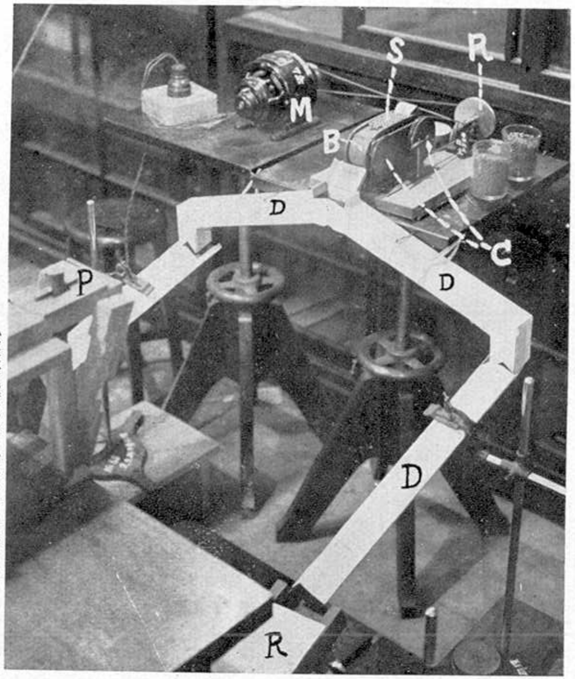
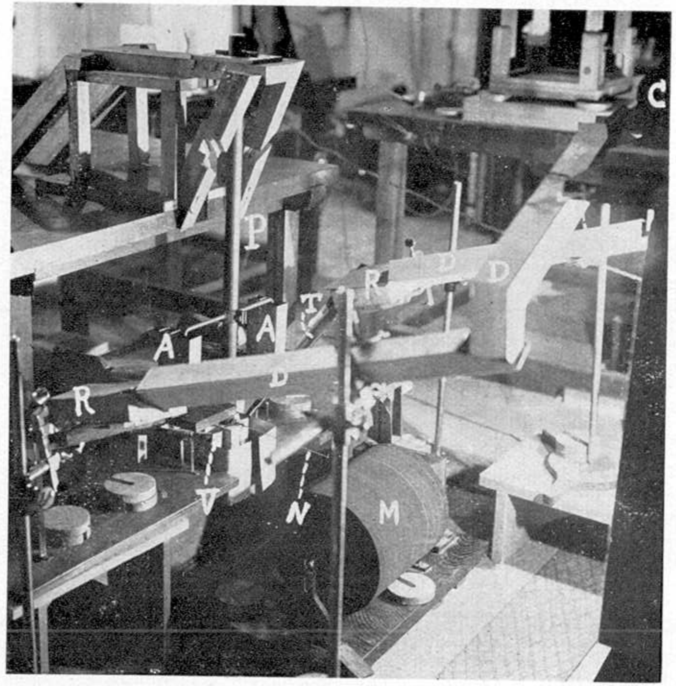


Fig. 4.



Terada and Nakaya : Experiments on the Effect of Impulses etc.

(震研彙報第五號、圖版、寺田、中谷)

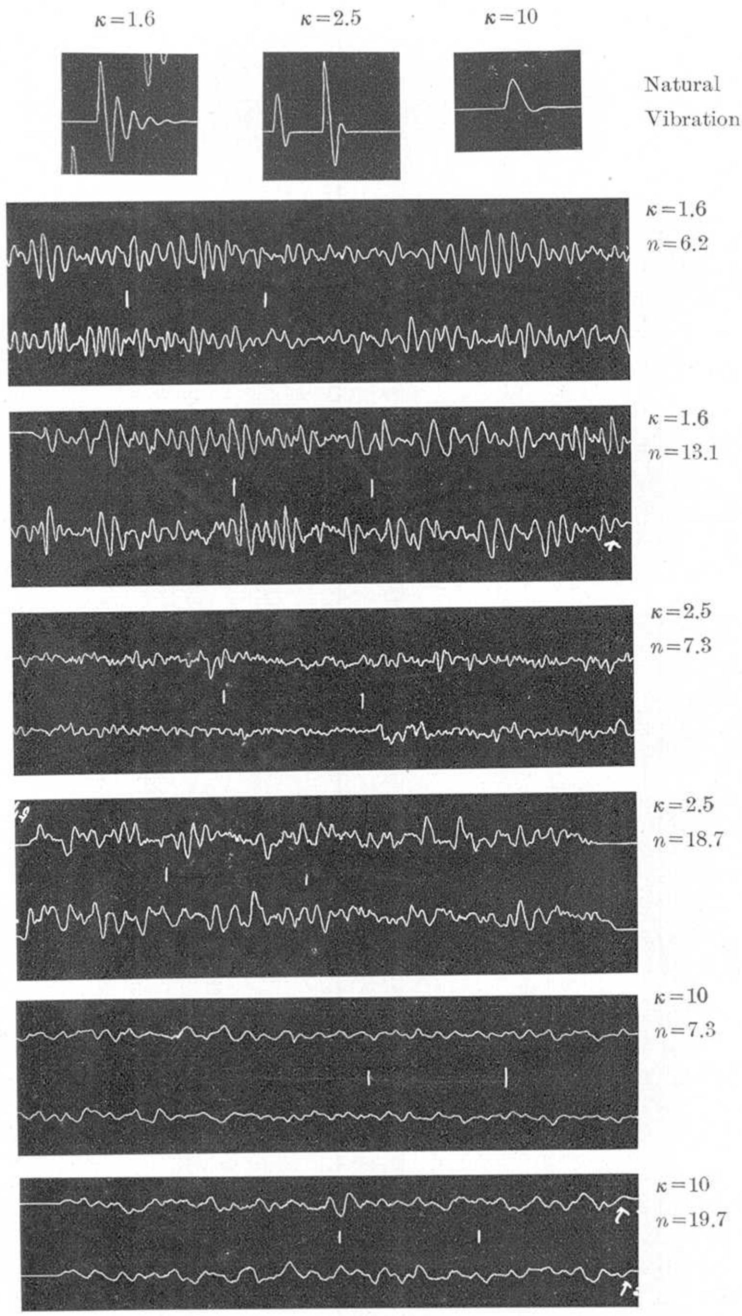


Fig. 11.

Terada and Nakaya: Experiments on the Effect of Impulses etc.

（震研彙報第五號、圖版、寺田、中谷）