Experimental Studies on Elastic Waves. (Part 2)

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彈性波の實驗（第二報）

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次に述べる彈性波の傳播に關する實驗は、此の彙報の第三號に述べた報告の續きであつて、其の後得られた實驗の結果をまとめたものである。實驗の方法は第一報に述べたものと大差無く、寒天を媒質として、其の表面を傳はる彈性波に就いて、種々の實驗をしたのである。

此の報告で取扱つて居る第一の問題は、彈性係數と密度とを異にする二つの層が重なつて居る時に、其の一番の表面を傳はる表面波の速度が、其の波長や、又層の厚さなどによつて、どんなに變化するかと云ふ事である。先づ固い寒天を容器の中に固めてから、其の上にやはらかい寒天を流して固めて層を作つた。表面には固まる時に、不規則な凹凸を生じるけれども、其處はさしみ庖丁で平に削り取つて仕舞ふ事にした。本文中の第一圖に示した樣に、寒天の一端は垂直に削り取つてあつて、振源から來る波が此處で反射して、表面に沿つて定常波が出來る樣にしてある。此の定常波の波長と、彈性波の源の振動數とを測り、夫等の値から表面波の傳播の速度を計算した。色々な層の厚さや波長に對する傳播の速度は本文中の第一表、第二表に示した通りで、波長を λ. 上層の厚さを H とすると、速度は $\frac{\lambda}{H}$ の函數として與へられ、$\frac{\lambda}{H}$ が大きい程速度が早く、$\frac{\lambda}{H}$ が大きくなるに從ひ下の媒質のみ場合の表面波の速度に、$\frac{\lambda}{H}$ が 0 に近づくに從ひ、上の媒質のみの場合の表面波の速度に、漸近的に近付く事が明にされた。之等の事は、當然彈性論の立場から期待される所であり、本彙報の第三號で妹澤所員の發表された計算とも大體一致した結果なのであるが、元來寒天の樣に彈性的には不完全な物質に彈性論が其儘の形で適用されるとも考へられないし、又實際の地殼を傳はる所謂「地震波」に就いても同樣の事が云へると思はれるのであつて、種々のモデルに就いて行つた寒天の實驗と、實際の地震記象と、彈性論の結果とを比較研究する事に依つて、始めて地殼の構造に關する正當な手がかりが得られるものと考へられる。併し此の點に就いて、現在の粗雜な實驗から決定的の議論をなす事は差ひかへて置く事にした。

第二の問題は、堀を掘る事によつて、媒質の表面を傳はる振動が遮斷されるかと云ふ問題である。之は實際問題として精密な測定等を行つて居る研究所等を、汽車、電車、自働車や其他のものから來る振動から遮斷したいと云ふ時等に屢々議論に上る所である。堀の深さや幅が、來る波長の數分の一のものでも、其の對岸の振幅は堀が無い時に較べて著しく減じる事は、第一報で報告して置いた通りである。其の後實驗した所に依ると、堀の兩岸の振幅の平均の比は、堀を深くすればする程小さくなるが、其の減少の割合は、始めの中

が急で後は次第にゆるくなるので、或る一定の深さになると、夫以上堀を深くしても大した効果が無いと云ふ事になつた。之は平均振幅に就いてであるが、對岸のきまつた點に就いて見ると、堀に近い點は淺い堀によつても其の振幅が非常に減るが、遠くの點では殆んど影響が無い。所が堀を段々深くして行くと、一旦非常に振幅が小さくなつた堀に近い點は、最早夫以上大した影響を受けないが、堀から遠い點は次第に其の振幅が小さくなつて行く。之等の結果を實際に適用しやうと云ふに就いては、實際の地面を傳はる表面波の振幅が深さと共に減少する割合が知られて居ないから、はつきりした事は云へないけれども、寒天に於けるよりは急な割合で減るのであらうと云ふ事は略想像される所である。若しさうだとすれば、問題になる樣な振動は、傳播速度が每秒數百米、周期が一秒の十分のいくつと云ふ位であるから、深さが十米内外の堀で充分所要の目的が達せられるものと考へられる。尙堀に近い所は淺い堀でもかなり振幅が減少するといふ實驗の結果から見ると、實際の堀は問題になつてゐる建物になるべく近く作るのが有効であらう。

The experimental investigation described in the following pages forms a part of our scheme of researches into different fields of phenomena concerning the propagation of elastic waves. The first report of the experiments has appeared in the last number of this Bulletin.(1) Since that time, some new results have been acquired of which the present paper is a report.

The general arrangements of the experiments are essentially the same as those described in the previous paper, but some modifications have since been introduced here and there when they seemed to be necessary or desirable.

Briefly, a mass of agar-agar was solidified in a rectangular vessel with a dimension of 80 cm. in length, 25 cm. in width, and 30 cm. in depth. In the solidified mass, a circular brass rod with its diameter 1 cm. and its length 24 cm. was imbedded horizontally at a certain depth from the surface. The rod was connected by means of a thin vertical rod to an eccentric wheel which is driven by an electric motor. The brass rod thus set in vertical vibration served as a source of elastic waves. The reflection of the elastic waves either at the bottom and the sides of the vessel could be practically excluded by spreading pads of cotton wool on these sides to a sufficient thickness, by means of which the waves were continuously damped away without any sensible reflection.

The vessel for containing the medium was so constructed that its side walls can be removed after the agar mass was solidified. Fine powders of aluminium were then blown onto the side face of the mass, which served to

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(1) T. Terada and C. Tsuboi, Bull. Earthq. Res. Inst., Vol. 3, (1927) 55.

indicate the path of the vibrational motion of each individual particle on the lateral surface of the medium. Unlike in the previous experiments, the modes of vibration of the medium thus revealed were observed directly through a telescope fitted with a micrometer gauge, by means of which the measurement were made of the dimensions of the trajectory of each individual particle of the medium.

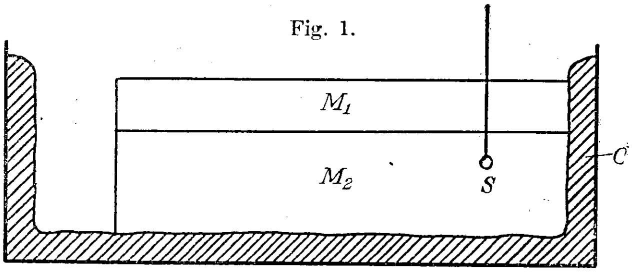
It will be not out of place here to remark that when the surface of the mass is sufficiently plane, the path of each particle of the medium as indicated by the aluminium powder is observed to be an ellipse whose major axis is vertical and remains in a ratio approximately 10 : 7 to the minor axis. The ratio of the two axes observed in this case is almost exactly what is required by the theory of the Rayleigh wave. On the other hand, the decrease in the amplitudes of vibration with depth from the surface was also seen in the previous paper to be almost exactly what is required by the same theory. Thus there remains hardly any doubt that what we are observing is nothing else than the Rayleigh wave. So far as the writer is aware, the positive identification of the Rayleigh wave in the case of the actual earthquake has not yet been made. The cause seems to be generally attributed to the imperfection of the vertical seismograph to record the motion of the earth. In these connections, it seems not uninteresting that we have demonstrated the positive existence of the Rayleigh wave experimentally.

10. Dispersion of the Surface Waves along the Surface of a Stratified Layer.

A somewhat concentrated solution of agar-agar was solidified in the rectangular vessel. The surface of the solidified mass became corrugated into an irregular wavy form owing to its own shrinkage during the course of solidification. For cutting off these wavy parts of the surface, use was made of a flat long knife, called "Sasimi-Bôtyô" specially deviced for the use in Japanese cookery. After cutting off these wavy parts, a more dilute solution of agar-agar than the first was poured upon the latter to be solidified into the form of a horizontal superficial layer. These two masses stick to each other so firmly that there is no slipping along the boundary between them. The uppermost surface was also shaved into a horizontally plane surface by cutting off the superficial wavy parts by the knife. The source of vibration was imbedded in the lower medium. For the present, it will not be of a very important difference, however, whether the source of vibration is imbedded in the lower medium or in the upper, because what we are observing is the surface wave at a distance sufficiently large from the source compared with the wave-length of the elastic wave.

As is shown in fig. 1, one end of the agar mass was cut down vertically so as to form a sharp edge or cliff. At this edge, the progressive wave was reflected backwards and consequently a stationary wave was generated along the surface of the medium.

Fig. 1.



M_1......Agar mass
M_2......Agar mass
S ......Source of vibration
C ......Cotton wool

The nodes and loops of the stationary wave appeared very distinctly so that the wave-length of the stationary wave could be measured at once. The product of twice the wave-length into the frequency of the vibration will give the velocity of propagation of the elastic wave.

By these processes, the velocities of propagation of the elastic waves were determined for different wave-lengths for a given thickness of the upper layer. After a series of these measurements were carried out, the thickness of the layer was diminished by 1 cm. by cutting off the surface with the flat long knife. With thus reduced thickness of the layer, the velocities of the wave were again determined for different wave-lengths and so on.

It is evident that the wave-velocity depends on the wave-length λ and the thickness of the superficial layer H only through the quotient $\frac{\lambda}{H}$ if we neglect the effect of the surface tension of the medium.

A few examples of the results of the measurements are shown in the following tables.

TABLE I.

Thickness of the upper layer H (cm.)	Frequency of Vibration f (per min.)	Wave Length λ (cm.)	$\frac{\lambda}{H}$	Velocity $v\left(\frac{\text{cm.}}{\text{sec.}}\right)$
5.5	1930	5.90	1.07	380
4.5	1950	5.86	1.30	381
3.5	1830	6.31	1.80	385
3.5	1720	7.30	2.06	413
2.5	1890	7.12	2.85	449
2.5	1720	8.70	3.48	499
1.5	2000	7.20	4.80	480

The upper medium......1.6% of agar-agar
The lower medium......2.4% of agar-agar

Fig. 2. The Velocity for Different $\frac{\lambda}{H}$

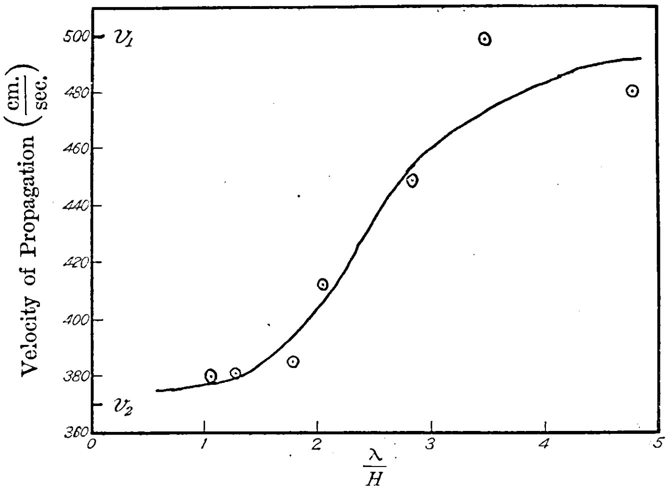
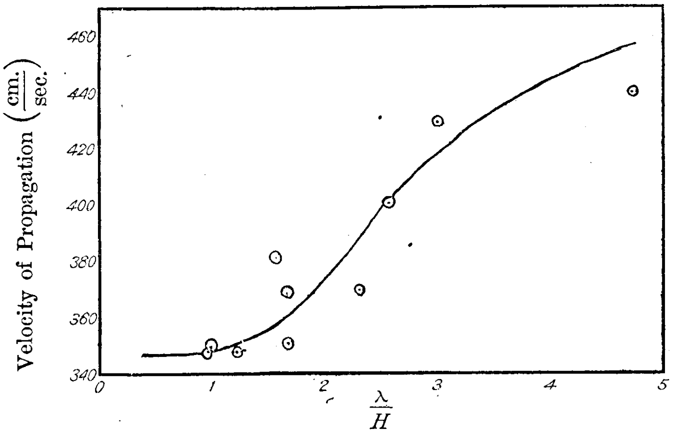


TABLE II.

H	f	λ	$\frac{\lambda}{H}$	v
5.5	1910	5.50	1.00	350
5.5	1980	5.28	0.96	348
4.5	1620	7.05	1.57	381
4.5	1900	5.50	1.22	348
3.5	1900	5.83	1.67	369
3.5	1790	5.88	1.68	351
2.5	1720	7.50	3.00	430
2.5	1890	6.40	2.56	403
2.5	1920	5.78	2.31	370
1.5	2000	7.10	4.73	440

The upper medium......1.8% of agar-agar
The lower layer ....2.4% of agar-agar

Fig. 3. The Velocity for Different $\frac{\lambda}{H}$

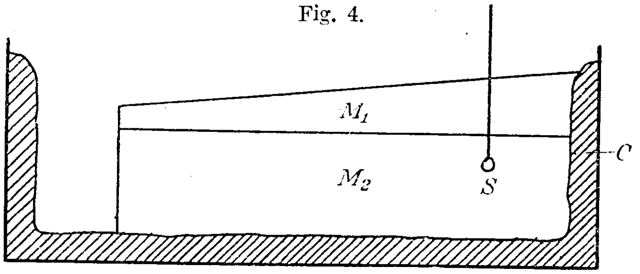


In the above results graphically represented in fig. 2 and 3, v_1 and v_2 are the velocities of the elastic waves in the case when the thickness of the upper layer tends to infinity or zero, or the wave-length to zero or infinity. In other words, v_1 and v_2 are the velocities of the Rayleigh wave on the surface of the single medium M_1 and M_2 respectively.

The medium of the lower layer in the two cases of the experiments was one and the same sample and it is seen in both cases that the velocities of the wave tend to one and the same value when the wave-length tends to infinity, as might be naturally expected.

When the thickness of the upper layer is not uniform along the path of the elastic wave but changes gradually as is shown in fig. 4, the velocity of propagation of the elastic wave is of different values from point to point. The wave-length of the stationary wave along the surface is the longer, the nearer is it to the edge. The elongation of the wave-length must mean so much increase in its velocity of propagation. An example of the measurements in such a case is summarised in the following table.

Fig. 4.



M_1......Agar mass
M_2......Agar mass
S ......Source of vibration
C ......Cotton wool

TABLE III.

Mean Depth H(cm.)	1.65	2.15	2.75	3.35	3.90
Wave-length λ (cm.)	7.0	6.5	6.2	4.9	4.8
λ/H	4.24	3.02	2.25	1.96	1.23
Velocity v (cm./sec.)	432	401	382	302	296

Frequency of vibration......1850 per min.

The general features of the relation inferred by these experimental results are in qualitative agreement with the outcomes of the elastic theory recently developed by K. Sezawa(1) of our Institute. Any detailed discussions, however, regarding, for example, the effect of the imperfection of the elasticity of the medium cannot be drawn in the present state of our experiment.

On the other hand, B. Gutenberg(2) pointed out the dispersion phenomena in the actual earthquakes, which shows in its features a substantial coincidence with the present experimental results.

It may of course not be justified to identify the actual earthquake "waves" at once with such a mathematically pure waves as treated in the present experiments or in the elastic theory, yet we may hope to get some clue for elucidating the physical configuration of the earth crust, on one hand by paying special attentions to dispersion phenomena of the earthquake "waves", and on the other hand, carrying out more or less systematic experiments on various classes of models of the crust.

11. The Effect of a Canal as a Screen for Waves.

The preliminary discussions of the said effect were given in the previous paper where the facts were emphasised that a canal of which both the width and the depth are a few tenth of the wave-length was found to be sufficiently effective in reducing sensibly the amplitudes of vibration on its opposite side. The efficacy of a given canal in this respect depends on its relative size to the wave-length of the incident wave, and is greatest at a certain definite value of the latter. In the present article, it will be studied how the efficacy of a canal in this respect dapends upon its depth.

A number of small concave mirrors were arranged on the surface of the medium along a straight line perpendicular to the length of the canal as is shown in fig. 5.

A light from a tungsten pointolite was projected on all of the mirrors and was reflected from them to make small bright images of the light source on a frosted plane glass screen placed in a suitable position. After the source was set into vibration and the motion of the medium has attained its stationary state, each of those spots describe one and the same track repeatedly. Photographs

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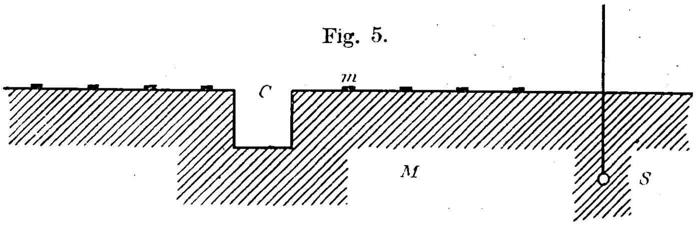
(1) K. Sezawa, Bull Earthq. Res. Inst., Vol. 3, (1927) 1.

(2) B. Guteuberg, Phys. Zeits., 25 (1924) 377.

Fig. 5.



C......Canal
M......Agar mass
m......Small concave mirror
S......Source of vibration

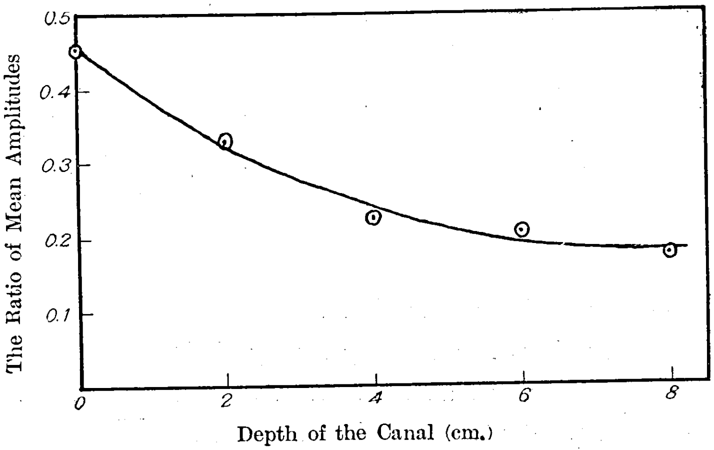
were taken of these paths of moving images by an ordinary camera. The measurements of the amplitudes of vibration of the spots were made with thus taken images on the photographic plates. The ratio of the mean amplitudes of vibration of both sides of the canal is very much affected by the depth of the canal. A few examples of the results of measurements are shown in the following table and figure.

TABLE V.
Ratio of the Mean Amplitudes of Both Sides of the Canal

Depth of the Canal		0 cm.	2	4	6	8
Experiment	1	0.481	0.402	0.250	0.303	0.220
	2	0.435	0.447	0.220	0.242	0.202
	3	0.417	0.195	0.157	0.148	0.081
	4	0.474	0.277	0.278	0.219	0.199
Mean		0.452	0.330	0.226	0.203	0.176

Width of the Canal......2 cm.

Fig. 6. The Decrease of the Ratio of Mean Amplitudes of Both Sides of the Canal with its Depth.



As can be seen from the above results, the ratio of the mean amplitudes decreases with increasing depth of the canal. But the decreases becomes comparatively slow after the depth has passed a certain value.

What has been observed in the above results is chiefly connected with the mean amplitudes on both sides of the canal. In the following, the modes will be studied in which the amplitudes at a point at a definite distance from the canal decrease with the increasing depth of the canal.

As seen with a telescope, the surface of the medium in motion appear, somewhat like the fig. 7. The thickness d of the parts in half shadow indicates the amplitudes of vibration at the surface. The distribution of the amplitudes of vibration thus manifested along the surface of the medium were surveyed by a micrometer gauge mounted in the telescope.

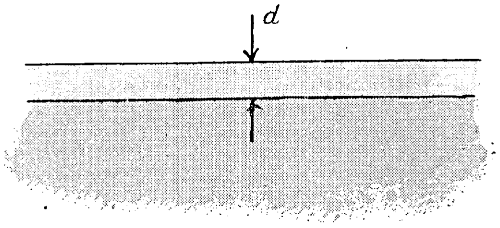


Fig. 7. The Surface of the Medium in Motion as seen with a Telescope.

In the following example, the modes can be seen in which different parts of the opposite side of the canal decrease in amplitude with increasing depth of the canal.

TABLE VI.

Amplitudes of Vibration (in arbitrary scale)

Depth of the Canal / Distance from the Canal	0	2	4	6	8
2 cm.	100	51	48	42	39
6	100	64	46	58	36
10	100	103	47	60	46

As can be seen from the results above, the amplitude in the point nearest to the canal is greatly decreased even by a very shallow canal while those in distant points are not so much affected. As the canal is gradually deepened, the amplitude in the point nearest to the canal which has once been greatly reduced suffers scarcely any further effect, while those at distant points are gradually reduced.

The general features of the relation implied in the above statements will be summarised in the following schematic figure.

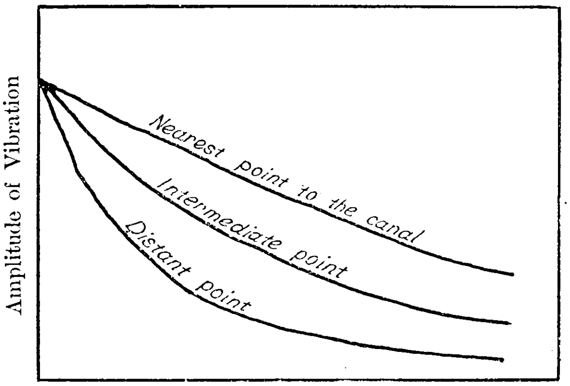


Fig. 8.

These features are quite analogous with those experienced in the familiar phenomena of sound shadow behind an obstacle. The canal in the present experiment may correspond to an obstacle for the sound wave. In the latter case, even a very narrow obstacle is sufficient to throw shadow just behind of it, but is incapable to produce any sensible effect upon the intensity of the sound at a distant point from itself.

It has not infrequently become the favorite topic among the physicists and the engineers in our country how will it be possible to screen from traffic and other artificial disturbances a research laboratory or an observatory of any kind constructed on the weak ground where different kinds of fine measurements are going on. If such disturbances are justified to be regarded as a kind of surface wave, the experimental results obtained above might afford some clue to the possibility of the attempt.

We have good reasons to expect that the amplitudes of vibration in the superficial soil decrease with depth more rapidly than in the agar-agar of the present model experiments. Therefore, if a canal is constructed on the superficial soil in the same relative scale of magnitude with the wave-length as in the present experiments, we may naturally expect its effect to be more efficient than in the present experimental case. The velocity of propagation of such disturbances being of an order of a few hundred meters per second and its period a few tenth of a second, the wave-length will be a few tens of a meter. Thus a canal whose width and depth are less than ten meters will be sufficient to produce a desired reduction in the amplitudes of vibration. And it might be added further that the nearer the canal is constructed to the laboratory in question, the more efficient is its effect.

In conclusion, the writer wishes to express his sincere thanks to Professor Torahiko Terada for the advices given and the interests taken throughout the course of the present experiment.

October, 1927.

(to be continued)