On the Forerunners of Earthquake-motions of Certain Earthquakes.

By

Takeo **MATUZAWA**, Keisuke **HASEGAWA** and Seizô **HAENO**.

Aru Zisin no Sindô no Sakigake no Bubun ni tuite.

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Aramasi no Kotogara (Abstract in Japanese).

(1) Kwantô Tihô ni okoru Zisin wo Tôkyô de kiroku suruto Suiheidô dewa itiban Hazime ni oyoso 1.5 Byô no Aida taihen tiisai Undô ga atte sorekara hakkiri sita Undô ni naru. Zyôgedô wa sono Hantai ni Hazime no Undô ga taihen ôkii.

(2) Sono hoka ni Syuyôdô no kuru Mae ni sarani hitotu no Nami no Atumari ga hakkirisuru Baai ga aru. Koto ni Zyôgedô de hakkiri suru. Nami no arawarekata wa Zisin no okoru Basyo de Tigai ga aru (Tôkyô de hakatte).

(3) Ue ni nobeta iroiro na Nami no okoru Zikan to Singenkyori tono Kwankei wo sirabeta.

(4) Korerano iroiro na Bubun no Nami no Syûki niwa medatta Kubetu wa tukerarenaiga Sindômen no Katayori niwa medatta Koto ga aru.

(5) Kono yôna iroiro na Nami no Kubetu wo tukeru Koto no dekinai yôna Zisin no okoru Tihô tono Aida ni daitai Sakai wo tukeru koto ga dekiru.

(6) Kwantô Tihô no Sita no aru Hukasa ni kyûni Bussei no kawaru Tokoro ga aruto suruto itiban Kantan ni mata Tugô yoku iroiro na Koto ga setumei dekiru.

(7) Kwantô Tihô ni okoru Zisin no Hukasa wa daitai 20 kilo kara 50 kilo gurai no Tokoro ni aru Koto ni naru.

(8) Tokubetu na Oto wo tomonau Zisin tono Aida ni medatta Kwankei ga mitomerareru.

(9) Kwantô Tihô ni aru hoka no Kwansokuten de hakatta mono demo daitai doyô na Koto ga mirareru ga kuwasii Koto wa sonouti ni hôkokusuru.

I. Introduction.

It is a remarkable fact that seismograms of certain earthquakes in the Kwantô Basin observed at Tôkyô frequently show a quite typical forerunner at the commencement of motions. As far as the present authors' knowledge, any remarks concerning the said motion have only been found in a paper by Professor A. Imamura[1] on the relation between the duration of the preliminary tremor and the epicentral distance of earthquakes.

At any rate, such a remarkable phenomenon suggests the existence of certain unusual subterranean structure in certain depth. Thus, such a case as this will furnish an excellent example for interpretting the seismogram.

II. Typical Characters of the Motion.

Before going into description of typical characters of the motion, it must be remarked that the mode of development of typical phases under consideration is different, as naturally be expected, according to the locality of the origin of earthquakes.

In the other alternative, it is also possible that among earthquakes occurring even in the same locality mechanism of occurrence may differ each other, thus resulting different manifestation of certain phases of motions. Hence it may be allowed that in the following description general tendency of the matter is described and discussed.

Characters in the vertical and horizontal component.

In illustration, some examples of typical seismogram obtained at Hongô, Tôkyô, will be shown. (Pl. VII, Figs. 1a, 1b, Pl. VIII, Figs. 2a, 2b, Pl. IX, Figs. 3a, 3b.) Glancing at the figures, it will at once be noticed that the appearance of both the horizontal and the vertical component seismogram is quite characteristic.

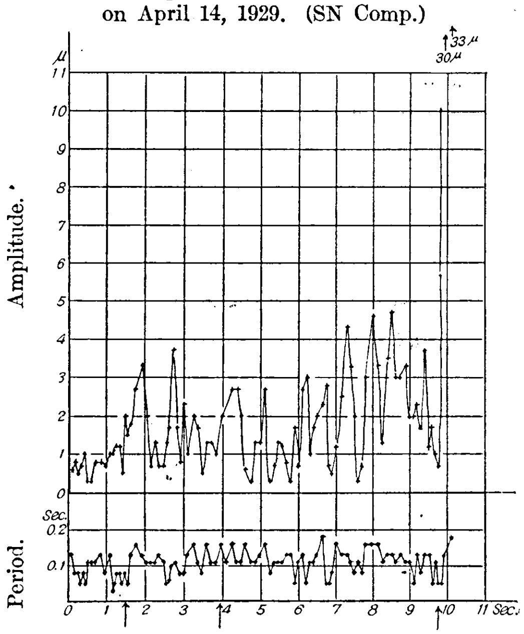
In the horizontal component, during a few seconds in the beginning the motion is quite small and then followed suddenly by somewhat distinct motions. On the other hand, in the vertical component the beginning phase is quite large compared with the second. In other words, the first phase is typically polarised in the vertical plane. After a few seconds from the commencement of the second phase, we can sometimes recognise distinctly the existence of the third phase. On the whole, the development of the third phase depends typically on the locality of origin of earthquakes as described later on. After this phase, the so-called S phase sets in.

(1) A. Imamura, Rep. I. E. I. C. **95** (1921), 103-117. If our understanding is not wrong, his case seems to include some cases of different kinds, such as the so-called Mohorovičić wave, P-PP phase and this case here stated, and was explained suggestively in a similar manner as that of Mohorovičić.

Periods of motion in each phase.

In the next place, it is necessary to investigate the period of motion in each phase for interpretting physically the nature of motions. Two examples of the result of analysis will be shown in illustration. (Figs. 4, 5.)

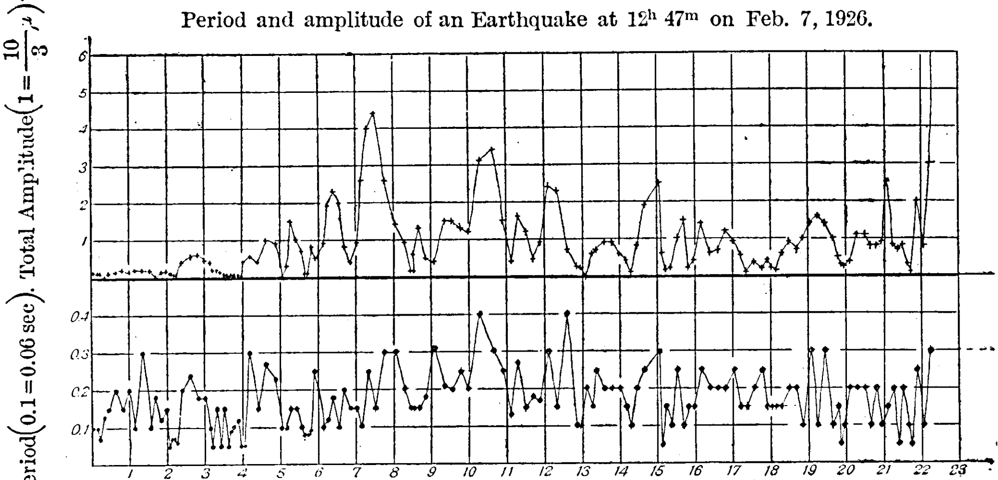
Period and Amplitude of an Earthquake at 2^h 8^m on April 14, 1929. (SN Comp.)



Abscissa: Time from commencement.

Fig. 4.

Period and amplitude of an Earthquake at 12^h 47^m on Feb. 7, 1926.



Abscissa: Time from commencement.

Fig. 5.

In short, it seems that there is no special relation among periods of each phase.

The time and distance relation of each phase.

Relations between the duration of each respective phase and the epicentral distance of earthquakes under consideration are especially noteworthy. (See Fig. 6.)

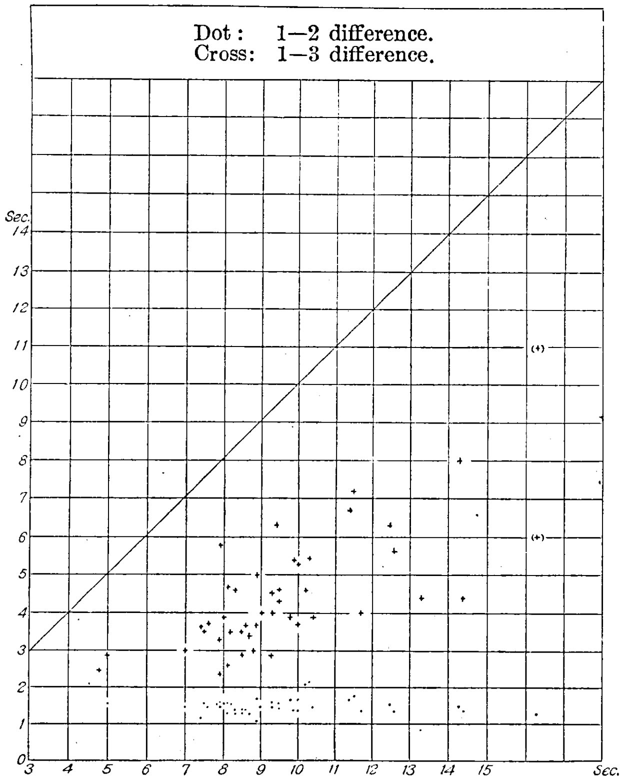


Fig. 6.

In the figure the duration of each phase is taken as ordinate, and the preliminary tremor (i. e. difference of time between commencement of the P and that of the S phase) is taken as abscissa. Recently, the net work of seismic observations in Japan has been much improved by the meteorological stations[1] and especially in the Kwantô district the precision of determination of epicentres of earthquakes has been remarkably improved by Professor Imamura.[2]

The reason of adopting the PS difference as abscissa in place of the epicentral distance is, however, mainly due to the following. First, in the beginning of this work, the broad aspect of the matter was very conveniently obtained and much time was saved by this simplification. Secondly, the depth of the origin of an earthquake cannot sometimes be plausibly determined. On the other hand, in this work it is important to investigate

(1) The Central Meteor. Obs. Tôkyô, Organisation of the Seismic observation in Japan, (1926).

(2) A. Imamura, Bull. Earthq. Res. Inst. Tôkyô, Imp. Univ. **1** (1926), 7-25; **3** (1927) 105-131.

the effect of the distance which is travelled by waves for inferring the nature of each phase. Thus, the PS difference was taken as a measure of the distance of the origin of earthquakes. Strictly speaking, it may not be legitimate to assume as above, but it may be allowed in the sense of the first approximation.

As apparent from the figure the matter is quite remarkable. The time difference of the first and the second phase is almost independent of the epicentral distance and moreover does not fluctuate much from the mean value. This point is important for elucidating the mechanism of the origin of the motion as discussed later on.

The commencement of the third phase is also seen in the figure. The value undergoes much fluctuation compared with the 1–2 difference. Usual difficulty of identification of this phase is one factor to cause such fluctuation, but other factors as discussed later may perhaps play a considerable rôle for this fact. At any rate, the existence of this phase is doubtless.

Space distribution of earthquakes showing the said characters.

The space distribution of earthquakes manifesting the said characters is also remarkable and notworthy. The earthquakes investigated are shown in the table (Table I a, I b.) and plotted in the map. (Fig. 7)

TABLE Ia.
(Refer to Fig. 7)

No.	Commencement at Tôkyô.					Time interval of each phase from commencement.			Epicentral distance in Km.	Nature of vertical motion.
	Y	M	D	H	M	1-2	1-3	1-S		U ; up. D : Down
(1)	1925	1	9	16	02	2.15(?)	5.45	10.3		D. Distinct
(2)	〃	〃	22	19	17	1.4	5.65	12.6		〃 〃
(3)	〃	〃	24	17	26	1.2	3.65	7.4		U. 〃
(4)	〃	2	13	16	13	1.5	5.8	7.9		U. 〃
(5)	〃	〃	14	09	42	1.45	4.6	9.5		〃 〃
(6)	〃	3	31	10	06	1.7	5.3	10.0		〃 〃
(7)	1926	1	14	13	08	2.8?	6.3	9.4	45.0	
(8)	〃	2	7	12	47	1.4	4.4	14.4	74.2	
(9)	〃	〃	8	14	47		3.0	8.8	55.5	
(10)	〃	〃	10	21	14	1.6	2.4	7.9	39.4	
(11)	〃	〃	13	23	58	1.4	4.6	8.3	39.4	U. Distinct

(12)	1926	2	22	10	20	1.55	4.0	9.3	65.6	
(13)	〃	〃	28	05	34	1.3	2.6	8.1	11.4	
(14)	〃	3	20	20	0	1.4	2.9	8.5	59.0	Moderate
(16)	〃	3	26	03	02	1.3		8.3	69.6	
(17)	〃	4	14	02	08	1.5	4.0	9.0	61.6	〃
(18)	〃	〃	22	18	18	1.55		7.85	54.5	
(19)	〃	〃	27	23	17	1.1	3.7	8.9	26.2	U. Moderate
(20)	〃	5	1	02	35	1.7	5.0	8.9	29.2	U. Distinct
(21)	〃	〃	11	06	51	1.5	3.0	7.0	36.4	
(22)	〃	〃	19	01	59	1.6	3.9	8.0	53.5	U. 〃
(23)	〃	〃	20	20	48	1.6	3.5	7.5	45.5	U. Moderate
(24)	〃	〃	23	23	34	1.4	3.7	8.6	43.5	
(25)	〃	6	22	16	13	1.3	3.4	8.7	55.5	U. Distinct
(26)	〃	7	30	0	41	1.5	3.3	7.9	49.5	〃 〃
(27)	〃	8	13	13	28	1.5	8.0	14.3		
(28)	〃	9	17	5	48	1.7	3.9	9.8		U(?)Moderate
(29)	〃	8	15	18	42	1.4	3.7	10.0		U. Distinct
(30)	〃	8	14	01	11	2.1	4.6	10.2		
(31)	〃	9	7	22	33	1.6	4.7	8.1		
(34)	〃	12	13	07	01	1.6	6.3	12.5		D. Distinct
(35)	1924	12	29	21	44	1.6	4.3	9.5		〃 〃
(36)	〃	10	23	21	47	0.9	4.4	13.3		〃 〃
(37)	〃	〃	3	01	31	1.6	4.5	9.3		U. 〃
(38)	〃	9	18	10	09	1.4	4.0	11.7		
(39)	〃	〃	14	19						
(40)	〃	〃	〃	13						
(41)	〃	〃	4	15	24	1.5	3.9	10.4		
(42)	〃	8	25	22						Moderate
(43)	〃	8	19						(Kasima-Nada	
(44)	〃	〃	〃						(NE of Tyôsi)	
(45)	〃	〃	20	05		1.3	6.0 11.0(?)	16.3		
(46)	〃	〃	17	11					(Twice in Kasimanada)	
(47)	〃	〃	17	7						
(48)	〃	〃	15	02	53					
(49)	〃	〃	〃	03	02					
(50)	〃	〃	〃	08	27					
(51)	〃	〃	6	23	22	1.8	7.2	11.5		D. Distnict
(52)	〃	〃	〃	12	40	1.6	2.5	4.8		
(53)	〃	7	14	21		1.6	3.5	8.2		
(54)	〃	〃	〃	02	19	1.5	2.9	9.3		
(55)	〃	6	24	03		1.7	6.7	11.4		

(56)	1924	7	4	06	30	1.3	3.5	8.5				
(57)	〃	6	23	07	31	1.4	5.4	9.9				
(58)	〃	〃	14	03	43	1.5	3.7	7.6				
									Epicentral distance			
									Tôkyô	Kamakura	Kiyosumi	
(59)	1924	5	31	21					(Twice in Kasimanada)			
(60)	〃	〃	21	(in the morning)		1.6	2.9	5.0				U. Distinct
(61)	〃	5	14	17	06							
						Duration of 1-S						
						Tôkyô	Kamakura	Kiyosumi				
(62)	1926	1	10	18	03	18.2			148			
(63)	〃	〃	〃	〃	35	19.0			156.8			
(64)	〃	〃	12	02	59	11.3	7.9		92.8	65.6		
(65)	〃	〃	13	06	59	5.6	5.7		34.0	21.7		
(66)	〃	〃	17	07	58	5.5	6.4					
(67)	〃	〃	28	01	36	17.1		12.2	128		93.5	
(68)	〃	〃	31	08	15	5.8	5.3		34.5	38.4		
(69)	〃	2	4	21	15	5.2	8.5	12.5	32.8	55.6	102	
(70)	〃	〃	9	23	12	8.4		7.7	56.8		45.6	
(71)	〃	〃	10	02	12	5.3	6.2	5.0	39.0	49.0	38.4	
(72)	〃	〃	〃	〃	13	5.1	6.2		40.0	47.3		
(73)	〃	〃	21	05	35	9.0		9.4	80.0		78.0	
(74)	〃	3	9	03	43	7.2	9.5	8.8	27.8	57.3	54.0	
(75)	〃	〃	13	12	57	11.5	10.7	7.8	94.6	84.0	19.0	
(76)	〃	4	11	15	26	8.3	3.6	8.1	64.0	8.9	69.0	
(77)	〃	〃	18	15	54	5.5	5.0	10.0	33.4	23.3	74.0	
(78)	〃	5	12	06	41	5.9	3.3	7.2	45.5	15.5	52.2	
(79)	〃	〃	21	04	11	6.4	7.4	10.0	36.7	74.5	113.0	
(80)	〃	〃	24	10	10	17.1	13.2	13.7	110.	60.0	72.3	
(81)	〃	6	28	18	50	12.4	10.2	9.6	71.2	36.2	34.5	
(82)	〃	7	11	08	01	15.0	17.4	14.8	115.6	153.5	115	
(83)	〃	〃	12	19	07	7.5	7.2	15.4	61.2	37.0	101	
(84)	〃	〃	14	04	07	15.4	15.2	20.3	72.7	91.2	139.5	
(85)	〃	7	26	09	31	11.6			106			
(86)	〃	7	30	0	41	7.7	7.4	14.4	58.5	37.8	101	
(87)	〃	5	9	13	29	13.1	14.2	11.2	104	132	85.6	
(88)	〃	〃	〃	14	09	17.8	15.1	11.9	133.5	164		

TABLE Ib.
(Refer to Fig. 7)

No.	Commencement at Tôkyô	Epicentre	Presence (+) or absence (−) of characteristics	Remark
1′	1918 4 19 8 20 P.M.	Middle of Musasi	+	
2′	1921 12 29 11 06 〃	NE of Musasi	+	
3′	1922 8 25 4 45 A.M.	SW of Hitati	+	
4′	1918 10 4 11 56 〃	SE of Kazusa	+	E of Musasi by Central Meteor. Obs.
5′	1919 1 24 0 26 P.M.	W of Kasumigaura	+	Mizukaidô 〃
6′	1916 8 21 11 32 〃	Near Ota-mati (Simotuke)	+	Kasimanada 〃
7′	1915 5 19 5 45 A.M.	Off the SE coast of Bôsô	+	Uraga-Suidô, but N of Kasumigaura may be plausible.
8′	〃 〃 〃 6 03 A.M.	E Coast of Kazusa	+	〃
9′	1919 5 20 9 45 A.M.	W of Kasumigaura	+	
10′	1923 9 2 11 46	$\lambda=140°$ 20′ $\varphi=34°$ 45′	−	
11′	〃 〃 1 14 22	139° 32′ 35° 7.5	+	Doubtfull.
12′	〃 〃 2 22 9	139° 42′.2 35° 7′.0	−	Doubtfull.
13′	〃 〃 〃 14 10	140 00 35 32.7	+	
14′	〃 〃 3 23 16	W of Sagami	+	
15′	〃 〃 〃 18 27	140 82 35 27	−	
16′	〃 〃 26 17 24	139 28 34 40.7	−	
17′	〃 10 4 0 54	139 04.6 35 18.6	−	
18′	〃 〃 5 22 5	139 07.3 35 24.6	+	Vaguely traced.
19′	〃 11 5 5 45	139 20.0 35 31.8	+	
20′	〃 〃 23 11 33	139 29.4 35 15.4	+	

From No. 1′ to No. 9′, adopted from Prof. Omori, "Seimographical observations in Tôkyô of the Earthquakes with sound, Bull. I.E.I.C. **11** (1923) 33-63.
From No. 10′ to No. 20′, conspicuous aftershocks of Kwantô Earthquake, Sept. 1,1923.

As to be seen from the map, earthquakes occurring in almost all parts of the Kwantô district show the characteristics more or less. On the contrary, earthquakes from the neighbouring ocean do not. In closer examination, however, the development of the character is remarkably dependent on the locality of the epicentre.

Distribution of Earthquakes

●: Earthquake which shows distinctly the characteristics
○: " " " vaguely " "
△: " " " does not show "

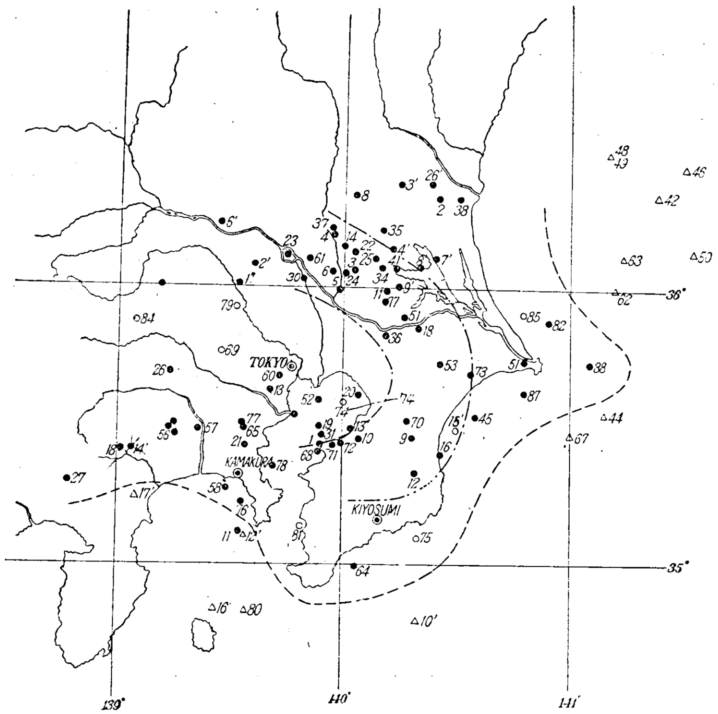


Fig. 7.

Earthquakes in which the development of each of the three phases is remarkably observed at Tôkyô are mainly those from the region bordered by chain lines in the map. (Refer to Figs. 1a, 1b, 2a, 2b, 3a, 3b, and Pl. X, Fig. 8.)

In certain earthquakes, the first phase is clearly seen, but the commencement of the third phase is uncertain. (Pl. XI, Figs. 9a, 9b, Pl. XII, 10a, 10b, Pl. XIII, Fig. 11.) In such cases it is usual that the magnitude of the preliminary part is remarkably small compared with the principal portion. Such a characteristic is usually observed in earthquakes occurring very close to Tôkyô as remarked by Professor Imamura.[1] For example, see Pl. XIII, XIV, Fig. 12. Even in such earthquakes of a very small horizontal motion and near origin the commencement of the vertical motion is quite large.

(1) A. Imamura, Rep. I. E. I. C. (in Japanese) **99** (1925).

In the above description, the case was limited only to that of observed at Hongô, Tôkyô. Such character seems, however, to be observed in certain other stations. For example, see Pl. XIV, Fig. 12b. In the above shown example of an earthquake (on Feb. 11, 1927) the 1–2 difference at Tôkyô is different from that of at Kamakura.

III. Origin of the Characteristics of the Motion.

From the facts as above stated we may be able to go into elucidation of the mechanism to produce such remarkable phenomena.

The fact that the 1–2 difference of time is almost independent of the distance of the seismic origin suggests that the occurrence of such earthquakes might proceed stepwise, giving stepwise arrival of seismic waves, as having been suggested by Professor K. Suyehiro.(1) According to this possible cause, each earthquake must necessarily come out from just the same mechanism at any occasion and at any place in the Kwantô Plain. The fluctuation of the time difference is too small to be expected as such. Moreover, the characteristic polarisation of the first phase is difficult to explain easily. If such is the case, the S phase must also reveal such a stepwise manifestation. Actually such fact is not usually observed. The third phase cannot be regarded as such, as apparent from the time relation. As shown in Fig. 12, comparison of observations at Tôkyô and Kamakura shows that the 1–2 difference of time is different from each other. Thus, explanation by such mechanism is too rush.

The case similar to that of the Mohorovičič wave is also a factor which is worth examining. But the time-distance relation in this case is not favourable for this hypothesis. The remarkable polarisation of the first phase cannot also be explained.

The dispersion of waves may not be the case, for the period of each phase is not essentially different from each other. Almost constant difference of the 1–2 duration is also unfavourable for this supposition. This will also contradict a supposition that the velocity of propagation of waves in certain medium might be different according to difference of polarisation.

Possibility of small commencement of motions due to scattering(2) cannot

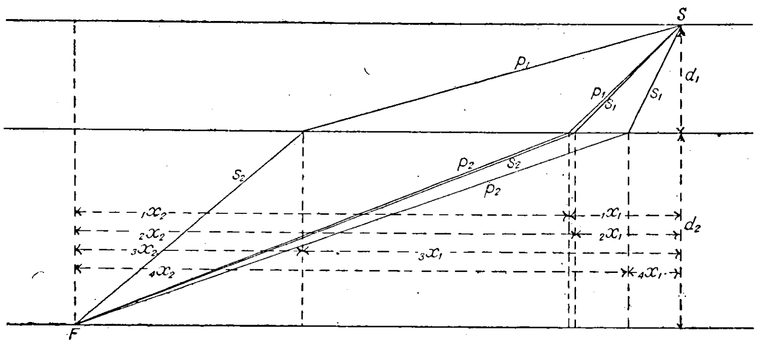
(1) K. Suyehiro, this Bulletin, **1** (1926) 59.
(2) T. Matuzawa, Jap. Journ. Astro. Geophys. **4** (1926) 32.

explain the initial predominance of the vertical motion.

Expected difference between the direct and the reflected waves at certain part of the earth's crust cannot explain the regularity of the time relation and the polarisation of waves.

Introducing the following hypothetical subterranean structure of the Kwantô district, the salient facts observed seem to be plausibly explained.

It is assumed that at certain depth there is a certain place where the properties of matter undergo sudden changes, and most earthquakes occur deeper than such a place. (Fig. 13)



F: Origin of an earthquake
S: Station

Fig. 13.

In such a case the P phase of the earthquake motion is modified at the place and transformed into two parts, i.e. the P and the S in the upper layer. The same is the case with respect to the S phase, except the case when its plane of vibration is parallel to the plane of discontinuity. Thus, three phases will be produced in the preliminary part.

It is essentially important to investigate the time and the amplitude relations of each phase.

The time-distance relation.

The time-distance relation can be determined if the following six quantities are known, i.e. the depth of the place of discontinuity, that of an earthquake, velocities of the P and the S waves in the upper and lower layers respectively. Thus the case turns to the problem of geometrical optics.

Referring to Fig. 13, expressions giving the relation between the epicentral distance ($h={}_ix_1+{}_ix_2$, $i=1, 2, 3, 4$) and travelled time τ_i are given by solving the following equations.

For the p_1, p_2 path,

$$\frac{{}_1x_1}{d_1}=\frac{1}{\sqrt{\left(\frac{p_2}{p_1}\right)^2\left(\frac{d_2}{{}_1x_2}\right)^2+\left(\frac{p_2}{p_1}\right)^2-1}},$$

$$\tau_1=\frac{\sqrt{d_1^2+{}_1x_1^2}}{p_1}+\frac{\sqrt{d_2^2+{}_1x_2^2}}{p_2}=\sqrt{d_2^2+{}_1x_2^2}\left(\frac{1}{p_2}+\frac{p_2(h-{}_1x_2)}{p_1^2\,{}_1x_2^2}\right), \tag{1}$$

for the s_1, p_2 path,

$$\frac{{}_2x_1}{d_1}=\frac{1}{\sqrt{\left(\frac{p_2}{s_1}\right)^2\left(\frac{d_2}{{}_2x_2}\right)+\left(\frac{p_2}{s_1}\right)^2-1}},$$

$$\tau_2=\frac{\sqrt{d_1^2+{}_2x_1^2}}{s_1}+\frac{\sqrt{d_2^2+{}_2x_2^2}}{p_2}=\sqrt{d_2^2+{}_2x_2^2}\left(\frac{1}{p_2}+\frac{p_2(h-{}_2x_2)}{s_1^2\,{}_2x_2^2}\right), \tag{2}$$

for p_1, s_2 path,

$$\frac{{}_3x_1}{d_1}=\frac{1}{\sqrt{\left(\frac{s_2}{p_1}\right)^2\left(\frac{d_2}{{}_3x_2}\right)^2+\left(\frac{s_2}{p_1}\right)^2-1}},$$

$$\tau_3=\frac{\sqrt{d_1^2+{}_3x_1^2}}{p_1}+\frac{\sqrt{d_2^2+{}_3x_2^2}}{s_2}=\sqrt{d_1^2+{}_3x_1^2}\left(\frac{1}{p_1}+\frac{p_1(h-{}_3x_1)}{s_3^2\,{}_2x_1^2}\right), \tag{3}$$

for the s_1, s_2 path,

$$\frac{{}_4x_1}{d_1}=\frac{1}{\sqrt{\left(\frac{s_2}{s_1}\right)^2\left(\frac{d_2}{{}_4x_2}\right)^2+\left(\frac{s_2}{s_1}\right)^2-1}},$$

$$\tau_4=\frac{\sqrt{d_1^2+{}_4x_1^2}}{s_1}+\frac{\sqrt{d_2^2+{}_4x_2^2}}{s_2}=\sqrt{d_2^2+{}_4x_2^2}\left(\frac{1}{s_2}+\frac{s_2(h-{}_4x_2)}{s_1^2\,{}_4x_2^2}\right). \tag{4}$$

Thus, the duration from the commencement of the motion to each phase is given by following forms:

$$T_1=\tau_2-\tau_1=f(d_1,\ d_2,\ p_1,\ p_2,\ p_2,\ s_1,\ h), \tag{5}$$

$$T_2=\tau_3-\tau_1=f(d_1,\ d_2,\ p_1,\ p_2,\ s_2,\ p_1,\ h), \tag{6}$$

$$T_3=\tau_4-\tau_1=f(d_1,\ d_2,\ p_1,\ p_2,\ s_2,\ s_1,\ h). \tag{7}$$

Thus, strictly speaking, the $T_i (i=1, 2, 3)$ and h relation is complicated, but it may be remarked that when $h \gg d_1$ and d_2, the relation tends to a linear one.

From observed data, the unknown quantities such as the depth, velocities, etc., may be determined. But the exact solution of the equations becomes extremely complicated. Thus, it is rather convenient and practical to resort to a tentative solution of the problem.

Velocities of the P and the S waves in the lower layer are tentatively assumed as $p_2=5.56$, $s_2=3.3$ which are usually assumed in the outer part of the globe. Our knowledge of elastic properties of materials such as loam, tuff, etc., which constitute the extreme surface of the Kwantô district is still vague. Taking into account of the results of Professor S. Kusakabe[1], the velocity of the S wave is tentatively assumed to be 2.5 km/sec. Assuming the same Poisson's ratio as in the lower layer, the P wave becomes 4.3 km/sec.

Assuming that the depth of the upper layer is 7 km and that of the earthquake from this layer is 13 km., the time-distance (epicentral) curve as shown in the figure (Fig. 14) is obtained. If the abscissa is taken as the duration of the preliminary tremor, the relation turns to as shown in Fig. 15. Numerical values calculated from such assumptions are tabulated in Table II.

Time-distance relation.
Broken line: *PS* and distance relation.[2]

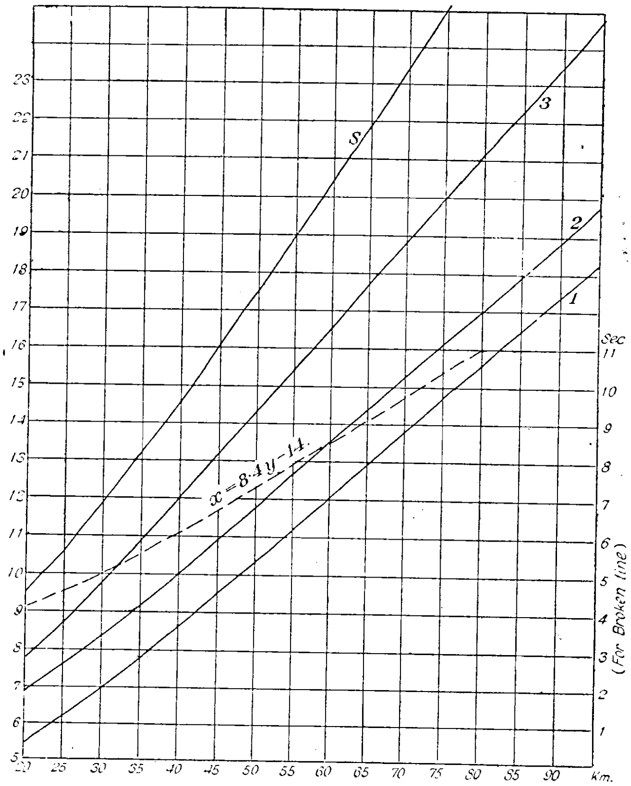


Fig. 14.

(1) S. Kusakabe, Pub. I. E. I. C. **14** (1902).
(2) The formula is of course for linear part not too close to the origin.

Duration of 1-2 and 1-3.

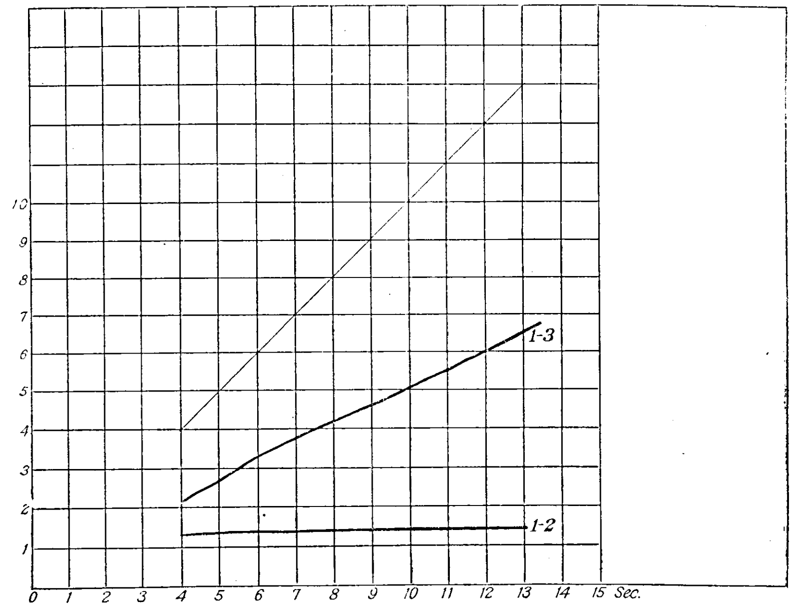


Fig. 15.

TABLE II.

(Refer to Fig. 13)

${}_1x_1$	${}_1x_2$	h_1	τ_1	${}_2x_1$	${}_2x_2$	h_2	τ_2
5.06	15	20.06	5.54	2.53	15	17.53	6.5
5.72	20	25.72	6.30	2.85	20	22.85	7.3
6.07	25	31.07	7.20	3.05	25	28.05	8.1
6.35	30	36.35	8.05	3.18	30	33.18	8.9
7.42	35	42.42	9.10	3.27	35	38.27	9.8
7.65	40	47.65	10.0	3.32	40	43.32	10.65
7.83	45	52.83	10.8	3.36	45	48.36	11.5
7.95	50	57.95	11.7	3.39	50	53.39	12.4
8.06	55	63.06	12.6	3.41	55	58.41	13.3
8.15	60	68.15	13.5	3.43	60	63.43	14.1
8.23	65	73.23	14.4	3.44	65	68.44	15.0
8.28	70	78.28	15.2	3.46	70	73.46	15.8
8.30	75	83.3	16.2	3.46	75	78.46	16.8
8.33	80	88.3	17.0	3.47	80	83.47	17.6

$_3x_1$	$_3x_2$	h_3	τ_3	$_4x_1$	$_4x_2$	h_4	τ_4
15	12.6	27.6	9.3	4.89	15	19.87	9.5
20	13.7	33.7	10.6	5.75	20	25.75	10.8
25	14.3	39.3	11.9	6.37	25	31.37	12.3
30	14.65	44.6	13.1	6.77	30	36.77	13.7
35	14.85	49.8	14.3	7.07	35	42.07	15.3
40	15.0	55.0	15.4	7.28	40	47.28	16.7
45	15.1	60.1	16.6	7.44	45	52.44	18.2
50	15.2	65.2	17.8	7.56	50	57.56	19.8
55	17.3	70.3	19.0	7.65	55	62.65	21.2
60	15.35	75.35	20.1	7.72	60	67.72	22.8
65	15.38	80.4	21.3	7.77	65	72.77	24.4
70	15.42	85.4	22.4	7.83	70	77.83	25.8
75	15.46	90.46	23.46	7.86	75	82.86	27.2

Comparing these figures (Figs. 14, 15) with Figs. 6, 16, we can see that the general tendency of the observed time relation is favourably explained not only qualitatively but also quantitatively.

In Fig. 16, the relation of the duration of the preliminary tremor to the epicentral distance of earthquakes occurring in the Kwantô district(1) observed at stations near by such as Tôkyô, Kamakura and Kiyosumiyama is investigated. Earthquakes selected are those of no.7 – no.26 and no.62 – no.88 in Table I, which were determined from observations at the stations above mentioned. Reports from other stations were referred to as far as possible. In determining the epicentre of earthquakes, the Omori's formula $y=7.44\,x$ was assumed. In this case, the rôle played by this formula is only to limit the probable position of the epicentre, as if a fish is driven to a place by the fisherman's net.(2) Hence it may not be seriously objectionable to utilise such formu-

(1) The estimated focal depth of earthquakes in the Kwantô district is situated at about 20-50 km.

(2) Let l be the distance between two stations, and p_1 be the distance from one station to a point orthogonally projected on the line l from the epicentre determined by using a formula $y=ax$, and p_2 be similar quantity determined by using another formula $y=bx$. If the duration of the preliminary tremor observed at each station be x_1 and x_2 respectively, the difference of p_1 and p_2 is given by

$$|p_1-p_2|=\left|\frac{(a^2-b^2)(x_1^2-x_2^2)}{-2l}\right|.$$

Hence, if distribution of stations be sufficiently dense, position of epicentre will be determined tolerably well, even if different formulæ were used.

Duration of PS and Distance.
Dot: At Tôkyô.
Cross: At Kamakura
Circle: At Kiyosumi
Dot in circle : measured by the present authors

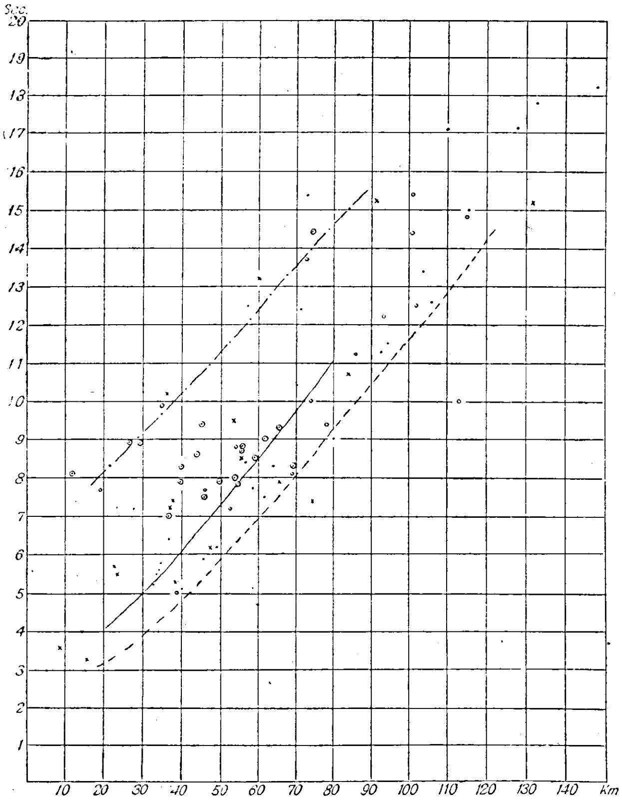


Fig. 16.
Full line : Calculated. Broken : Lower limit
Chain line : approximate upper limit.

la for studying the relation.

With regard to the coincidence of the calculated values to the observed, it is important to investigate the degree of freedom of choice of the six elements for giving such a solution.

If we assume that the observed three curves (1–2, 1–3, 1–S, and the distance relation) are straight lines, we may have six constants (including zero) as parameters determining the straight lines. On the other hand, in the hypothetical case here introduced, the T_i and h relation is also approximately linear, and the six parameters determing the linear relation of T_i and h are respectively functions of six quantities such as d_1, d_2, p_1, p_2, s_1, s_2. Thus, the problem is uniquely determined.

In the actual case, the relations are not linear exactly. Hence the problem is not so simple as this. But it may safely be said that the diversity of possible other solutions may be much limited, if we try to explain the case by assuming such a hypothetical structure of earth's crust as here proposed.

The way of fluctuation of observed values of each 1–2, 1–3, and 1–S difference is also favourable for this hypothesis. As apparent from the figure (Fig. 6) the fluctuation is smallest in the phase 1–2. This may be naturally understood considering that this difference is due to the p_2p_1 and p_2s_1 difference. Hence this is mainly due to the event in the upper layer, thus giving small fluctuations even if the depth of the origin of earthquakes are distributed in a wide range. On the contrary, the 1–3 difference concerns equally with the events in both the upper and the lower layer, and the 1–S difference is mainly due to the event in the lower layer. Hence the remarkable fluctuation in these phases may be naturally expected according to the fluctuation of the depth of earthquakes.

In passing, a remark will be added on the depth of local earthquakes occurring in the Kwantô district. As to be seen from Fig. 16, the distribution of points showing the 1–S difference is densely confined between the two lines one of which is that expected from the hypothesis, and the other is situated lower than the former by about 1.5 sec. This may perhaps be due to the effect of missing some part of the 1–2 phase owing to the smallness of earthquakes. The points denoted by the dot in the circle were determined by one of the present authors and the others are adopted from those determined by Mr. Ch. Yasuda.[1] The former are distributed above the line calculated from the hypothesis with only one exception. The said difference 1.5 sec is difficult to explain by the effect of the focal depth, for even if we choose the depth very shallow, the amount cannot be expected. Considering these effects, our hypothesis seems also favourable in this respect. It may rather be said that most earthquakes in the Kwantô district originate deeper than twenty kilometers, the majority of cases being included near this value.

It seems that there is an upper limit of the 1–S difference of earthquakes in the Kwantô district. The line limiting the upper part (the chain line in Fig. 16) is situated above the full line by about 4 sec., thus the focal depth of such earthquakes is expected about 30 km deeper than that specified by the full line. Hence the depth of origins of such earthquakes is about 50 km.

Amplitude of the motions.

In the next place, it is necessary to investigate the relation of magnitude of motions in each phase expected from our hypothesis. This problem is that of the reflection and refraction of elastic waves at a boundary of two layers and was already solved by C. G. Knott.[2] As far as our results concern, there is no clue for estimating the density in each layer. Hence numerical calcul-

(1) A. Imamura and Ch. Yasuda, Proc. Imp. Acad. **3** (1927) 279; this Bull. **3** (1927) 105.
(2) C. G. Knott, Phil. Mag. **48** (1899) 64–97.

ation cannot be carried out precisely. But to obtain a broad aspect of matter assuming that the density in each layer is equal with each other, the ratio of magnitude in the first and the second phase is estimated as shown in the table. (Table III.)

TABLE III.

Epicentral dist. in kilometre.	30	40	50	60	P denotes the dilatational motion. S denotes the distortional motion. v in suffix denotes the vertical component. h in suffix denotes the horizontal component.
P_v / S_v	4.3	3.8	3.6	3.6	
S_h / P_h	.65	.61	.60	.60	

Thus, it may be seen that P_v/S_v is rather smaller than the observed value. The S_h/P_h is remarkably smaller than the observed value. But this fact may not be a fatal contradiction to our hypothesis from following reasons.

It is quite natural that the elastic constants decreases gradually from deep place toward the surface of the earth's crust. Thus, the trajectly of the propagation of waves may be concave toward the surface and the angle of emergence will gradually approach the right angle. In such case the vertical component of the dilatational motion will become larger and that of the distorsional motion will be smaller.

With regard to the horizontal component of motion, just the inverse relation may hold.

Another factor to be considered concerns with the mode of development of motions in each phase when the elastic constants in the medium undergo gradual changes.

Using customary notations, equations of motion in the medium in which the constants change in the z direction are given by

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+\mu)\frac{\partial\Delta}{\partial x}+\mu\nabla^2 u+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\frac{\partial\mu}{\partial z},$$

$$\rho\frac{\partial^2 v}{\partial t^2}=(\lambda+\mu)\frac{\partial\Delta}{\partial y}+\mu\nabla^2 v+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\frac{\partial\mu}{\partial z},$$

$$\rho\frac{\partial^2 w}{dt^2}=(\lambda+\mu)\frac{\partial\Delta}{\partial z}+\mu\nabla^2 w+2\frac{\partial w}{\partial z}\frac{\partial\mu}{\partial z}+\Delta\frac{\partial\lambda}{\partial z}.$$

In the simplest case when a plane wave is propagated in the z direction and displacements are proportional to e^{ipt}, u and w are functions of z only and $v=0$.

The equations degenerate to

$$\frac{\partial^2 u}{\partial z^2}+\frac{1}{\mu}\frac{\partial \mu}{\partial z}\frac{\partial u}{\partial z}+\frac{\rho p^2}{\mu}u=0, \quad \text{for the distortional wave}$$

and

$$\frac{\partial^2 w}{\partial z^2}+\frac{1}{\lambda+2\mu}\frac{\partial}{\partial z}(\lambda+2\mu)\frac{\partial w}{\partial z}+\frac{\rho p^2}{\lambda+2\mu}w=0, \quad \text{for the dilatational wave.}$$

Thus, for studying these two equations, it is sufficient to investigate the character of a function represented by a differential equation of the following form:

$$\frac{d^2 y}{dz^2}+\frac{1}{a}\frac{da}{dz}\frac{dy}{dz}+\frac{b}{a}y=0,$$

where a stands for $\lambda+2\mu$ for the dilatational motion and μ for the distortional.

The normal form of the equation is obtained by putting $y=\frac{U}{\sqrt{a}}$,

$$\frac{d^2 U}{dz^2}+IU=0, \quad \text{where} \quad I=\frac{b}{a}+\frac{1}{4}\frac{1}{a^2}\left(\frac{da}{dz}\right)^2-\frac{1}{2}\frac{1}{a}\frac{d^2 a}{dz^2}.$$

Thus it may be seen that the amplitude of motion varies inversely proportional to $\sqrt{a}$, viz., square root of elastic constants.

From our experience of observation of velocity of seismic waves it seems that the poisson's ratio becomes gradually smaller toward the surface, i.e. the rigidity increases rapidly compared with the incompressibility. Such regions are, however, far deeper parts of earth's crust than that here concerned.

In passing, the behaviour of U will be examined. As well known from the theory of differential equation, when $I<0$, U does not become zero more than once with regard to z. In other words, the motion is aperiodic with regard to space. Such case occurs when the last term of I becomes exceedingly large, i.e. when the rate of change of medium is very rapid.

On the other hand, when $I>0$, U is periodic with regard to z. A formal solution of U will easily be obtained as follows, by putting

$$I=p^2\left\{\frac{\rho}{a}+\frac{1}{4p^2}\frac{1}{a_2}\left(\frac{da}{dz}\right)^2-\frac{1}{2p^2}\frac{1}{a}\frac{d^2 a}{dz^2}\right\}=\lambda\Omega(z), \quad \text{where } \lambda=p^2,$$

the solution of the differential equation is transformed into that of the integral equation of the Volterra's type

$$Q_1(z)=U(z)+\lambda\int_{z_0}^{z}\Omega(t)(z-t)U(t)dt,$$

where Q_1 is a polynomial of the first degree such that $A(x-B)$. Thus A, B

and z_0 are arbitrary constants, but in the solution of the equation, the number of arbitrary constants to be determined degenerates to two.

A solution in series is

$$U(z)=U_0(z)+\lambda U_1(z)+\lambda^2 U_2(z)+\ldots\ldots+\lambda^n U_n(z)+\ldots\ldots$$

where

$$U_0(z)=Q_1(z),$$

$$U_1(z)=(-1)\int_{z_0}^{z} K(z,\ t)U_0(t)dt,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$U_n(z)=(-1)^n\int_{z_0}^{z} K_{n-1}(z,\ \sigma)Q_1(\sigma)d\sigma,$$

$$K(z,\ t)=\Omega(t)(z-t),$$

$$K_1(z,\ \sigma)=\int_{\sigma}^{z} K(z,\ t)K(t,\ \sigma)dt,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$K_{n-1}(z,\ \sigma)=\int_{\sigma}^{z} K(z,\ t)K_{n-2}(t,\ \sigma)dt.$$

In special case when $\Omega(z)$ is a constant, U is of course a trigonometric function. When $\Omega(z)=z^m$ (1)

$$U=\sqrt{z}\left\{AJ_{\frac{1}{m+2}}\left(\frac{2\sqrt{\lambda}}{m+2}z^{\frac{m+2}{2}}\right)+BY_{\frac{1}{m+2}}\left(\quad ,, \quad\right)\right\},$$

and when $\lambda\Omega(z)=e^{2z}$,

$U=AJ_0(e^z)+BY_0(e^z)$, where A and B are arbitrary constants.

Hence the behaviour of U is generally very complicated. But roughly speaking, it may be seen that the amplitude of U diminishes gradually with distance when I increases with distance, viz., when the wave is propagated in a medium, the elastic constants of which decrease gradually with distance. But it must be remembered that the principal factor determining the amplitude of motion is the inverse of the square root of elastic constants.

Returning to the original problem, next point to be examined concerns with the distribution of earthquakes showing the characteristics above stated. The mode of development of each phase is as described in the foregoing section. Earthquakes from the neighbouring Pacific do not show the character-

(1) After this paper was written, Mr. K. Sezawa read a paper on July 5, 1927 in which a similar special case was treated.

istics. (See Fig. 7) This may perhaps be due to an effect that waves undergo much modification during its propagation through much heterogeneous part between land and ocean bed. It seems also that the mechanism of occurrence of earthquakes is much different in accordance with the difference of origin of earthquakes in land and ocean. If an earthquake occurs gradually, then the motions observed will also be gradual. Hence, characteristics of motions in each phase will hardly be identified.

Development of each phase of earthquakes from southern part of the Bôsô Peninsula and Sagami Province is also less distinct. According to authorities in geology,(1) these regions have undergone much crustal revolutions since geological ages of somewhat younger age. Thus seismic waves from these regions will also undergo much modification.

One more remark will be added to the mode of development of the third phase. If a distortional wave is incident at a boundary with its plane of vibration parallel to it, the dilatational wave is not usually excited. Thus closer examination of this phase will furnish some clue to the mechanism of occurrence of earthquakes. This effect will also induce fluctuations of the time and the distance relation.

IV. Earthquakes Accompanied with Sound.

Studying a paper by Professor F. Omori(2) "Seismographical Observations in Tôkyô of the Earthquakes with Sound," we can find that earthquakes which were classified by him as those accompanied with a detonative sound were of just the same character as those studied by us. The manifestation of the large initial motion in the vertical component is quite the same. With regard to the horizontal motion, he remarked the forerunner only in the case of earthquake at $0^h\ 26^m\ 07^s$ p.m. Jan. 24, 1919. But closer examination of the records illustrated by him we can find it, though not distinct owing to the slow rate of driving of the recording drum. The space distribution of such earthquakes is also nearly the same as those studied here by us. Thus it may be supposed that elastic waves produced by such earthquakes from such localities are observed at Tôkyô without being much modified during its propagation,

(1) For examples, N. Yamasaki, Journ. Fac. Sci. Imp. Univ. Tôkyô, [ii] **2** (1926) 79-119; H. Yabe and R. Aoki, Ann. Rep. Work, Saitô Gratitude Foundation, **1** (1923/24) 70-83.

(2) F. Omori, Bull. I. E. I. C. **11** (1923) 33-63.

resulting in manifestations of simple sound and typical appearance of each phases. In our opinion, even the earthquakes with rushing sound as classified by Professor Omori are not essentially different from those studied by us. The initial vertical motion was not large compared with that of the principal part, and yet was almost largest in the preliminary part. It may be rather natural to expect such small varieties, considering that the earthquake is a quite complicated phenomenon.

V. A Remark on Extremely Deep Earthquakes.

Existence of earthquakes of extremely deep origin (even deeper than 300 km) has been suggested by certain authors such as Professor Turner,(1) Professor T. Shida,(2) et. al. Recently Mr. K. Wadati(3) examined carefully such earthquakes in Japan and obtained very probable value.

On the other hand, it is generally believed that there is a layer where certain properties of matter undergo discontinuous change or at least very rapid change at a depth of about 50 km under land area.

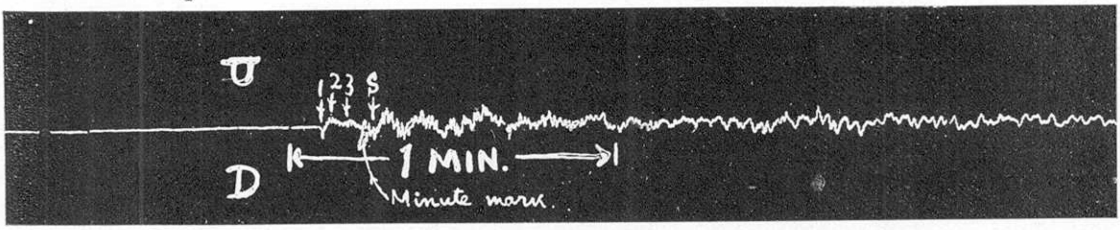
Taking the above two facts for granted, it is naturally supposed that for such earthquakes the manifestation of some phases of waves as discussed in the previous sections might be expected. We have not yet sufficient data for such case. But closer examination of the record of earthquake on July 27, 1926 observed in our institute (Pl. XV Fig. 17) seems to reveal such characteristics. In the figure, three steps marked 1, 2 and 3 are almost doubtlessly identified. As we have no knowledge about the time-distance relation, we are not yet in the position to reduce numerical relations. And yet the 1–2 interval is about 8 sec., hence if in the upper layer the velocities of P and S be 5.6 and 3.3 km/sec respectively then the depth of the upper layer is estimated as about 60 km.

Estimation of depth of earthquake origin by the knowledge of the angle of emergence of waves determined from seismograms leads sometimes to quite erroneous idea. If the trajectory of waves as discussed above is taken into account, such case may naturally be explained.

June, 1927.

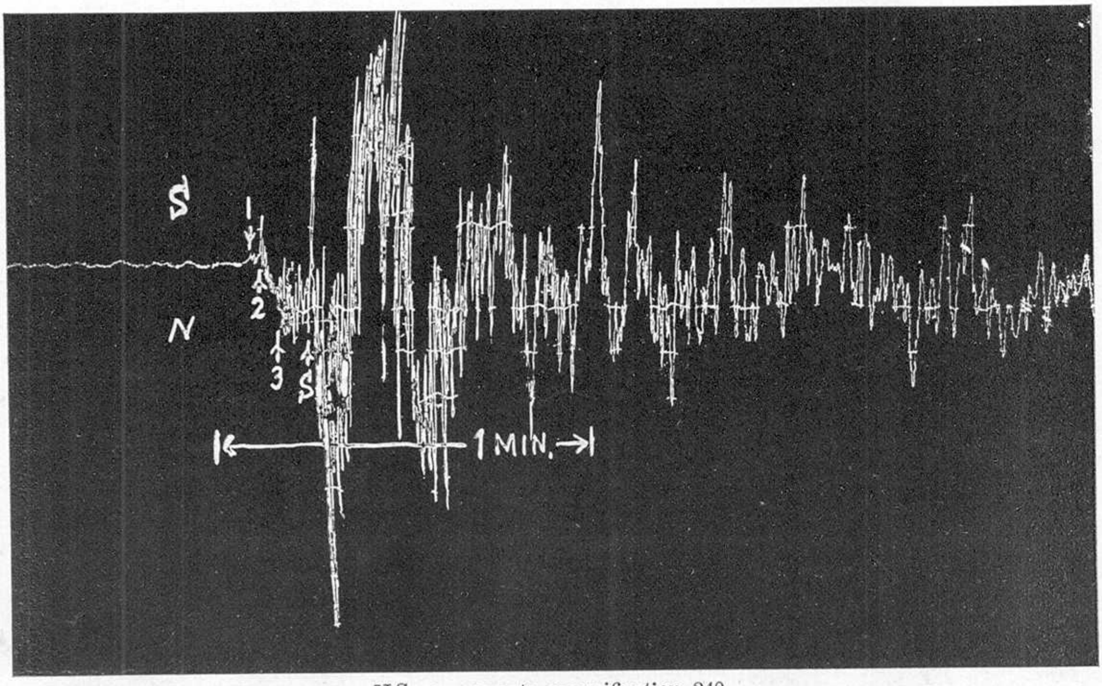
(1) The International Seismological Summary.
(2) T. Shida, Chikyû (The Globe) (in Japanese) 7, (1927) 87–89.
(3) K. Wadati, Journ. Meteor. Soc. Japan [ii] **5** (1927).
(4) Do.

Earthquake near Ôtaki, Kazusa Province at 10^h 20^m on Feb. 22, 1926.



Vertical component: magnification 60.

Fig. 1 a.

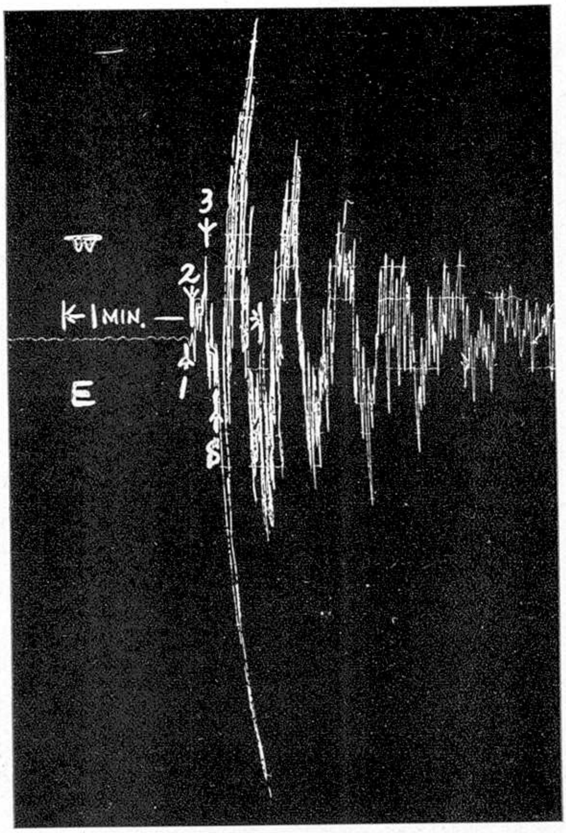


NS component: magnification 240.

Fig. 1 b.

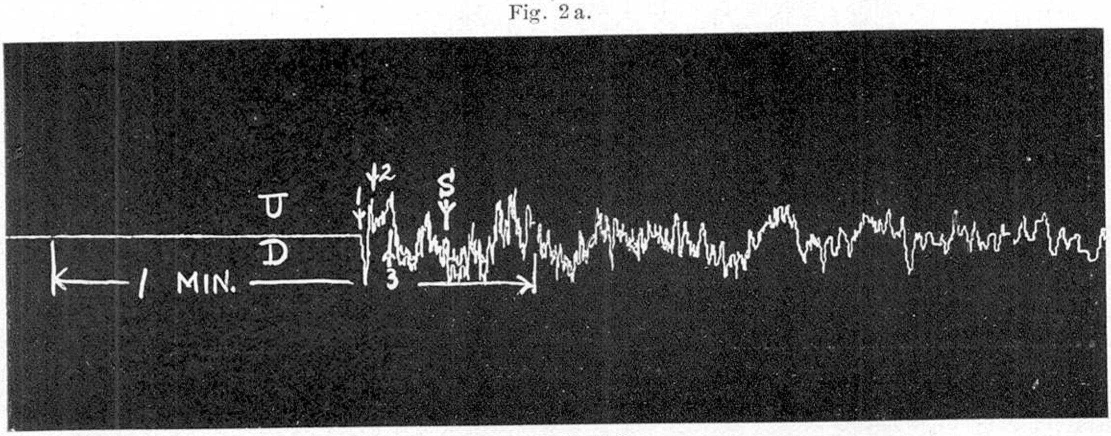
(震研彙報第四號、圖版、松澤)

Earthquake at 16^h 02^m on Jan. 9, 1925.



magnification 120.

Fig. 2 a.

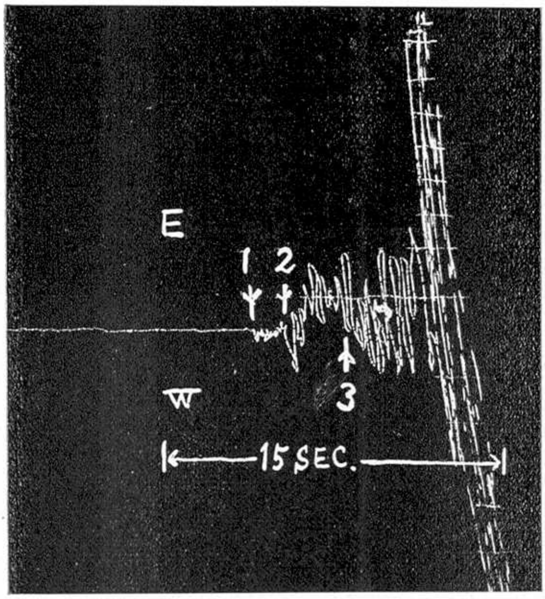


magnification 60.

Fig. 2 b.

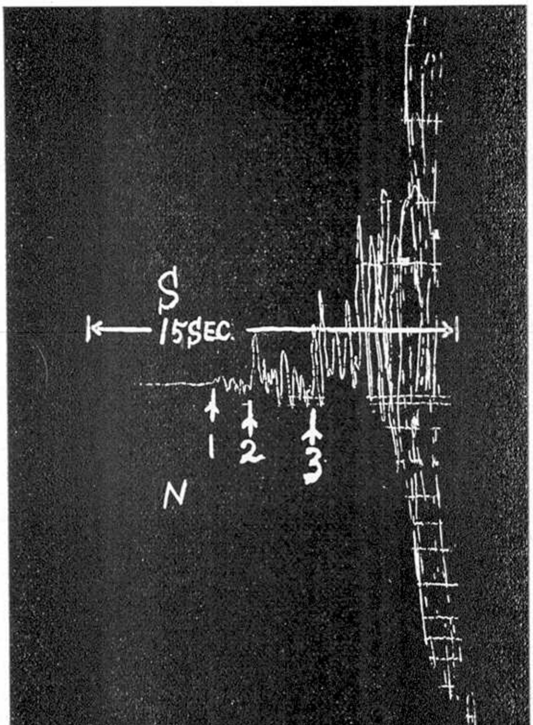
(震研彙報第四號、圖版、松澤)

Earthquake at 23^h 58^m on Feb 13, 1926.



Magnification 600.

Fig. 3 a.

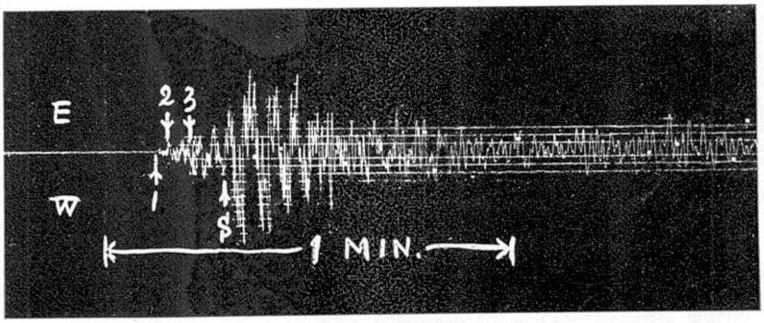


Magnification 600.

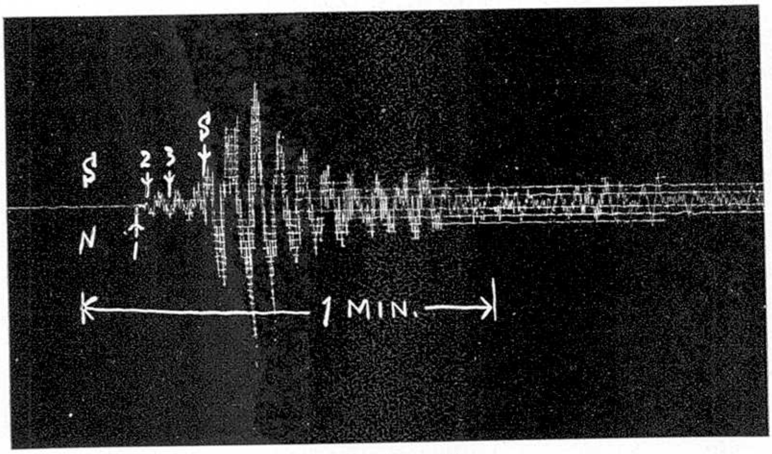
Fig. 3 b.

(震研彙報第四號、圖版、松澤)

Earthquake near Inbanuma on Feb. 25, 1927.



Magnification 120.



Magnification 120.

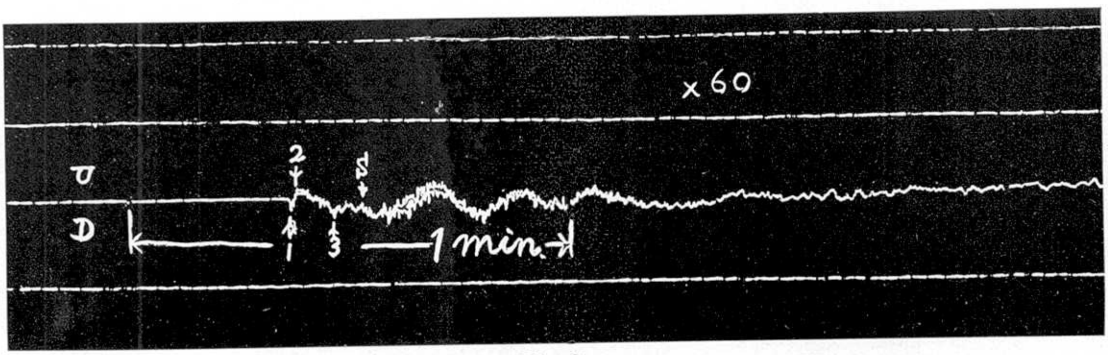


Fig. 8.

(震研彙報第四號、圖版、松澤)

Earthquake near Hatiôji at 0^h 41^m on July 30,1926.
(magnification 600)

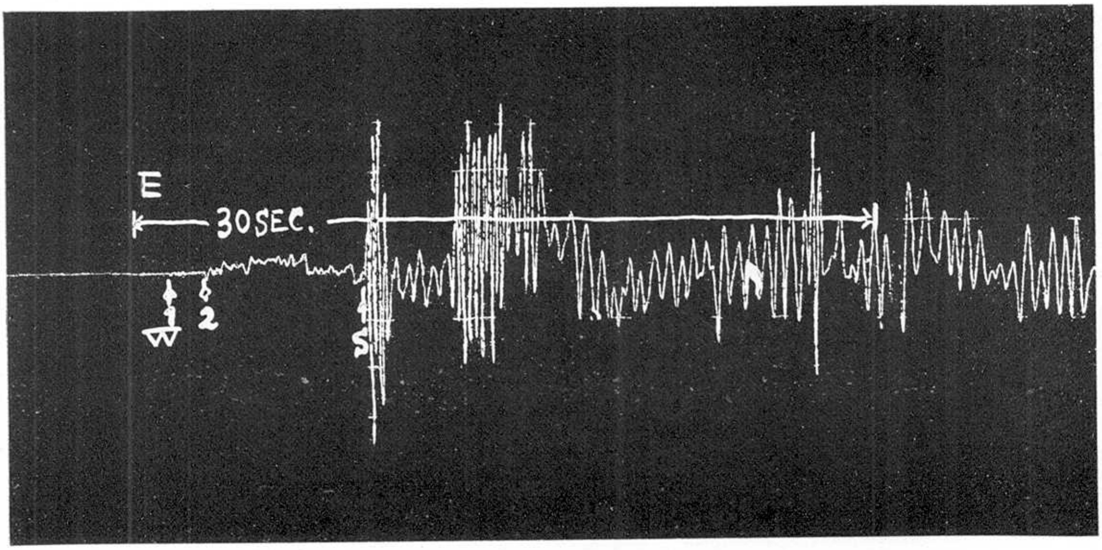


Fig. 9 a.

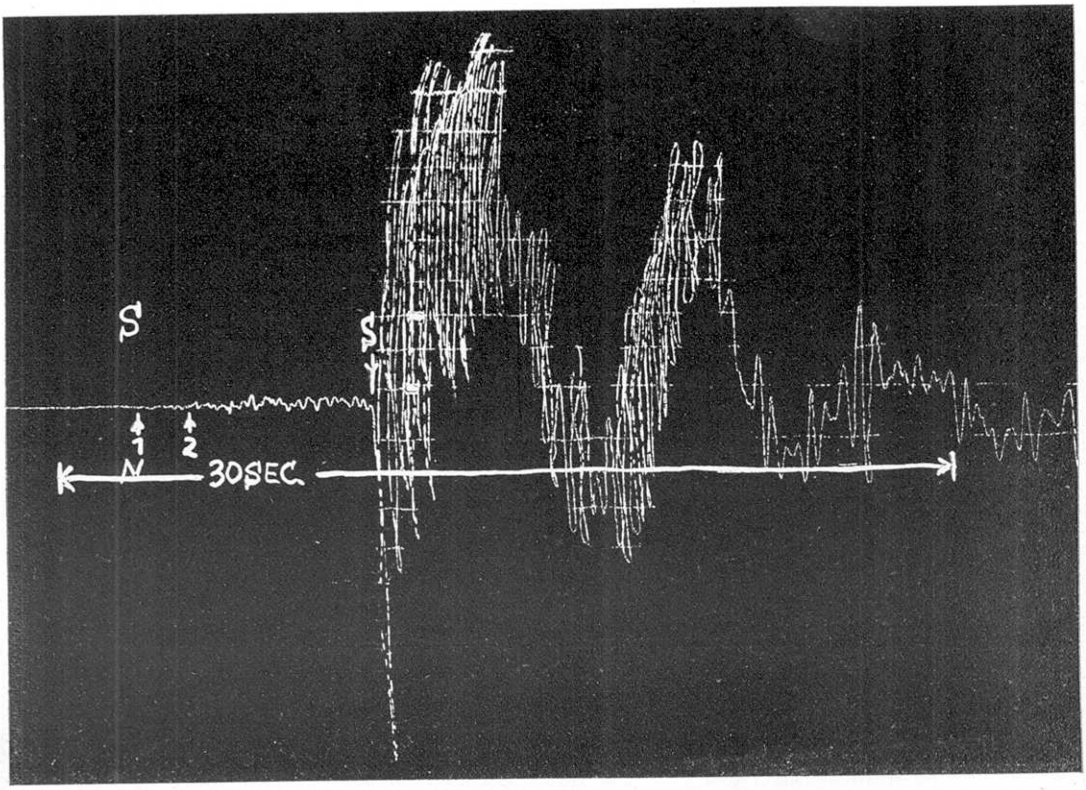


Fig. 9 b.

（震研彙報第四號、圖版、松澤）

Earthquake near Mt. Tukuba at $18^h\ 42^m$ on Aug. 15, 1926.
(magnification 300)

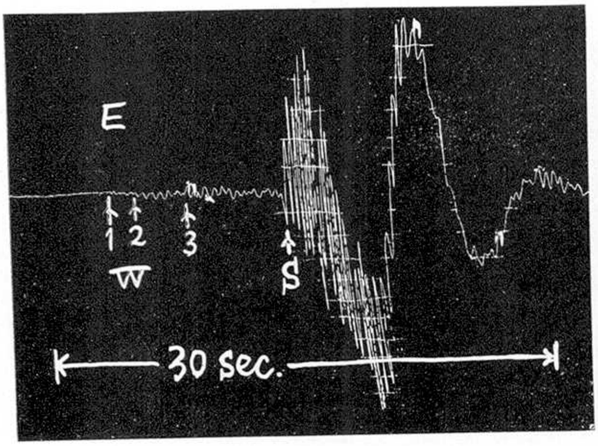


Fig. 10 a.

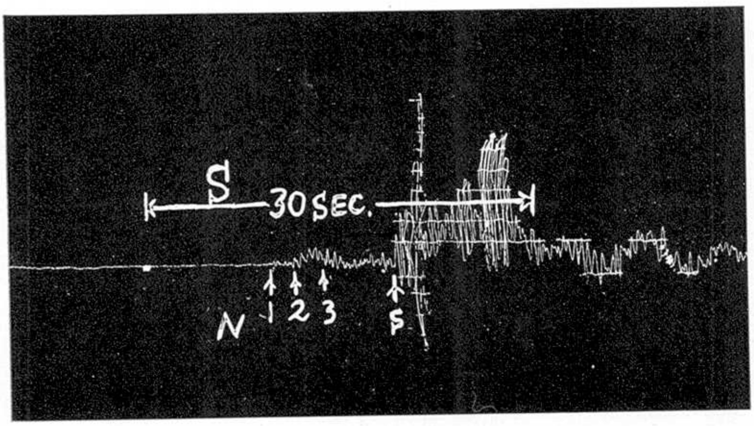


Fig. 10 b.

(震研彙報第四號、圖版、松澤)

Earthquake near Kanagawa Feb. 11, 1927.

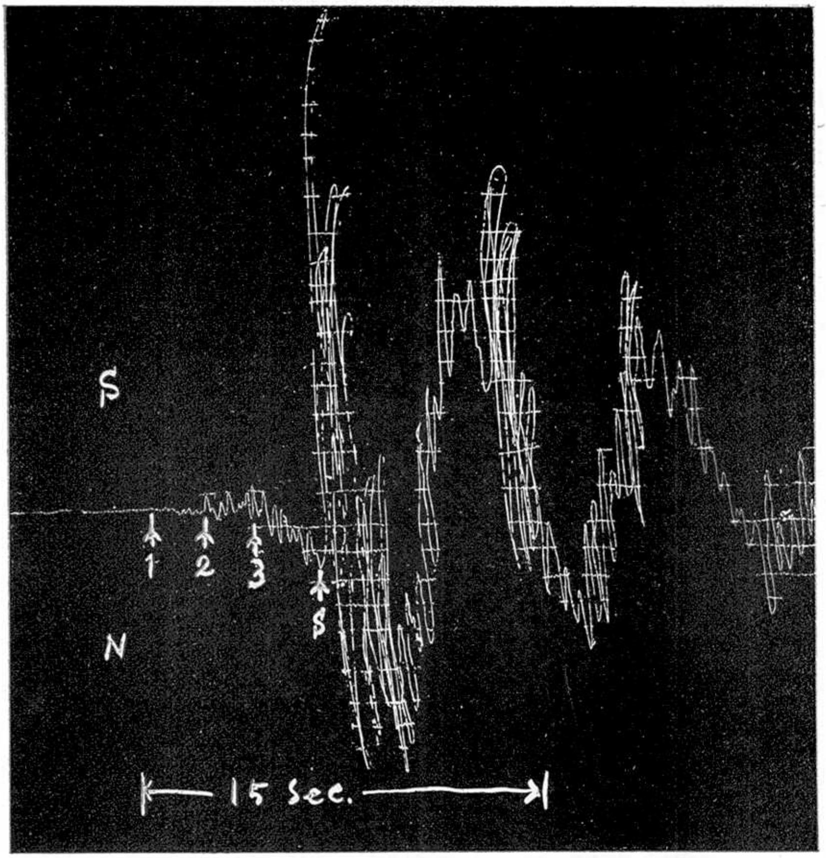


Fig. 12a. Observed at Tôkyô.

Earthquake at the Southern foot of Mt. Huji at $13^h\ 28^m$ Aug. 13, 1926 (magnification 300)

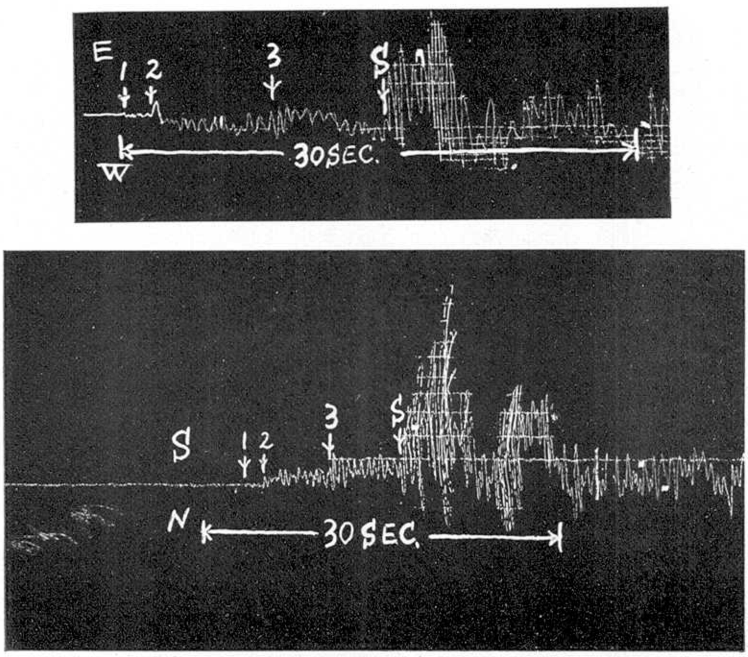
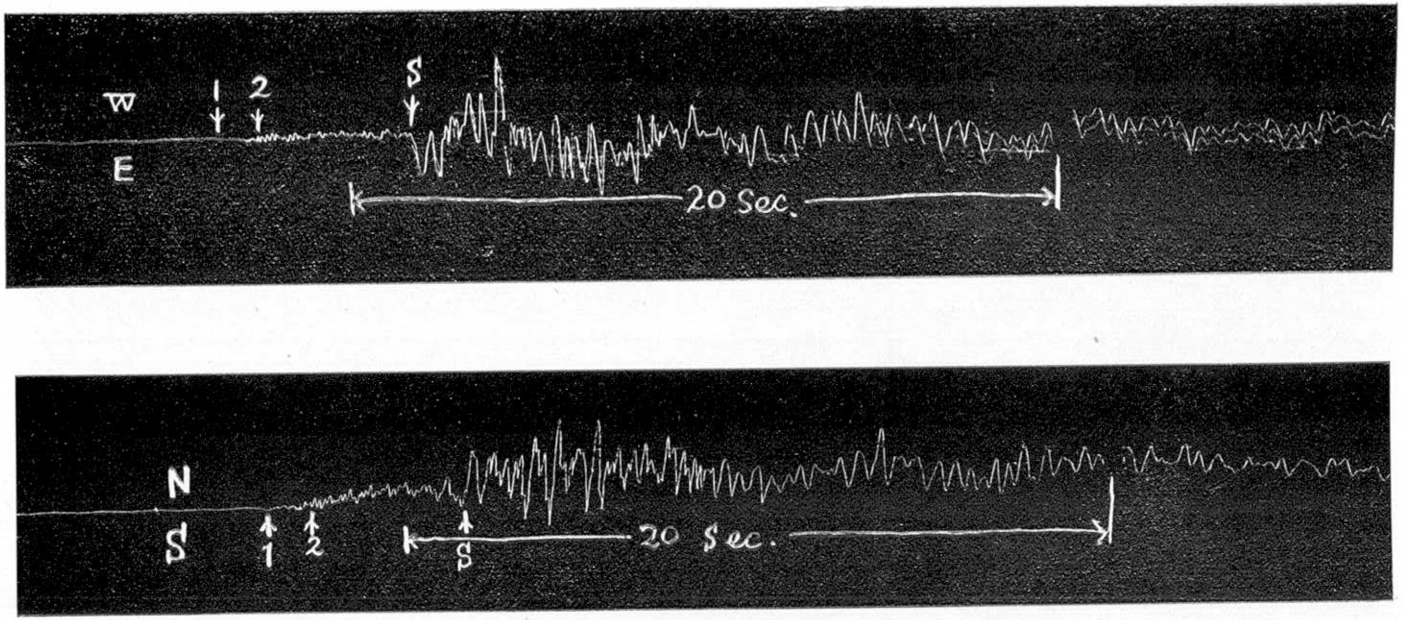


Fig. 11.

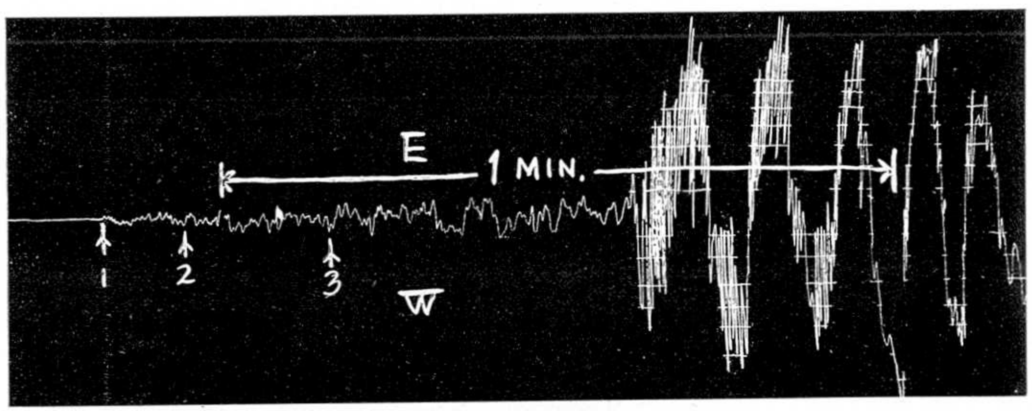
(震研彙報第四號、圖版、松澤)



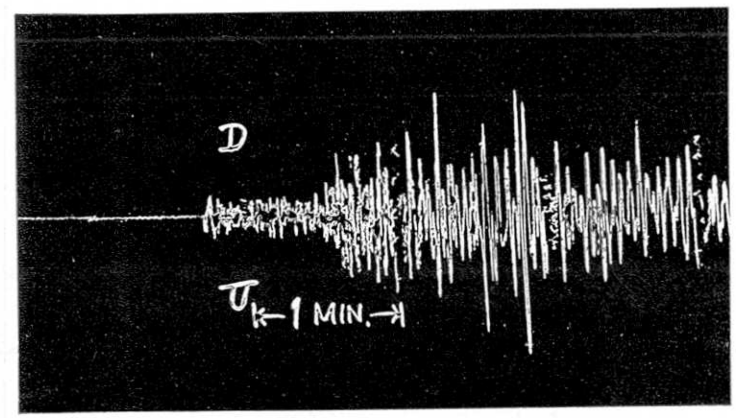
Observed at Yuigahama, Kamakura:
magnification 150.

Fig. 12 b

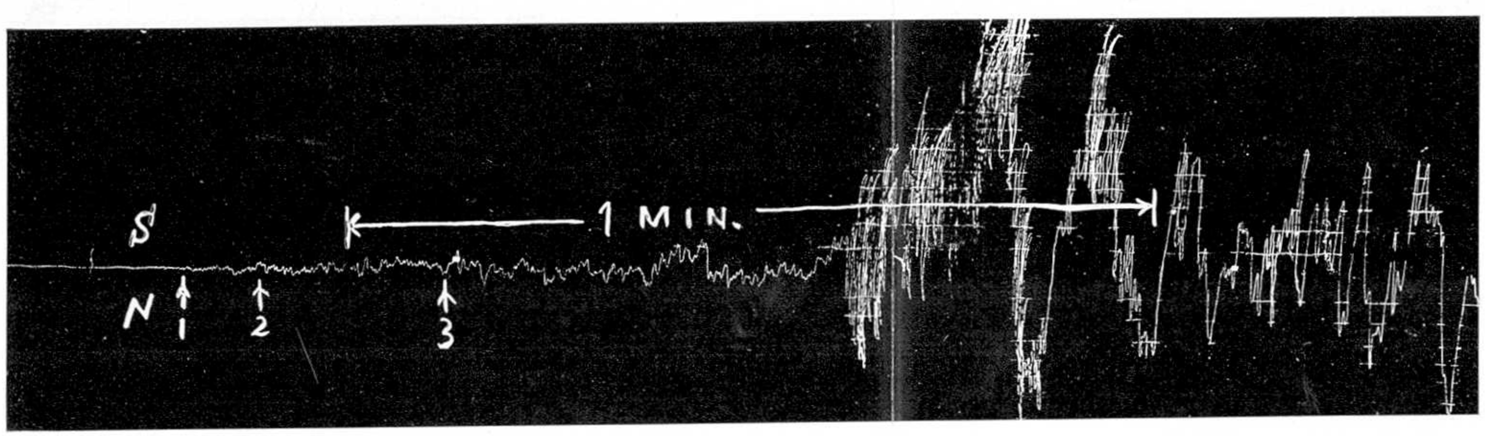
Earthquake of very deep origin on July 27, 1926.



Magnification 300.



Vertical component, magnification 238.



Magnification 300.

Fig. 17.