Experimental Studies on Elastic Waves (Part 1)

By

Torahiko TERADA and Chûji TSUBOI

Earthquake Research Institute

弾性波の實驗　(第一報)

所員　寺田寅彦
所員　坪井忠二

地震學上の諸問題で、實驗物理的の研究を要するものが多數にあるに拘らず、從來此の方面の研究が餘り進んで居ない様に見える。地震波傳播に關する諸問題も其例である。此方面の理論的の諸問題は從來偶然有數な數理物理學者の興味をひいた爲、可也な發展をしたやうであるが、元來簡單な彈性方則に基礎を置いた理論を、地殼の如きものに應用した場合に、如何なる程度まで適用され得るかといふ事に就いては疑を挿む餘地がある。それで若し不完全な彈性を有する物質に就いて、純實驗的に波動傳播に關する諸現象を研究して、一方では此れを從來の理論と對照し、又一方では地震波の實測の結果と比較すれば、此れによつて地殼の彈性に關して何等かの新しい知識を得る事が出來る見込があるかと思はれる。

又應用地震學の方面でも、例へば地震による建物の搖動に關する諸問題、地表の微動に對する溝や堀の效果などでも、理論的には甚だ困難なものが、容易に實驗によつて解決せらるゝ場合が多からうと思ふ。

以上の樣な考から彈性波に關する實驗を始める事にしたのであるが、未だ始めたばかりで、豫備的の粗雜な方法により大體の見當をつけたに過ぎない。

以下に報告する實驗の諸考案は主に坪井君のものであり、其の遂行は同君と助手矢田君によつてなされたものである。それで以下の報告も坪井君の筆を煩はし、自分は唯玆で紹介の辭を述べるに止めた。　　寺田寅彦

此の實驗の目的に副ふものとして、第一に用ひた媒質は寒天である。先づ實驗の準備として、寒天の彈性の常數の大體の値を測つたが、其の結果、ヤングの彈性率は 2.0×10^5 c.g.s. ポアソンの比は 0·47 といふ値を得た。之から計算すると、每秒數十回振動する源から生ずる彈性波の波長は、數センチメートル乃至十數センチメートルの桁になるので、實驗に使ふ寒天の容器は、之に適當する大きさに作つた。振動の源としては、鐵の球を寒天の中に埋めたものを使ひ、其の上に近付けた電磁石に交流を通して、その球を振動させる樣にした。併し、之では一定の振動數のものだけしか得られないので、寒天の中に埋めた眞鍮の棒を電氣モートルに取付けたエキセントリック、ホイールで動かす樣にし、電氣抵抗でモートルの廻轉數を加減する事によつて、種々の振動數の振動を得る工夫もした。

かうして生じた振動を觀測するには、寒天の表面に小さい鏡を並べて之に光をあてて、鏡から反射して出來る像の振動する有様を見る方法を採つた。寒天の容器の側面や底から波が反射するのを防ぐ爲には、其處に綿を充分厚く敷いて、之によつてダンプさせる方法を採り、尙實際、反射が此れによつて防がれると云ふ事も實驗で確めた。

始め注目したのは表面波であつて、深さが增すと共に、其の振幅は、理論が要求するのと殆んど同じ樣に減少して居るのを見た。寒天を切つて崖を作ると、其の緣で波が反射して、定常波が出來る事、併し、反射が不完全である爲に節にあたる所も相當の振幅を持つて居る事などを明にし、此の振幅から反射の良否の程度を定め得る事を示した。次に堀を掘つて、其の對岸の振動を減少させる事を調べ、波長の數分の一の幅や深さの堀によつても、對岸の振幅を著しく減少させる事が出來る事を示した。しかも、一定の大きさの堀は、或るきまつた波長の波をさへぎるのに都合がよい事、即對岸の振幅が、此の波長に對して極小になる事が明になつた。同時に、前に述べた堀の緣での反射の程度が、此の波長に對して極大になる事が解つたので、堀が波をさへぎるメカニズムが多少明にされた。それから、媒質に半ば埋まつて居る物體が、來る波に適應して如何に動くかと云ふ事も實驗したが、未だはつきりした結論は得られて居ない。

1. Introduction.

Among the diverse fields of researches in the domain of seismology, those quarters which may be effectively explored within the walls of a physical laboratory by the hands of experimental physicists, seem to have been unduly neglected. The cause of the neglect seems, in our opinion, to have been an accidental one, neither due to the essential futility of the attempt nor due to any practical difficulty of the method. Late Professor Kusakabe's research on the elastic properties of rocks may be mentioned as a beautiful example of the possibilities in this direction.

Problems of seismology awaiting thorough experimental studies are numerous. However, those connected with the propagation of elastic waves seem to form a category which is one of the most inviting and most promising. The problems of seismic waves have hitherto been the favourable topics of many eminent mathematical physicists and we owe, indeed, a lot of interesting acquirements of modern seismology to their labours. We may, however, scarcely assume that the mathematical theory of elasticity based on the most simple assumption on the elastic property of matter may be applied in all its consequence to the actual wave phenomena in the earth crust, especially because we are still in profound ignorance respecting the elastic properties and the structure even of its most superficial layer. In view of these considerations, it seems not quite superfluous to carry out a more or less systematic study on the various phenomena of elastic waves, from a purely experimental point of view, especially with a substance which may show marked deviations from the perfect elasticity. We may compare the results of such experiments, on one hand, with the outcomes of the theories and, on the other hand, with the facts of observations of actual earthquakes. By these means, it may be hoped, we may come across some clues for elucidating the actual properties and structures of the crust. Moreover, by means of various model experiments, we may solve comparatively easily a number of problems concerning the nature and mechanism of earthquake origins which may appear to surpass the power of mathematical analysis. Ample fields of useful researches are also in prospect in the domain of practical seismology.

Our present attempt is merely a preparation for the first step in the direction above pointed out. The experiments(1) which will be briefly described in the following are of a preliminary nature carried out for the purpose of sheer orientation.

T. Terada.

2. Physical Properties of Agar-agar.

As a suitable substance for studying the propagation of elastic waves, agar-agar was firstly chosen. Besides its deviation from the perfect elasticity, this substance has many favourable properties for the objects of the present experiment.

In order to determine the proper size of the vessel for containing this medium, its elastic constants(2) are needed. For this purpose, Young's modulus and Poisson's ratio were determined, both of which are easily measurable.

With the sample of agar-agar to be used in the determination, a test piece was made in the form of a circular cylinder. For this purpose, a solution of agar-agar was poured into a hollow circular cylinder of glass of a suitable size, placed upon a plane glass plate, and was there solidified. The solidified mass was taken out of the glass cylinder and placed on the brass

---

(1) The designs of the apparatuses and the methods of experiments as well as the discussions of the results are chiefly due to Tsuboi. The experiments were carried out by the able assistance of Mr. Y. Yada of the Institute.

(2) It is of course doubtful whether the usual word "elastic constant" may have any strict meaning for such a substance as agar-agar. Here, the word is used only in its approximate sense.

plate of the measuring apparatus. The arrangement for measuring the elastic constants is shown in Fig. 1, in which S is a spherometer, G a thin glass plate, K the agar cylinder, W weights, and A aluminium plate.

A suitable series of weights was placed on the aluminium pan below. The corresponding depression in height and the lateral expansion at about the middle height of the agar cylinder were measured by the spherometer and a micrometer screw gauge respectively. The micrometer screw gauge is not shown in the figure. From these two readings, Young's modulus and Poisson's ratio were calculated. Their values, for example, for a sample of agar-agar of 1.2 % are:

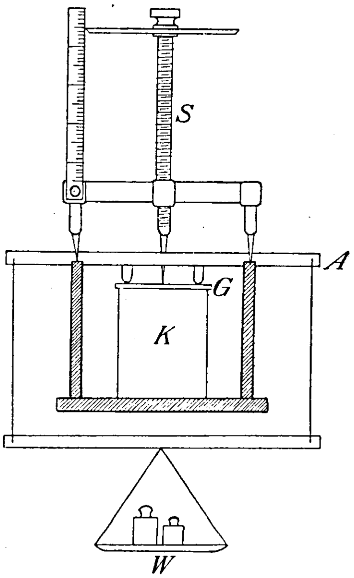


Fig. 1.

$$\text{Young's modulus} = E = 2.0 \times 10^5 \text{ c.g.s.}$$
$$\text{Poisson's ratio} = \sigma = 0.47.$$

These values, of course, depend on temperature,⁽¹⁾ but this will not matter much in the present degree of accuracy of our experiment. Then, from the ordinary formula, the velocities of elastic waves were estimated; the results are:

$$\text{Longitudinal Wave} \quad V_l = \sqrt{\frac{E}{\rho}\,\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}} = 1096\,\frac{\text{cm.}}{\text{sec.}}$$

$$\text{Transverse Wave} \quad V_t = \sqrt{\frac{E}{\rho}\,\frac{1}{2(1+\sigma)}} = 261\,\frac{\text{cm.}}{\text{sec.}}$$

where ρ is the density of the medium which is approximately 1.1 in this case.

The velocities of the elastic waves being of such magnitude as obtained above, their wave lengths will range from a few cm. to 10 cm., according as the frequency of the vibration is several ten times in a second. Thus a vessel 50 cm. in length will contain from 5 to 10 waves. The actual vessel used in the present experiment was 50 cm. in length, 25 cm. in depth, and 20 cm. in width.

---

(1) H. J. Poole, Trans. Faraday Soc., **22** (1926), 82.

In summer, agar is liable to putrefaction but this can be easily avoided by mixing a small quantity of mercury bichloride in the medium.

From the surface of the solidified agar, the constituent water constantly evaporates, thus causing slight changes in the values of the elastic constants and the density of the medium. Many trials have been made to keep away this evaporation. The surface of the medium was oiled, for example, but the attempt proved to be unsuccessful for the purpose. The effect of evaporation of the constituent water on the elastic constants and the density of the medium is, however, so small that it will matter little for the present purpose of our experiments.

3. Source of Vibration.

A small electromagnet fed by an alternating current of 50 cycles was used to excite the origin of the vibration in the medium. An iron sphere about 1 cm. in diameter which was imbedded at a certain depth from the surface of the medium, was set in vertical vibration under the influence of this electromagnet. The frequency of the vibration of the sphere is 50 or 100 times in a second according as it is magnetised or not respectively. By this method, however, a vibration of a definite frequency can only be produced. When the frequency of vibration is needed to be varied, as was actually the case, another method should be devised. For this purpose, an eccentric wheel driven by a *D.C.* shunt motor was used. A brass rod of about 1 cm. in diameter which was horizontally imbedded in the medium was set in vertical vibration by the eccentric wheel. (Fig. 2.) The lateral vibration of this source was avoided by means of a suitable guide. By adjusting the electric resistance, potentiometrically connected in the circuit of the motor as is shown in Fig. 2, the revolution of the motor could be varied at will from 0 to 2000 times per minutes. The constancy of the revolution velocity of this kind of electric motor proved to be a very favourable property to be used as a source of vibration.

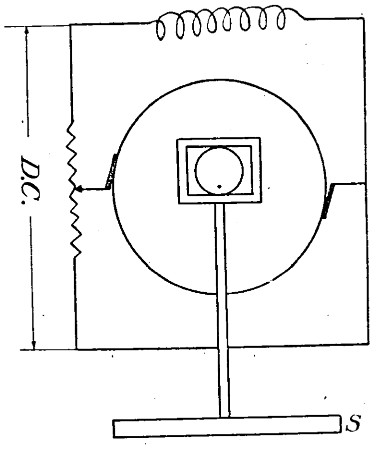


Fig. 2.

4. Method of Observation.

An optical method was used for observing the vibrations thus produced in the medium. The general scheme of arrangements is shown in Fig. 3, in which *S* is a light source, *M* reflecting mirrors, *m* small mirrors, *K* the agar mass, *L* a lens, and *T* a frosted glass plate. A number of small mirrors *m* were placed on the surface of the medium along a straight line and illuminated by light from a point source. Light reflected by these mirrors was forcussed by means of the lens *L* on the screen *T*. The image due to each mirror vibrates according to the vibration of the medium. The mirrors placed on the surface of the medium were a few mm. square in their area and a few tenth of mm. in their thickness, so that it is almost at once evident that they follow the vibration of the medium faithfully. Instead of the plane mirrors, concave ones may conveniently be used, for they dispence of the use of a lens for forcussing the images. As the source of light, a tungsten pointolite of Tôkyô Electric Company was used.

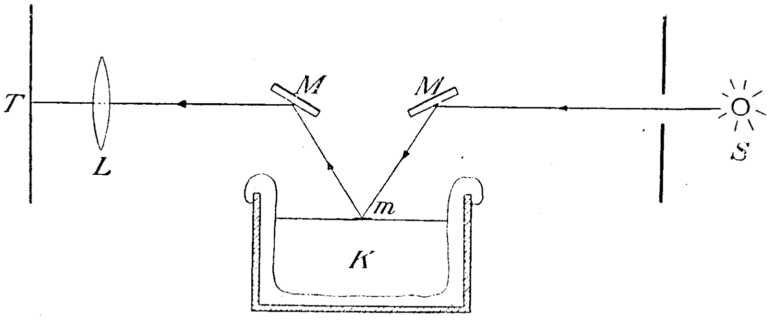


Fig. 3.

After the vibration has attained the stationary state, the image by each mirror will repeatedly describe one and the same path. This path could be traced with a pencil upon a tracing paper. For the observation of transient phenomena, however, the screen was replaced by a photographic film wound around a cylindrical drum. By rotating the drum, the motion of the image of the mirror was recorded on the film. If, in Fig. 3, the elastic wave propagates perpendicularly to the plane of the paper, the rotation axis of the small mirrors would be parallel to the plane of this paper, so that the images vibrate perpendicularly to this plane. The amplitude of the vibration of the image by the mirror is proportional to its inclination.

5. Boundary Conditions.

It is of utmost importance in the present experiment to get free from the reflection of the waves at the sides and at the bottom of the vessel. For this purpose, pads of cotton wool were spread at the sides and at the bottom of the vessel to a sufficient thickness. A sheet of absorbent cotton was further spread upon the ordinary cotton. By these arrangements, the abrupt change in the density at the boundary of the medium was practically excluded.

That no sensible reflection of elastic wave takes place at the sides or at the bottom of the vessel was ascertained by the following experiment. Two mirrors were placed on the surface of the agar mass at different distances from the source of vibration, and the elastic wave was suddenly made to propagate from the source. If a sensible reflection of the wave is actually present, some different phase, other than the direct progressive wave from the source, must appear in the records of the vibrations of the images of these mirrors. But this was not the case. In the neighbourhood of the walls of the vessel, however, the motion of the medium is somewhat constrained, so that the mirrors should be placed at a sufficient distance from the walls of the vessel to study the unconstrained motion of the medium.

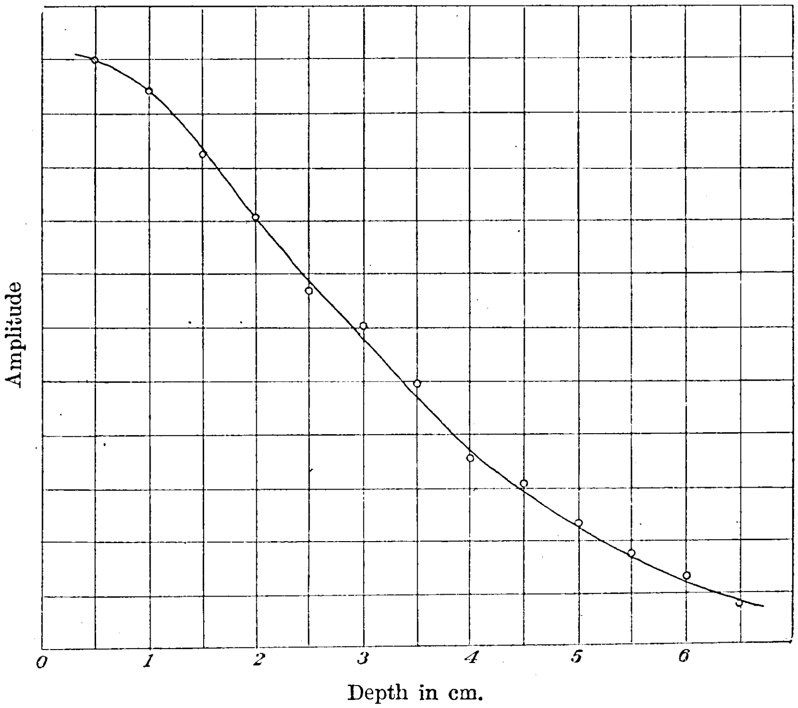
6. The Variation of the Amplitude of Vibration with Depth.

Whether the wave now under discussion is really a surface wave or not is the next question to be determined. For this purpose, the variation of the amplitude of vibration with depth was investigated. A vessel was specially constructed for this purpose, of which the side walls could be removed after the medium was solidified. After removing the side walls, pieces of very thin and short hair wire were stuck horizontally on the side face of the medium at every half cm. of depth. Their amplitudes of vibration were measured by a telescope fitted with a micrometer gauge. The results of the measurements are shown in Table 1 and graphically in Fig. 4.

TABLE 1.

Depth (cm.)	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
Amplitude	100	94.2	82.2	70.5	57.0	50.0	39.8	25.8	21.3	13.3	7.0	3.0	0.8

Fig. 4.



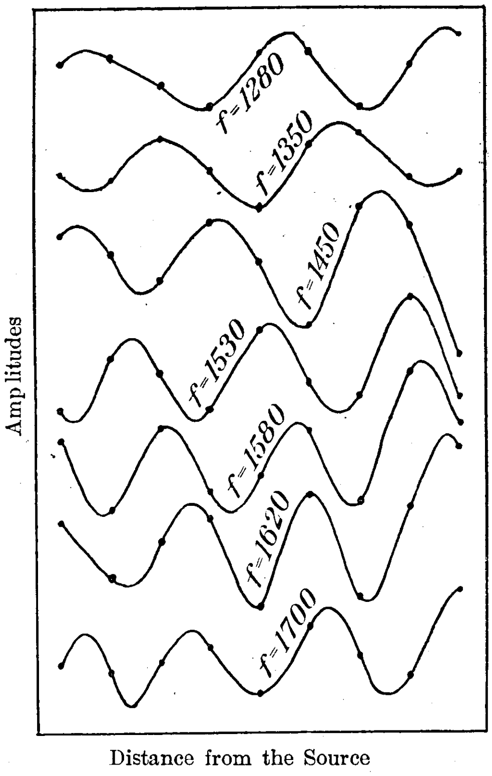
In the table, the amplitude of vibration at the depth of 0.5 cm. was taken to be 100, and the amplitudes at any depth were represented in their percentage values as compared with that at 0.5 cm. The relation between the depth and the amplitude of vibration above obtained is exactly what is expected if the wave generated is a surface one. The depth of the medium in this case was about 20 cm., whereas the amplitude of vibration at the depth of 10 cm. is practically zero. This fact is sufficient to prove that the bottom has no sensible influence on the propagation of elastic waves along the surface of the medium.

7. Experiments Regarding the Reflection of Waves at the Edge of a Cliff.

The source of the waves in this case was line-shaped, vibrating vertically by means of an eccentric wheel. The edge of the cliff was parallel to the source of vibration. Small mirrors were placed at every cm. from the source up to the edge of the cliff and their amplitudes of vibrations were measured.

By changing the frequency of the vibration, the amplitude distribution is consequently altered. The results shown in Fig. 5; are typical of the several cases examned. The distance between the source and the edge of the cliff in this case was 23 cm.

Fig. 5.



As is evident from the result, the stationary wave is taking place. Suppose the progressive wave to be rapresented by

$$y_1=A\sin(x-vt)$$

and the retrogressive wave by

$$y_2=B\sin(x+vt)$$

where x-axis is parallel to the direction of the propagation of the wave, v the velocity of the wave, and t the time. At the edge of the cliff, the wave is imperfectly reflected, so that $A=B+C$ where $C>0$.

The resultant motion of the waves will be

$$y=C\sin(x-vt)+2B\sin x\cos vt.$$

Therefore the amplitude of the resultant vibration will depend on the value of x through the relation

$$y_0=C+2B|\sin x|$$

The nodes of vibration are not exactly at rest.

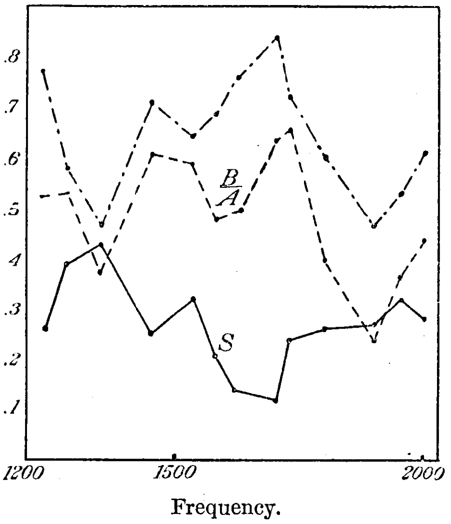
8. Experiments Regarding the Effect of a Canal as a Screen for Waves.

In connection with the experiments of the preceeding article, the effect of a canal as a screen for waves was investigated. This problem has some practical interest because a hint may be drawn from the results of the experiments regarding the possibilities of screening off the actual waves on ground due to earthquakes or traffic disturbances by a suitable canal.

A rectangular canal which was 5 cm. deep and 2 cm. wide was dug parallel to the direction of the line source so as to obstruct the propagation of the waves. In the opposite side of the canal, the amplitudes of vibration is greatly reduced.

The ratio S of the mean amplitudes on both sides of the canal varies according to the frequency of vibration. The curve in full line of Fig. 6 shows the way in which this ratio varies with the frequency of vibration.

Fig. 6.



As can be seen from the figure, the ratio S takes its minimum value when the frequency of vibration is 1700 per minutes. Below this frequency, the ratio decreases with the increasing frequency, and after this minimum value has been passed, however, the ratio increases with the frequency.

Like in the case of the cliff described in the preceeding article, the elastic wave is here also imperfectly reflected at the edge of the canal. The resultant amplitudes of the progressive and retrogressive waves are

$$y_0 = C + 2B|\sin x|$$

using the same notation as the preceeding article.

In the experiments, C and $C+2B$ are directly measurable, so that the amplitude of the incident wave $A=B+C$ will at once be deduced.

The ratio $\frac{B}{A}$ will, at any rate, be a measure of the efficacy of the edge of the canal in reflecting the incident wave. The curve in broken line of Fig. 6 shows the ratio $\frac{B}{A}$ thus obtained as the function of the frequency of the vibration.

As can be seen from the figure, the ratio $\frac{B}{A}$ takes its maximum value at the same frequency at which the amplitude of vibration at the opposite side of the canal takes its minimum value.

The curve in chain line of Fig. 6 shows the relative value of the reflecting power $\frac{B}{A}$ as compared to the sum $S+\frac{B}{A}$. Some irregularities seen in the $\frac{B}{A}$ curve are here very much smoothed out.

A part of the energy of the incident wave is transmitted to the opposite side of the canal and another part is reflected backwards at the edge of the cliff. The proportion in which the energy of the incident wave is divided into these two parts depends on the relative magnitude of the canal with respect to the wave length, and there is a definite optimum width and depth of the canal for screening off a wave of a definite wave length.

How this optimum size of the canal is determined for a given wave length is not yet certain.

9. The Vibration of Wooden Block Half Imbedded in the Medium.

Rectangular pieces of wood of different sizes were half imbedded on the surface of the medium. Minute mirrors were stuck on these pieces. By observing the vibrations of the images of these mirrors, the mode was studied in which such an imbedded block behaves under the action of an incident wave. One of the record thus taken is reproduced in Fig. 7. The series of small spots in the figure are time marks indicating every $\frac{1}{50}$ of a second. As can be seen from the figure, the vibration of the wooden pieces with its own rocking period predominates at the beginning of the vibration. After some time, this vibration continuously damps away, and the state of forced oscillation comes in turn. When the incident wave ends, the piece comes to rest after some vibrations with its own rocking period. These experiments are intimately connected with the practical problems regarding the rocking vibration of the pedestal of a seismograph or that of a rigid building imbedded in a weak ground in the case of an earthquake. The experiments are now going on to study what change in the mode of vibration takes place when the wave length is gradually changed relative to the magnitude of the wooden piece.

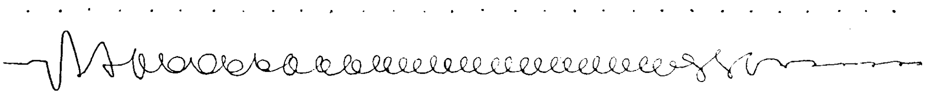


Fig. 7.

(to be continued.)